WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)

परीक्षार्थी प्रवेश–पत्र के अनुसार भरें

विषय Subject : MATHEMATICS

विषय कोड Subject Code : 041

परीक्षा का दिन एवं तिथि
Day & Date of the Examination : MONDAY 14.03.2016

उत्तर देने का माध्यम
Medium of answering the paper : ENGLISH

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाएं
Write code No. as written on the top of the question paper

Code Number	Set Number
65/1/S	① ● ③ ④

अतिरिक्त उत्तर–पुस्तिका (ओं) की संख्या
No. of supplementary answer-book(s) used: NIL

विकलांग व्यक्ति : हाँ / नहीं
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किसी शारीरिक अक्षमता से प्रभावित हों तो संबंधित वर्ग में ✓ का निशान लगाएं।
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B	D	H	S	C	A

B = दृष्टिहीन, D = मूक व बधिर, H = शारीरिक रूप से विकलांग, S = स्पास्टिक, C = डिस्लेक्सिक, A = ऑटिस्टिक

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Whether writer provided : Yes / No — No

यदि दृष्टिहीन हैं तो उपयोग में लाए गये सॉफ्टवेयर का नाम :
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प्रत्येक अक्षर के एक खाने में लिखें तथा नाम के प्रत्येक भाग के बीच एक खाना खाली छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।

Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

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Section-A

1. $\vec{r}\cdot\hat{n} = d$

$\hat{n}$ - Unit vector perpendicular to the plane

d - Distance of the plane from origin.

$\vec{r}\cdot(2\hat{i} - 3\hat{j} + 6\hat{k}) = 4$

$\therefore \dfrac{\vec{r}\cdot(2\hat{i} - 3\hat{j} + 6\hat{k})}{\sqrt{2^2+3^2+6^2}} = \dfrac{4}{\sqrt{2^2+3^2+6^2}} \Rightarrow \vec{r}\cdot\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{4}{7}$ (Distance of plane from origin $= \frac{4}{7}$)

$\vec{r}\cdot(6\hat{i} - 9\hat{j} + 18\hat{k}) = -30$

$\therefore \dfrac{\vec{r}\cdot(6\hat{i} - 9\hat{j} + 18\hat{k})}{\sqrt{6^2+9^2+18^2}} = \dfrac{-30}{\sqrt{6^2+9^2+18^2}} \Rightarrow \vec{r}\cdot\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{-10}{7}$ (Distance of plane from origin $= \frac{10}{7}$ in the direction opposite to the unit vector)

$\therefore$ Distance between the planes $= \frac{4}{7} - \left(\frac{-10}{7}\right) = \frac{14}{7} = \underline{\underline{2 \text{ units}}}$

2. $\bar{a} - \sqrt{2}\bar{b}$ is a unit vector

$|\bar{a} - \sqrt{2}\bar{b}| = 1$

$\therefore |\bar{a} - \sqrt{2}\bar{b}|^2 = 1$

$|\bar{a}|^2 + |\sqrt{2}\bar{b}|^2 - 2\sqrt{2}\,\bar{a}\cdot\bar{b} = 1$ $\quad (|\bar{a}| = |\bar{b}| = 1)$ $\quad$ (θ: Angle between vectors $\bar{a}$ & $\bar{b}$)

$1 + 2 - 2\sqrt{2}\,\bar{a}\cdot\bar{b} = 1$ $\quad \bar{a}\cdot\bar{b} = \frac{1}{\sqrt{2}} \Rightarrow |\bar{a}||\bar{b}|\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \therefore \boxed{\theta = 45^\circ}$ (Angle between $\bar{a}$ & $\bar{b}$ is 45°)

3. $|\bar{a}| = \frac{1}{2}$ $|\bar{b}| = \frac{4}{\sqrt{3}}$ $|\bar{a}\times\bar{b}| = \frac{1}{\sqrt{3}}$ $|\bar{a}\cdot\bar{b}| = ?$

$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$ (θ: Angle between vectors $\bar{a}$ & $\bar{b}$).

