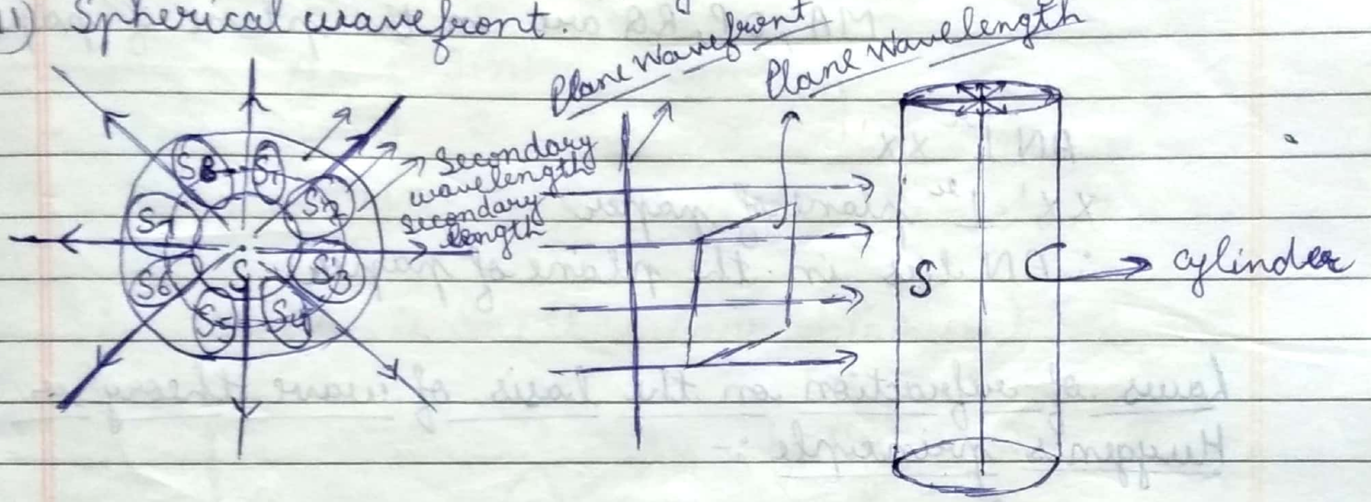
Chapter-10

Wave Optics

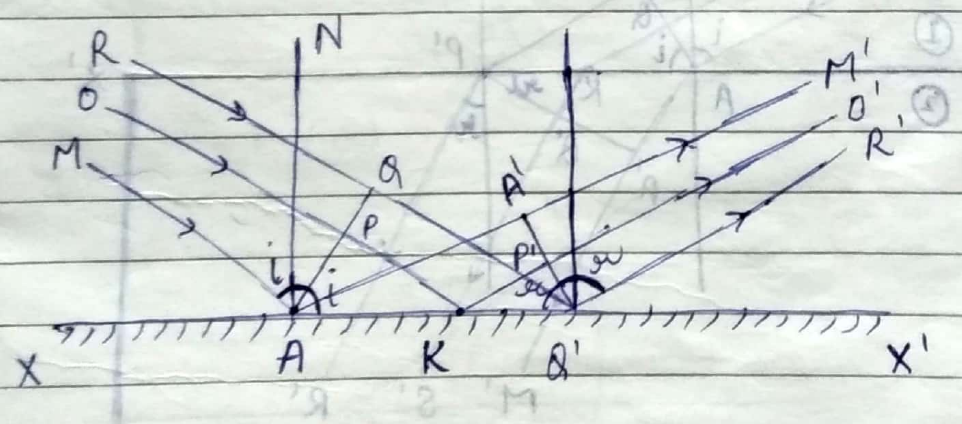
Chapter : Huygen's Principle

Wavefront :- The locus of all points having same phase

(i) Plane wavefront (ii) Cylindrical wavefront
(iii) Spherical wavefront.



Laws of Reflection of Light on the basis of wave theory or Hugen's Principle :



Time taken by a wave to reach from P - P' via K can be given as

$$t = \frac{PK}{c} + \frac{KP'}{c} = \frac{AK\sin i}{c} + \frac{KQ'\sin r}{c}$$

$$= \frac{AK\sin i + (AQ' - AK)\sin r}{c}$$

$$= \frac{AK(\sin i - \sin r) + AQ'\sin r}{c}$$

For time t to remain constant we must have

$$AK(\sin i - \sin r) = 0$$

$$\sin i = \sin r$$

$\boxed{i = r}$ which is the I law of reflection of light.

