1.1 Electric Charge

Charge is an intrinsic property associated with elementary particles of matter due to which it produces and experiences electric and magnetic effect. Benjamin Franklin introduced two types of charges namely positive charge and negative charge based on **frictional (statical) electricity** produced by rubbing two unlike objects like amber and wood.

Transference of electrons is the cause of frictional electricity.

Basic Properties of Electric Charge

(i) **Additivity of Electric Charges** Charges are scalars and they add up like real numbers. It means if a system consists of n charges $q_1, q_2, q_3, \ldots, q_n$, then total charge of the system will be $q_1 + q_2 + \ldots + q_n$.

(ii) **Conservation of Electric Charges** The total charge of an isolated system is always remains conserved i.e. initial and final charge of the system will be same.

(iii) **Quantisation of Electric Charges** Charge exists in discrete amount rather than continuous value and hence, they are said to be quantised.

Mathematically, charge on an object,

$$q = \pm ne.$$

where, n is an integer and e = electronic charge $= 1.6 \times 10^{-19}$C.

Units of Charge

(i) **SI unit** coulomb (C)

(ii) **CGS system**

(*a*) electrostatic unit (esu) or stat-coulomb (stat-C)

(*b*) electromagnetic unit (emu) or ab-C (ab-coulomb)

1 ab-C = 10 C, 1 C $= 3 \times 10^9$ stat-C

Conductors and Insulators

Those substances which readily allow the passage of electricity through them are called **conductors**, e.g. metals, the earth whereas those substances which offer high resistance to the passage of electricity are called **insulators,** e.g. plastic rod and nylon.

1.2 Coulomb's Law

It states that the electrostatic force of attraction or repulsion acting between two stationary point charges is given by $F = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}$

where, q_1 and q_2 are the stationary point charges and r is the separation between them in air or vacuum.

Also, K= $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N-m^{2}C^{-2}

where, ε_0 = permittivity of free space

$= 8.85419 \times 10^{-12}$ C^2/N-m^2

In vector form, $\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{|\mathbf{r}|^2} \hat{\mathbf{r}}$ or $|\mathbf{F}| = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}$

Absolute Permittivity of Medium (Dielectric Constant)

The force between two charges q_1 and q_2 located at a distance r in a medium other than free space may be expressed as $F = \dfrac{1}{4\pi\varepsilon} \cdot \dfrac{q_1 q_2}{r^2}$

where, ε is absolute permittivity of the medium.

$$\frac{F_{\text{vacuum}}}{F_{\text{medium}}} = \frac{\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}}{\dfrac{1}{4\pi\varepsilon} \cdot \dfrac{q_1 q_2}{r^2}} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

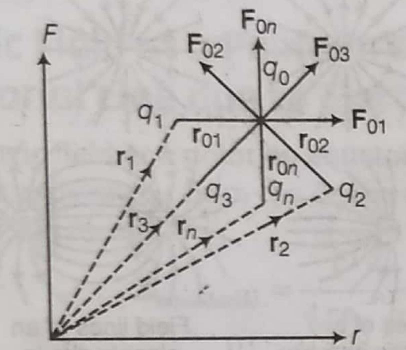
The ratio of force between two changes in vacuum and force acting between when they are shifted in a medium is called relative permittivity (ε_r) of the medium also called dielectric constant of the medium.

1.3 Electrostatic Forces

Electrostatic forces (Coulombian forces) are conservative forces. i.e. the work done against these forces does not depend upon the path followed.

Principle of Superposition of Electrostatic Forces

This principle states that the net electric force experienced by a given charge particle q_0 due to a system of charged particles is equal to the vector sum of the forces exerted on it due to all the other charged particles of the system. The force between two charges is not affected by the presence of other charges.



Superposition of electrostatic forces

i.e. $\mathbf{F}_0 = \mathbf{F}_{01} + \mathbf{F}_{02} + \mathbf{F}_{03} + \ldots + \mathbf{F}_{0n}$

$$\mathbf{F}_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_0}{|\mathbf{r}_{01}|^3}\mathbf{r}_{01} + \frac{q_2 q_0}{|\mathbf{r}_{02}|^3}\mathbf{r}_{02} + \ldots + \frac{q_n q_0}{|\mathbf{r}_{0n}|^3}\mathbf{r}_{0n}\right]$$

where, $\mathbf{r}_{01} = \mathbf{r}_0 - \mathbf{r}_1$, $\mathbf{F}_{01}$ = force on q_0 due to q_1.