$\frac{1}{\sqrt{3}} = \frac{1}{2}\times\frac{4}{\sqrt{3}}\sin\theta$

$\sin\theta = \frac{1}{2}$ $\therefore \theta = 30^\circ$

$|\bar{a}\cdot\bar{b}| = |\bar{a}||\bar{b}|\cos\theta$

$= \frac{1}{2}\times\frac{4}{\sqrt{3}}\times\frac{\sqrt{3}}{2} = 1$

$\therefore |\bar{a}\cdot\bar{b}| = 1$

4. $A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$

$kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$

But given $kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$

$\begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$ Equating individual terms,

$2k = -8$ $\underline{\underline{k = -4}}$

$3k = 4a$

$3\times(-4) = 4a$ $\quad a = -3$

$-5k = 5b$

$b = -k = 4$

5. $|AB| = |A||B|$ (Provided A & B are square matrices)

$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1\times(-1) - 2\times 3 = -7$

$|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = 1\times(-2) - 3\times(-4) = 10$

$\therefore |AB| = -7\times 10 = -70$

6. $|A| = 5$

$|AA^T| = |A||A^T|$ (As A is a square matrix)

$= |A||A|$ (As $|A| = |A^T|$)

$= |A|^2 = 25$

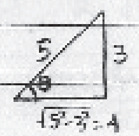
Section-B

7. To prove: $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Proof: Let $\sin^{-1}\left(\frac{3}{5}\right) = \theta$

$\therefore \sin\theta = \frac{3}{5}$

$\therefore \tan\theta = \frac{3}{4}$ (From triangle ①)



Triangle ①

$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right)$

Now $2\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$ $\left\{2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{1-\frac{9}{16}}\right) = \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) = \tan^{-1}\left(\frac{3\times 8}{7}\right) = \tan^{-1}\left(\frac{24}{7}\right)$

$\therefore 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$ $\left\{\tan^{-1}a - \tan^{-1}b = \tan^{-1}\left(\frac{a-b}{1+ab}\right)\right\}$

$= \tan^{-1}\left(\frac{24\times 31 - 17\times 7}{7\times 31 + 24\times 17}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}1 = \frac{\pi}{4}$ Hence Proved

2. Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x}\, dx$ — ①

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\, dx$ $\left\{ \int_0^a f(x)dx = \int_0^a f(a-x)dx \right\}$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x}\, dx$ — ②

Adding ① & ②

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x}\, dx = \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x}$ $\left\{ \sin^2 x + \cos^2 x = 1 \right\}$ $\left\{ \cos(A-B) = \cos A \cos B + \sin A \sin B \right\}$

$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x \cdot \frac{1}{\sqrt{2}} + \sin x \cdot \frac{1}{\sqrt{2}}} = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x \cos\frac{\pi}{4} + \sin x \sin\frac{\pi}{4}}$

$= \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos\left(x-\frac{\pi}{4}\right)}$

Put $x - \frac{\pi}{4} = t$

$dx = dt$

For $x = 0$, $t = \frac{-\pi}{4}$, for $x = \frac{\pi}{2}$, $t = \frac{\pi}{4}$

$$I = \frac{1}{2\sqrt{2}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec t \, dt$$

$$= \frac{1}{2\sqrt{2}} \log\left|\sec t + \tan t\right|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$= \frac{1}{2\sqrt{2}} \left(\log\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right| - \log\left|\sec\left(\frac{-\pi}{4}\right) + \tan\left(\frac{-\pi}{4}\right)\right| \right)$$

$$= \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$$

$$= \frac{1}{2\sqrt{2}} \log\left|\frac{(\sqrt{2}+1)^2}{2-1}\right| = \frac{1}{\sqrt{2}} \log\left|\sqrt{2}+1\right|$$

9. Let $I = \int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

$$= \int \log(\log x)\, dx + \int \frac{dx}{(\log x)^2}$$

$$= I_1 + I_2 + C \text{ (Say)} \qquad \{C = \text{Arbitrary constant}\}$$

$I_1 = \int \log(\log x)\, dx$ & $I_2 = \int \frac{dx}{(\log x)^2}$

Consider $I_1 = \int \log(\log x) \times 1 \, dx$

$= x\log(\log x) - \int x \times \frac{1}{\log x} \times \frac{1}{x} dx$ {Applying Integration by parts}

$= x\log(\log x) - \int \frac{dx}{\log x} \times 1$

$= x\log(\log x) - \left[\frac{x}{\log x} - \int x \times \frac{-1}{(\log x)^2} \times \frac{1}{x} dx\right]$ {Applying Integration by parts}

$I_1 = x\log(\log x) - \frac{x}{\log x} - \int \frac{dx}{(\log x)^2}$

But $\int \frac{dx}{(\log x)^2} = I_2$

$\therefore \underline{I} = I_1 + I_2 + C = x\log(\log x) - \frac{x}{\log x} + C$

10. Let $I = \int \frac{(1-\sin x)dx}{\sin x(1+\sin x)}$

$= \int \frac{(1-\sin x)(1-\sin x)\,dx}{\sin x(1+\sin x)(1-\sin x)}$ {Multiplying numerator & Denominator by $1-\sin x$}

$= \int \frac{(1-\sin x)^2 dx}{\sin x(1-\sin^2 x)} = \int \frac{(1+\sin^2 x - 2\sin x)dx}{\sin x \cos^2 x}$

