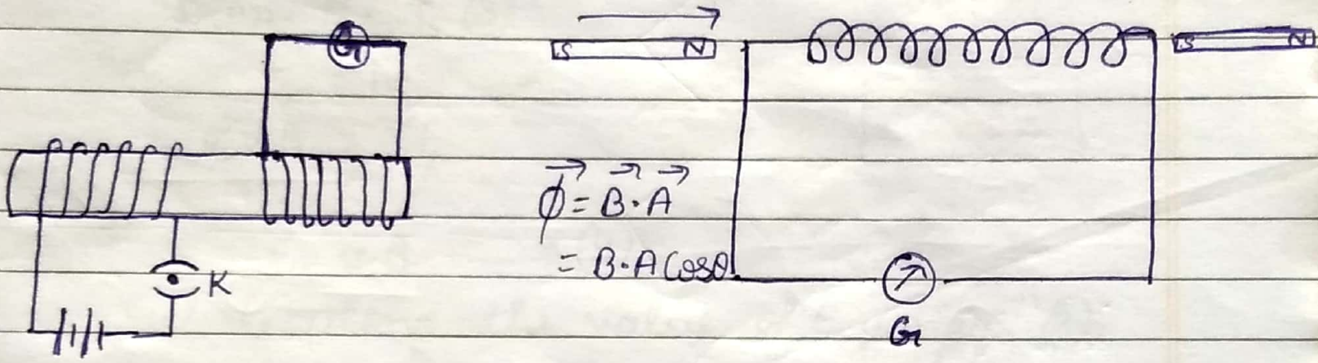
Chapter-6

Unit-4. Electromagnetic Induction & AC

Faraday's Experiment :-



Faraday's Laws of electromagnetic induction :-

It can be stated as follows:-

First Law :- Whenever the magnetic flux linked with a closed circuit changes, an emf (& hence a current) is induced in it which last only so long as the change in flux is taken place. This phenomenon is called electromagnetic induction.

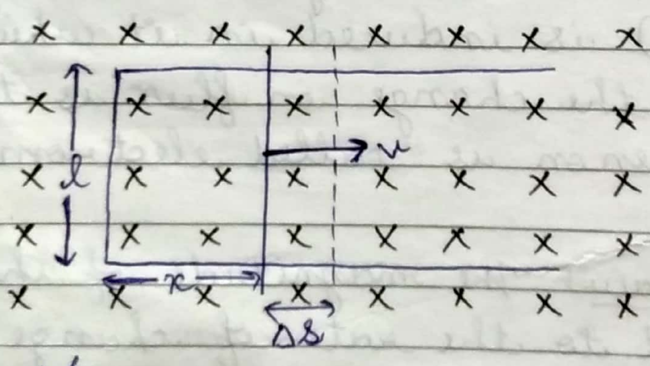
Second Law :- The magnitude of the induced emf is equal to the rate of change of magnetic flux linked with the closed circuit. Mathematically

$$|\mathcal{E}| = \frac{d\phi}{dt}$$

Lenz's Law :- This law states that the direction of the induced current is such that it opposes the cause which produces it, i.e. it opposes the ~~cause which~~ change in magnetic field.

To show that Lenz's Law is in agreement with the law of conservation of energy :-

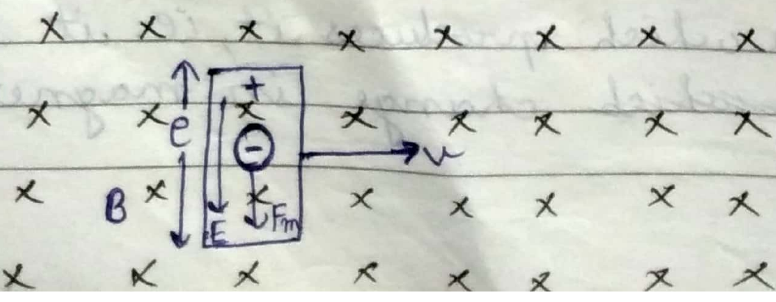
Motional Emf from Faraday's Law :-



$$e = \frac{d\phi}{dt}$$

$$= \frac{-B \cdot \Delta A}{\Delta t} \Rightarrow -Bl\frac{\Delta x}{\Delta t}$$

$$e = -Blv$$



Vivek.

