

# SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

MATHEMATICS (041)

Time allowed: 3 hours

Maximum Marks: 100

## General Instructions:

- (i) All questions are compulsory.
- (ii) This question paper contains 29 questions.
- (iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
- (iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
- (v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
- (vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

### SECTION-A

Questions from 1 to 4 are of 1 mark each.

1. What is the principal value of  $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$ ?
2. A and B are square matrices of order 3 each,  $|A| = 2$  and  $|B| = 3$ . Find  $|3AB|$
3. What is the distance of the point (p, q, r) from the x-axis?
4. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = 3x^2 - 5$  and  $g : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $g(x) = \frac{x}{x^2+1}$ . Find  $g \circ f$

### SECTION-B

Questions from 5 to 12 are of 2 marks each.

5. How many equivalence relations on the set  $\{1,2,3\}$  containing (1,2) and (2,1) are there in all? Justify your answer.
6. Let  $l_i, m_i, n_i$ ;  $i = 1, 2, 3$  be the direction cosines of three mutually perpendicular vectors in space. Show that  $AA' = I_3$ , where  $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ .
7. If  $e^y (x + 1) = 1$ , show that  $\frac{dy}{dx} = -e^y$
8. Find the sum of the order and the degree of the following differential equations:

$$\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$$

9. Find the Cartesian and Vector equations of the line which passes through the point  $(-2, 4, -5)$  and parallel to the line given by  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{8-z}{-6}$
10. Solve the following Linear Programming Problem graphically:  
 Maximize  $Z = 3x + 4y$   
 subject to  
 $x + y \leq 4, x \geq 0$  and  $y \geq 0$
11. A couple has 2 children. Find the probability that both are boys, if it is known that (i) one of them is a boy (ii) the older child is a boy.
12. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which its area increases, when side is 10 cm long.

**SECTION-C**

Questions from 13 to 23 are of 4 marks each.

13. If  $A + B + C = \pi$ , then find the value of

$$\begin{vmatrix} \sin(A + B + C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A + B) & -\tan A & 0 \end{vmatrix}$$

**OR**

Using properties of determinant, prove that

$$\begin{vmatrix} b + c & a - b & a \\ c + a & b - c & b \\ a + b & c - a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

14. It is given that for the function  $f(x) = x^3 - 6x^2 + ax + b$  Rolle's theorem holds in  $[1, 3]$  with  $c = 2 + \frac{1}{\sqrt{3}}$ . Find the values of 'a' and 'b'
15. Determine for what values of x, the function  $f(x) = x^3 + \frac{1}{x^3}$  ( $x \neq 0$ ) is strictly increasing or strictly decreasing

**OR**

Find the point on the curve  $y = x^3 - 11x + 5$  at which the tangent is  $y = x - 11$

16. Evaluate  $\int_0^2 (x^2 + 3) dx$  as limit of sums.
17. Find the area of the region bounded by the y-axis,  $y = \cos x$  and  $y = \sin x, 0 \leq x \leq \frac{\pi}{2}$
18. Can  $y = ax + \frac{b}{a}$  be a solution of the following differential equation?  

$$y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \dots\dots\dots (*)$$
  
 If no, find the solution of the D.E. (\*).

**OR**

Check whether the following differential equation is homogeneous or not

$$x^2 \frac{dy}{dx} - xy = 1 + \cos\left(\frac{y}{x}\right), x \neq 0$$

Find the general solution of the differential equation using substitution  $y=vx$ .

19. If the vectors  $\vec{p} = a\hat{i} + \hat{j} + \hat{k}$ ,  $\vec{q} = \hat{i} + b\hat{j} + \hat{k}$  and  $\vec{r} = \hat{i} + \hat{j} + c\hat{k}$  are coplanar, then for a, b, c  $\neq 1$  show that

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

20. A plane meets the coordinate axes in A, B and C such that the centroid of  $\Delta ABC$  is the point  $(\alpha, \beta, \gamma)$ . Show that the equation of the plane is  $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$
21. If a 20 year old girl drives her car at 25 km/h, she has to spend Rs 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to Rs 5/km. She has Rs 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. Write any one value reflected in the problem.
22. The random variable X has a probability distribution P(X) of the following form, where k is some number:
- $$P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$
- (i) Find the value of k (ii) Find  $P(X < 2)$  (iii) Find  $P(X \leq 2)$  (iv) Find  $P(X \geq 2)$
23. A bag contains  $(2n + 1)$  coins. It is known that 'n' of these coins have a head on both its sides whereas the rest of the coins are fair. A coin is picked up at random from the bag and is tossed. If the probability that the toss results in a head is  $\frac{31}{42}$ , find the value of 'n'.

