TMA-101 **1032** Printed Pages : 4

Paper Code & Roll No. to be filled in your Answer Book

Roll No. | | | | | | | | | | | |

B. Tech. I Year, I Sem.

Odd Semester Examination-2015

Mathematics-I

Time : 3 Hrs.] *[Max. Marks :100*

Note: Answer Any Four (4x5=20)

Q.1.1: Transform $\begin{pmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{pmatrix}$ into a unit matrix by using elementary transformations.

Q.1.2: Find the value of λ *and* μ so that the equations:

$$\begin{aligned} 2x+3y+5z&=9 \\ 7x+3y-2z&=8 \\ 2x+3y+\lambda z&=\mu \end{aligned}$$ have

(i) no solution

(ii) a unique solution

(iii) an infinite number of solutions.

Q.1.3: Find the characteristic equation of the matrix A.

$$A = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$$

Q.1.4: Show the matrix $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is a unitary matrix if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$.

Q.1.5: Write matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ -2 & 5 & 3 \\ -1 & 6 & 3 \end{pmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

Note : Answer Any Four (4x5=20)

Q.2.1: If $y = a\cos(\log x) + b\sin(\log x)$, Prove that

$$(x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

Q.2.2: If $z = e^{ax+by}.f(ax-by)$, *prove that* $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$

Q.2.3: State and prove Euler's theorem on Homogeneous function.

Q.2.4: If $u = f(r)$ and $x = r\cos\theta,\ y = r\sin\theta$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

Q.2.5: If $u = \sin^{-1}\left(\frac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}\right)$, Show that $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$

Note : Answer Any Two (2x10=20)

Q.3.1: If $y_1 = \frac{x_2 x_3}{x_1},\ y_2 = \frac{x_1 x_3}{x_2},\ y_3 = \frac{x_1 x_2}{x_3}$. Show that the Jacobian y_1, y_2, y_3 *with respect to* x_1, x_2, x_3 *is* 4.

Q.3.2: If $u = x + y^2, v = y + z^2, w = z + x^2$. prove that

$$\frac{\partial x}{\partial u} = \frac{1}{1 + 8xyz}$$

Q.3.3: The period T of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Find the maximum error in T due to possible errors upto 1% in l and 2.5% in g.

Note : Answer Any Two (2x10=20)

Q.4.1: Change the order of integration and evaluate

$$\int_0^2 \int_{2-\sqrt{4-y^2}}^{2+\sqrt{4-y^2}} dx\,dy$$

Q.4.2: A loop of the curve $y^2 = x^2\left(1-x^2\right)$ is rotated about the y-axis. Find the volume generated.

Q.4.3: Show that : $\int_0^{\pi/2} \sin^p\theta \cos^q\theta\, d\theta = \dfrac{\Gamma\frac{p+1}{2}\,\Gamma\frac{q+1}{2}}{2\,\Gamma\frac{p+q+2}{2}}$

Note : Answer Any Two (2x10=20)

Q.5.1: A particle moves along the curve $\bar{r} = \left(t^3-4t\right)\hat{i} + \left(t^2+4t\right)\hat{j} + \left(8t^2-3t^3\right)\hat{k}$, where t is the time. Find the magnitude of the tangential components of its acceleration at t=2.

Q.5.2: Find the divergence and curl of $\bar{v} = (xyz)\hat{i} + \left(3x^2y\right)\hat{j} + \left(xz^2-y^2z\right)\hat{k}$ at $(2,-1,1)$

Q.5.3: The vector field $\bar{F} = x^2\hat{i} + z\hat{j} + yz\hat{k}$ is defined over the volume of the cuboids given by $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. enclosing the surface S. Evaluate the surface integral $\iint_S \bar{F}.d\bar{S}$

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