TMA-201 71 Printed Pages : **5**

Paper Code & Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B.Tech. (II - Sem.)

Even Semester Examination - 2016

MATHEMATICS-II

[Time : 3 Hours] *[Maximum Marks :100]*

Note : Attempt **all** questions. The choice of questions is internal as indicated.

Q1. Attempt any **four** of the following : (5x4=20)

(a) Find the value of λ for which the differential equation $(xy^2+\lambda x^2y)dx+(x+y)x^2dy=0$ is exact. Solve the equation for this value of λ.

(b) Solve : $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\sin(\log x^2)$

(c) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\tan x$

(d) Solve the differential equation by removing the first derivative.

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$$

(e) The equations of Motions of a particle are given by $\frac{dx}{dt}+\omega y=0$ & $\frac{dy}{dt}-\omega x=0$

Find the path of the particle and show that it is a circle.

(f) Solve : $(D^3-D)\,y = 2x+1+4\cos x+2e^x$

Q2. Attempt any **four** of the following: (5x4=20)

(a) Find the Laplace transforms of $\frac{\cos at-\cos bt}{t}$

(b) Find the Laplace transform of the following periodic function $F(t)=\begin{cases} t & , \quad o<t<\pi \\ n-t & , \quad n<t<2\pi \end{cases}$

(c) Evaluate: $\int_0^\infty te^{3t}\sin t\,dt$

(d) Find the inverse Laplace transform of $\log\left\{\sqrt{\frac{s+1}{s-1}}\right\}$

(e) State Convolution theorem and hence find $L^{-1}\left\{\frac{1}{s^2(s^2+1)}\right\}$

(f) Solve the differential equation using Laplace transform method

$$\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=0,$$

where $y=1$,

$$\frac{dy}{dt}=2 \text{ and } \frac{d^2y}{dt^2}=2 \text{ at } t=0$$

Q3. Attempt any **two** of the following : (10x2=20)

(a) (i) Test the convergence or divergence of the series $\sum_{n=1}^{\infty}\frac{2n^2+3n}{5+n^5}$

(ii) Test the convergence of the series

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$$

(b) Discuss the convergence of the series

$$\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+........\infty$$

(c) (i) Show that the given series is Uniformly Convergent.

$$\sum_{n=1}^{\infty}\frac{\sin(x^2+nx)}{n(n+2)} \text{ for all real } x$$

(ii) Examine the convergence of the series

$$2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\ldots\ldots\ldots\infty$$

Q4. Attempt any **two** of the following: (10x2=20)

(a) Find a Fourier series of a function. $f(x)=x-x^2,\ -\pi\le x\le\pi$ .Hence show that

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots.=\frac{\pi^2}{12}$$

(b) Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x\partial y^2}+2\frac{\partial^3 z}{\partial y^3}-\sqrt{(x+2y)}+e^{x+y}$$

(c) Find the half range sine series for the function $f(x)=(2x-1)$; $0<x<1$

Q5. Attempt any **two** of the following: (10x2=20)

(a) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y=a\sin\frac{\pi x}{l}$ from which it is released at a time t = 0. Show that the displacement of any point at a distance 'x' from one end at time 't' is given by

$$y(x,t)=a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$$

(b) Solve the following equation by the method of Separation of Variables

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}, \text{ where } u(o, y) = 8e^{-3y}$$

(c) Find the temperature in a bar of length 2 whose ends are kept at zero temperature and lateral surface insulated if the initial temperature is $\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right)$.

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