Similarly, $\mathbf{r}_{0n} = \mathbf{r}_0 - \mathbf{r}_n$; $\mathbf{F}_{0n}$ = force on q_0 due to q_n

$$\therefore \quad \mathbf{F}_0 = \frac{q_0}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_{0i}|^3}\mathbf{r}_{0i}\right]$$

Net force in terms of position vector,

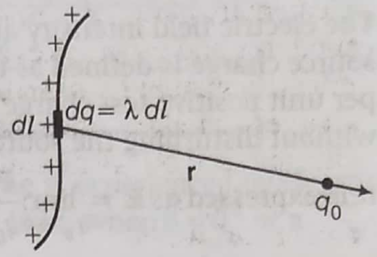
$$\mathbf{F}_0 = \frac{q_0}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|^3}(\mathbf{r}_0 - \mathbf{r}_i)\right]$$

Electrostatic Force due to Continuous Charge Distribution

The region in which charges are closely spaced in a continuous manner is said to have continuous distribution of charge. It is of three types (i) Linear charge distribution, (ii) Surface charge distribution and (iii) Volume change distribution.

(i) Force on a charge due to linear charge distribution (charge distributed along a line) is give by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda\, dl}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$



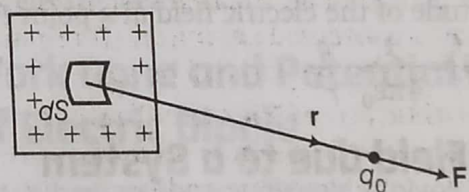
where λ is called linear charge density (charge per unit length) and dl is a short length element of linear charge distribution.

(ii) Force due to surface charge distribution (charge distribution over a plane surface) is given by

$$dq = \sigma\, dS$$

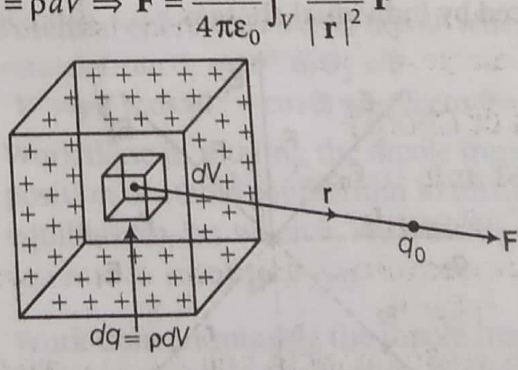
$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\sigma\, dS}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$

where σ is called surface charge density (charge per unit area) and dS is a small surface element.



(iii) Force due to volume charge distribution (charge distributed over a volume) is given by

$$dq = \rho dV \Rightarrow \mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho dV}{|\mathbf{r}|^2}\hat{\mathbf{r}}$$



where σ is called volume charge density and dV is a small volume element.

1.4 Electric Field

The space arround a charge in which its effect can be felt significantly i.e. the area which appears attraction or repulsion force on another charge is called electric field.

Electric Field Intensity

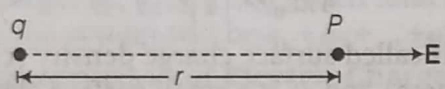
The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

It is expressed as $\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0}$

Here, $q_0 \to 0$, i.e. the test charge q_0 must be small, so that it does not produces its own electric field. SI unit of electric field intensity ($\mathbf{E}$) is $\mathbf{NC}^{-1}$ and it is a vector quantity.

Electric Field due to a Point Charge

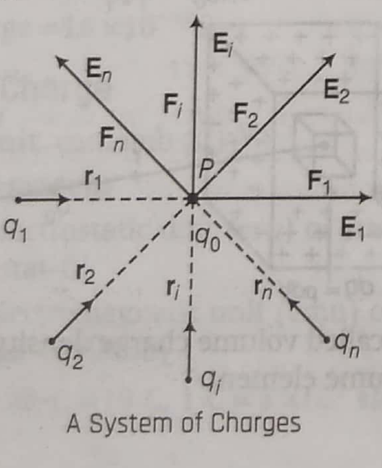
Electric field intensity at point P is, $\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{|\mathbf{r}|^2} \hat{\mathbf{r}}$



The magnitude of the electric field at a point P is given by $|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$

Electric Field due to a System of Charges

Electric field due to a system of charges at a point is equal to the vector sum of electric fields produced by individual charges.



