TMA-101

Roll No.

Odd Semester Examination, 2019-20

B. Tech: CSE/ECE/EEE/MECHANICAL/CIVIL (1st Semester)

Mathematics -1

(AS PER OLD SYLLABUS UPTO 2017-2018)

Time: 3:00hr **M.M:100**

Total no. of printed pages: 2

Note: Attempt all questions.

Q.1 *Attempt any four parts.* 5*4

(a) *Reduce the matrix to normal form and find its rank:*

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$$

(b) *Find the value of k such that the system of equations*

$$2x + 3y - 2z = 0$$
$$3x - y + 3z = 0$$
$$7x + ky - z = 0$$

has non − trivial solutions.

(c) *Verify Cayley Hamilton theorem for the matrix*

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(d) *Find the eigen values and eigen vectors of the matrix*

$$A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$$

(e) Examine the following system of vectors for linear dependence. If dependent, find the relation between them:

$X_1 = (1, 2, 4)$, $X_2 = (2, -1, 3)$, $X_3 = (0, 1, 2)$, $X_4 = (-3, 7, 2)$

(f) *Prove the characteristic roots of a hermitian matrix are all real.*

Q.2 *Attempt any four parts.* 5*4

(a) If $y = \sin(m\sin^{-1}x)$, prove that

$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$

(b) *If* $u = x^x y^y z^z$ *show that at* $x = y = z$,

$$\frac{\partial^2 z}{\partial x \partial y} = -(x\log ex)^{-1}$$

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(c) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$

(d) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ prove that

$$x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$$

(e) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ about (1,1)upto second degree terms by taylors theorem.

(f) If $u = f(r)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$

Q.3. Attempt any two parts: 10*2

(a) Examine the function $f = x^3 + y^3 - 63(x+y) + 12xy$ for maximum and minimum values.

(b) If $x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, find $\frac{\partial(r,\theta,\varphi)}{\partial(x,y,z)}$

(c) (i) A balloon is in the form of right circular cylinder of radius 1.5m and length 4m and is surmounted by hemispherical ends. If the radius is increased by 0.01m and the length by 0.05m, find the percentage change in the volume of the balloon.

(ii) Find the possible percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if r_1, r_2, r_3 are each in error by plus 1.2% .

Q.4 Attempt any two parts: 10*2

(a) (i) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma m+n}$

(ii) Evaluate $\int_0^1 \frac{dx}{\sqrt{-\log x}}$

(b) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dydx$ by changing the order of integration.

(c) Find the area of one of the loops of $x^4 + y^4 = 2a^2xy$, by converting into polar coordinates.

Q.5 Attempt any two parts: 10*2

(a) Prove that $(y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ is both solenoidal and irrotational.

(b) Verify Stoke's theorem for a vector field defined by $\bar{F} = (x^2 - y^2)i + 2xyj$ in rectangular xy plane bounded by the lines $x = 0, x = a, y = 0, y = b$.

(c) Evaluate $\iint_S \bar{A}.\hat{n}\, ds$ where $\bar{A} = (x + y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.