#### SECTION-D

Questions from 24 to 29 are of 6 marks each

24. Using properties of integral, evaluate  $\int_0^{\pi} \frac{x}{1+\sin x} dx$

OR

Find:  $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$

25. Does the following trigonometric equation have any solutions? If Yes, obtain the solution(s):

$$\tan^{-1} \left( \frac{x+1}{x-1} \right) + \tan^{-1} \left( \frac{x-1}{x} \right) = -\tan^{-1} 7$$

OR

Determine whether the operation \* define below on  $\mathbb{Q}$  is binary operation or not.

$$a * b = ab + 1$$

If yes, check the commutative and the associative properties. Also check the existence of identity element and the inverse of all elements in  $\mathbb{Q}$ .

26. Find the value of  $x$ ,  $y$  and  $z$ , if  $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$  satisfies  $A' = A^{-1}$

OR

Verify:  $A(\text{adj } A) = (\text{adj } A)A = |A| |I$  for matrix  $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

27. Find  $\frac{dy}{dx}$ , if  $y = e^{\sin^2 x} \left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}$
28. Find the shortest distance between the line  $x - y + 1 = 0$  and the curve  $y^2 = x$
29. Define skew lines. Using only vector approach, find the shortest distance between the following two skew lines:

$$\vec{r} = (8 + 3\lambda) \hat{i} - (9 + 16\lambda) \hat{j} + (10 + 7\lambda) \hat{k}$$
$$\vec{r} = 15 \hat{i} + 29 \hat{j} + 5 \hat{k} + \mu (3 \hat{i} + 8 \hat{j} - 5 \hat{k})$$

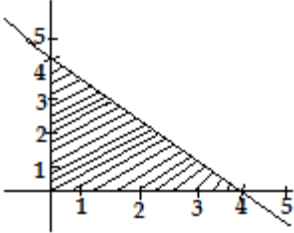
# SAMPLE QUESTION PAPER

CLASS-XII (2016-17)

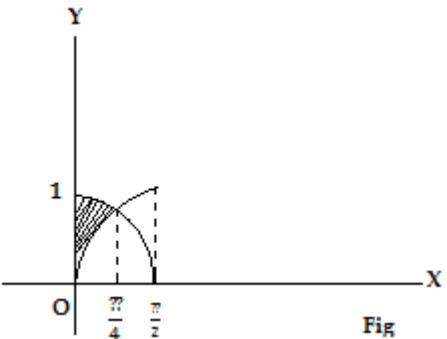
MATHEMATICS (041)

## Marking Scheme

1.	$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = \tan^{-1}\left(-\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$	1
2.	$ 3AB  = 3^3  A   B  = 27 \times 2 \times 3 = 162$	1
3.	Distance of the point (p, q, r) from the x-axis $=$ Distance of the point (p, q, r) from the point (p,0,0) $= \sqrt{q^2 + r^2}$	1
4.	$\text{gof}(x) = \text{g}\{f(x)\} = \text{g}(3x^2 - 5) = \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1} = \frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$	1
5.	Equivalence relations could be the following: $\{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ and (1) $\{(1,1), (2,2), (3,3), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2)\}$ (1) So, only two equivalence relations.(Ans.)	2
6.	$AA' = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \longrightarrow (1)$ because $l_i^2 + m_i^2 + n_i^2 = 1$ , for each $i = 1, 2, 3 \longrightarrow 1/2$ $l_i l_j + m_i m_j + n_i n_j = 0$ ( $i \neq j$ ) for each $i, j = 1, 2, 3 \longrightarrow 1/2$	2
7.	On differentiating $e^y (x + 1) = 1$ w.r.t. x, we get $e^y + (x + 1) e^y \frac{dy}{dx} = 0 \longrightarrow (1)$ $\Rightarrow e^y + \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -e^y \longrightarrow (1)$	2
8.	Here, $\left\{ \frac{d^2y}{dx^2} + (1 + x) \right\}^3 = -\frac{dy}{dx} \longrightarrow (1)$ Thus, order is 2 and degree is 3. So, the sum is 5 $\longrightarrow (1)$	2
9.	Here, $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{-6}$ is same as $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z-8}{6}$ Cartesian equation of the line is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6} \longrightarrow (1)$ Vector equation of the line is $\vec{r} = (-2\hat{i} + 4\hat{j} - 5\hat{k}) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k}) \longrightarrow (1)$	2

