Ans 29.

6

Let x and y be the decision variables where,

x represents the packets of Screw A and

y represents the packets of Screw B.

$Z = 70x + 100y$ {to be maximised}

Subject to constraints,

$4x + 6y \le 240$

$6x + 3y \le 240$

$x \ge 0, \quad y \ge 0$ (non-negative constraints)

Converting above inequalities into equalities,

$4x + 6y = 240$

$\Rightarrow 2x + 3y = 120$

x	0	60
y	40	0

$6x + 3y = 240$

$\Rightarrow 2x + y = 80$

x	0	40
y	80	0

$x = 0$ and $y = 0$ (non negative constraints)

Consider a test point (0,0)

$2x + 3y \le 120$

$0 + 0 \le 120$

$0 < 120$

which is true

$2x + y \le 80$

$0 + 0 \le 80$

$0 < 80$

which is true

Corner points — Z = 70x + 100y. (sorry)

O (0,0) — Z = 0+0 = 0

A (0,40) — Z = 0 + 100(40) = 4000 → maximum value

B (30,20) — Z = 70(30) + 100(20) = 3800

C (40,0) — Z = 70(40) + 100(0) = 2400

The factory owner must produce 0 packets of Screw A and 40 packets of Screw B to maximise his profit.

His maximum profit = 4000 paise or Rs 40

graph ⇒

Corner points	Z = 70x + 100y
O (0,0)	Z = 0+0 = 0
A (0,40)	Z = 70(0) + 100 (40) = 4000
B (30,20)	Z = 70 (30) + 100 (20) = 4100 → maximum value
C (40,0)	Z = 70 (40) + 100 (0) = 2800

The factory owner must produce 30 packets of Screw A and 20 packets of Screw B to maximise his profit.

His maximum profit = 4100 paise or Rs. 41

Ans 28. Equation of line in cartesian form :-

6

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$$

$\Rightarrow$ $x = 3\lambda+2$
$y = 4\lambda-1$ $\Rightarrow$ $P(3\lambda+2, 4\lambda-1, 2\lambda+2)$
$z = 2\lambda+2$

$1\frac{1}{2}$

Equation of plane in cartesian form :-

$$x-y+z = 5.$$

The line and plane intersect so the point of line must satisfies the equation of the plane.

$$x-y+z = 5$$

$$\Rightarrow (3\lambda+2)-(4\lambda-1)+(2\lambda+2) = 5$$

$$3\lambda+2-4\lambda+1+2\lambda+2 = 5$$

$$\lambda+5 = 5$$

$$\boxed{\lambda = 0}$$

$1\frac{1}{2}$

So, the required point $P(3(0)+2, 4(0)-1, 2(0)+2)$

$= P(2, -1, 2)$

1

Distance of the point Q(−1, −5, −10) from P(2, −1, 2)

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$d = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2}$$

$$d = \sqrt{9+16+144}$$

$$d = \sqrt{169}$$

$d = 13$ units Ans

2

Ans 27.

6

$$\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x}\,dx$$

Put $\sin x - \cos x = t$

$$(\cos x + \sin x)\,dx = dt$$

Also $(\sin x - \cos x)^2 = t^2$

$$\sin^2 x + \cos^2 x - 2\sin x\cos x = t^2$$

$$1 - \sin 2x = t^2$$

$$\sin 2x = 1 - t^2$$

1

limits, when, $x = 0$, $t = -1$

$x = \pi/4$, $t = 0$

$\frac{1}{2}$

$$\int_{-1}^{0} \frac{dt}{16+9(1-t^2)}$$

$$= \int_{-1}^{0} \frac{dt}{16+9-9t^2}$$

$$= \int_{-1}^{0} \frac{dt}{25-9t^2}$$

$$= \int_{-1}^{0} \frac{dt}{-9(t^2-25/9)}$$

$$= \frac{-1}{9}\int_{-1}^{0} \frac{dt}{t^2-(5/3)^2}$$

$$= \frac{-1}{9}\left[\frac{1}{2\times\frac{5}{3}} \log\left|\frac{t-5/3}{t+5/3}\right|\right]_{-1}^{0}$$

$$= \frac{-1}{9}\left[\frac{3}{10} \log\left|\frac{3t-5}{3t+5}\right|\right]_{-1}^{0}$$

$$= \frac{-1}{\cancel{9}_3}\times\frac{\cancel{3}}{10}\left[\log\left|\frac{3(0)-5}{3(0)+5}\right| - \log\left|\frac{3(-1)-5}{3(-1)+5}\right|\right]$$

$$= \frac{-1}{30}\left[\log\left|\frac{-5}{5}\right| - \log\left|\frac{-8}{2}\right|\right]$$

$$= \frac{-1}{30}\left[\log|-1| - \log|-4|\right] = \frac{-1}{30}\left[\log\left|\frac{-1}{-4}\right|\right] = \frac{-1}{30}\log\left(\frac{1}{4}\right)$$

or $\frac{1}{15}\log 2$ Ans

2

$1\frac{1}{2}$

Ans 26.