$= \int \frac{dx}{\sin x \cos^2 x} + \int \frac{\sin x}{\cos^2 x}dx - 2\int \frac{dx}{\cos^2 x}$

$= \underbrace{\int \frac{\sin x\, dx}{(1-\cos^2 x)\cos^2 x}}_{= I_1} + \underbrace{\int \frac{\sin x}{\cos^2 x}dx}_{= I_2} - 2\tan x + C$ (Say) $\{\int \sec^2 x\,d$

$I_2 = \int \frac{\sin x}{\cos^2 x}dx$

Put $\cos x = t$

$-\sin x\,dx = dt$

$$\therefore \frac{I}{2} = \int \frac{-dt}{t^2} = \frac{1}{t} + c_1 = \sec x + c_1$$

$$I_1 = \int \frac{\sin x \, dx}{(1-\cos^2 x)(\cos^2 x)}$$

Put $\cos x = u$

$$-\sin x \, dx = du$$

$$I_1 = \int \frac{-du}{(1-u^2)u^2} = \int \frac{du}{(u^2-1)u^2} = \int \frac{(u^2-(u^2-1))du}{(u^2-1)u^2}$$

$$= \int \frac{du}{u^2-1} - \int \frac{du}{u^2}$$

$$= \frac{1}{2}\log\left|\frac{u-1}{u+1}\right| + \frac{1}{u} + c'$$

$$\left\{ \int \frac{du}{u^2-1} = \int \frac{du}{(u+1)(u-1)} = \frac{1}{2}\int \frac{((u+1)-(u-1))du}{(u+1)(u-1)} = \frac{1}{2}\log\left|\frac{u-1}{u+1}\right| \right\}$$

$$I_1 = \frac{1}{2}\log\left|\frac{\cos x - 1}{\cos x + 1}\right| + \sec x + c'$$

$$\therefore I = \int \frac{(1-\sin x)dx}{\sin x(1+\sin x)} = \frac{1}{2}\log\left|\frac{\cos x - 1}{\cos x + 1}\right| + 2\sec x - 2\tan x + k$$

where k is an arbitrary constant.

$x = am^2$

$ay^2 = (am^2)^3$

$ay^2 = a^3m^6$

$y^2 = a^2m^6$

$\therefore\ y = \pm am^3$

Considering $(x, y) = (am^2, am^3)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dm}}{\frac{dx}{dm}}$$

$$\frac{dy}{dm} = \frac{d}{dm}(am^3) = 3am^2$$

$$\frac{dx}{dm} = \frac{d}{dm}(am^2) = 2am$$

$$\therefore\ \left.\frac{dy}{dx}\right]_{(am^2, am^3)} = \frac{3am^2}{2am} = \frac{3m}{2} = \text{Slope of tangent at } (am^2, am^3)$$

$$\text{Slope of normal at } (am^2, am^3) = \frac{-1}{\left.\frac{dy}{dx}\right]_{(am^2, am^3)}} = \frac{-1}{\frac{3m}{2}} = \frac{-2}{3m}$$

$\therefore$ Equation of normal to the curve at (am^2, am^3)

$$y - am^3 = \frac{-2}{3m}(x - am^2)$$

$$3my - 3am^4 = -2x + 2am^2$$

$$3my + 2x = 3am^4 + 2am^2$$ is the required equation.

12) $f(x) = \begin{cases} k\sin\left(\frac{\pi}{2}(x+1)\right) & x \le 0 \\ \frac{\tan x - \sin x}{x^3} & x > 0 \end{cases}$

$f(x)$ is continuous at $x = 0$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

Now $\lim_{x\to 0^-} f(x) = f(0) = \lim_{x\to 0^-} k\sin\left(\frac{\pi}{2}(x+1)\right) = k\sin\frac{\pi}{2} = k$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\tan x - \sin x}{x^3} = \lim_{x\to 0^+} \frac{\sin x}{x}\left(\frac{\sec x - 1}{x^2}\right)$$

$$= \lim_{x\to 0^+} \frac{\sin x}{x} \cdot \lim_{x\to 0^+} \frac{1-\cos x}{\cos x \times x^2}$$

$$= \frac{\lim_{x\to 0^+} \frac{\sin x}{x} \times \lim_{x\to 0^+} \frac{2\sin^2\frac{x}{2}}{4\times\left(\frac{x}{2}\right)^2}}{\lim_{x\to 0^+} \cos x}$$