<p><b>10.</b></p>	<p>The feasible region is a triangle with vertices  <math>O(0,0)</math>, <math>A(4,0)</math> and <math>B(0,4)</math></p> $Z_0 = 3 \times 0 + 4 \times 0 = 0$ $Z_A = 3 \times 4 + 4 \times 0 = 12$ $Z_B = 3 \times 0 + 4 \times 4 = 16$ <p>Thus, maximum of <math>Z</math> is at <math>B(0,4)</math> and the maximum value is 16 <math>\longrightarrow \frac{1}{2}</math></p> 	<p>2</p>
<p><b>11.</b></p>	<p>Sample space = <math>\{ B_1B_2, B_1G_2, G_1B_2, G_1G_2 \}</math>, <math>B_1</math> and <math>G_1</math> are the older boy and girl respectively.</p> <p>Let <math>E_1</math> = both the children are boys;  <math>E_2</math> = one of the children is a boy ;  <math>E_3</math> = the older child is a boy</p> <p>Then, (i) <math>P(E_1/E_2) = P\left(\frac{E_1 \cap E_2}{E_2}\right) = \frac{1/4}{3/4} = \frac{1}{3} \longrightarrow (1)</math></p> <p>(ii) <math>P(E_1/E_3) = P\left(\frac{E_1 \cap E_3}{E_3}\right) = \frac{1/4}{2/4} = \frac{1}{2} \longrightarrow (1)</math></p>	<p>2</p>
<p><b>12.</b></p>	<p>Here, <math>\text{Area}(A) = \frac{\sqrt{3}}{4} x^2</math>, where '<math>x</math>' is the side of the equilateral triangle <math>\longrightarrow \frac{1}{2}</math></p> <p>So, <math>\frac{dA}{dt} = \frac{\sqrt{3}}{2} \times \frac{dx}{dt} \longrightarrow (1)</math></p> <p><math>= \frac{\sqrt{3}}{2} (10) (2) = 10\sqrt{3} \text{ cm}^2/\text{sec} \longrightarrow \frac{1}{2}</math></p>	<p>2</p>
<p><b>13.</b></p>	<p>As <math>A + B + C = \pi</math>,</p> $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix} = \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix} \longrightarrow (2)$ $= 0 \times \begin{vmatrix} 0 & \tan A \\ -\tan A & 0 \end{vmatrix} - \sin B \times \begin{vmatrix} -\sin B & \tan A \\ -\cos C & 0 \end{vmatrix} + \cos C \times \begin{vmatrix} -\sin B & 0 \\ -\cos C & -\tan A \end{vmatrix}$ $= 0 - \sin B \tan A \cos C + \cos C \sin B \tan A = 0 \text{ (Ans.)} \longrightarrow (2)$ <p style="text-align: center;"><b>OR</b></p>	<p>4</p>

	<p>Let <math>\Delta = \begin{vmatrix} b+c &amp; a-b &amp; a \\ c+a &amp; b-c &amp; b \\ a+b &amp; c-a &amp; c \end{vmatrix}</math></p> <p>Applying <math>C_1 \rightarrow C_1 + C_3</math>, we get <math>\Delta = (a+b+c) \begin{vmatrix} 1 &amp; a-b &amp; a \\ 1 &amp; b-c &amp; b \\ 1 &amp; c-a &amp; c \end{vmatrix} \longrightarrow (1)</math></p> <p>Applying <math>R_2 \rightarrow R_2 - R_1</math>, and <math>R_3 \rightarrow R_3 - R_1</math>, we get</p> $\Delta = (a+b+c) \begin{vmatrix} 1 & a-b & a \\ 0 & 2b-a-c & b-a \\ 0 & 2a+b+c & c-a \end{vmatrix} \longrightarrow (1)$ <p>Expanding <math>\Delta</math> along first column, we have the result <math>\longrightarrow (2)</math></p>	4
14.	<p>Since Rolle's theorem holds true, <math>f(1) = f(3)</math></p> <p>i.e., <math>(1)^3 - 6(1)^2 + a(1) + b = (3)^3 - 6(3)^2 + a(3) + b</math></p> <p>i.e., <math>a + b + 22 = 3a + b</math></p> $\Rightarrow a = 11 \longrightarrow (2)$ <p>Also, <math>f'(x) = 3x^2 - 12x + a</math> or <math>3x^2 - 12x + 11</math></p> <p>As <math>f'(c) = 0</math>, we have</p> $3\left(2 + \frac{1}{\sqrt{3}}\right)^2 - 12\left(2 + \frac{1}{\sqrt{3}}\right) + 11 = 0$ <p>As it is independent of <math>b</math>, <math>b</math> is arbitrary. <math>\longrightarrow (2)</math></p>	4
15.	<p>Here, <math>f'(x) = 3x^2 - 3x^{-4} = \frac{3(x^6 - 1)}{x^4} \longrightarrow (1)</math></p> $= \frac{3(x^4 + x^2 + 1)}{x^4} (x + 1)(x - 1)$ <p>Critical points are <math>-1</math> and <math>1 \longrightarrow (1)</math></p> <p><math>\Rightarrow f'(x) &gt; 0</math> if <math>x &gt; 1</math> or <math>x &lt; -1</math>; and <math>f'(x) &lt; 0</math> if <math>-1 &lt; x &lt; 1</math></p> $\left\{ \because \frac{3(x^4 + x^2 + 1)}{x^4} \text{ always + ive} \right\}$ <p>Hence, <math>f(x)</math> is strictly increasing for <math>x &gt; 1 \longrightarrow (1)</math></p> <p>or <math>x &lt; -1</math>; and strictly decreasing for</p> $(-1, 0) \cup (0, 1) [1] \longrightarrow (1)$ <p style="text-align: center;"><b>OR</b></p> <p>Here, <math>\frac{dy}{dx} = 3x^2 - 11 \longrightarrow \frac{1}{2}</math></p> <p>So, slope of the tangent is <math>3x^2 - 11</math></p>	4