6

201

Circle,

$x^2 + y^2 = 32$ —— (i)

centre (0,0)

radius = $4\sqrt{2}$ cm.

line, $y = x$ —— (ii)

On solving (i) and (ii) eq.

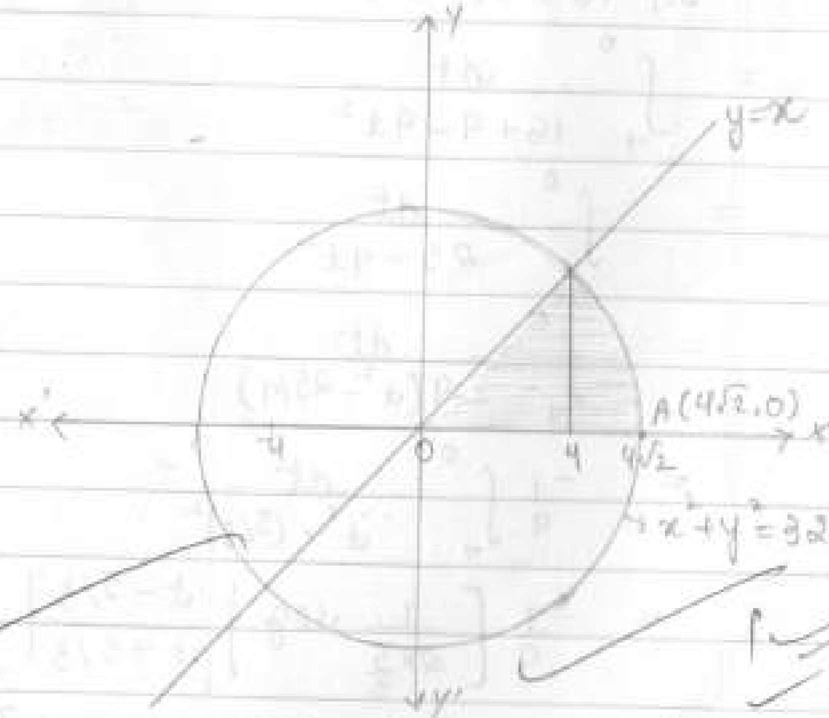
$x^2 + x^2 = 32$

$2x^2 = 32$

$x^2 = 16$

$x = \pm 4$

$y = \pm 4$



Required Area :-

$$\int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$$

1 + 1

$$= \left(\frac{x^2}{2}\right)_0^4 + \left(\frac{x}{2}\sqrt{32 - x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right)_4^{4\sqrt{2}}$$

$$= \left(\frac{x^2}{2}\right)_0^4 + \left(\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right)_4^{4\sqrt{2}}$$

2

$$= \left(\frac{16}{2} - \frac{0}{2}\right) + \left(\frac{4\sqrt{2}}{2}\sqrt{32-32} + 16\sin^{-1}\frac{4\sqrt{2}}{4\sqrt{2}} - \left[\frac{4}{2}\sqrt{32-16} + 16\sin^{-1}\frac{4}{4\sqrt{2}}\right]\right)$$

$$= (8-0) + \left(0 + 16\sin^{-1}(1) - 2\sqrt{16} - 16\sin^{-1}\frac{1}{\sqrt{2}}\right)$$

$$= 8 + (16 \times \pi/2 - 2 \times 4 - 16 \times \pi/4)$$

$$= 8 + 8\pi - 8 - 4\pi$$

$$= 4\pi \text{ units Ans}$$

Ans 25.

(b)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

By elementary row transformation.

$$A = IA$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

$$I = A^{-1} A$$

Therefore, $A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$

Ans 24. To show : R is an equivalence relation

6

Solution.

For Reflexive :-

$(a, a) : a, a \in A$

$a R a \quad \forall a \in A$

$|a-a|$ is divisible by 4

0 is divisible by 4

which is true

$\Rightarrow$ R is a reflexive relation.

