1.1 Electrostatic Potential

The electrostatic potential at any point in an electric field is equal to the amount of work done in bringing the unit positive test charge without acceleration from infinity to that point.

Electrostatic potential, $V = \dfrac{\text{Work done } (W)}{\text{Charge } (q_0)}$

Its SI unit is volt (V) and $1\text{ V} = 1\text{ JC}^{-1}$ and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.
It is a scalar quantity.

NOTE Electrostatic potential is a state dependent function as electrostatic forces are conservative forces.

Electrostatic Potential Difference

The electrostatic potential difference between two points in an electric field is defined as the amount of work done in moving a unit positive test charge from one point to the other point against electrostatic force without any acceleration (i.e. the difference of electrostatic potentials of the two points in the electric field).

$$V_B - V_A = \frac{W_{AB}}{q_0}$$

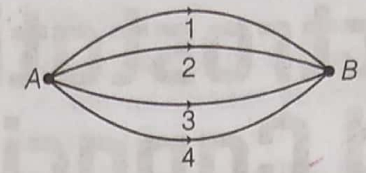
where, W_{AB} is work done in taking charge q_0 from A to B against of electrostatic force.

Also, the line integral of electric field from initial position A to final position B along any path is termed as potential difference between two points in an electric field, i.e.

$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

NOTE As, work done on a test charge by the electrostatic field due to any given charge configuration is independent of the path, hence potential difference is also same for any path.

For the diagram given below, potential difference between points A and B will be same for any path.



Electrostatic Potential due to a Point Charge

Electrostatic potential due to a point charge q at any point P lying at a distance r from it is given by $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{r}$

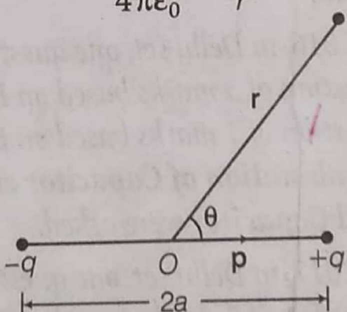
The potential at a point due to a positive charge is **positive** while due to negative charge, is **negative**.

- When a positive charge is placed in an electric field, it experiences a force which drives it from points of higher potential to the points of lower potential. On the other hand, a negative charge experiences a force driving it from lower potential to higher.

Electrostatic Potential due to Electric Dipole

- Electrostatic potential due to an electric dipole at any point P inclined at an angle θ whose position vector is r w.r.t. mid-point of the dipole is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2}$$



Electrostatic potential at point P due to a short dipole ($a << r$) is given by $V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$

where, θ is the angle between $\mathbf{r}$ and $\mathbf{p}$.

Important Results

- The electrostatic potential on the perpendicular bisector due to an electric dipole is zero.
- Electrostatic potential at any point P due to a system of n point charges $q_1, q_2, \ldots, q_n$ whose position vectors are $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ respectively, is given by

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|}$$

where, $\mathbf{r}$ is the position vector at point P w.r.t. the origin.

- Electrostatic potential due to a thin charged spherical shell carrying charge q and radius R respectively, at any point P lying

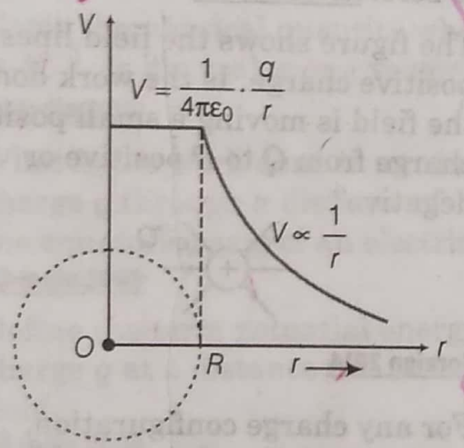
(i) inside the shell is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$, for $r < R$

(ii) on the surface of shell is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$, for $r = R$

(iii) outside the shell is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$, for $r > R$

where, r is the distance of point P from the centre of the shell.

- Graphical representation of variation of electric potential due to a charged shell at a distance r from centre of shell is given below:



Variation of potential due to charged shell with distance r from its centre

1.2 Equipotential Surface

A surface which has same electrostatic potential at every point on it, is known as equipotential surface.

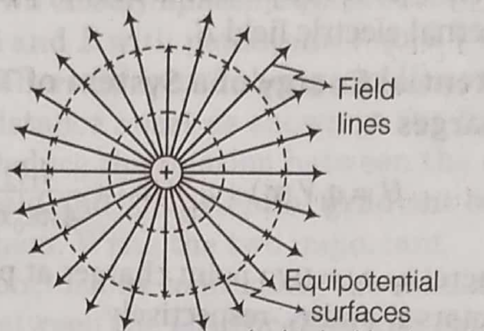
For a single charge q, the potential is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

The shape of equipotential surface due to

(i) line charge is cylindrical

(ii) Point charge is spherical



Different properties of equipotential surface are given below:

(i) Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point which is not possible.