$$F_m = evB \sin 90^\circ$$

$$F_m = evB \longrightarrow (1)$$

$$e = El \longrightarrow (2)$$

$$F_e = qE \longrightarrow (3)$$

$e = El$

$Fe = qE$

$F_m = Fe$

$\cancel{q}vB = \cancel{q}E$

$E = vB.$

$\boxed{e = vBl}$

From (1) & (3)

$$F_m = Fe$$

$$\cancel{q}vB = \cancel{q}E$$

$$E = vB \longrightarrow (4)$$

Substituting the value of E in eq.^{n} (2)

$$\boxed{e = vB.l}$$

<u>Relation between induced charge and change in magnetic flux</u> :-

$$|e| = \frac{d\phi}{dt}$$

$$i = \frac{\Delta q}{\Delta t} = \frac{e}{R}$$

$$\frac{\Delta q}{\Delta t} = \frac{\Delta \phi}{\Delta t} \frac{1}{R}$$

$$\boxed{\Delta \phi = R \cdot \Delta q}$$

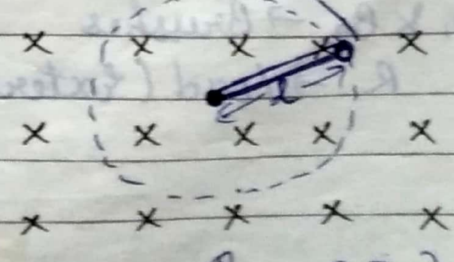
$\Delta A = \pi l^2$

$$e = \frac{\Delta \phi}{\Delta t}$$

$$= \frac{B \Delta A}{\Delta t}$$

$$= \frac{B \pi l^2}{T}$$

$$= \frac{B \pi l^2}{2\pi/\omega} \Rightarrow e = \frac{1}{2} B l^2 \omega$$



$\therefore \left(\omega = \frac{\theta}{t} = \frac{2\pi}{t}\right)$

Methods of Generating induced emf :