$APQ \perp^r$ plane paper.

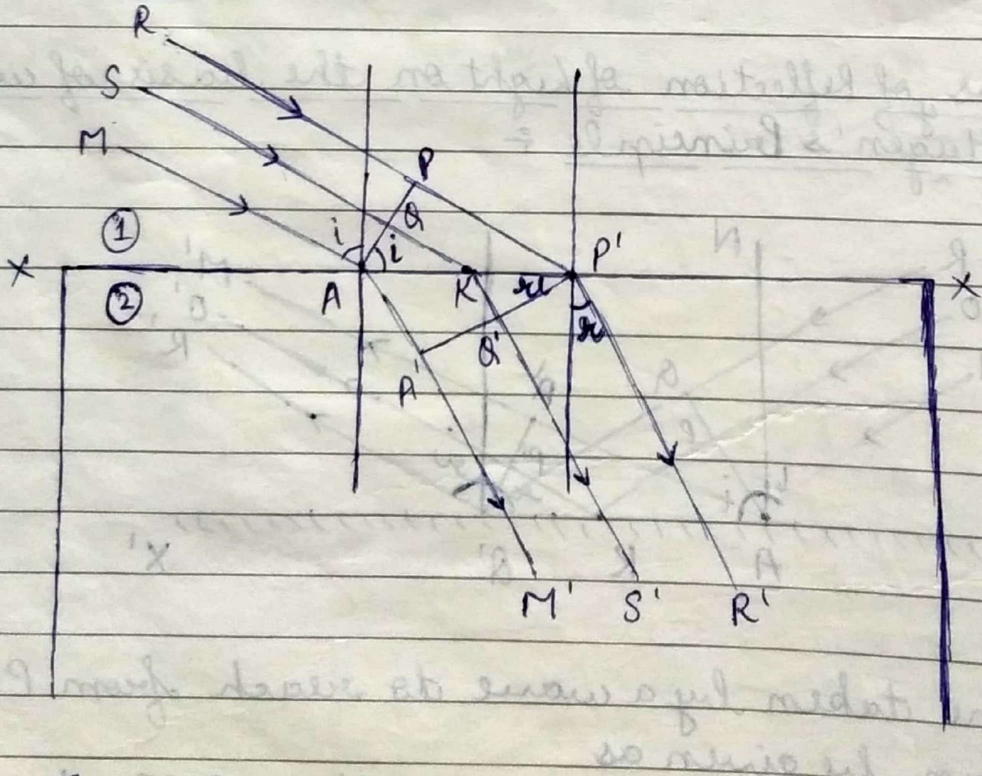
MA, OP, RQ are in the plane of paper.

$AN \perp^r XX'$

$XX' \perp^r$ plane of paper

$\therefore$ AN lies in the plane of paper.

Laws of refraction on the basis of wave theory or Huygen's principle :-



Time taken by the wavelength from point Q to Q' is

$$t = \frac{QK}{c} + \frac{KQ'}{v}$$

$$= \frac{AK \sin i}{c} + \frac{KP' \sin r}{v}$$

$$= \frac{AK \sin i}{c} + \frac{(AP' - AK)\cdot \sin r}{v}$$

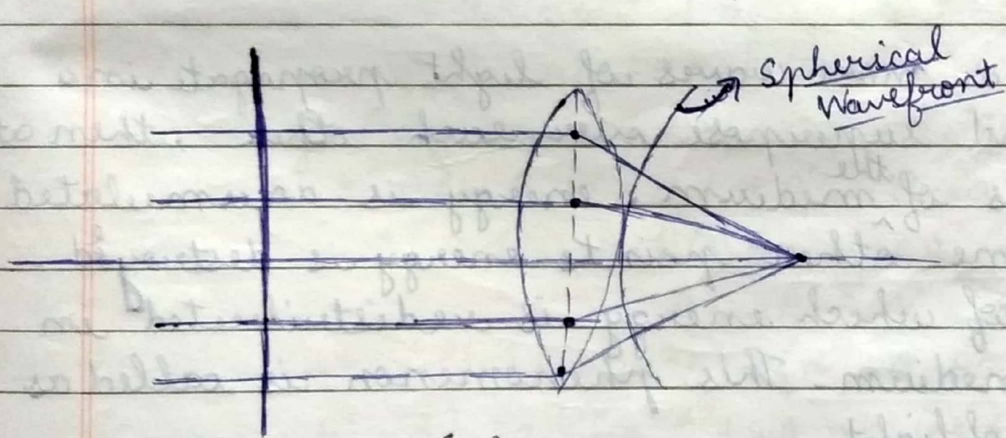
$$= AK\cdot\left(\frac{\sin i}{c} - \frac{\sin r}{v}\right) + \frac{AP' \sin r}{v}$$