For Symmetric relation :-

let $(a,b) \in R \ \forall \ a,b \in A$

$a R b \ \forall \ a,b \in A$

$|a-b|$ is divisible by 4

$\Rightarrow$ $|b-a|$ is divisible by 4

$\Rightarrow$ $b R a \ \forall \ b,a \in A$

$\Rightarrow$ $(b,a) \in R$

$\Rightarrow$ R is an symmetric relation since $(a,b) \in R$ also $(b,a) \in R$

For transitive relation :-

let $(a,b) \in R$ and $(b,c) \in R \ \forall \ a,b,c \in A$

$a R b$ and $b R c \ \forall \ a,b,c \in A$

$|a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$|a-b| = 4\lambda$ — (i)

$|b-c| = 4\mu$ — (ii)

(i) + (ii)

$|a-b+b-c| = 4(\lambda+\mu)$

$|a-c| = 4(\lambda+\mu)$

$\Rightarrow$ $a R c \ \forall \ a,c \in A$

$\Rightarrow \quad (a, c) \in R$

$\rightarrow$ R is an transitive relation since $(a,b) \in R$, $(b,c) \in R$ and also $(a,c) \in R$

Hence R is reflexive, symmetric and transitive so it is a equivalence relation.

The set of all elements related to A are:-

$R = \{(1,5),(5,1),(1,9),(9,1),(1,1)\}$.

Equivalence class $[2] = \{2, 6, 10\}$

Ans 23. Let X denote the larger of the two numbers.

$X = 2, 3, 4, 5$

$P(X=2) = \frac{2}{20} = \frac{1}{10}$

$P(X=3) = \frac{4}{20} = \frac{2}{10}$

$P(X=4) = \frac{6}{20} = \frac{3}{10}$

$P(X=5) = \frac{8}{20} = \frac{4}{10}$

Probability Distribution :-

X	2	3	4	5
P(X)	1/10	2/10	3/10	4/10

Mean

$E(X) = \Sigma p_i x_i$

$\Sigma p_i x_i = 2 \times \frac{1}{10} + 3 \times \frac{2}{10} + 4 \times \frac{3}{10} + 5 \times \frac{4}{10}$

$\Sigma p_i x_i = \frac{2}{10} + \frac{6}{10} + \frac{12}{10} + \frac{20}{10} = \frac{40}{10} = \boxed{4}$ ans

$\frac{1}{2} + \frac{1}{2}$

Variance

$\Sigma p_i (x_i^2) - (\Sigma p_i x_i)^2$

$\sigma = \left(4 \times \frac{1}{10} + 9 \times \frac{2}{10} + 16 \times \frac{3}{10} + 25 \times \frac{4}{10}\right) - (16)$

$= \left(\frac{4}{10} + \frac{18}{10} + \frac{48}{10} + \frac{100}{10}\right) - (16)$

$= \left(\frac{170}{10}\right) - 16$

$= 17 - 16 = \boxed{1}$ Ans

$\frac{1}{2} + 1$

Standard deviation $\sqrt{1} = 1$ ans

Ans 22 Let E_1 = "Event that the girl threw 3, 4, 5 or 6"

E_2 = "Event that the girl threw 1, 2."

and A = "Event that the girl gets exactly one tail"

4

$$P(E_1) = \frac{4}{6} = \frac{2}{3}, \quad P(E_2) = \frac{2}{6} = \frac{1}{3}$$

$$P(A/E_1) = \frac{1}{2}, \quad P(A/E_2) = 3/8$$

Now, $$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$P(E_1/A) = \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{2}{3}\times\frac{1}{2} + \frac{1}{3}\times\frac{3}{8}}$$

$$P(E_1/A) = \frac{\frac{2}{6}}{\frac{2}{6}+\frac{1}{8}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \frac{\frac{1}{3}}{\frac{8+3}{24}}$$

$$P(E_1/A) = \frac{1}{3}\times\frac{24}{11} = \boxed{\frac{8}{11}}$$ Ans

Ans 21:

$\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$

$\vec{a_1} = 4\hat{i} - \hat{j} + 0\hat{k}$

$\vec{b_1} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$

$\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{b_2} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

Shortest distance $= \left| \dfrac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

Now,

$\vec{a_2} - \vec{a_1} = (\hat{i} - \hat{j} + 2\hat{k}) - (4\hat{i} - \hat{j} + 0\hat{k})$

$= -3\hat{i} + 0\hat{j} + 2\hat{k}$

$= -3\hat{i} + 2\hat{k}$

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$\vec{b_1} \times \vec{b_2} = \hat{i}(-10+12) - \hat{j}(-5+6) + \hat{k}(4-4)$ $|\vec{b_1} \times \vec{b_2}| = \sqrt{4+1+0} = \sqrt{5}$.