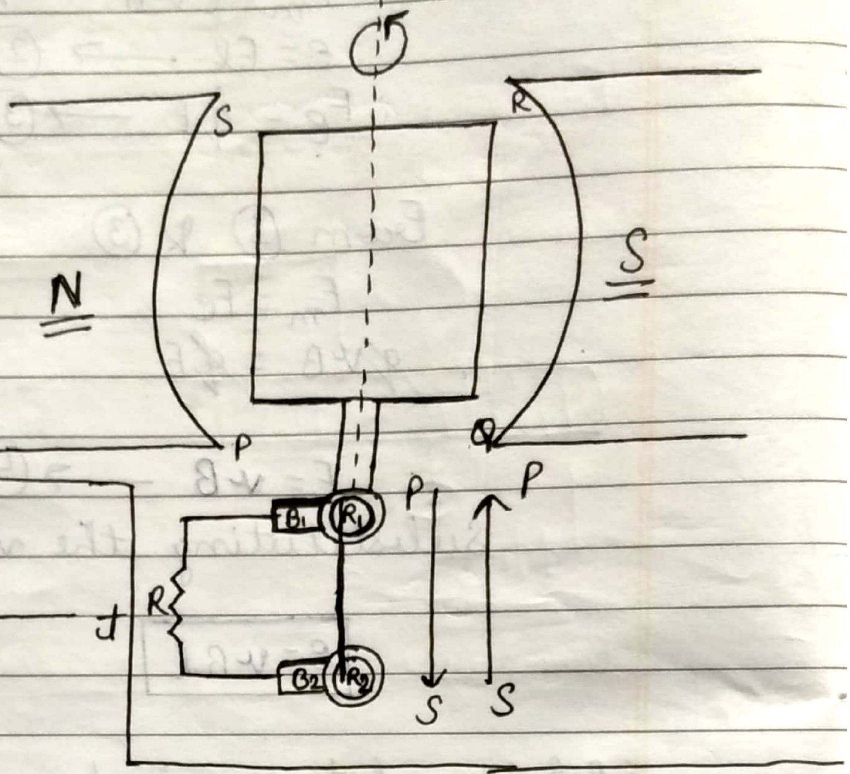
A.C Generator :-

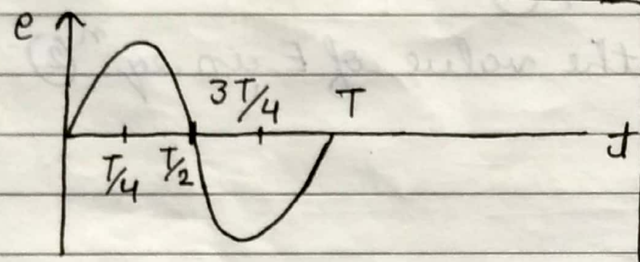
$t = T/4$ $S \to P$

$t = T/2$ 0

$t = 3T/4$ $P \to S$

$t = T$ 0





$\omega = \frac{\theta}{T} \Rightarrow \theta = \omega t \to (1)$

$\phi = NBA \cos\theta$

$\phi = NBA \cos \omega t$

$\frac{d\phi}{dt} = e = -NBA\omega \sin \omega t$

$e = NBA\omega \sin \omega t$

$e_0 = NBA\omega = e_{max}$

$$\boxed{\therefore\ e = e_0 \sin \omega t}$$

$\omega = \frac{\theta}{T}$

$\theta = \omega T$

$\phi = NBA\cos\theta$

$\phi = NBA\cos\omega t$

$\frac{d\phi}{dt} = e$

$e = -NBA\omega \sin\omega t$

$e = NBA\omega\sin\omega t$

PQRS = Coil

N, S = Poles of magnets

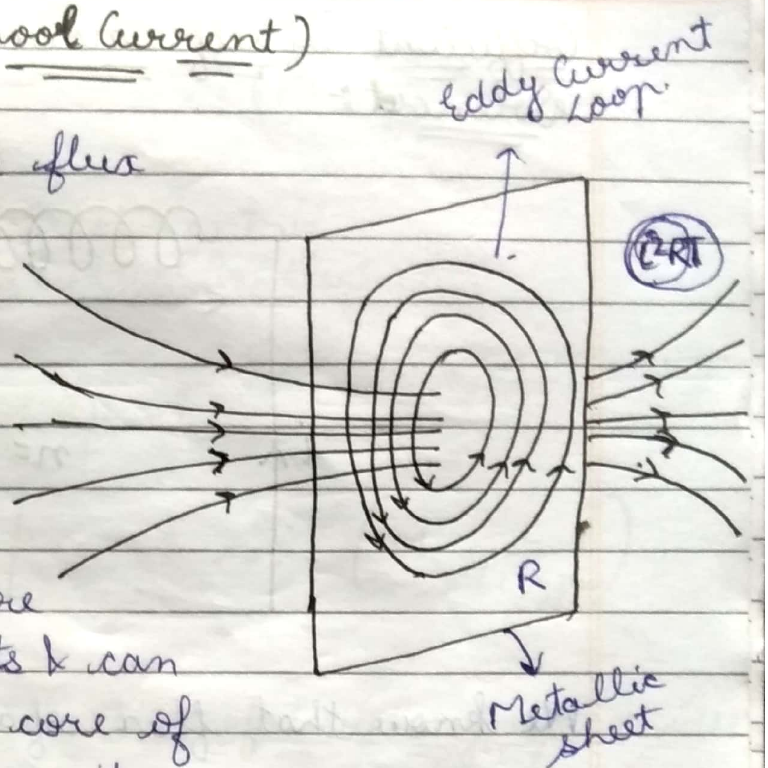
R_1 & R_2 → Metallic Rings

B_1 & B_2 → Brushes

R → Load (External)

Eddy Current :- (Whirlpool Current)

When changeᵍ magnetic flux is linked with a metallic shield or conductor then currents are produced over the surface of conductor in the form of concentric loops. These ~~currents~~ circuits are known as Eddy Currents & can be minimised in the core of transformers by making it laminated core. These current are unwanted current.