For time t to remain constant we must have

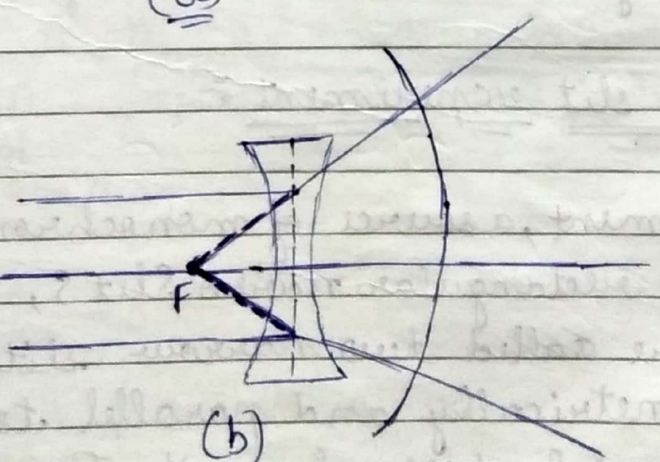
$$AK\left[\frac{\sin i}{c} - \frac{\sin r}{v}\right] = 0$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{c}{v} = \mu \text{ (Say) Constant}$$

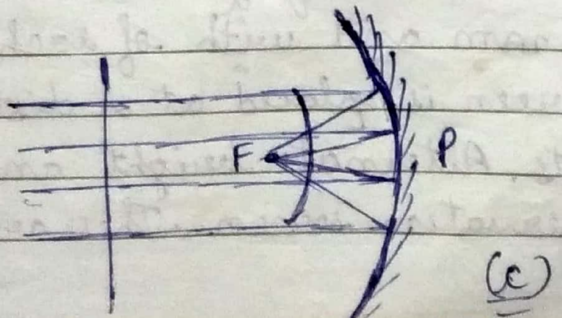
$\therefore$ Which is Snell's Law (Proved).



(a)



(b)



(c)

Interference of Light :-

When two or more waves of light propagate in a medium and superpose over each other, then at some points of the medium, energy is accumulated and at some other points energy is destroyed. As a result of which energy is redistributed in the whole medium. This phenomenon is called as interference of light.

Young's Double slit experiment :-

In this experiment, a source of monochromatic light illuminates a rectangular narrow Slit S, about 1 mm wide, S_1 & S_2 are ~~called~~ two narrow slits which are arranged symmetrically and parallel to the slit S at a distance of about 10cm from it. The seperation b/w S_1 & S_2 is $\approx$ 2 mm and width of each slit is = 0·3 mm. An observation screen is placed at a distance of $\approx$ 2 m from the two slits. Alternate bright and dark bands appear on the observation screen. These are called

$y =$

interference fringes. When one of the slit's S_1 or S_2 is closed, bright and dark fringes disappear & the intensity of light becomes uniform.

* Coherent sources :- Defination
* Theory of interference fringes:-

$y_1 = a_1 \sin \omega t$

$y_2 = a_2 \sin(\omega t + \phi)$

Displacement, after superposition, can be given as.

$y = y_1 + y_2$ ------ from principle of superposition.

$= a_1 \sin\omega t + a_2 \sin(\omega t + \phi)$

$= a_1 \sin\omega t + a_2 [\sin \omega t \cos\phi + \cos \omega t \sin \phi]$

$y = (a_1 + a_2 \cos\phi) \cdot \sin \omega t + a_2 (\sin\phi \cdot \cos \omega t)$

$= A \cos\theta \cdot \sin \omega t + A(\sin\theta \cdot \cos \omega t) = A \sin(\omega t + \theta)$

where, $A\cos\theta = a_1 + a_2 \cos\phi$

$A \sin\theta = a_2 \sin\phi$

$A^2 (\sin^2\theta + \cos^2\theta) = (a_1 + a_2 \cos\phi)^2 + a_2^2 \sin^2\phi$

$A^2 = a_1^2 + a_2^2 \cos^2\phi + 2a_1 a_2 \cos\phi + a_2^2 \sin^2\phi$

$a_1^2 + a_2^2(\cos^2\phi + \sin^2\phi)$

$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\theta$

$$\boxed{A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}}$$

Amplitude of resultance wave.

