Paper Code & Roll No. to be filled in your Answer Book

Roll No.

Odd Semester Examination-2016

B.Tech. (Semester-I)

MATHEMATICS - I

[Time : 3 Hours] [Maximum Marks :100]

Note : Attempt **all** questions.

1. Attempt **any four** questions : [5x4=20]

(a) Using elementary transformations find the inverse

of $A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

(b) Find the values of a,b, for which the following systems has a

(i) unique solution

(ii) no solution, and

(iii) infinite solutions

3x-2y+z=b,

$5x-8y+9z=3$

$2x+y+az=-1$

(c) Examine the following vectors for linear dependence and independence. If dependent, find relation between them

$U = t^3 + 4t^2 - 2t + 3$

$V = t^3 + 6t^2 - t + 4$

$W = 3t^3 + 8t^2 - 8t + 7$

(d) Verify Cayley-Hamilton theorem and hence find the inverse of the following matrix :

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$

(e) Diagonalize the following matrix :

$$\begin{bmatrix} 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(f) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

2. Attempt **any four** questions : [5 x 4 =20]

(a) If $y=\sin(m \sinh^{-1} x)$, prove that :

$$(1+x^2)\, y_{n+2} + (2n+1)xy_{n+1} + (n^2+m^2)y_n = 0.$$

(b) If $\theta = t^n e^{-\frac{r^2}{4t}}$ find the value of n for which

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

(c) If $u = x\Phi\left(\frac{y}{x}\right) + \Psi\left(\frac{y}{x}\right)$ show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial x^2} = 0$$

(d) If $u = u\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ show that

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$$

(e) Expand $e^x \log(1+y)$ in the neighbourhood of the point (0, 0) up-to 3rd degree term.

(f) If $x = u\cos\alpha - v\sin\alpha$, $y = u\sin\alpha + v\cos\alpha$, prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}$

3. Attempt **any two** questions : [10 x 2 = 20]

(a) If $u^3 + v^3 + w^3 = x + y + z$,

$u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and

$u + v + w = x^2 + y^2 + z^2$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$

(b) In a plane triangle ABC, find the maximum value of $\cos A \cos B \cos C$.

(c) In estimating the number of bricks in a pile which is measured to be ($5m \times 10m \times 5m$) the count of bricks is taken as 100 bricks per m^3. Find the error in the cost when the tape is stretched 2% beyond the standard length. The cost of bricks is Rs.2000 per thousand bricks.

4. Attempt **any two** questions : [10 x 2 = 20]

(a) (i) Evaluate $\iint x^2\,dx\,dy$, over the area outside $x^2 + y^2 - ay = 0$ and inside $x^2 + y^2 - 2ay = 0$.

(ii) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.

(b) Establish the relation between Beta and Gamma functions and hence evaluate $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}}$

(c) (i) Find the area outside the circle r = a and inside the cardioid r = a(1+ cos θ).

(ii) Evaluate $\iiint \frac{dxdxdz}{(a^2-x^2-y^2-z^2)}$, for all positive values of the variables for which the expression is real.

5. Attempt **any four** questions : [5 x 4 =20]

(a) Find the value of a and b such that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2 y + z^3 = 4$ cut orthogonally at (1, -1, 2).

(b) Find the directional derivative of $\varphi = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the normal to the surface $x \log z - y^2 = -4$ at (−1, 2,1).

(c) Show that $\text{div}(\text{grad } r^n) = n(n+1)r^{n-2}$

(d) Determine the constants a,b,c s[illegible]t $\vec{F}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4y+2z)\hat{k}$ is irrotational. Hence find the [illegible]r potential.

(e) Evaluate $\iint_s \vec{F}.d\hat{s}$ where $\vec{F}=z\hat{i}+x\hat{j}-3zy^2\hat{k}$ S is the surface of the cylinder $x^2+y^2=16$ inc[illegible] in the first octant between $z=0$ and $z=5$.

(f) Verify Stoke's theorem for the vector [illegible] $\vec{F}=(2x-y)\hat{i}-yz^2\hat{j}-y^2z\hat{k}$ over the upper surface of $x^2+y^2+z^2=1$ bounded by its proje[illegible] on xy plane.

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