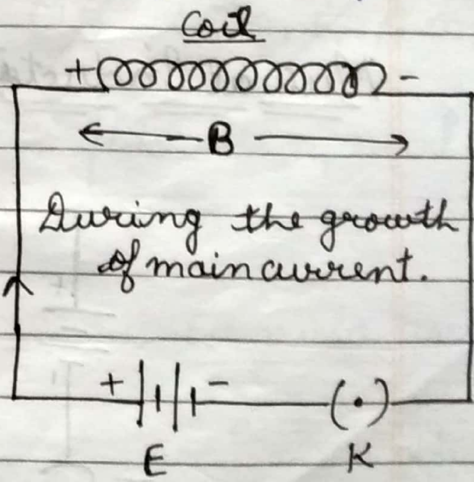
Self Induction :-

It is the phenomenon of production of induced emf in a coil when a changing current passing through it.

$\phi \propto i$

$\phi = Li$, $L = \frac{\phi}{i}$

L → coffecient of a self induction of a coil.



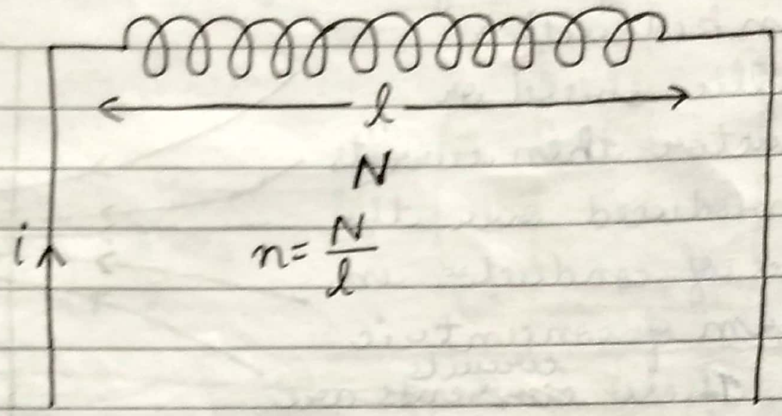
$$\frac{d\phi}{dt} = L\frac{di}{dt}$$

$$\boxed{e = -L\frac{di}{dt}}$$

$$\boxed{L = \frac{-e}{di/dt}}$$

~~S.I.~~ S.I. unit of L is HENRY

Coefficient of self induction of a long straight Solenoid :-



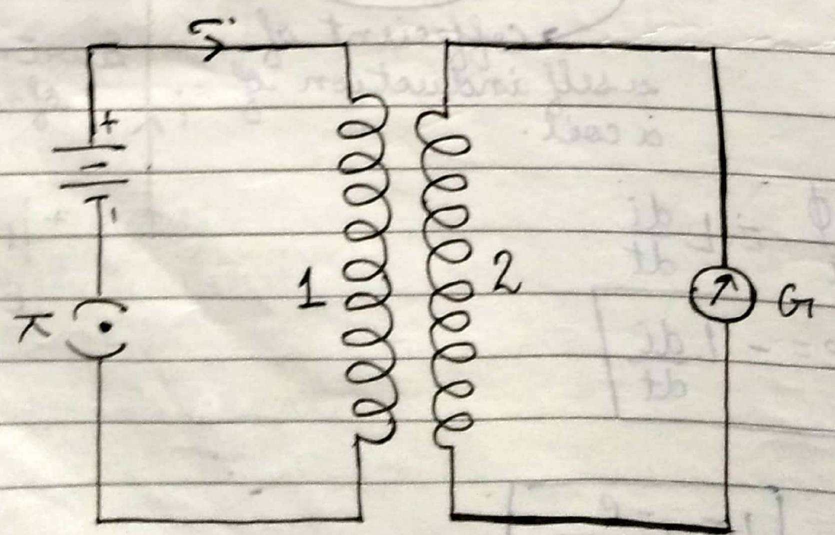
We know that flux of a coil, $\phi = Li \rightarrow (1)$