	<p>Slope of the given tangent line is 1.</p> <p>Thus, <math>3x^2 - 11 = 1 \longrightarrow (1)</math></p> <p>that gives <math>x = \pm 2</math></p> <p>When <math>x = 2, y = 2 - 11 = -9</math></p> <p>When <math>x = -2, y = -2 - 11 = -13</math></p> <p>Out of the two points <math>(2, -9)</math> and <math>(-2, -13) \longrightarrow (2)</math></p> <p>only the point <math>(2, -9)</math> lies on the curve</p> <p>Thus, the required point is <math>(2, -9) \longrightarrow \frac{1}{2}</math></p>	
16.	<p>Here, <math>f(x) = x^2 + 3, a = 0, b = 2</math> and <math>nh = b - a = 2 \longrightarrow (1)</math></p> $\int_0^2 (x^2 + 1) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+(n-1)h)] \longrightarrow (1)$ $= \lim_{h \rightarrow 0} h [3 + 1^2 h^2 + 3 + 2^2 h^2 + 3 + \dots + (n-1)^2 h^2 + 3]$ $= \lim_{h \rightarrow 0} h [3n + h^2 \{1^2 + 2^2 + 3^2 + \dots + (n-1)^2\}]$ $= \lim_{h \rightarrow 0} [3nh + h^3 \{ \frac{(n-1)n(2n-1)}{6} \}]$ $= \lim_{h \rightarrow 0} [3nh + \{ \frac{(nh-h)nh(2nh-h)}{6} \}] \longrightarrow (1)$ $= \lim_{h \rightarrow 0} [3 \times 2 + \{ \frac{(2-h)2(4-h)}{6} \}]$ $= 6 + \frac{16}{6}, \text{ i.e., } \frac{26}{3} \longrightarrow (1)$	4
17.	<p>The rough sketch of the bounded region is shown on the right. <math>\longrightarrow (1)</math></p> <p>Required area = <math>\int_0^{\pi/4} \cos x dx - \int_0^{\pi/4} \sin x dx \longrightarrow (1)</math></p> $= (\sin x + \cos x) \Big _0^{\pi/4} \longrightarrow (1)$ $= \sin \frac{\pi}{4} + \cos \frac{\pi}{4} - \sin 0 - \cos 0$ $= \frac{2}{\sqrt{2}} - 1, \text{ i.e., } (\sqrt{2} - 1) \text{ sq units } \longrightarrow (1)$  <p style="text-align: center;"><b>Fig</b></p>	4
18.	$y = ax + \frac{b}{a} \dots (1)$ <p>gives <math>\frac{dy}{dx} = a \longrightarrow \left(1 \frac{1}{2}\right)</math></p> <p>Substituting this value of 'a' in (1), we get</p>	4



$$y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \longrightarrow \left(1 \frac{1}{2}\right)$$

Thus,  $y = ax + \frac{b}{a}$  is a solution of the following differential equation  $y = x \frac{dy}{dx} + \frac{b}{\frac{dy}{dx}} \longrightarrow 1$

OR

Given differential equation can be written as

$$\frac{dy}{dx} = \frac{1+xy + \cos\left(\frac{y}{x}\right)}{x^2} = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right] \dots\dots(1)$$

$$\text{Let } F(x,y) = \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{x^2}\right].$$

$$\text{Then } F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} + \left[\frac{1 + \cos\left(\frac{\lambda y}{\lambda x}\right)}{(\lambda x)^2}\right]$$

$$= \frac{y}{x} + \left[\frac{1 + \cos\left(\frac{y}{x}\right)}{\lambda^2 x^2}\right] \neq F(x,y)$$