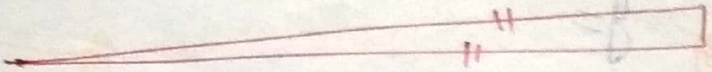
(i) Condition of Constructive interference:-

$\phi = 2n\pi$ where $n = 0, 1, 2, 3, \ldots$

$\frac{2\pi}{\lambda} x = 2n\pi$

$$\boxed{x = n\lambda}$$

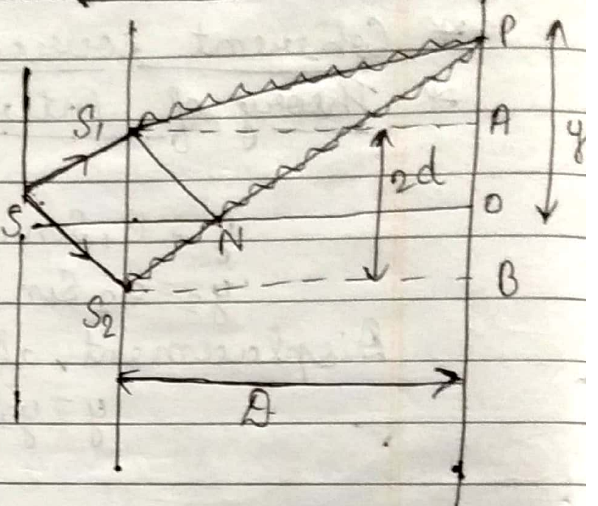
(ii) Condition of destructive interference:-



$$\phi = (2n+1)\frac{\pi}{2} \quad \text{where } n = 0, 1, 2, 3, ---$$

$$\frac{2\pi}{\lambda} x = (2n+1)\frac{\pi}{2} \Longrightarrow \boxed{x = (2n+1)\frac{\lambda}{2}}$$

Young's Double Slit Experiment:



$$y = A \sin(\omega t + \theta)$$

$$x = S_2P - S_1P \longrightarrow (3)$$

$$S_1P^2 = S_1A^2 + AP^2$$

$$S_1P^2 = D^2 + (y-d)^2$$

$$S_2P^2 = S_2B^2 + BP^2$$

$$S_2P^2 = D^2 + (y+d)^2$$

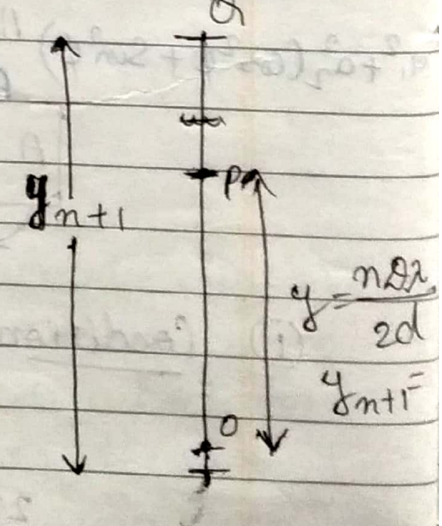
$$\therefore S_2P^2 - S_1P^2 = (y+d)^2 - (y-d)^2$$

$$(S_2P - S_1P)(S_2P + S_1P) = 4yd \qquad (\because S_1P \approx S_1A) \quad (\because S_2P \approx S_2B)$$

$$x(D + D) = 4yd$$

$$\boxed{y = \frac{Dx}{2d}} \longrightarrow (4)$$

Thickness of dark fringe:



$$y_n = \frac{D}{2d} \cdot n\lambda$$

$$y_{n+1} = (n+1)\frac{D\lambda}{2d}$$

$\therefore$ Thickness of dark fringe is

$$\beta = y_{n+1} - y_n$$

$$\boxed{\beta = \frac{D\lambda}{2d}} \longrightarrow (5)$$

Thickness of Bright Fringe :-

$$y = \frac{Dx}{2d}$$

For n^{th} dark fringe

$$x = (2n+1)\frac{\lambda}{4}$$

$$\therefore y'_n = \frac{D}{2d}(2n+1)\frac{\lambda}{4}$$

$$y'_{n-1} = \frac{D}{4d}[2(n-1)+1]\lambda$$

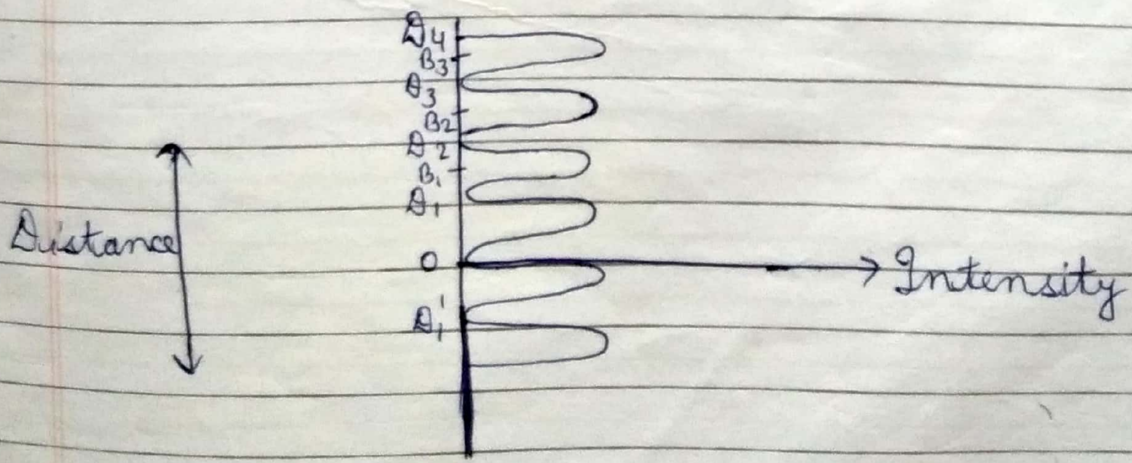
Thickness of bright fringe $\beta' = y'_n - y'_{n-1} = \frac{D}{4d}(2n+1)\lambda - \frac{D}{4d}[2n-1]\lambda$

$$= \frac{D\lambda}{4d}[\cancel{2n}+1-\cancel{2n}+1]$$

$$\boxed{\beta' = \frac{D\lambda}{2d}} \rightarrow 6.$$

From eq 5 & 6 we see that the thickness of dark and bright fringes are same.

Graph for intensity of fringes ~~as~~ vs. distance :-



Whenever there is change in flux lingked with a closed circuit of coil an induced emf or current is produced in it

The induced produced in the coil is directly proportional to the rate of change of flux passing through it.

Mathematically, $e = \frac{-d\phi}{dt}$

(-) ve sign signifies that the direction of induced current opposes the cause which produces in it.

Ray Optics Derivations :

* $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ and $m = -\frac{v}{u} = \frac{I}{O}$

* $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and $m = \frac{v}{u} = \frac{I}{O}$

* $\mu = \frac{1}{\sin i}$

* $\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{R}$

* $\frac{1}{v} - \frac{\mu}{u} = \frac{1-\mu}{R}$

* $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

* $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

* $P = \frac{1}{f}$ ex: $f = 20$ then $P = \frac{100}{20}$ D

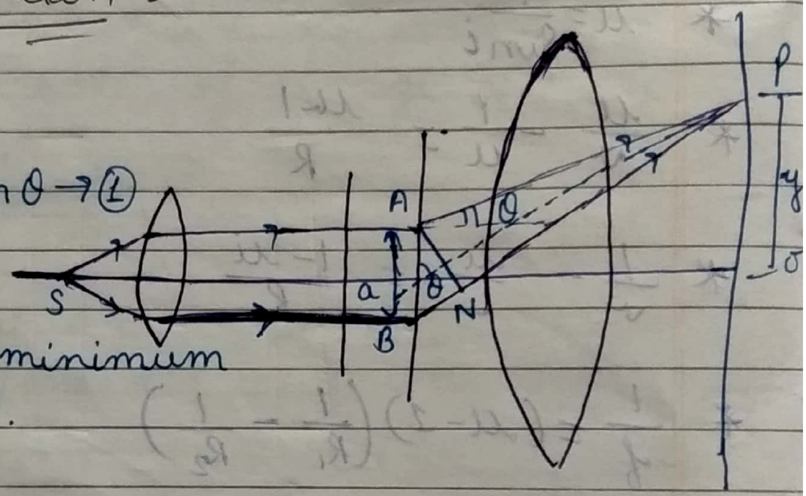
* Combination of ~~lens~~ lenses & mirrors } Numericals.