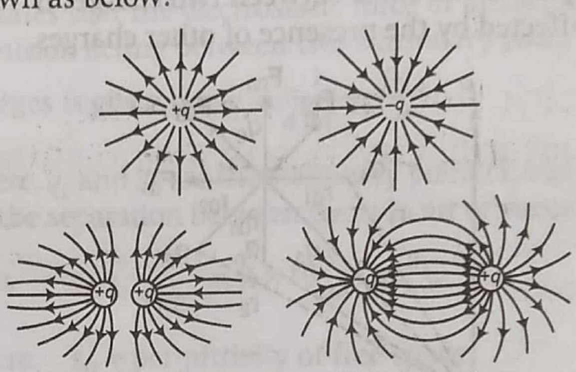
A System of Charges

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots + \mathbf{E}_n = \sum_{i=1}^{n} \mathbf{E}_i$$

$$\Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|r_i|^2} \hat{\mathbf{r}}_i$$

Electric Field Lines

Electric field lines are a way of pictorially mapping the electric field intensity around a configuration of charge(s). These lines start from positive charge and end on negative charge. The tangent on these lines at any point gives the direction of field at that point. Electric field lines due to positive and negative point charges and their combinations are shown as below:



Field lines of two equal positive charges

Field lines of an electric dipole

Different electric field lines

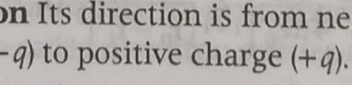
1.3 Electric Dipole

Two point charges of equal magnitude and opposite in sign separated by a small distance altogether form an electric dipole.

Electric Dipole Moment

The strength of an electric dipole is measured by a vector quantity known as electric dipole moment ($\mathbf{p}$) which is the product of the charge (q) and separation between the charges ($2l$).

$$\mathbf{p} = q \times 2l \Rightarrow |\mathbf{p}| = q\,(2l)$$

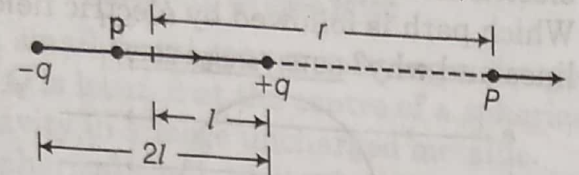


- **Direction** Its direction is from negative charge ($-q$) to positive charge ($+q$).
- **SI unit** Its SI unit is C-m.

Electric Field at a Point on the Axial Line due to Electric Dipole

The electric field intensity at a point on axial line of the dipole at a distance r from the centre of the dipole is given by the formula.

$$\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\mathbf{p}r}{(r^2 - l^2)^2}$$



When $l << r$,

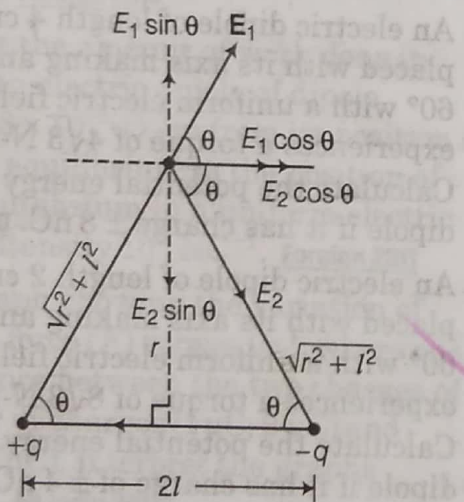
$$\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\mathbf{p}}{r^3} \Rightarrow |\mathbf{E}_{\text{axial}}| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2|\mathbf{p}|}{r^3}$$

Electric Field at a Point on the Equatorial Line due to Electric Dipole

The electric field at a point on equatorial line of the dipole at a distance r from the centre of the dipole is given by the formula.