Now $\lim_{x\to 0} \frac{\sin x}{x} = 1$ & $\lim_{x\to 0}\left(\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\right)^2 = 1$

$$\therefore \lim_{x\to 0^+} f(x) = \frac{1\times\frac{1}{2}}{1} = \frac{1}{2}$$

$\therefore \boxed{k = \frac{1}{2}}$ $\{$As $f(x)$ is continuous at $x=0\}$

13. Consider $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$

$$= \tan^{-1}\left(\frac{1-\sqrt{\frac{1-x^2}{1+x^2}}}{1+\sqrt{\frac{1-x^2}{1+x^2}}}\right)$$

Put ~~that~~ $x^2 = \cos 2\theta$ $\therefore \theta = \frac{1}{2}\cos^{-1}x^2$

$$\therefore \tan^{-1}\left(\frac{1-\sqrt{\frac{1-x^2}{1+x^2}}}{1+\sqrt{\frac{1-x^2}{1+x^2}}}\right) = \tan^{-1}\left(\frac{1-\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}}{1+\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}}\right)$$

$1-\cos 2\theta = 2\sin^2\theta$

$1+\cos 2\theta = 2\cos^2\theta$

$$\therefore \tan^{-1}\left(\frac{1-\sqrt{\frac{1-x^2}{1+x^2}}}{1+\sqrt{\frac{1-x^2}{1+x^2}}}\right) = \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)$$

$$= \frac{\pi}{4}-\theta$$

We want: $\dfrac{d}{d(\cos^{-1}x^2)}\left(\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)\right) = \dfrac{d}{d\left(\frac{2\theta}{\theta}\right)}\left(\frac{\pi}{4}-\theta\right) = \dfrac{\frac{d}{d\theta}\left(\frac{\pi}{4}-\theta\right)}{\frac{d}{d\theta}(2\theta)}$

$$= \frac{1}{2}\frac{d}{d\theta}\left(\frac{\pi}{4}-\theta\right) = \frac{-1}{2}$$

14. A plane which passes through $A(3,2,1)$, $B(4,2,-2)$ & $C(6,5,-1)$ is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix}$$

$$0 = (x-3)\left(0\times(-2)-(-3)\times 3\right) - (y-2)\left(1\times(-2)-(-3)\times 3\right) + (z-1)\left(1\times 3 - 3\times 0\right)$$

$$9(x-3) - 7(y-2) + 3(z-1) = 0$$

$$9x - 7y + 3z = 27 - 14 + 3 = 16$$

$\therefore$ Plane passing through points A, B, C is $9x-7y+3z=16$

Now A, B, C & $D(\lambda,5,5)$ are coplanar

$\therefore$ D lies on the plane of A, B, C

$\therefore$ $9\lambda - 7\times 5 + 3\times 5 = 16$

$$9\lambda = 36$$

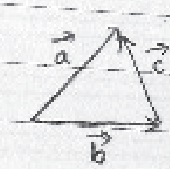
$$\lambda = 4$$

15. $\vec{a} = \vec{b} + \vec{c}$

Here $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$

$\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$



$\vec{a} = \vec{b} + \vec{c}$

$\therefore\ p\hat{i} + q\hat{j} + r\hat{k} = s\hat{i} + 3\hat{j} + 4\hat{k} + 3\hat{i} + \hat{j} - 2\hat{k}$

$p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

Equating components,

$p = s+3$

$q = 4$

$r = 2$

Area of triangle $= 5\sqrt{6}$

But Area of triangle $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$$5\sqrt{6} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p & 4 & 2 \\ p-3 & 2 & 4 \end{vmatrix} = \frac{1}{2}\left|10\hat{i} - (2p+6)\hat{j} + (12-p)\hat{k}\right|$$

$\therefore\ 600 = 100 + (2p+6)^2 + (12-p)^2$

$500 = 4p^2 + 36 + \cancel{24p} + 144 + p^2 - \cancel{24p} \Rightarrow 5p^2 = 320$

$500 - 180 = 320$

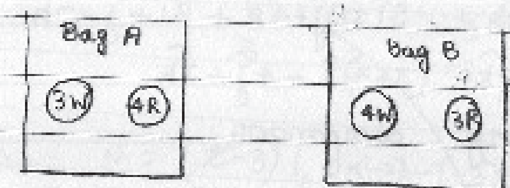
$p^2 = 64$

$p = \pm 8$

If $p = 8$, $s = p - 3 = 5$

If $p = -8$, $s = p - 3 = -11$

16.