(ii) Equipotential surfaces are closely spaced in the region of strong electric field and *vice-versa*.

(iii) Electric field is always normal to equipotential surface at every point of it and directed from one equipotential surface at higher potential to the equipotential surface at lower potential.

(iv) Work done in moving a test charge from one point of equipotential surface to other is zero.

Relation Between Electric Field and Potential Gradient

Relation between electric field and potential gradient is given by

$$E = -\frac{dV}{dr} \text{ i.e. } E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

where, negative sign indicates that the direction of electric field is from higher potential to lower potential, i.e. in the direction of decreasing potential.

NOTE (i) Electric field is in the direction of which the potential decreases steepest.

(ii) Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

Potential Energy in an External Field

(i) **Potential Energy of a Single Charge** Potentia energy of a single charge q at a point with position vector $\mathbf{r}$, in an external field is $qV(\mathbf{r})$,

where $V(\mathbf{r})$ is the potential at that point due to external electric field E.

(ii) **Potential Energy of a System of Two Charges**

$$U = q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 \mathbf{r}_{12}}$$

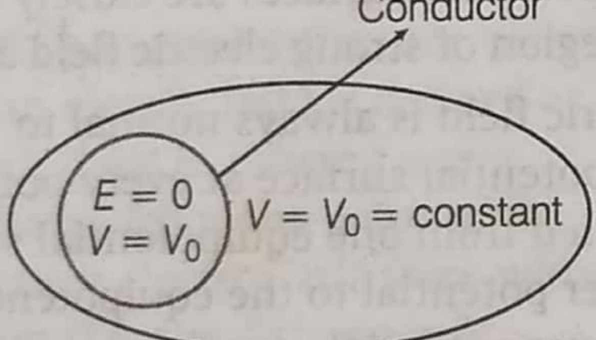
where, q_1, q_2 = two point charges at position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively

$V(\mathbf{r}_1)$ = potential at $\mathbf{r}_1$ due to the external field

$V(\mathbf{r}_2)$ = potential at $\mathbf{r}_2$ due to the external field

1.3 Electrostatic Shielding

- The process which involves the making of a region free from any electric field is known as electrostatic shielding.



It happens due to the fact that no electric field exist inside a charged hollow conductor. Potential inside a shell is constant. In this way, we can also conclude that the field inside the shell (hollow conductor) will be zero.

2.1 Conductors and Insulators

Conductor contains a large number of free charge carriers to conduct electricity while insulator does not contain any free charge carriers to conduct electricity.

Examples of conductors are metals and graphite. Examples of insulators are plastic rod and nylon.

NOTE Inside a conductor, the electrostatic field is zero.

Free Charges and Bound Charges Inside a Conductor

(*i*) In a metal, the outer (valence) electrons are free to move. These electrons are free for moving within the metal but not free to leave the metal. These free electrons are free charges inside a conductor and are the cause of conducting the electricity by conductors.

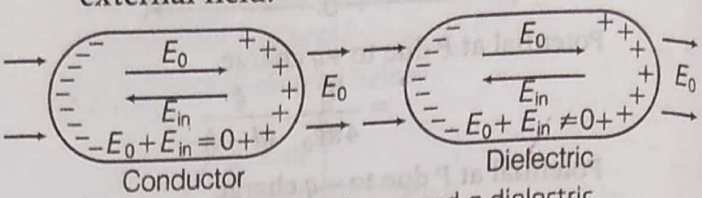
(*ii*) The bound charges are those positive ions which are made up of nuclei and the bound electrons remain in their fixed positions.

NOTE (*i*) Inside a conductor, the electric field is zero.
(*ii*) The interior of a conductor can have no excess charge in static situation.
(*iii*) Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

2.2 Dielectrics and Polarisation

(*i*) When a conductor is placed in an external electric field, the free charge carriers adjust itself in such a way that the electric field due to induced charges and external field cancel each other and the net field inside the conductor is zero.

In case of dielectric however, the opposing field so induced does not exactly cancel the external field.



Behaviours of a conductor and a dielectric in an external electric field

(*ii*) A net dipole moment is developed by an external field in either case, whether polar or non-polar dielectric. The dipole moment per unit volume is called **polarisation** and it is denoted by *p*.

$$\mathbf{p} = X_e \mathbf{E}$$

where, X_e is called **electric susceptibility** of the dielectric medium.