VKS

Diffraction of Light :

Phenomenon of bending of light around the edge of an obj./obstacles of the size of wavelength of light.

Fraunhoffer diffraction :-



In ΔABN

$\sin\theta = \frac{BN}{AB} = BN = AB\sin\theta \rightarrow (1)$

If $BN = \lambda$ and $\theta = \theta_1$
then first secondary minimum will be observed at P.

$\therefore \sin\theta_1 = \frac{\lambda}{a}$

Similarly $\sin\theta_2 = \frac{\lambda x}{a}$ for II secondary min

$\sin\theta_A = \frac{n x}{a}$ for n^{th} dark $\rightarrow (2)$ (where $n=1,$

From figure as ~~θx~~ θ_n is very small $\sin\theta_n \approx \tan\theta_n$

$\therefore \tan\theta_n = \frac{y_n}{D}$

As θ_n is very small

$\therefore \sin\theta_n \approx \tan\theta_n$

$\therefore \frac{y_n}{D} = \frac{n\lambda}{a}$

$y_n = \frac{nD\lambda}{a}$

Thus for $(n-1)^{th}$ dark

$$y_{(n-1)} = (n-1)\frac{D\lambda}{a}$$

$\therefore$ Thickness of bright fringe

$$\boxed{\beta = y_n - y_{n-1} = \frac{D\lambda}{a}} \rightarrow (4)$$

For first sec. max for bright we have

$$BN = \frac{3\lambda}{2} \text{ and } \theta = \theta_1'$$

$\therefore$ From (1) $\sin\theta_1' = \frac{3\lambda}{2a}$

Similarly for 2^{nd} bright $\sin\theta_2' = \frac{5\lambda}{2a}$

For n^{th} bright $\sin\theta_n' = (2n+1)\frac{\lambda}{2a} \rightarrow (5)$

(where $n = 1, 2, 3 \ldots$)

As θ_n' is very small $\sin\theta_n' \approx \tan\theta_n' = \frac{y'_n}{D} \rightarrow (6)$

2, 3---)

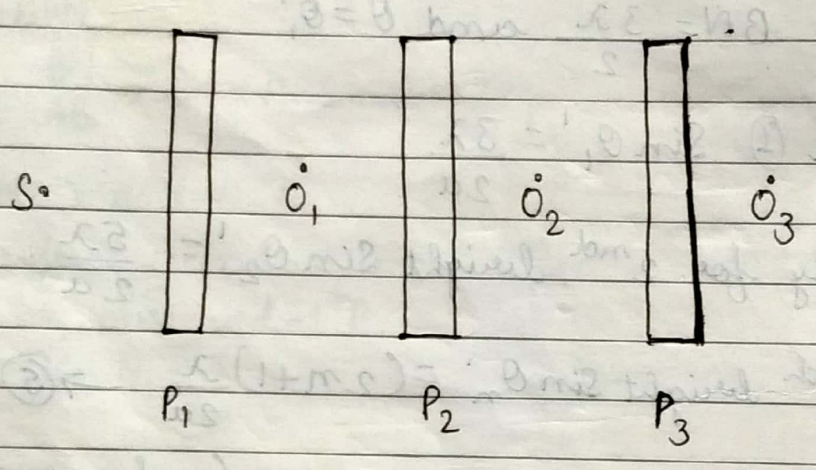
From (5) & (6)

$$y_n' = (2n+1)\frac{D\lambda}{2a} \rightarrow (7)$$

$\therefore$ Thickness of

Problem 25.

Three identical polaroid sheets P_1, P_2 and P_3 are oriented so that the axes of P_2 & P_3 are inclined at angles of 60° and 90° respectively, with respect to the axis of P_1. A monochromatic source, S, of intensity I_0, is kept in front of the polaroid sheet P_1. Find the intensity of the light, as observed by observers O_1, O_2 & O_3 positioned as shown below.



Ans. Intensity observed by observer $O_1 = \frac{I_0}{2}$

Intensity observed by $O_2 = \frac{I_0}{2}\cos^2 60^\circ = \frac{1}{4} \times \frac{I_0}{2}$

$= \frac{I_0}{8}$

Intensity observed by $O_3 = \frac{I_0}{8}\cos^2(90^\circ - 60^\circ)$

$= \frac{I_0}{8}\cos^2(30^\circ)$

$= \frac{I_0}{8} \times \left\{\frac{\sqrt{3}}{2}\right\}^2 = \frac{3I_0}{32}$