Let E_1: Event that the balls are drawn from bag A

E_2: Event that the balls are drawn from Bag B

C: Event that 2 white balls & 1 red ball are drawn

$P(E_2 | C) = ?$

Now, $P(E_1) = \frac{1}{2} = P(E_2)$

$P(C|E_1) = \frac{3}{7} \times \frac{2}{6} \times \frac{4}{5} \times {}^3C_1 = \frac{4}{35} \times 3$ (multiplied by 3 because they can be chosen in any order)

$P(C|E_2) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times {}^3C_1 = \frac{6}{35} \times 3$

By Bayes theorem,

$$P\left(\frac{E_2}{C}\right) = \frac{P(E_2)\,P(C|E_2)}{P(E_1)P(C|E_1) + P(E_2) \times P(C|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{35} \times 3}{\frac{1}{2} \times \frac{4}{35} \times 3 + \frac{1}{2} \times \frac{6}{35} \times 3} = \frac{6}{10} = \frac{3}{5}$$

17. Let the length of the plot be l.
Let the breadth of the plot be b.

New $l \times b = A$ (A = Area of the plot)

$(l-50)(b+50) = A$

$(l-10)(b-20) = A - 5300$

$lb - 50b + 50l - 2500 = A$

$\therefore -b + l = 50$ ① ($A = lb$)

$lb - 10b - 20l + 200 = A - 5300$

$10b + 20l = 5500$

$b + 2l = 550$ ②

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} l \\ b \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$$\underset{A}{\|} \quad \underset{X}{\|} \quad \underset{B}{\|}$$

$AX = B$

$\therefore X = A^{-1} B$

When $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

($ad - bc = 1 \times 1 - (-2) \times 1 = 3 \neq 0$
$\therefore A^{-1}$ exists)

Here $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{1+2} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1} B = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 550 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 600 \\ 450 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$\therefore \begin{bmatrix} l \\ b \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$l = 200$ m

$b = 150$ m

He wants to donate the plot to the school because he wants rural places to become developed & he is thereby showing his kind heartedness & his intention to help the society to develop by producing more literates. Children should have an opportunity to learn.

18.

$$2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$$

$$\frac{dy}{dx} = \frac{-2ye^{x/y}}{y - 2xe^{x/y}}$$

$$\frac{dx}{dy} = \frac{y - 2xe^{x/y}}{-2ye^{x/y}} = F(x,y) \text{ (say)}$$

$$F(\lambda x, \lambda y) = \frac{\lambda y - 2\lambda x e^{\frac{\lambda x}{\lambda y}}}{-2\lambda y e^{\lambda x/\lambda y}} = \lambda^0 F(x,y)$$

$\therefore$ The equation is a homogeneous differential equation

Put $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\therefore v + y\frac{dv}{dy} = \frac{y - 2vye^{v}}{-2ye^{v}} = \frac{-1}{2}e^{-v} + v \Rightarrow y\frac{dv}{dy} = \frac{-1}{2}e^{-v}$$

$$e^{v}dv = \frac{-1}{2}\frac{dy}{y}$$

Integrating

$$e^{v} = \frac{-1}{2}\log y + C$$

$$\therefore\ e^{x/y} = \frac{-1}{2}\log y + C$$ is the required solution to the differential equation.

19. $$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^{3}$$

$$\frac{dy}{dx} - \frac{y}{(x+1)} = e^{3x}(x+1)^{2}$$

This equation is of the form $\frac{dy}{dx} + Py = Q$ $\{P, Q = \text{Functions of } x \text{ alone}\}$

linear differential equation.

$$\therefore \text{Integrating factor} = e^{\int P dx} = e^{-\int \frac{dx}{x+1}} = e^{-\log(x+1)} = \frac{1}{x+1}$$