$\vec{b_1} \times \vec{b_2} = 2\hat{i} - \hat{j} + 0\hat{k}$

1

Now,

$$d = \left| \frac{(-3\hat{i}+0\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j}+0\hat{k})}{\sqrt{4+1+0}} \right|$$

$$d = \left| \frac{-3\times 2 + 0\times -1 + 2\times 0}{\sqrt{5}} \right|$$

$$d = \left| \frac{-6+0+0}{\sqrt{5}} \right|$$

$$\boxed{d = \frac{6}{\sqrt{5}}} \text{ units Ans}$$

1

Ans 20. Let $\vec{d}$ be $x\hat{i}+y\hat{j}+z\hat{k}$

4

Now, $\vec{d} \perp \vec{c}$

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}+\hat{j}-\hat{k}) = 0$$

$$3x + y - z = 0 \quad \text{—— (i)}$$

$\frac{1}{2}$

Also, $\vec{d} \perp \vec{b}$

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}-4\hat{j}+5\hat{k}) = 0$$

$$x - 4y + 5z = 0 \quad \text{—— (ii)}$$

$\frac{1}{2}$

Also, $\vec{d} \cdot \vec{a} = 21$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (4\hat{i} + 5\hat{j} - \hat{k}) = 21$$

$$4x + 5y - z = 21 \quad \text{—— (iii)}$$

Solving (i) and (iii) equation

$$3x + y - z = 0$$
$$-4x + 5y - z = 21$$
$$-x - 4y = -21$$
$$\Rightarrow \quad x + 4y = 21 \quad \text{—— (iv)}$$

Multiply (i) by 5 and add to (ii) eq.

$$15x + 05y - 5z = 0$$
$$+x - 4y + 5z = 0$$
$$16x + y = 0 \quad \text{—— (v)}$$

Solving (iv) and (v) eq.

$$x + 4y = 21$$
$$-64x + 4y = 0$$
$$-63x = 21$$

$$x = \frac{-21}{63} = \frac{-7}{21} = \frac{-1}{3}$$

1

2

Putting value of x in eq. (iv).

$$x + 4y = 21$$

$$\frac{-1}{3} + 4y = 21$$

$$4y = 21 + \frac{1}{3}$$

$$4y = \frac{63+1}{3}$$

$$4y = \frac{64}{3}$$

$$y = \frac{\cancel{64}}{\cancel{12}} = \frac{3\cancel{2}}{\cancel{6}} = \frac{16}{3}$$

Also, $3x + y - z = 0$

$$\cancel{3}\left(\frac{-1}{\cancel{3}}\right) + \frac{16}{3} = z$$

$$-1 + \frac{16}{3} = z$$

$$\frac{-3+16}{3} = z \quad , \quad z = \frac{13}{3}$$

Therefore, $\boxed{\vec{d} = \frac{-1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}}$ Ans

$$= \frac{1}{3}(-\hat{i} + 16\hat{j} + 13\hat{k}) \text{ Ans.}$$

$\cancel{2}\frac{1}{2}$

$1\frac{1}{2}$

1

Ans 19.

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

on comparing the above equation with the standard linear equation

$$\frac{dy}{dx} + Py = Q$$

we get, $P = 2\tan x$, $Q = \sin x$

Therefore, $I.F. = e^{\int P dx}$

$$I.F. = e^{2\int \tan x \, dx}$$

$$= e^{2[\log(\sec x)]}$$

$$= e^{\log(\sec x)^2} = \sec^2 x$$

Now,

$$y \cdot I.F. = \int Q \times I.F. \, dx.$$

$$y \cdot \sec^2 x = \int \sin x \times \sec^2 x \, dx$$

$$y \cdot \sec^2 x = \int \sin x \times \frac{1}{\cos^2 x} dx$$

$$y \cdot \sec^2 x = \int \tan x \cdot \sec x \, dx$$

Put $\sec x = t$

$(\sec x \tan x) dx = dt$

$y \cdot \sec^2 x = \int \cancel{t}\, dt$

$y \cdot \sec^2 x = t + C$

$y \cdot \sec^2 x = \sec x + c$

Now when $y = 0$, $x = \pi/3$.

$0 = \sec \frac{\pi}{3} + c$, $0 = +2 + c$

$+1\frac{1}{2}$

$c = -2$

Therefore,

Solution = $y \cdot \sec^2 x = \sec x - 2$.

or. $y = \frac{1}{\sec x} - \frac{2}{\sec^2 x}$

$\frac{1}{2}$

or. $y = \sec x^{-1} - 2(\sec^2 x)^{-1}$ Ans.

