

TMA-301

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Paper Code & Roll No. to be filled in your Answer Book

Roll No.

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Odd Semester Examination-2016

B.Tech. (Semester-III)**MATHEMATICS-III**

[Time : 3 Hours]

[Maximum Marks : 100]

Note : Attempt **all** questions.1. Attempt **any four** questions : [5×4=20](a) Test the analyticity of the function $w = \log z$.

(b) Examine the nature of the function

$$f(z) = \frac{x^2 y^2 (x + iy)}{x^4 + y^{10}}, z \neq 0; f(0) = 0$$

(c) Using Cauchy integral formula evaluate the following integral

$$\oint_C \frac{e^{2z} dz}{z - \pi i}, \text{ where } C \text{ is the ellipse } |z-2| + |z+2| = 6$$

(d) Find the Laurent's series expansion of $\frac{1}{(z+1)(z+2)^2}$ in the region $2 < |z-1| < 3$ 3.

(e) Evaluate the following integral $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta}$

(f) Using Cauchy residue theorem evaluate the integral

$$\int_C \frac{z^2 - 2z dz}{(z+1)^2(z^2 + 4)}, \text{ where } C \text{ is the circle } |z|=5$$

2. Attempt **any two** questions : [10×2=20]

(a) For a distribution, the mean is 10, variance is 16, γ_1 is 1 and β_2 is 4. Find the first four moments about the origin.

(b) Find the normal equations and hence fit the curve of the form $y = ax + \frac{b}{x}$ for the following data

x	1	2	3	4	5	6	7	8
y	5.4	6.2	8.2	10.3	12.6	14.8	17.2	19.5

(c) Obtain the rank correlation co-efficient for the following data :

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

3. Attempt any two questions : [10×2=20]

(a) (i) Six dice are thrown 729 times how many times do you expect at least three dice to show a five or six.

(ii) The mean height of 500 students is 151 cm and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students have heights between 120 cm and 155 cm.

(b) The following are the mean lengths and ranges of lengths of a finished product from 10 samples each of size 5. The specification limits for length are 200 ± 5 cm. Construct \bar{X} and R-chart and examine whether the process is under control and state your recommendations.

Sample no.	1	2	3	4	5	6	7	8	9	10
Mean \bar{X}	201	198	202	200	203	204	199	196	199	201
Range R	5	0	7	3	3	7	2	8	5	6

(c) Draw a trend line by the method of semi averages to the following data :

years	1991	1992	1993	1994	1995	1996	1997	1998	1999
sales	32	38	50	48	54	42	48	72	82

4. Attempt any four questions : [5×4=20]

(a) Show that the following two sequences have convergence of the second order with the same limit \sqrt{a}

$$x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2} \right) \text{ and } x_{n+1} = \frac{1}{2} x_n \left(3 - \frac{x_n^2}{a} \right)$$

(b) Prove that $1 + \left(\frac{\delta^2}{2} \right) = \sqrt{1 + \delta^2} \mu^2$

(c) Find the missing value from the following data :

x	1	1.5	2	2.5	3	3.5	4
y	6	10	20	15	5

(d) From the table, find the number of students who obtained marks between 36 to 45 :

marks	30-40	40-50	50-60	60-70	70-80
no of students	25	35	22	11	7

(e) Using Newton's divided difference formula find the value of $f(8)$ and $f(12)$:

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

- (f) find the interpolating polynomial which satisfies the following data and hence evaluate $f(9.5)$ from the given table :

x	7	8	9	10
f(x)	3	1	1	9

5. Attempt **any two** questions : [10×2=20]

- (a) Solve the following system by Gauss-Seidel iteration method :

$$2x+10y+z=13, \quad 2x+2y+10z=104, \quad 10x+y+z=12$$

- (b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by trapezoidal rule, Simpson's 3/8 rule and 1/3 rule, by dividing the interval into 9 parts. Compare the results with its actual value.

- (c) Solve $\frac{dy}{dx} = yz + x, \frac{dz}{dx} = xz + y$, given that $y(0)=1, z(0)=-1$ for $y(0.1), z(0.1)$ and $y(0.2), z(0.2)$, using Runge-Kutta method.

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