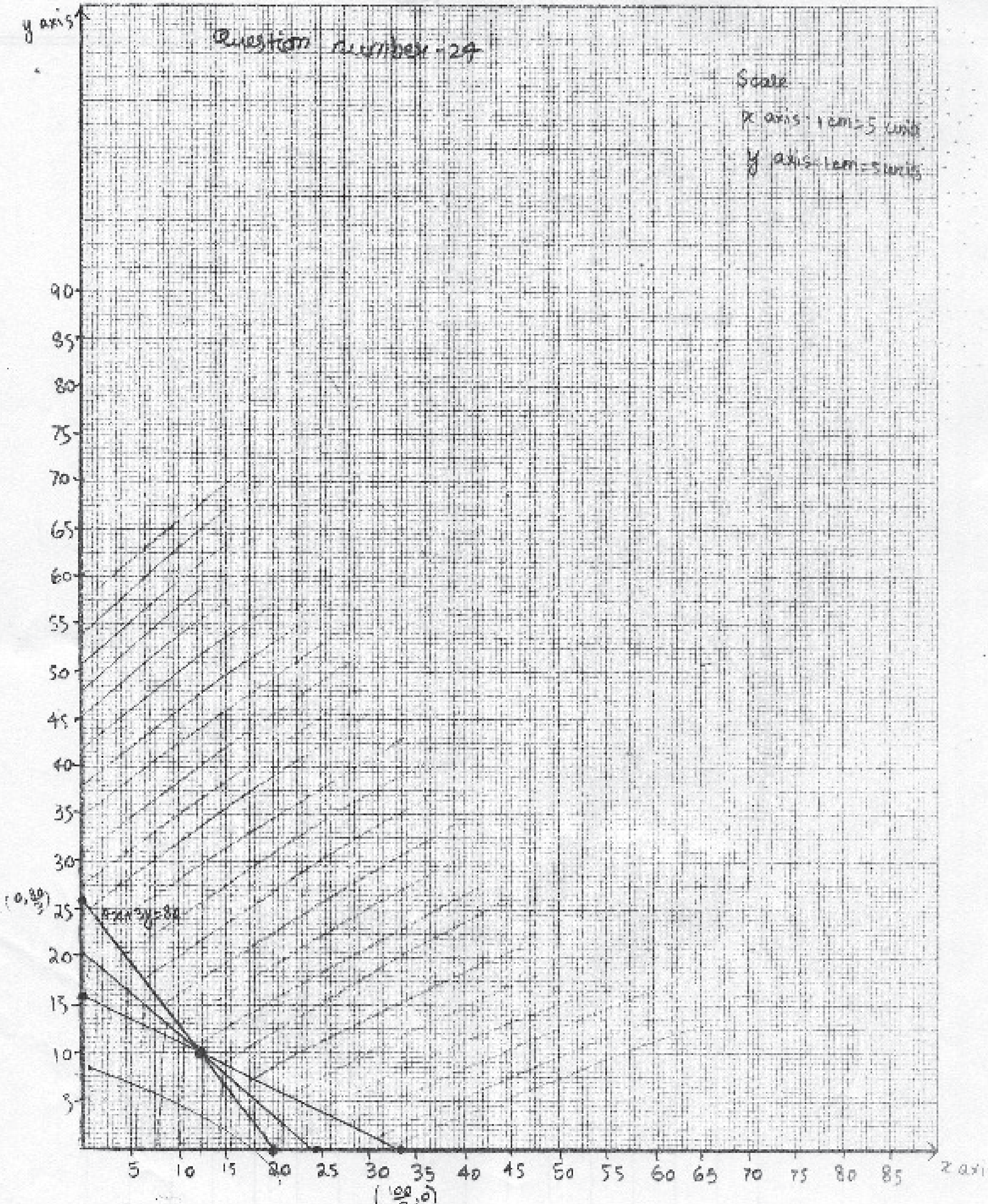
Solution to the differential equation:

$$y \times I.F = \int Q \times I.F\, dx + C$$

$$\therefore \frac{y}{x+1} = \int e^{3x}(x+1)^{2} \times \frac{1}{(x+1)}\, dx + C$$

$$= \int x e^{3x} dx + \int e^{3x} dx + C = \frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x} dx + \int e^{3x} dx + C$$

$$\frac{y}{x+1} = \frac{xe^{3x}}{3} + \frac{2}{9}e^{3x} + C$$ is the solution to the required differential equation



Question number -29
Scale
x axis 1cm=5 units
y axis 1cm=5 units
y axis
x axis
(0, 80/3)
(100/3, 0)
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5

Section-C

24. Let the number of units of ~~Vitamin A~~ food F_1 = x

Let the number of units of ~~Vitamin Minerals~~ food F_2 = y

~~$x \geq 80$~~ ~~$x \geq 0$~~ $x \geq 0$ $y \geq 0$

~~$y \geq 100$~~ ~~$y \geq 0$~~ $4x + 3y \geq 80$ $4x+3y=80$

$3x + 6y \geq 100$ $3x+6y=100$

$Z = 5x + 6y$ Minimize Z $8x+6y=160$

$5x = 60$

$x = 12$

$y = \frac{32}{3}$

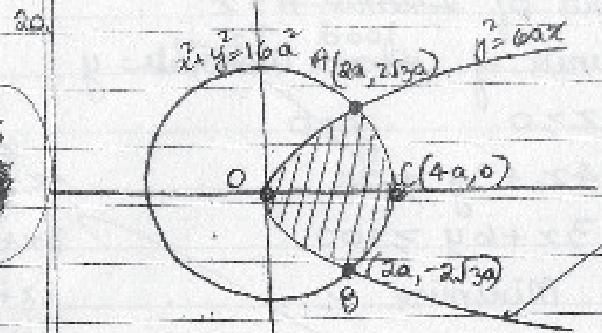
(x,y)	Z
$\left(0, \frac{80}{3}\right)$	₹.160
$\left(\frac{100}{3}, 0\right)$	₹.166.67
$\left(12, \frac{32}{3}\right)$	₹.124

Since the region is unbounded, we should check if any point other than $\left(12, \frac{32}{3}\right)$ is in common to region shaded, & $5x+6y \leq 124$

Clearly, from the graph, only $\left(12, \frac{32}{3}\right)$ lies on both regions.