Hence, the given D.E. is not a homogeneous equation.  $\longrightarrow (1)$

Putting  $y = vx$  and  $\frac{dy}{dx} = v + x \frac{dv}{dx}$  in (1), we get

$$v + x \frac{dv}{dx} = v + \frac{1 + \cos v}{x^2}$$

$$\Rightarrow \frac{dv}{1 + \cos v} = \frac{1}{x^3} dx$$

$$\Rightarrow \sec^2\left(\frac{v}{2}\right) dv = \frac{2}{x^3} dx \longrightarrow (1)$$

Integrating both sides, we get

$$2 \tan \frac{v}{2} = -\frac{1}{x^2} + C \longrightarrow 1 \frac{1}{2}$$

$$\text{or } 2 \tan \frac{y}{2x} = -\frac{1}{x^2} + C \longrightarrow \frac{1}{2}$$

4

19. Since the vector  $\vec{p}$ ,  $\vec{q}$  and  $\vec{r}$  are coplanar

$$\therefore [\vec{p}, \vec{q}, \vec{r}] = 0$$

$$[\vec{p} \quad \vec{q} \quad \vec{r}] = 0 \longrightarrow (1)$$

$$\text{i.e., } \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{matrix} \longrightarrow (1)$$

$$\text{or } \begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$$

4

	$\Rightarrow a(b-1)(c-1) - 1(1-a)(c-1) - 1(1-a)(b-1) = 0$ <p>i.e., <math>a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0 \longrightarrow (1)</math></p> <p>Dividing both the sides by <math>(1-a)(1-b)(1-c)</math>, we get</p> $\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$ <p>i.e., <math>-\left(1 - \frac{1}{1-a}\right) + \frac{1}{1-b} + \frac{1}{1-c} = 0</math></p> <p>i.e., <math>\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1 \longrightarrow (1)</math></p>									
20.	<p>We know that the equation of the plane having intercepts a, b and c on the three coordinate axes is <math>\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \longrightarrow (1)</math></p> <p>Here, the coordinates of A, B and C are (a,0,0), (0,b,0) and (0,0,c) respectively.</p> <p>The centroid of <math>\Delta ABC</math> is <math>\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) \longrightarrow (1)</math></p> <p>Equating <math>\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)</math> to <math>(\alpha, \beta, \gamma)</math>, we get <math>a = 3\alpha, b = 3\beta</math> and <math>c = 3\gamma \longrightarrow (1)</math></p> <p>Thus, the equation of the plane is <math>\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1</math></p> <p>or <math>\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3 \longrightarrow (1)</math></p>	4								
21.	<p>Let the distance covered with speed of 25 km/h = x km</p> <p>and the distance covered with speed of 40 km/h = y km <math>\left(\frac{1}{2}\right)</math></p> <p>Total distance covered = z km</p> <p>The L.P.P. of the above problem, therefore, is <math>\longrightarrow (1)</math></p> <p>Maximize <math>z = x + y</math></p> <p>subject to constraints</p> $\left. \begin{aligned} 4x + 5y &\leq 200 \\ \frac{x}{25} + \frac{y}{40} &\leq 1 \end{aligned} \right\} \longrightarrow (1)$ <p><math>x \geq 0, y \geq 0 \longrightarrow (1)</math></p> <p>Any one value <math>\longrightarrow \left(\frac{1}{2}\right)</math></p>	4								
22.	<p>Here,</p> <table style="margin-left: 40px;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>P(X)</td> <td>k</td> <td>2k</td> <td>3k</td> </tr> </table> <p>(i) Since <math>P(0) + P(1) + P(2) = 1</math>, we have</p>	X	0	1	2	P(X)	k	2k	3k	4
X	0	1	2							
P(X)	k	2k	3k							