2.3 Capacitor

A capacitor is a device which is used to store electrostatic potential energy or charge. It comprises of two conductors separated by an insulating medium.

Capacitance of a Conductor

If charge q is given to an insulated conductor, it leads to increase its electric potential by V such that $q \propto V \Rightarrow q = CV$

where, C is known as capacitance of a conductor. The capacitance depends on the shape, size and geometry of conductor, nature of surrounding medium and presence of other conductor in the neighbourhood of it.

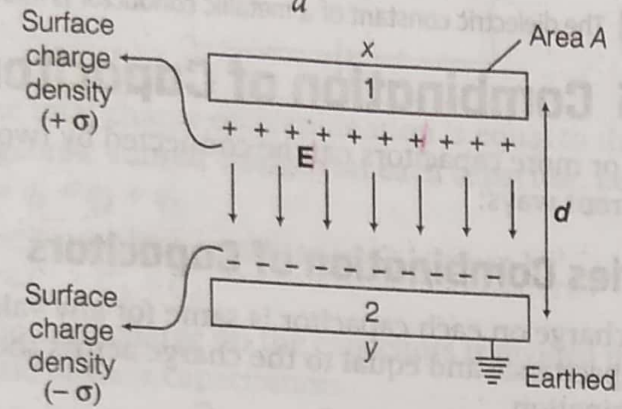
Its SI unit is farad (F). Here, 1 farad $= \frac{1 \text{ coulomb}}{1 \text{ volt}}$

Farad is a very large unit of capacitance. So, μF is usually taken.

Parallel Plate Capacitor

The most common among all capacitors is parallel plate capacitor. It comprises of two metal plates of area A and separated by distance d filled with air or some other dielectric medium. The capacitance of air filled parallel plate capacitor is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$



When a dielectric of dielectric constant K is filled fully between the plates, then

$$C = \frac{KA\varepsilon_0}{d} = KC_0$$

2.4 Dielectric

When a dielectric slab is introduced between the plates of charged capacitor or in the region of electric field, an electric field E_p induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence, net electric field inside the dielectric gets reduced to $(E_0 - E_p)$, where, E_0 is external electric field. The ratio of applied external electric field and reduced electric field is known as **dielectric constant** K of dielectric medium, i.e. $K = \dfrac{E_0}{E_0 - E_p}$

Dielectric Constant

If C_{vacuum} be capacity of a condenser with vacuum or air between its plates and $C_{\text{dielectric}}$ be the capacity with dielectric between the plates, the dielectric constant K is defined as $K = \dfrac{C_{\text{dielectric}}}{C_{\text{vacuum}}}$.

Dielectric constant is also known as **specific inductive capacity** of the dielectric.

Dielectric Strength

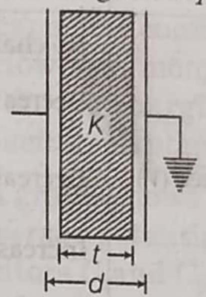
The dielectric strength is equal to that maximum value of electric field that can exist in a dielectric without causing the breakdown of its insulating property.

Dielectric and Capacitor

With the advent of dielectric in capacitor, net electric field inside the dielectric gets reduced, consequently potential difference across (charges disconnected) capacitor decreases. Hence, capacitance of capacitor increases as $C \propto \dfrac{1}{V}$ and new capacitance becomes KC_0.

Some Important Points

- The capacitance of a parallel plate capacitor partially filled with a dielectric medium of dielectric constant K is given by

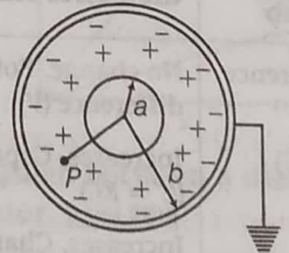


$$C = \frac{\varepsilon_0 A}{(d - t + t/K)}$$

$C = \dfrac{\varepsilon_0 A}{d-(t-\frac{t}{k})}$

where, A = area of each plate
d = separation between two plates
t = thickness of dielectric medium
K = dielectric constant of dielectric medium

- Capacitance of spherical capacitor

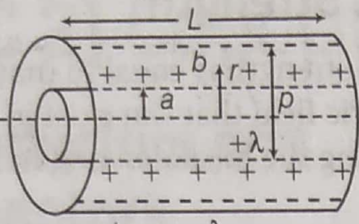


$$C = 4\pi\varepsilon_0 k\left(\frac{ab}{b-a}\right)$$

- Capacitance of an isolated spherical conductor of radius r is given by

$$C = 4\pi\varepsilon_0 r$$

(i) Capacitance of cylindrical capacitor



$$C = \frac{2\pi\varepsilon_0 KL}{\log_e(b/a)}$$

where,

a = radius of inner coaxial cylinder

b = radius of outer coaxial cylinder

L = length of coaxial cylinder

9ii0 Introduction of dielectric medium into the (charges disconnected) parallel plate capacitor leads to change in physical quantities as listed below.