$\therefore$ The Minimum cost of the diet = ₹. 124

with 12 units of food F_1 & $\frac{32}{3}$ units of food F_2



Solving the two curves

$x^2+6ax=16a^2$

$\left(\frac{x}{a}\right)^2+6\left(\frac{x}{a}\right)-16=0$

$\left(\frac{x}{a}+8\right)\left(\frac{x}{a}-2\right)=0$

$x=-8a$ or $x=2a$
×

∴ $x=2a$

$y=\pm 2\sqrt{3}a$ $\left(y^2=12a^2 \quad y=\pm 2\sqrt{3}a\right)$

∴ Required area = 2× Area OACO

$= 2\times\int (x_1-x_2)\,dy$

$= 2\times\int_0^{2\sqrt{3}a}\sqrt{16a^2-y^2}\,dy - 2\times\int_0^{2\sqrt{3}a}\frac{y^2}{6a}\,dy$

$= 2\times\left[\frac{y}{2}\sqrt{16a^2-y^2}+\frac{16a^2}{2}\sin^{-1}\frac{y}{4a}\right]_0^{2\sqrt{3}a} - \left[\frac{y^3}{9a}\right]_0^{2\sqrt{3}a}$

$= 2\sqrt{3}a\times 2a+16a^2\sin^{-1}\frac{\sqrt{3}}{2}-0-\frac{8\times\cancel{3}\sqrt{3}a^2}{\cancel{9}3}$

$= 4\sqrt{3}a^2 - \frac{8\sqrt{3}}{3}a^2 + \frac{16a^2 \times \pi}{3}$ $\left\{ \int \sqrt{A^2 - x^2}\,dx = \frac{x}{2}\sqrt{A^2 - x^2} + \frac{A^2}{2}\sin^{-1}\frac{x}{A} \right\}$

$= \frac{4}{\sqrt{3}}a^2 + \frac{16a^2\pi}{3}$ sq units

21. $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

$f(x)$ is increasing strictly in an interval if $f'(x) > 0$ in that interval

$f(x)$ is strictly decreasing in an interval if $f'(x) < 0$ in that interval.

$f'(x) = 4x^3 - 24x^2 + 44x - 24$

$= 4(x^3 - 6x^2 + 11x - 8)$

21. Maximum & Minimum values of $f(x) = \sec x + \log \cos^2 x$

~~$= \sec x + 2\log \cos x$~~

$f'(x) = 0$

$\therefore \sec x \tan x + \frac{2}{\cos x}(-\sin x) = 0 \qquad x \neq \frac{\pi}{2}$

$\therefore \tan x(\sec x - 2) = 0$

$\therefore x = \pi$ or $x = \frac{\pi}{3}, \frac{5\pi}{3}$

Now $f''(x) < 0$ for maximum

$f''(x) > 0$ for minimum

$f''(x) = \sec^2 x(\sec x - 2) + \tan^2 x \sec x$

$f''(\pi) = 1 \times -3 + 0 = -3 < 0$

$f''\left(\frac{\pi}{3}\right) = f''\left(\frac{5\pi}{3}\right) = 0 + 3 \times 2 = 6 > 0$

Function attains maximum value at $x = \pi$ & minimum

$\therefore$ value at $x = \frac{\pi}{3}$ & $\frac{5\pi}{3}$

$f(\pi) = -1$

$f\left(\frac{\pi}{3}\right) = 2 - 2\log 2$ ~~$2\log 5$~~

But when $x = \frac{\pi}{2}$ $\therefore$, function becomes undefined.

~~Minimum & Maximum do not exist as Minimum~~

2 - 2log2

22. TP: $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

Proof : $C_1 \rightarrow C_1 - 2C_3$

$$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ac \\ a^2+b^2 & c^2 & ab \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_2$

Now taking $a^2+b^2+c^2$ common from C_1

$$\Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ac \\ 1 & c^2 & ab \end{vmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2-a^2 & ac-bc \\ 0 & c^2-a^2 & ab-bc \end{vmatrix}$$

Taking $(a-b)$ common from R_2 & $(c-a)$ common from C_3

$$\Delta = (a^2+b^2+c^2)(a-b)(c-a)\begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & a+c & -b \end{vmatrix}$$

Expanding along C_1

$$\Delta = (a^2+b^2+c^2)(a-b)(c-a)\left(-(a+b)\times b + (a+c)c\right)$$

$$= (a^2+b^2+c^2)(a-b)(c-a)(b^2 - c^2 + bc - ba)$$

$$= (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$$

H.P.