	$k + 2k + 3k = 1$ <p>i.e., <math>6k = 1</math>, or <math>k = \frac{1}{6}</math> <math>\longrightarrow</math> (1)</p> <p>(ii) <math>P(X &lt; 2) = P(0) + P(1) = k + 2k = 3k = \frac{1}{2}</math>; <math>\longrightarrow</math> (1)</p> <p>(iii) <math>P(X \leq 2) = P(0) + P(1) + P(2) = k + 2k + 3k = 6k = 1</math> <math>\longrightarrow</math> (1)</p> <p>(iv) <math>P(X \geq 2) = P(2) = 3k = \frac{1}{2}</math> <math>\longrightarrow</math> (1)</p>	
23.	<p>Let the events be described as follows:</p> <p><math>E_1</math> : a coin having head on both sides is selected.</p> <p><math>E_2</math> : a fair coin is selected.</p> <p>A : head comes up in tossing a selected coin</p> $P(E_1) = \frac{n}{2n+1}; P(E_2) = \frac{n+1}{2n+1}; P(A/E_1) = 1; P(A/E_2) = \frac{1}{2} \longrightarrow (2)$ <p>It is given that <math>P(A) = \frac{31}{42}</math>. So,</p> $P(E_1) P(A/E_1) + P(E_2) P(A/E_2) = \frac{31}{42}$ $\Rightarrow \frac{n}{2n+1} \times 1 + \frac{n+1}{2n+1} \times \frac{1}{2} = \frac{31}{42} \longrightarrow (1)$ $\Rightarrow \frac{1}{2n+1} \left[ n + \frac{n+1}{2} \right] = \frac{31}{42}$ $\Rightarrow 42(3n + 1) = 62(2n + 1)$ $\Rightarrow 2n = 20, \text{ or } n = 10 \longrightarrow (1)$	4
24.	$I = \int_0^\pi \frac{x}{1+\sin x} dx = \int_0^\pi \frac{\pi-x}{1+\sin(\pi-x)} dx \quad (1)$ $= \pi \int_0^\pi \frac{1}{1+\sin x} dx - \int_0^\pi \frac{x}{1+\sin x} dx$ $\Rightarrow 2I = \pi \int_0^\pi \frac{1}{1+\sin x} dx \quad (1)$ $\Rightarrow \frac{\pi}{2} \int_0^\pi \frac{1}{1+\cos\left(\frac{\pi}{2}-x\right)} dx$ $\Rightarrow \frac{\pi}{2} \int_0^\pi \frac{1}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)} dx$ $\Rightarrow \frac{\pi}{4} \int_0^\pi \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right) dx \quad (1)$ $\Rightarrow I = \frac{\pi}{4} \left[ -2\tan\left[\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] \right]_0^\pi \quad (2)$ $\Rightarrow I = \frac{\pi}{4} [2 - (-2)] = \pi \quad (1)$ <p style="text-align: center;"><b>OR</b></p>	6

	<p>Let <math>I = \int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \int \frac{\tan x \sec^2 x}{\tan^3 x + 1} dx</math> (½)</p> <p>On substituting <math>\tan x = t</math> and <math>\sec^2 x dx = dt</math>, we get (1)</p> <p><math>I = \int \frac{t}{t^3 + 1} dt = \int \frac{t}{(t+1)(t^2 - t + 1)} dt</math> (½)</p> <p><math>= -\frac{1}{3} \int \frac{1}{t+1} dt + \frac{1}{3} \int \frac{t+1}{t^2 - t + 1} dt</math></p> <p><math>= -\frac{1}{3} \log t + 1  + \frac{1}{6} \int \frac{(2t-1)+3}{t^2 - t + 1} dt</math> (1)</p> <p><math>= -\frac{1}{3} \log t + 1  + \frac{1}{6} \int \frac{2t-1}{t^2 - t + 1} dt + \frac{1}{2} \int \frac{1}{t^2 - t + 1} dt</math></p> <p><math>= -\frac{1}{3} \log t + 1  + \frac{1}{6} \log t^2 - t + 1  + \frac{1}{2} \int \frac{1}{(t - \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} dt</math></p> <p><math>= -\frac{1}{3} \log t + 1  + \frac{1}{6} \log t^2 - t + 1  + \frac{1}{\sqrt{3}} \tan^{-1}(\frac{2t-1}{\sqrt{3}})</math> (2)</p> <p><math>= -\frac{1}{3} \log \tan x + 1  + \frac{1}{6} \log \tan^2 x - \tan x + 1  + \frac{1}{\sqrt{3}} \tan^{-1}(\frac{2 \tan x - 1}{\sqrt{3}}) + c</math> (1)</p>	6
25.	<p><math>\tan^{-1}(\frac{x+1}{x-1}) + \tan^{-1}(\frac{x-1}{x}) = -\tan^{-1} 7</math></p> <p><math>\Rightarrow \tan^{-1} \left[ \frac{(\frac{x+1}{x-1}) + (\frac{x-1}{x})}{1 - (\frac{x+1}{x-1})(\frac{x-1}{x})} \right] = -\tan^{-1} 7</math> , if <math>(\frac{x+1}{x-1})(\frac{x-1}{x}) &lt; 1</math> ....(*) (2)</p> <p><math>\Rightarrow \tan^{-1} \left[ \frac{x(x+1) + (x-1)^2}{(x-1)x - (x+1)(x-1)} \right] = -\tan^{-1} 7</math></p> <p><math>\Rightarrow \frac{(x^2+x) + (x^2+1-2x)}{(x^2-x) - (x^2-1)} = \tan [-\tan^{-1} 7]</math></p> <p><math>\Rightarrow \frac{2x^2 - x + 1}{-x + 1} = -7</math> (1)</p> <p><math>\Rightarrow 2x^2 - 8x + 8 = 0</math></p> <p><math>\Rightarrow (x - 2)^2 = 0</math></p> <p><math>\Rightarrow x = 2</math> (1)</p> <p>Let us now verify whether <math>x = 2</math> satisfies the condition (*)</p> <p>For <math>x = 2</math>,</p> <p><math>(\frac{x+1}{x-1})(\frac{x-1}{x}) = 3 \times \frac{1}{2} = \frac{3}{2}</math> which is not less than 1</p> <p>Hence, this value does not satisfy the condition (*) (1)</p> <p>i.e., there is no solution to the given trigonometric equation. (1)</p> <p style="text-align: center;"><b>OR</b></p> <p>Given * on <math>\mathbb{Q}</math>, defined by <math>a*b = ab+1</math></p> <p>Let, <math>a \in \mathbb{Q}, b \in \mathbb{Q}</math> then</p> <p><math>ab \in \mathbb{Q}</math></p>	6