$+\frac{1}{2}$

Ans 18.

4

$$\int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)}\,dx$$

Put $\sin x = t$

$\cos x\,dx = dt$ $\frac{1}{2}$

$$2\int \frac{dt}{(1-t)(1+t^2)}$$

Now by partial fraction,

$$\frac{1}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$$

$1 = A(1+t^2) + (Bt+C)(1-t)$

$1 = A + At^2 + Bt - Bt^2 + C - Ct$

$A + C = 1$ — (i)

$A - B = 0$ — (ii)

$B - C = 0$ — (iii)

(i) + (iii)

$A + C + B - C = 1$

$A + B = 1$ — (iv)

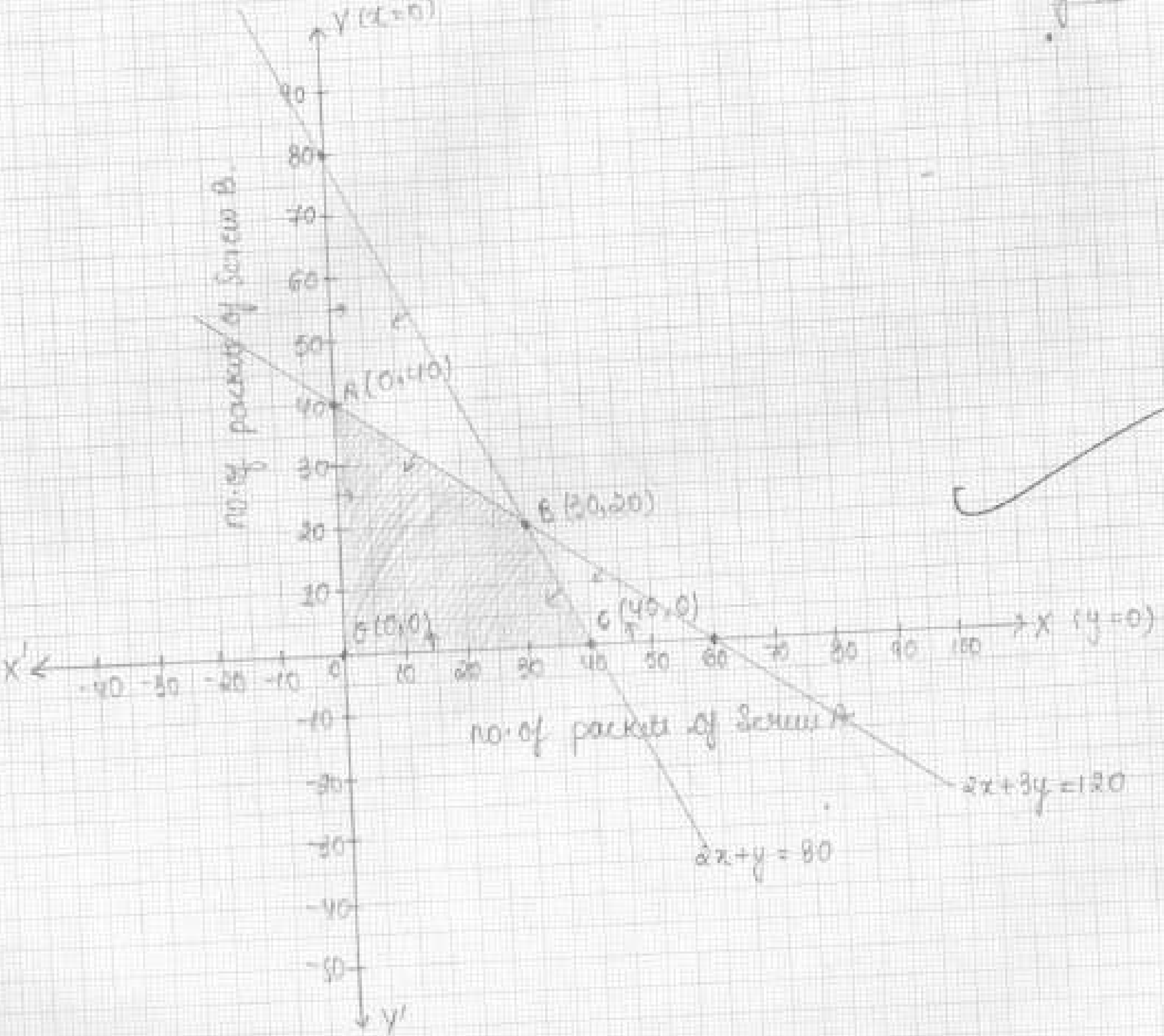
$A - B = 0$ — (ii)

$2A = 1$

$A = 1/2$, $B = 1/2$, $C = 1/2$ $1\frac{1}{2}$

Ans 29.