Before introduction of dielectric slab	After introduction of dielectric slab
Charge (q)	No change (charge − q)
Electric field (E)	Decreases $\left(E' = \frac{E}{K}\right)$
Potential difference (V)	Decreases $\left(V' = \frac{V}{K}\right)$
Capacitance (C)	Increases ($C' = KC$)
Electrostatic energy (U)	Decreases $\left(U' = \frac{U}{K}\right)$

(iii) Introduction of dielectric medium in a charged capacitor connected with a battery.

Before introduction of dielectric slab	After introduction of dielectric slab
Potential difference (V)	No change, Potential difference (V)
Capacitance (C)	Increases, Capacitance ($C' = KC$)
Charge (q)	Increases, Charge $q' = qK$
Electric field (E)	No change, Electric field $E' = E$
Electrostatic potential energy (U)	Increases, Electrostatic energy $U' = KU$

Conductor and Capacitor

When a metallic conducting slab is partially filled in a capacitor, then capacitance of conductor becomes

$$C = \frac{\varepsilon_0 A}{d - t}$$

where, t = thickness of metallic plate and d = separation between two plates.

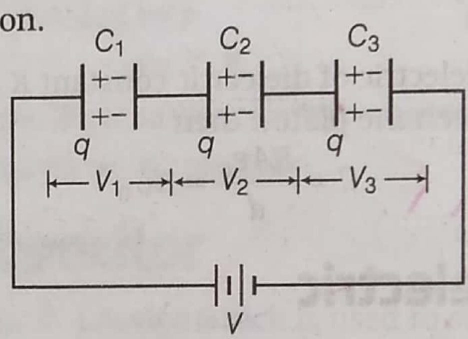
NOTE The dielectric constant of a metallic conductor is infinity.

2.5 Combination of Capacitors

Two or more capacitors can be connected by two different ways:

Series Combination of Capacitors

The charge on each capacitor is same for any value of capacitance and equal to the charge across the combination.



The potential difference across the combination is equal to the algebraic sum of potential difference across each capacitor, i.e. $V = V_1 + V_2 + V_3$

The potential is divided across capacitors in inverse ratio of their capacitances, i.e.

$$V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

The equivalent capacitance is given by

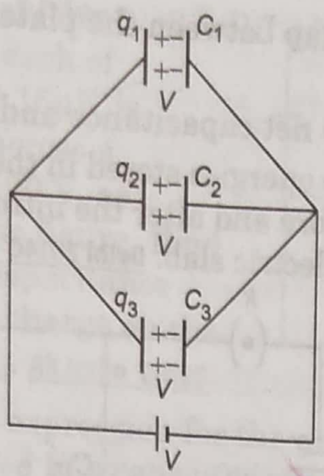
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The equivalent capacitance of n identical capacitors connected in series each of capacitance C is given by

$$C_s = \frac{C}{n}$$

Parallel Combination of Capacitors

The potential difference across each capacitor is same for any value of capacitance and equal to the potential difference across the combination.



The total charge on combination is equal to the algebraic sum of charges on each capacitor, i.e.

$q = q_1 + q_2 + q_3$

The equivalent capacitance (C) is given by

$C = C_1 + C_2 + C_3$

The total charge on the capacitors is divided in the ratio of their capacitances,

i.e. $q \propto C \Rightarrow q_1 : q_2 : q_3 = C_1 : C_2 : C_3$

The equivalent capacitance of n identical capacitors connected in parallel combination is $C_p = nC$.

2.6 Energy Stored in a Capacitor

Electrostatic energy stored in a (parallel plate) capacitor is given by

$$U = \frac{1}{2}CV^2 = \frac{q^2}{2C} = \frac{1}{2}qV$$

where, q = charge on capacitor,
C = capacitance,
V = potential difference across capacitor.

The energy stored per unit volume in an electric field E is known as **energy density**. It is given by

$$U_E = \frac{1}{2}\varepsilon_0 E^2$$

Common Potential

When two capacitors of different potentials are connected by a conducting wire, then charge flows from capacitor at high potential to the capacitor at low potential. This flow of charge continues till their potentials become equal, this equal potential is called common potential.

Common potential, $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$