	<p>and <math>(ab+1) \in \mathbb{Q}</math></p> <p><math>\Rightarrow a*b = ab+1</math> is defined on <math>\mathbb{Q}</math></p> <p><math>\therefore *</math> is a binary operation on <math>\mathbb{Q}</math> <span style="float: right;">(1)</span></p> <p><b>Commutative:</b> <math>a*b = ab+1</math></p> <p style="margin-left: 40px;"><math>b*a = ba+1</math></p> <p style="margin-left: 80px;"><math>= ab+1 \quad (\because ba = ab \text{ in } \mathbb{Q})</math></p> <p><math>\Rightarrow a*b = b*a</math></p> <p>So <math>*</math> is commutative on <math>\mathbb{Q}</math> <span style="float: right;">(1)</span></p> <p><b>Associative:</b> <math>(a*b)*c = (ab+1)*c = (ab+1)c+1</math></p> <p style="margin-left: 40px;"><math>= abc+c+1</math></p> <p><math>a*(b*c) = a*(bc+1)</math></p> <p style="margin-left: 40px;"><math>= a(bc+1)+1</math></p> <p style="margin-left: 40px;"><math>= abc+a+1</math></p> <p><math>\therefore (a*b)*c \neq a*(b*c)</math></p> <p>So <math>*</math> is not associative on <math>\mathbb{Q}</math> <span style="float: right;">(1)</span></p> <p><b>Identity Element :</b> Let <math>e \in \mathbb{Q}</math> be the identity element, then for every <math>a \in \mathbb{Q}</math></p> <p style="margin-left: 40px;"><math>a*e = a</math> and <math>e*a = a</math></p> <p style="margin-left: 40px;"><math>ae+1 = a</math> and <math>ea+1 = a</math></p> <p><math>\Rightarrow e = \frac{a-1}{a}</math> and <math>e = \frac{a-1}{a}</math> <span style="float: right;">(1)</span></p> <p><math>e</math> is not unique as it depend on 'a', hence identity element does not exist for <math>*</math> <span style="float: right;">(1)</span></p> <p><b>Inverse:</b> since there is no identity element hence, there is no inverse. <span style="float: right;">(1)</span></p>	6
26.	<p>The relation <math>A' = A^{-1}</math> gives <math>A'A = A^{-1}A = I</math> <span style="float: right;">(1)</span></p> <p>Thus, <math display="block">\begin{bmatrix} 0 &amp; x &amp; x \\ 2y &amp; y &amp; -y \\ z &amp; -z &amp; z \end{bmatrix} \begin{bmatrix} 0 &amp; 2y &amp; z \\ x &amp; y &amp; -z \\ x &amp; -y &amp; z \end{bmatrix} = \begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix} \quad \left(1 \frac{1}{2}\right)</math></p> <p style="margin-left: 40px;"><math>\Rightarrow \begin{bmatrix} 0 + x^2 + x^2 &amp; 0 + xy - xy &amp; 0 - xz + xz \\ 0 + xy - xy &amp; 4y^2 + y^2 + y^2 &amp; 2yz - yz - yz \\ 0 - zx + zx &amp; 2yz - yz - yz &amp; z^2 + z^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math></p> <p style="margin-left: 40px;"><math>\Rightarrow \begin{bmatrix} 2x^2 &amp; 0 &amp; 0 \\ 0 &amp; 6y^2 &amp; 0 \\ 0 &amp; 0 &amp; 3z^2 \end{bmatrix} = \begin{bmatrix} 1 &amp; 0 &amp; 0 \\ 0 &amp; 1 &amp; 0 \\ 0 &amp; 0 &amp; 1 \end{bmatrix}</math> <span style="float: right;">(2)</span></p> <p style="margin-left: 40px;"><math>\Rightarrow 2x^2 = 1; 6y^2 = 1 \text{ and } 3z^2 = 1</math></p> <p style="margin-left: 40px;"><math>\Rightarrow x = \pm \frac{1}{\sqrt{2}}; y = \pm \frac{1}{\sqrt{6}}; z = \pm \frac{1}{\sqrt{3}}</math> <span style="float: right;">(1 <math>\frac{1}{2}</math>)</span></p> <p style="text-align: center;">OR</p>	6

