BAST-301

Roll No.

Odd Semester Examination, 2019-20

B. Tech: EEE/ MECHANICAL/CIVIL (3rd Semester)

Mathematics – III

Time: 3:00 hrs. **Max. Marks:100**

Total no. of printed pages: 2

Note : Attempt ALL questions.

Q1. Attempt **any four** of the following **4X5=20**

a. Using Fourier integral representation, show that :
$\int_0^\infty \frac{\omega \sin x\omega}{1+\omega^2} d\omega = \frac{\pi}{2} e^{-x}, x > 0$

b. Find Fourier sine transform of $\frac{e^{-ax}}{x}$, a > 0. Hence find Fourier Sine transform of $\frac{1}{x}$.

c. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, if\ |x| < 1 \\ 0, \quad if\ |x| > 1 \end{cases}$, and use it to evaluate
$\int_0^\infty \left\{\frac{x \cos x - \sin x}{x^3}\right\} \cos\frac{x}{2} dx.$

d. If $f_c(p) = \frac{1}{2} \tan^{-1}\left\{\frac{2}{p^2}\right\}$, then find *f(x)*.

e. The temperature u in the semi-infinite rod $0 \le x \le \infty$ is determined by the differential equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ subject to conditions
(i) u = 0 when t = 0, x ≥ 0
(ii) $\frac{\partial u}{\partial x} = -\mu\ (a\ constant) when\ x = 0\ and\ t > 0$

Making use of cosine transform, show that $u(x,t) = \frac{2\pi}{\mu} \int_0^\infty \frac{\cos px}{p^2} (1 - e^{-kp^2 t})\, dp.$

f. Find the Fourier cosine transform of $F(x) = \begin{cases} \cos x, & 0 < x < a \\ 0 & x > a \end{cases}$

Q2. Attempt **any four** of the following **4X5=20**

a. Find the Laplace Transformation of the function
$f(t) = te^{-t} \sin 2t$

b. Find the Laplace transform of the function $f(t) = \frac{e^{-4t}\sin 3t}{t}$

c. Find the Laplace transform of the following periodic function of Period $\frac{2\pi}{\omega}$ is define as

$$f(t) = \begin{cases} \cos \omega t & 0 \le t \le \frac{\pi}{\omega} \\ 0 & \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$$

d. Find the Inverse Laplace Transform of the $\cot^{-1}\left(\frac{s+3}{2}\right)$.

e. Find the Inverse Laplace Transform of the following by Convolution Theorem $\left\{\frac{1}{s^3(s^2+1)}\right\}$.

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f. Solve the following differential equations using Laplace Transform

$y'' + 9y = 6\cos 3t \quad given\ that\ y(0) = 2\ , y'(0) = 0.$

Q3. Attempt **any two** of the following **2x10=20**

a. The first four moments about the working mean 28.5 of a distribution are 0.294, 7.144, 42.409 and 454.98. Comment upon the Skewness and Kurtosis.

b. Fit a straight line to the following data by least square method:

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

c. In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible:

Variance of x = 9, regression equations : 8x – 10y = – 66 and 40x – 18y = 214. What were (a). the mean value of x and y, (b). the standard deviation of y and the coefficient of correlation between x and y?

Q4. Attempt **any two** of the following **2x10=20**

a. (i) Find the missing value of the following data :

x	0	1	2	3	4
f(x)	1	3	9	?	81

(ii)The population of a town in the decennial census was given below. Estimate the population for the year 1895.

Year (x)	1891	1901	1911	1921	1931
Population y (in thousands)	46	66	81	93	101

(b) Find $f'(10)$ from the following data:

x	3	5	11	27	34
f(x)	-13	23	899	17315	35606

(c) Find the root of the equation $x^3 - 5x - 11 = 0$ by the method of iteration correct to three decimal places.

Q5. Attempt **any two** of the following **2x10=20**

a. *Evaluate* $\int_0^6 \frac{dx}{1+x^2}$ *by using:*

ii. *Simpson's one – third rule.*

iii. *Simpson's three – eighth rule.*

b. Given $\frac{dy}{dx} = y - x, y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ *correct to four decimal places by Runge kutta method of order four.*

c. Find $y(2)$ *if* $\frac{dy}{dx} = \frac{1}{2}(x + y)$ and $y(0) = 2,\ y\ (0.5) = 2.636,\ y(1) = 3.595$ *and* $y(1.5) = 4.968$ by Milne Predictor Corrector method.