As $(b-c)(a+b+c) = ab + b^2 + bc - ac - bc - c^2$

$b^2 - c^2 + ab - ac$

25. Equation of plane containing two parallel lines has DRs of perpendicular

$(\vec{a_2}-\vec{a_1})\times\vec{b}$ $\vec{b}$ = DRs of the line

$\vec{a_1}$ = Point (Position vector on line 1)

$\vec{a_1}$ = Point (Position vector) on line 2

$\vec{a_1} = \hat{i}-\hat{j}$

$\vec{a_2} = 2\hat{j}-\hat{k}$

$\vec{b} = 2\hat{i}-\hat{j}+3\hat{k}$

$\vec{a_2}-\vec{a_1} = -\hat{i}+3\hat{j}-\hat{k}$

$$(\vec{a_2}-\vec{a_1})\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 3 & -1\\ 2 & -1 & 3\end{vmatrix} = 8\hat{i}+\hat{j}-5\hat{k}$$

$\therefore$ Equation of plane $= 8x+y-5z = 8\times1+1\times(-1)-5\times0$

$8x+y-5z = 7$

Now consider the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{5}$

$8\times3+1\times1+-5\times5=0$

$\therefore$ DRs of line & perpendicular to plane are perpendicular.

& Point on line $=(2,1,2)$ $8\times2+1-5\times2=7$ also satisfies the equation of plane. $\therefore$ The line is contained in the plane.

26) $f(x) = 4x^2+12x+15$

Let $f(x)=y$

$y = 4x^2+12x+15$

$4x^2+12x+(15-y)=0$

$$x = \frac{-12 \pm \sqrt{144-16(15-y)}}{8} = \frac{-12+4\sqrt{9-15+y}}{8}$$

$$x = \frac{\sqrt{y-6}-3}{2}$$

Consider $g(x) = \frac{\sqrt{x-6}-3}{2}$

Now $fog(x) = f(g(x)) = f\left(\frac{\sqrt{x-6}-3}{2}\right) = \frac{4}{4}(\sqrt{x-6}-3)^2 + 6(\sqrt{x-6}-3)+15$

$$= x-6+9-6\sqrt{x-6}+6\sqrt{x-6}-18+15$$

$$= x$$

$gof(x) = g(f(x)) = g(4x^2+12x+15) = \frac{\sqrt{4x^2+12x+9}-3}{2} = \frac{2x+3-3}{2} = x$

$\therefore fog(x) = gof(x) = x$

By definition, g is the inverse of f.
As inverse exists, f is invertible &

$\Rightarrow f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$

$f^{-1}(31) = \frac{\sqrt{31-6}-3}{2} = \frac{5-3}{2} = 1$

$f^{-1}(87) = \frac{\sqrt{87-6}-3}{2} = \frac{9-3}{2} = 3$

23.

X=	1	2	3	4	5	6
P(X=)	$\frac{10}{20} = \frac{1}{2}$	$\frac{6}{20} = \frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$	0	0

(as 5 & 6 can't be minimum values)

$P(X=1) = \frac{^5C_2}{^6C_3} = \frac{10}{20}$ (Numbers from 2-6 can be chosen (Any two)

$P(X=2) = \frac{^4C_2}{^6C_3} = \frac{6}{20}$ (Numbers from 3-6 can be chosen) (Any two)

$P(X=3) = \frac{^3C_2}{^6C_3}$ $\frac{3}{20}$ (Any 2 out of 4,3,6 can be chosen)

$P(X=2) = \frac{^2C_2}{^6C_3}$ $\frac{1}{20}$ only 5,6 can be chosen

$$\text{Mean} = E(x) = \sum x_i p_i$$

$$= \frac{10}{20}\times 1 + \frac{6}{20}\times 2 + \frac{3}{20}\times 3 + \frac{1}{20}\times 4 + 0 + 0$$

$$= \frac{10}{20} + \frac{12}{20} + \frac{9}{20} + \frac{4}{20} = \frac{35}{20} = 1.75$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$= \frac{10}{20}\times 1 + \frac{6}{20}\times 4 + \frac{3}{20}\times 9 + \frac{1}{20}\times 16 - \frac{49}{16}$$

$$= \frac{10+24+27+16}{20} - \frac{49}{16} = \frac{77}{20} - \frac{49}{16}$$

$$= \frac{7}{4}\left(\frac{11}{5} - \frac{7}{4}\right)$$

$$= \frac{7}{4} \times \frac{9}{20}$$

$$\sigma^2 = \frac{63}{80}$$

$$\begin{array}{r} 44 \\ -35 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 39 \\ 27 \\ 16 \\ \hline 77 \end{array}$$