$$\phi = NBA$$
$$= (nl)(\mu_0 ni)A \longrightarrow (2)$$

Equating (1) & (2)

$$Li = \mu_0 n^2 l A i$$
$$\boxed{L = \mu_0 n^2 l A}$$

Mutual Inductance (M) :-



$\phi_{21} \propto i_1$

$\phi_{21} = M i_1$ → Coefficient of Mutual Induction

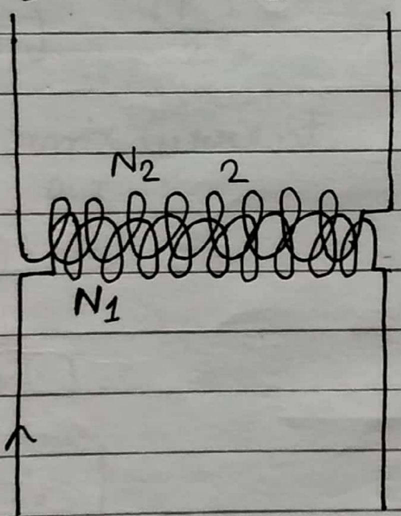
$\therefore \left(M = \frac{\phi_{21}}{i_1}\right)$

$$\frac{d\phi_{21}}{dt} = M\frac{di_1}{dt}$$

$$\boxed{e = M\frac{di_1}{dt}}$$

$\therefore \left(M = \frac{-e}{di/dt}\right)$

Coefficient of Mutual Induction of two long solenoids:



$\phi_{21} = M i_1 \rightarrow (1)$

$\phi_{21} = N_2 B_1 A$

$= N_2(\mu_0 n_1 i_1)A$

$\phi_{21} = (n_2 l)(\mu_0 n_1 i_1 A) \rightarrow (2)$

Equating eq. (1) & (2)

$M\cancel{i_1} = \mu_0 n_1 \cancel{i_1} A n_2 l$

$$\boxed{M = \mu_0 n_1 n_2 l A}$$

Ex 36 (S.L. Arora)

Formula used

$e = -L\frac{dI}{dt}$

Ans. $l = 30\,cm = 0.30\,m$

$A = 25\,cm^2 = 25 \times 10^{-4}\,m^2$, $N = 500$

Alternating current :- (~~After this topic~~)

Av. Value of A.C. $(I_m \times I_n)$

In case of ordinary current

$dq = i\,dt$

$dq = i_0 \sin$

RMS Value of AC :-

$$H \int dH = \int_0^T i^2 R\,dt = \int_0^T i_0^2 \sin^2 \omega t\, R\,dt$$

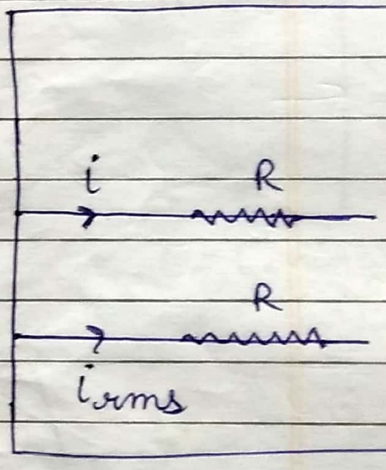
$$= i_0^2 R \int_0^T \sin^2 \omega t\, dt$$

$$= i_0^2 R \int_0^T \left(\frac{1-\cos 2\omega t}{2} \right) dt$$

$$= \frac{i_0^2 R}{2} \left[\{t\}_0^t - \left\{ \frac{\sin 2\omega t}{2\omega} \right\}_0^T \right]$$

$$= \frac{i_0^2 R}{2} \left[t - \frac{1}{2\omega} \left\{ \sin 2 \cdot \frac{2\pi}{T} T - \sin 2 \cdot \frac{2\pi}{T} \cdot 0 \right\} \right]$$

$$= \frac{i_0^2 R}{2} [T] = \frac{i_0^2 R T}{2} \rightarrow (1)$$



In case of rms Current

$$H = i_{rms}^2 R \cdot T$$

Equating (1) & (2)

