TMA-201 1001

Even Semester Examination, 2017-18

B.Tech. (SEMESTER-II)

MATHEMATICS-II

Time: 03:00 Hours **Max Marks : 100**

Note : Attempt all questions. The choice of questions is internal as indicated

Q1. Attempt **any four** of the following : [4x5=20]

(a) Solve $(D^2-2D+1)y = xe^x \sin x$

(b) Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = x^2 \log x$

(c) Solve the following simultaneous equations

$$\frac{dx}{dt} + 2x - 3y = t, \quad \frac{dy}{dt} - 3x + 2y = e^{2t}$$

(d) Solve using method of variation of parameters

$$(D^2-1)y = 2(1-e^{-2x})^{-1/2}$$

(e) Solve $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$

(f) Solve $x^2y\,dx - (x^3+y^3)dy = 0$

Q2. Attempt **any four** of the following : [4x5=20]

(a) Find the Laplace transformation of the function

$$F(t) = t^2e^{2t}\sin t$$

(b) Find the inverse Laplace transformation of the function

$$f(p) = \frac{p^2}{(p^2+a^2)(p^2+b^2)}$$

(c) Evaluate the following integral $\int_0^\infty \cos x^2 dx$

(d) State and prove Convolution theorem.

(e) Solve the following simultaneous differential equation using Laplace transformation:

$$\frac{dx}{dt} + y = \sin t \ , \qquad \frac{dy}{dt} + x = \cos t$$

Given that x=2, y=0 at t=0.

(f) Find the Laplace transform of

$$F(t) = \begin{cases} t & ,0 < t < a \\ 2a - t & ,a < t < 2a \end{cases}$$

$$with \ \ F(t + 2a) = F(t)$$

3. Attempt **any two** of the following: [2x10=20]

(a) Test the convergence of the series

(i) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

(ii) $\Sigma(1 + \frac{1}{\sqrt{n}})^{-n^{3/2}}$

(b) Discuss the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots\ldots\ldots\ldots\ldots$$

(c) Discuss the convergence of the exponential series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots$$

Also find the region of uniform convergence of this series.

4. Attempt **any two** of the following: [2x10=20]

(a) Expand $f(x) = x \sin x$ as a cosine series in $(0, \pi)$ and show that

$$1 + \frac{2}{1.3} - \frac{2}{3.5} + \frac{2}{5.7} + \ldots\ldots\ldots\ldots = \frac{\pi}{2}$$

(b) (i) Solve the partial differential equation:

$$(D-D'-1)(D-D'-2)z=\sin(2x+3y)$$ (7)

(ii) Find the particular integral of:

$$(D-3D'-2)^2 z = 2e^{2x}\tan(y+3x)$$ (3)

(c) Find the Fourier series for the function

$$f(x)=\begin{cases}-\pi & ,-\pi<x<0\\ x & ,0<x<\pi\end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.................=\frac{\pi^2}{8}$

5. Attempt **any two** of the following: [2x10=20]

(a) A tightly stretched string with fixed end points x=0 and x=l is initially in a position given by $y(x,0)=A\sin(\pi x/l)$ .If it is released from rest from this position, find the displacement y at any distance x from one end at any time t.

(b) Find the temperature in a bar of length 2 whose ends are kept at zero and internal surface insulated if the initial temperature is $\sin\frac{\pi x}{2}+3\sin\frac{5\pi x}{2}$.

(c) Using method of separation of variables, solve:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$$

$$u(0,y)=4e^{-y}-e^{-5y}$$

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Additional Choices Question No. 3 as follows:

Q.3 (e) *Obtain a Fourier expression for* $f(x) = x^3$ *for* $-\pi < x < \pi$ (10)

(f) Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = e^{2x-y} + Cos(x + 2y)$ (10)

(g.) (i) Classify the partial differential equation

$2\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + 3\frac{\partial^2 u}{\partial y^2} = 0$ (4)

(ii) Find the Laplace transform of $\frac{Cosat - Cosbt}{t}$ (6)