Scale
x-axis : 1 cm = 10 units
y-axis : 1 cm = 10 units



$$2\int \frac{1/2}{1-t} + \frac{1/2\,t + 1/2}{1+t^2}\,dt$$

$$2\times\frac{1}{2}\int\frac{dt}{1-t} + 2\times\frac{1}{2}\int\frac{t+1}{t^2+1}\,dt$$

$$\int\frac{dt}{1-t} + \int\frac{t\,dt}{t^2+1} + \int\frac{dt}{t^2+1}$$

$$\int\frac{dt}{1-t} + \frac{1}{2}\int\frac{2t\,dt}{t^2+1} + \int\frac{dt}{t^2+1}$$

$$-\log(1-t) + \frac{1}{2}\log(t^2+1) + \tan^{-1}t + c.$$

$1\frac{1}{2}$

$$-\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + c.$$

$$\Rightarrow \frac{1}{2}\log(1+\sin^2 x) - \log(1-\sin x) + \tan^{-1}(\sin x) + c \quad \underline{\text{Ans}}.$$

$$= \log(1+\sin^2 x)^{1/2} - \log(1-\sin x) + \tan^{-1}(\sin x) + c.$$

$$= \log\left|\frac{\sqrt{1+\sin^2 x}}{1-\sin x}\right| + \tan^{-1}(\sin x) + c.$$

$\frac{1}{2}$

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0412403

Ans 17°

(4)

Let, length = breadth = x

height = y

volume = $x \times x \times y$

$K = x^2 y$

$y = \frac{K}{x^2}$

According to question,

$S = x^2 + 4xy$

$S = x^2 + 4x \times \frac{K}{x^2} = x^2 + \frac{4K}{x}$

1

$\frac{ds}{dx} = 2x - \frac{4K}{x^2} = \frac{2x^3 - 4K}{x^2}$

$\frac{1}{2}$

Put $\frac{ds}{dx} = 0$ for critical points

$\frac{2x^3 - 4K}{x^2} = 0$

$2x^3 - 4K = 0$

$x^3 = 2K$

$x = (2K)^{1/3}$

201

Now, $\frac{d^2S}{dx^2} = 2 - \frac{4K(-2)}{x^3} = 2 + \frac{8K}{x^3}$

$\left(\frac{d^2S}{dx^2}\right)_{(x=(2K)^{1/3})} = 2 + \frac{8K}{2K} = 2+4 = 6$ $\frac{1}{2}$

$\frac{d^2S}{dx^2} > 0$ hence it is minimum. $\frac{1}{2}$

Now, $y = \frac{K}{x^2}$

$y = \frac{K}{(2K)^{2/3}} = \frac{K \cdot K^{-2/3}}{(2)^{2/3}} = \frac{K^{1/3}}{2^{2/3}}$

$y = \frac{2}{2}\left(\frac{K^{1/3}}{2^{2/3}}\right)$ $y = K^{1/3} \times 2^{-2/3}$

$y = 2^{-1}(2K^{1/3})$

$y = \frac{1}{2}(2K)^{1/3}$

$\boxed{y = \frac{1}{2}x}$ $\frac{1}{2}$ Hence Proved.

Value :- Helping in nature

Support to middle class people

cooperative & concern towards poor.

1

Ans 16.

$$f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$$

$$f'(x) = \frac{4x^3}{4} - 3x^2 - 10x + 24$$

$\frac{1}{2}$

$$f'(x) = x^3 - 3x^2 - 10x + 24$$

$$f'(x) = (x-2)(x^2 - x - 12)$$

$$f'(x) = (x-2)(x^2 - 4x + 3x - 12)$$

$$f'(x) = (x-2)[x(x-4) + 3(x-4)]$$

$$f'(x) = (x-2)(x+3)(x-4)$$

1

Put $f'(x) = 0$

$$(x-2)(x+3)(x-4) = 0$$

$$x = 2, -3, 4$$

$\frac{1}{2}$

$$\underset{-\infty}{\leftarrow} \; - \; \underset{-3}{|} \; + \; \underset{2}{|} \; - \; \underset{4}{|} \; + \; \underset{\infty}{\rightarrow}$$

$f'(x) > 0$ strictly increasing

$f'(x) < 0$ strictly decreasing

Intervals	$f'(x)$ sign	nature
$(-\infty, -3)$	$-ve$	decreasing
$(-3, 2)$	$+ve$	increasing
$(2, 4)$	$-ve$	decreasing
$(4, \infty)$	$+ve$	increasing