$$\mathbf{E}_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-\mathbf{p}}{(r^2 + l^2)^{3/2}}$$

If $l << r$,
$$|\mathbf{E}_{\text{equatorial}}| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|\mathbf{p}|}{r^3}$$

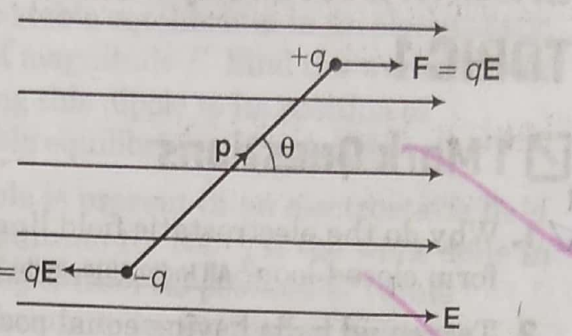


Torque on An Electric Dipole Placed in a Uniform Electric Field

Consider an electric dipole consisting of two charges $+q$ and $-q$ placed in a uniform external electric field intensity E. The dipole makes an angle θ with the direction of electric field, then torque acting on the dipole is given by the formula.

$$\tau = pE\sin\theta$$

In vector form, $\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$



- **Minimum torque** is experienced by electric dipole in electric field, when $\theta = 0°$ or π
$$\tau = \tau_{\min} = 0$$
- **Maximum torque** $\tau = \tau_{\max}$,
when $\sin\theta = 1 \Rightarrow \theta = \pi/2$
$$\tau_{\max} = pE$$
- Dipole is in stable equilibrium in uniform electric field when angle between **p** and **E** is $0°$ and in unstable equilibrium when angle is $180°$.
- There exists a net force and torque on electric dipole when placed in non-uniform electric field.

Work Done and Potential Energy of Electric Dipole

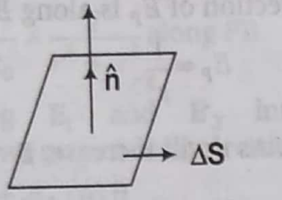
- When an electric dispole is placed in electric field then work is done in rotating it. In rotating the electric dipole from θ_1 to θ_2 is
$$W = pE\,(\cos\theta_1 - \cos\theta_2).$$
- Potential energy of electric dipole when it rotates from $\theta_1 = 90°$ to $\theta_2 = \theta$
$$W = pE\,(\cos 90° - \cos\theta) = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$$
- Work done in rotating the dipole from the position of stable equilibrium to unstable equilibrium, i.e. when $\theta_1 = 0°$ and $\theta_2 = \pi$.
$$W = 2pE$$
- Work done in rotating the dipole from the position of stable equilibrium to the position in which dipole experiences maximum torque, i.e. when $\theta_1 = 0°$ and $\theta_2 = 90°$.
$$W = pE$$

Area Vector

It is the vector associated with every area element of a closed surface and taken in the direction of the outward normal. Consider the diagram given alonside

$$\Delta \mathbf{S} = |\Delta \mathbf{S}| \hat{\mathbf{n}} = (\Delta S) \hat{\mathbf{n}}$$

Here, $\Delta \mathbf{S}$ is the area vector in the direction of the unit vector $\hat{\mathbf{n}}$ normal to the surface area ΔS.



Representation of area vector

2.1 Electric Flux

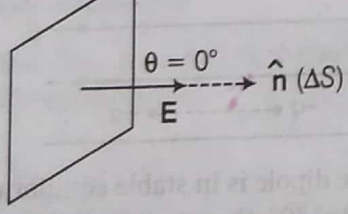
Electric flux linked with any surface is proportional to the total number of electric field lines that normally pass through that surface. It is a scalar quantity.

SI unit of electric flux is $\text{N-m}^2\text{C}^{-1}$ or JmC^{-1} or Vm.

CGS unit of electric flux is dyne-cm^2/ stat-C.

