TMA-201 16 Printed Pages : 5

Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B. Tech. (2ND Semester)

Examination, 2015

Mathematics-II

Time: 3.00 Hrs] **[Max. Marks: 100**

Note: (i) Attempt all the questions.

(ii) The choice of questions is internal as indicated.

Q1. Attempt any four of the following: (5×4 = 20)

(a) Solve:

$$\left(3y-2xy^3\right)dx+\left(4x-3x^2y^2\right)dy$$

(b) Solve:

$$\frac{d^2y}{dx^2}=\left[1-\left(\frac{dy}{dx}\right)^2\right]^{1/2}$$

(c) Solve $\frac{d^2y}{dx^2}+y=\sec x$ by using method of variation of parameters.

(d) Solve:

$$\left(D^2-4D+4\right)y=8x^2e^{2x}\sin 2x\ ;\ D\equiv\frac{d}{dx}$$

(e) Solve:

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=(\log x).\sin(\log x)$$

(f) Solve:

$$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$$

Q2. Attempt any four of the following: (5×40)

(a) Find the Laplace transform of the funn

$F(t)=te^{-t}\sin 2t$

(b) Draw the graph of the following periodic fun and find its Laplace transform, where

$$F(t)=\begin{cases}t & ;\ 0<t\le a\\(2a-t) & ;\ a<t\le 2a\end{cases}$$

(c) Find the inverse Laplace transform

$\cot^{-1}\left(\frac{s+3}{2}\right)$.

(d) A periodic function of period $\frac{2\pi}{w}$ is defined as:

$$F(t)=\begin{cases} E\sin\omega t \ ; & 0\le t\le \frac{\pi}{w} \\ 0 \ ; & \frac{\pi}{w}\le t\le \frac{2\pi}{w} \end{cases}$$

where E and w are constants. Find its Laplace transform.

(e) Use convolution theorem find:

$$L^{-1}\left\{\frac{s}{\left(s^2+1\right)\left(s^2+4\right)}\right\}$$

(f) Solve the differential equation using Laplace transform method:

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=e^t\cdot\sin t$$

where $y(0)=0$, $y'(0)=1$.

Q3. Attempt any two of the following: (10×2=20)

(a) (i) Discuss the convergence of the series:

$$\sum_{n=1}^{\infty}\frac{n}{n+1}$$

(ii) Test the convergency of the series:

$$\frac{\sqrt{2}-\sqrt{1}}{1}-\frac{\sqrt{3}-\sqrt{2}}{2}+\frac{\sqrt{4}-\sqrt{3}}{3}-\frac{\sqrt{5}-\sqrt{4}}{4}+\ldots$$

(b) Test the series:

$$\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\ldots\ldots\infty$$

(c) Define the uniform convergence. Show the sequence $\{S_n\}$, where $S_n(x)=\frac{nx}{1+(nx)^2}$ ot uniformly convergent on any interval containino.

Q4. Attempt any two of the following: (10×2)

(a) Prove that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos x}{n^2}$, :e show that $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots=\frac{\pi^{12}}{12}$.

(b) Solve the partial differential equation:

$$\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=e^{x+y}+\cos(x+2y)+e^{2x-}$$

(c) Find a Fourier series expansion for the function $f(x) = x\sin x$ in $-\pi < x < \pi$. Hence show that

$$\frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots\ldots = \frac{\pi}{4}.$$

Q5. Attempt any two of the following: (10×2 = 20)

(a) Using the method of separation of variable, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$. Where $u(x, t) = 6e^{-3x}$, when $t = 0$.

(b) If a string of length 'l' is initially at rest in equilibrium position and each of its point is given the velocity $\left(\frac{\partial u}{\partial t}\right)_{t=0} = b\sin^3\left(\frac{\pi x}{l}\right)$, find the displacement $u(x, t)$.

(c) Solve completely the equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ representing the vibrations of a string of length 'l', fixed at both ends, given that $u(0, t) = 0$, $u(l, t) = 0$, $u(x, 0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$; $0 < x < l$.

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