

Odd Semester Examination 2017 – 18

B Tech III Semester

TCS – 301 (CSE) - DISCRETE STRUCTURES

[Time: 3 Hours]

[Maximum Marks: 100]

Attempt all questions. All questions are compulsory.

1. Attempt any four questions. (5×4=20)

- For any Set A & B, Prove that $P(A \cap B) = P(A) \cap P(B)$
- Explain Pigeonhole principle. If 9 books are to be kept in 4 shelves, show that there must be at least one shelf which contains at least 3 books.
- Let $A = \mathbb{Z}^+$, the set of positive integers, and let $R = \{(a, b) \in A \times A \mid a \text{ divides } b\}$. Is R symmetric or antisymmetric. Also relate symmetric and antisymmetric properties of any relation to the properties of its matrix.
- Prove that $A - (B \cup C) = (A - B) \cap (A - C)$.
- How many one-one functions are there from a set with m elements to one with n elements?

2. Attempt any four questions. (5×4=20)

- Use quantifiers to express the statement "There is a woman who has taken a flight on every airline in the world."
- Show that the set of N Natural numbers is a semi group under the operation $x * y = \max(x, y)$.
- Construct the truth table for $\{(p \rightarrow q) \wedge (q \rightarrow r)\} \rightarrow [p \rightarrow r]$
- Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.
 - If $(G, *)$ is a group then prove that $(a * b)^{-1} = a^{-1} * b^{-1}$ where a, b ∈ G

3. Attempt any two questions. (10 ×2=20)

- What is meant by Hasse Diagram? Draw the Hasse diagram of relation R on A where $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)\}$
- What is complemented lattice and bounded Lattice? Show that lattice is distributive if and only if $(a * b) \text{ XOR } (b * c) \text{ XOR } (c * a) = (a \text{ XOR } b) * (b \text{ XOR } c) * (c \text{ XOR } a)$
- Is $x * y = x - y$, a binary operation over the set of natural numbers. Is it commutative or associative, Justify your answer for all cases.

4. Attempt any two questions. (10 ×2=20)

- Differentiate between tautology and contradiction.
- Which of the two lattices (S_m, \mid) for $m = 30$ and 45 are complemented? Are these lattices distributive?
- Let $A = \{1, 2, 3, 4\}$ and $R = \{(2, 1), (2, 3), (3, 2), (3, 3), (2, 2), (4, 2)\}$. Define R^+ to be the reflexive closure of R and R_s to be the symmetric closure of R. Prove or disprove the symmetric closure of R^+ is the same relation as the transitive closure of R_s .

5. Attempt any two questions. (10 ×2=20)

- Prove that by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer.
- Find all the partitions corresponding to a set of four elements. Draw the diagram of the corresponding lattice.
- In any Boolean algebra show that $a = 0 \iff ab + db = b$.