Different Conditions for the Electric Flux Linked with a Surface

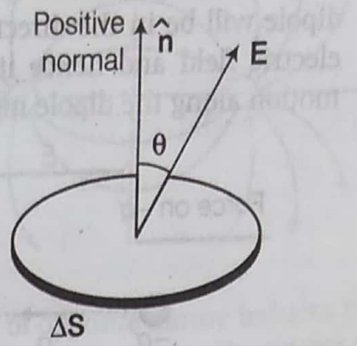
(*i*) When surface is held normal to the direction of uniform electric field **E**, then $\Delta\phi_E = E\Delta S$



Electric flux through normal area

(*ii*) When area vector of surface makes an angle θ with the direction of uniform electric field **E** , then $\Delta\phi_E = E\Delta S \cos\theta$.

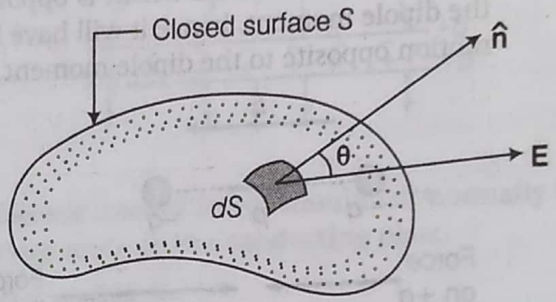
In vector form, $\Delta\phi_E = \mathbf{E} \cdot \Delta\mathbf{S}$



Eelectric Flux through an angle θ

(*iii*) Closed surface *S* lying inside the non-uniform electric field **E**

The total electric flux linked with the closed surface *S* is $\phi = \oint_S E \cdot dS$



Electric flux through a closed surface S

The surface integral of electric field over the closed surface represents total electric flux linked with the surface.

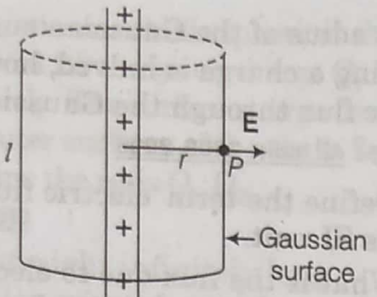
2.2 Gauss' Theorem

The total electric flux linked with closed surface *S* is $\phi_E = \oint \mathbf{E} \cdot d\mathbf{S} = \dfrac{q}{\varepsilon_0}$

where, *q* is the total charge enclosed by the closed Gaussian (imaginary) surface.

Applications of Gauss' Theorem

(*i*) **Electric field due to infinitely long uniformly charged wire** with linear charged density λ. We have considered cylindrical Gaussian surface.



From Gauss' law,

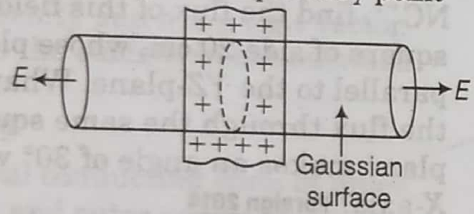
$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

$$\Rightarrow \quad E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

or $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Here, r is the perpendicular distance from the charged wire.

(*ii*) (a) Electric field due to a thin infinite plane sheet of charge with uniform surface charge density σ at any nearby point



$$E = \frac{\sigma}{2\varepsilon_0}$$

[for infinite plane sheet of charge]

and $$E = \frac{\sigma}{\varepsilon_0}$$

[for near charged conducting surface]

(b) Electric field intensity due to two equally and oppositely charged parallel plane sheet of charge at any point

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{[between the two plates]}$$

and $$E = 0 \quad \text{[outside the plates]}$$

(*iii*) **Electric field due to a thin charged spherical shell of radius *R* at a distance *r* from its centre.**

To find the field at a distance r from the centre of the spherical shell, we consider a spherical Gaussian surface of radius r centered at the shell and then Gauss' law is applied.

(a) **For point lying outside the shell** ($r > R$)

Since **E** and $d\mathbf{S}$ are in the same direction,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E \times 4\pi r^2 = \frac{\sigma \times (4\pi R^2)}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

(b) **Field at a point inside the shell** ($r < R$)

Here, the charge inside the Gaussian surface shell

As, $q = 0 \quad \Rightarrow \quad \mathbf{E} = 0$

(c) **Field at a point on the surface** ($r = R$)

On putting $r = R$,

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{\sigma}{\varepsilon_0}$$

where, surface charge density,

$$\sigma = \frac{q}{4\pi R^2}$$