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TMA-101 1066

Odd Semester Examination 2017

B.TECH. (SEMESTER-I)

MATHEMATICS - I

Time : 03:00 Hours **Max Marks : 100**

Note : Attempt **all** questions.

1. Attempt **any FOUR** of the following: 5x4=20

(a) Find the rank of the matrix A by reducing it to normal form where

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$$

(b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. Hence find A^{-1}

(c) Show that the vectors $X_1 = (1, 2, 3)$, $X_2 = (3, -2, 1)$ and $X_3 = (1, -6, -5)$ are linearly dependent. Find the relation between them.

(d) Define Unitary & Hermitian matrices with an example of each.

(e) Determine for what value of λ and μ the following equations : $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$

have (i) a Unique solution (ii) No Solution (iii) Infinite no. of solutions.

(f) Find the eigen values and corresponding eigen vectors for the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

2. Attempt **any FOUR** of the following: 5x4=20

(a) If $y = a\cos(\log x) + b\sin(\log x)$, prove that : $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

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(b) If $y = \tan^{-1} x$, prove that : $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$. Hence find y_n at $x = 0$

(c) If $z = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that $x\frac{dz}{dx} + y\frac{\partial z}{\partial y} = \sin 2z$ & $x^2\frac{\partial^2 z}{\partial x^2} + 2xyx^2\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 2\cos 3z \sin z$.

(d) If $u = e^{xyz}$, then find the value of $\frac{\partial^3}{\partial x\partial y\partial z}$.

(e) Expand $e^x \sin y$ in powers of x& y by Taylor's theorem up to and including 3rd degree terms.

(f) If $F = f(y-z, z-x, x-y)$ prove that : $\frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} = 0$

3. Attempt **any TWO** of the following: (10x2=20)

(a) If $x + y + z = u$, $y + z = uv$, $z = uvw$,, then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

(b) The period of the simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$, find the maximum error in T due to the gpossible error up to 1% in l and 2.5% in g .

(c) Find the point upon the plane $ax + by + cz = p$, at which the function $f = x^2 + y^2 + z^2$ has a minimum value and find this minimum f .

4. Attempt **any TWO** of the following: (10x2=20)

(a) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.

(b) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = a$.

(c) Define Gamma and Beta functions and prove that $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$

5. Attempt **any TWO** of the following: (10x2=20)

(a) Use divergence theorem to evaluate the surface integral $\iint_S (x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$ where S is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant.

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(b) Find the directional derivative of $\phi = (x^2 + y^2 + z^2)^{\frac{-1}{2}}$ at the point P(3,1,2) in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.

(c) Verify Stoke's theorem for a vector field defined by $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ in the rectangle in XY- plane bounded by lines x = 0, x = a, y = 0 and y = b.

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