	<p>Here, <math> A  = \begin{vmatrix} 1 &amp; -1 &amp; 2 \\ 3 &amp; 0 &amp; -2 \\ 1 &amp; 0 &amp; 3 \end{vmatrix} = 1(0+0) + 1(9+2) + 2(0-0) = 11</math> (1)</p> <p><math>\Rightarrow  A I = \begin{bmatrix} 11 &amp; 0 &amp; 0 \\ 0 &amp; 11 &amp; 0 \\ 0 &amp; 0 &amp; 11 \end{bmatrix}</math> .....(1) (1/2)</p> <p><math>\text{adj } A = \begin{bmatrix} 0 &amp; 3 &amp; 2 \\ -11 &amp; 1 &amp; 8 \\ 0 &amp; -1 &amp; 3 \end{bmatrix}</math> (2)</p> <p>Now, <math>A(\text{adj } A) = \begin{bmatrix} 1 &amp; -1 &amp; 2 \\ 3 &amp; 0 &amp; -2 \\ 1 &amp; 0 &amp; 3 \end{bmatrix} \begin{bmatrix} 0 &amp; 3 &amp; 2 \\ -11 &amp; 1 &amp; 8 \\ 0 &amp; -1 &amp; 3 \end{bmatrix} = \begin{bmatrix} 11 &amp; 0 &amp; 0 \\ 0 &amp; 11 &amp; 0 \\ 0 &amp; 0 &amp; 11 \end{bmatrix}</math> (1)</p> <p>and <math>(\text{adj } A)A = \begin{bmatrix} 0 &amp; 3 &amp; 2 \\ -11 &amp; 1 &amp; 8 \\ 0 &amp; -1 &amp; 3 \end{bmatrix} \begin{bmatrix} 1 &amp; -1 &amp; 2 \\ 3 &amp; 0 &amp; -2 \\ 1 &amp; 0 &amp; 3 \end{bmatrix} = \begin{bmatrix} 11 &amp; 0 &amp; 0 \\ 0 &amp; 11 &amp; 0 \\ 0 &amp; 0 &amp; 11 \end{bmatrix}</math> (1)</p> <p>Thus, it is verified that <math>A(\text{adj } A) = (\text{adj } A)A =  A I</math> (1/2)</p>	6
27.	<p>Putting <math>x = \cos 2\theta</math> in <math>\left\{ 2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right\}</math>, we get (1)</p> $2 \tan^{-1} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$ <p>i.e., <math>2 \tan^{-1} \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}} = 2 \tan^{-1}(\tan \theta) = 2\theta = \cos^{-1} x</math> (2)</p> <p>Hence, <math>y = e^{\sin^2 x} \cos^{-1} x</math></p> <p><math>\Rightarrow \log y = \sin^2 x + \log (\cos^{-1} x)</math></p> <p><math>\Rightarrow \frac{1}{y} \times \frac{dy}{dx} = 2 \sin x \cos x + \frac{1}{\cos^{-1} x} \times \frac{-1}{\sqrt{1-x^2}} = \sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}}</math> (2)</p> <p><math>\Rightarrow \frac{dy}{dx} = e^{\sin^2 x} \cos^{-1} x \left[ \sin 2x - \frac{1}{\cos^{-1} x \sqrt{1-x^2}} \right]</math> (1)</p>	6
28.	<p>Let <math>(t^2, t)</math> be any point on the curve <math>y^2 = x</math>. Its distance (S) from the line <math>x - y + 1 = 0</math> is given by <math>\frac{1}{2}</math></p> $S = \left  \frac{t^2 - t - 1}{\sqrt{1+1}} \right  \frac{1}{2}$ <p><math>= \frac{t^2 - t + 1}{\sqrt{2}} \quad \{ \because t^2 - t + 1 = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4} &gt; 0 \}</math> (1)</p> <p><math>\Rightarrow \frac{dS}{dt} = \frac{1}{\sqrt{2}} (2t-1)</math> (1)</p> <p>and <math>\frac{d^2S}{dt^2} = \sqrt{2} &gt; 0</math> (1)</p> <p>Now, <math>\frac{dS}{dt} = 0 \Rightarrow \frac{1}{\sqrt{2}} (2t-1) = 0</math>, i.e., <math>t = \frac{1}{2}</math> (1)</p> <p>Thus, S is minimum at <math>t = \frac{1}{2}</math></p>	6