(a) Strictly increasing = $(-3, 2) \cup (4, \infty)$ 1

(b) Strictly decreasing = $(-\infty, -3) \cup (2, 4)$ 1

Ans 15. 4

$y = \sin(\sin x)$

$\frac{dy}{dx} = \cos(\sin x) \cdot \cos x \Rightarrow \cancel{\cos x} = \cos(\sin x) = \frac{1}{\cos x} \cdot \frac{dy}{dx}$ 1

Now, $\frac{d^2y}{dx^2} = -\cos(\sin x)\sin x + \cos x(-\sin(\sin x)\cos x)$

$\frac{d^2y}{dx^2} = -\sin x \cos(\sin x) - \cos^2 x \sin(\sin x).$ 2

$\frac{d^2y}{dx^2} = -\sin x \times \frac{1}{\cos x} \times \frac{dy}{dx} - \cos^2 x\, y.$

$\frac{d^2y}{dx^2} = -\tan x \frac{dy}{dx} - \cos^2 x\, y.$

$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0.$ 1

Hence Proved

Ans 14.

(4)

$x = a(2\theta - \sin 2\theta)$

$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$

$\frac{dx}{d\theta} = 2a(1 - \cos 2\theta)$ 1

$y = a(1 - \cos 2\theta)$

$\frac{dy}{d\theta} = a(0 + 2\sin 2\theta)$

$\frac{dy}{d\theta} = 2a \sin 2\theta$ 1

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{2a \sin 2\theta}{2a(1 - \cos 2\theta)} = \frac{2 \sin\theta \cos\theta}{2 \sin^2\theta}$

$\frac{dy}{dx} = \cot\theta$ 1

$\left(\frac{dy}{dx}\right)_{(\theta = \pi/3)} = \cot \frac{\pi}{3}$

$= \cot(60^\circ)$

$= \boxed{\frac{1}{\sqrt{3}}}$ Ans 1

Ans 13.

4

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$$\begin{vmatrix} 0 & 1 & 1+3x \\ 3y & 1 & 1 \\ -3z & 1+3z & 1 \end{vmatrix}$$

1

taking 3 common from C_1

$$3\begin{vmatrix} 0 & 1 & 1+3x \\ y & 1 & 1 \\ -z & 1+3z & 1 \end{vmatrix}$$

1

$R_2 \to R_2 - R_1$

$$3\begin{vmatrix} 0 & 1 & -1+3x \\ y & 0 & -3x \\ -z & 1+3z & 1 \end{vmatrix}$$

1

Expanding along R_1

$3[-1(y-3xz)+(1+3x)(y+3zy)]$ 1

$3[-y+3xz+y+3zy+3xy+9xyz]$

$3[\ 9xyz+3xz+3zy+3xy)$

$9(3xyz+xy+yz+zx)$ Ans 1

Ans 12.

2

Let E_1 = "Event of obtaining the sum 8"

$E_1 = \{(2,6)(6,2)(3,5)(5,3)(4,4)\}$

and F = "Event that red die result in a number less than 4."

$$F = \begin{Bmatrix} (1,1) & (2,1) & (3,1) & (4,1) & (5,1) & (6,1) \\ (1,2) & (2,2) & (3,2) & (4,2) & (5,2) & (6,2) \\ (1,3) & (2,3) & (3,3) & (4,3) & (5,3) & (6,3) \end{Bmatrix}$$

$$P(E/F) = \frac{P(E\cap F)}{P(F)} \text{ or } \frac{n(E\cap F)}{n(F)}$$

1

$E \cap F = \{(6,2),(5,3)\}$

$$n(E\cap F) = \frac{2}{36}, \quad n(F) = \frac{18}{36}$$

$$P(E/F) = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \boxed{\frac{1}{9}} \text{ Ans}$$

1

Ans 11. let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

9

$|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$= 4\hat{i} + 8\hat{j} + 4\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{16+64+16} = \sqrt{96} = 4\sqrt{6}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$

$4\sqrt{6} = \sqrt{14} \times \sqrt{14} \times \sin\theta$

$4\sqrt{6} = 14 \sin\theta$

$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$

$\theta = \sin^{-1}\left(\frac{2\sqrt{6}}{7}\right)$ Ans.

1

Ans 10.

