BSCT-201 **1001**

Even Semester Examination 2018-19

B.TECH. (SEMESTER-II)

(New Syllabus)

MATHEMATICS – II

(Common for All Branches)

Time: 03:00 Hours **Max Marks : 100**

Note: Students need to attempt **all** questions as per instructions given below. Each question carries **equal** marks.

1. This part **contains 6 questions** each of **5 marks**. Student need to attempt **any four.** **[4×5=20]**

(a) Solve the equation $3x^4p^2 - xp - y = 0$

(b) Solve $y(1 + x\,y)dx + x(1 - xy)dy = 0$

(c) Solve the differential equation.

$$\frac{d^2y}{dx^2} + \frac{1}{x^{\frac{1}{3}}}\frac{dy}{dx} + \left(\frac{1}{4x^{\frac{2}{3}}} - \frac{1}{6x^{\frac{4}{3}}} - \frac{6}{x^2}\right) = 0$$

(d) Find the analytic function if.

$u - v = (x - y)(x^2 + 4xy + y^2)$

(e) Find by double integration the area enclosed by the curve $9xy = 4$ and the line $2x + y = 2$

(f) Use Cauchy integral formula to evaluate.

$$\oint_C \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)} dz$$

Where C is the circle $|z| = 3$

2. This part **contains 6 questions** each of **5 marks**. Student need to attempt **any four** :

[4×5=20]

(a) Prove that u = $x^2 - y^2 - 2xy - 2x + 3y$ is harmonic function find a function v such that

f(z) = u+iv is analytic also express f(z) in term of z.

(b) Find the bilinear transformation which maps the points z = 1 , i ,2+i in the z-plane onto the points ω = i , 1 , ∞ in the ω-plane.

(c) Evaluate $\iint_R y^2 dxdy$ over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$

(d) Change the order of integration is I = $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.

(e) Suppose $\vec{F}(x, y, z) = x^3\vec{i} + y\vec{j} + z\vec{k}$ is the force field , find the work done by $\vec{F}$ along the line from the (1 , 2 , 3) to (3 , 5 , 7)

(f) Show that the function e^z has an isolated essential singularity at z = ∞

3. This part **contains 3 questions** each of 10 marks. Student need to attempt **any two:** **[2×10=20]**

(a) (i). Show that the function f(z) = $\frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, z ≠ 0; f(0) = 0 , satisfies the C-R equation at z = 0, but is not analytic there.

(ii). Find the analytic function whose imaginary part is $e^x(x\cos y - y\sin y)$

(b) (i). Apply the variation of parameters to solve :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$$

(ii). Prove that $\int_{-1}^{1} [P_n(x)]^2 \; dx = \frac{2}{2n+1}$

(c) (i). Determine the poles of the following function and residue at each pole $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and hence evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$; where C $|z| = 3$

(ii) Evaluate the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at z = 1 ,2 , 3 and ∞ and show that their sum is zero

4. This part **contains 3 questions** each of 10 marks. Student need to attempt **any two :** **[2×10=20]**

(a) Apply calculus of residue to prove that :

$$\int_0^{2\pi} \frac{\cos 2\theta d\theta}{1-2a\cos\theta+a^2} = \frac{2\pi a^2}{1-a^2}, \qquad (a^2 < 1)$$

(b) Find the series solution of equation near $x = 0$

$$2x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + 3y = 0$$

(c) Evaluate $\iint (x+y)^2 dydx$ *over* the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

5. This part **contains 3 questions** each of 10 marks. Student need to attempt **any two :** **[2×10=20]**

(a) $(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx} + y\cos x = 0$ given $y = x$ is solution

(b) Evaluate $\iiint (x^2y^2 + y^2z^2 + z^2x^2)\, dxdydz$ over the volume of the sphere $x^2 + y^2 + z^2 = a^2$

(c) State and prove Laurent's theorem

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