2

$y = ae^{bx+5}$

$\frac{dy}{dx} = ae^{bx+5}(b)$

$\frac{dy}{dx} = ab\,e^{bx+5}$

$\frac{dy}{dx} = by$

$\frac{1}{y} \cdot \frac{dy}{dx} = b$

1

Differentiating again w.r.t x

$$\frac{y\frac{d^2y}{dx^2} - \frac{dy}{dx}\times\frac{dy}{dx}}{y^2} = 0$$

$$\therefore\ y\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 = 0 \quad \underline{\text{Ans}}$$

1

Ans 9.

2

$$\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}\,dx$$

$$\int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x}\,dx \qquad \frac{1}{2}$$

$$\int \frac{dx}{\cos^2 x}$$

$$\int \sec^2 x\,dx \qquad 1$$

$$\Rightarrow \tan x + c \quad \underline{\text{Ans}} \qquad \frac{1}{2}$$

Ans 8°

2

$$C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$$

$$C'(x) = 0.005(3x^2) - 0.02(2x) + 30$$

$$C'(x) = 0.015x^2 - 0.04x + 30$$

when $x = 3$.

$$C'(3) = 0.015(3)^2 - 0.04(3) + 30$$

$$= 0.015(9) - 0.04(3) + 30$$

$$= 0.135 - 0.12 + 30$$

$$= 30.015 \quad \text{Ans.}$$

1

Ans 7°

2

$$\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$$

$$y = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right)$$

1

$$y = \tan^{-1}\left(\frac{\cos x/2}{\sin x/2}\right)$$

$$y = \tan^{-1}(\cot x/2)$$

$$y = \tan^{-1}\left[\tan(\pi/2 - x/2)\right]$$

$$y = \frac{\pi}{2} - \frac{x}{2}$$

$\frac{1}{2}$

$$y = \frac{\pi}{2} - \frac{x}{2}$$

Differentiating w.r.t. x

$$\frac{dy}{dx} = 0 - \frac{1}{2}$$

$$\frac{dy}{dx} = -\frac{1}{2} \quad \text{Ans}$$

$\frac{1}{2}$

Ans 6. ②

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$

$$|A| = 14 - 12 = 2$$ $\frac{1}{2}$

$|A| \neq 0$ hence inverse exists

Now, $C_{11} = 7$, $C_{12} = +4$

$C_{21} = +3$, $C_{22} = 2$

$$\text{Adj}(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\,\text{adj}(A) = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \quad \text{Ans}$$

$\frac{1}{2}$

$2A^{-1} = 9I - A$

$2 \times \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ Hence Proved.

Ans 5. 2

$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$

R.H.S. $\sin^{-1}(3x - 4x^3)$

Put $x = \sin\theta$ $\frac{1}{2}$

$\sin^{-1}(3\sin\theta - 4\sin^3\theta)$ 1

$\sin^{-1}(\sin 3\theta)$

3θ

$3\sin^{-1}x$ $\frac{1}{2}$

RHS = LHS

Hence Proved

$\frac{-1}{2} \le x \le \frac{1}{2}$

$\frac{-1}{2} \le \sin\theta \le \frac{1}{2}$

$\sin^{-1}\left(\frac{-1}{2}\right) \le \theta \le \sin^{-1}\left(\frac{1}{2}\right)$

$-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$

$\left[\frac{\pi}{3}, \frac{\pi}{3}\right] \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Ans 4.

$a \circ b = (a * b) + 3$

$5 \circ 10 = (5 * 10) + 3$

$= 10 + 3$

$= 13$ Ans

1

Ans 5.

$|\vec{a}| = |\vec{b}|$

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$\frac{9}{2} = |\vec{a}|\,|\vec{a}| \cos 60^\circ$

$\frac{9}{2} = |\vec{a}|^2 \times \frac{1}{2}$

$9 = |\vec{a}|^2$

$|\vec{a}| = 3$.

Also $|\vec{a}| = |\vec{b}|$

$|\vec{b}| = 3$

$|\vec{a}| = |\vec{b}| = 3$ Ans

1

Ans 2: A is a skew symmetric matrix

(1) $\Rightarrow A' = -A$

$$A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$$

$A' = -A$

Now,

$$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix}$$

On comparing we get,

$$\boxed{\begin{matrix} b = 3 \\ a = -2 \end{matrix}}$$ Ans.

1

Ans 1.

1

$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

$\tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3})$

$\tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$

$\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3} - \pi$

$\frac{\pi}{2} - \pi$

$\frac{\pi - 2\pi}{2} = -\frac{\pi}{2}$ Ans

As we know that

$\tan^{-1}x + \cot^{-1}x = \pi/2.$

1

100/100

One Hundred only

Evaluated St. according to the marking scheme

1