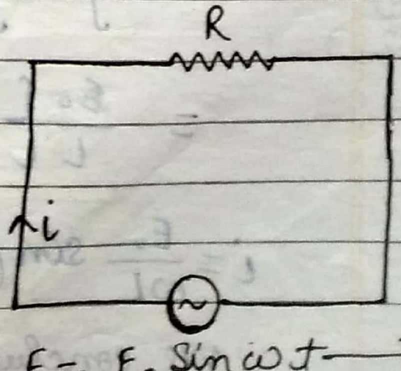
$$i_{rms}^2 R \cdot T = \frac{i_0^2 R T}{2}$$

$$\boxed{i_{rms} = \frac{i_0}{\sqrt{2}}}$$

$$\boxed{i_v = 0.707\, i_0}$$

→ ~~Virtal~~ Virtual Current

A.C. Through a pure resistor :-



$$i = \frac{E}{R} = \frac{E_0 \sin \omega t}{R} = \frac{E_0}{R} \sin \omega t$$

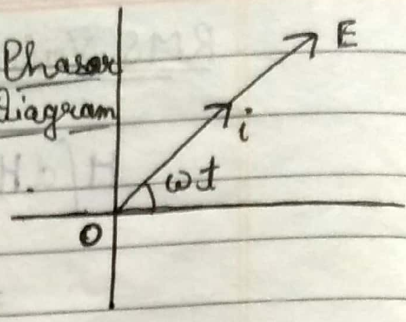
$$= i_0 \sin \omega t \rightarrow (2)$$

$$E = E_0 \sin \omega t \longrightarrow (1)$$

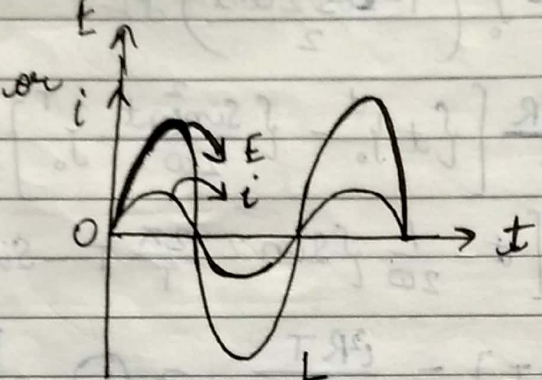
Comparing ① & ②, we conclude that

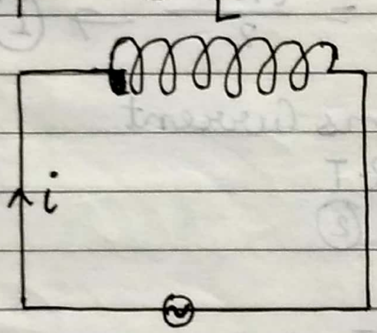
E and i remain in same phase.

Phasor Diagram



A.C. through pure inductor :-





$E = E_0 \sin \omega t \longrightarrow$ ①

$$E = L\frac{di}{dt}$$

$$E_0 \sin\omega t = L\frac{di}{dt}$$

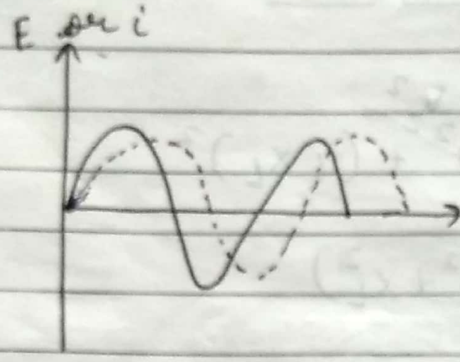
$$di = \frac{E_0}{L}\sin\omega t \, dt$$

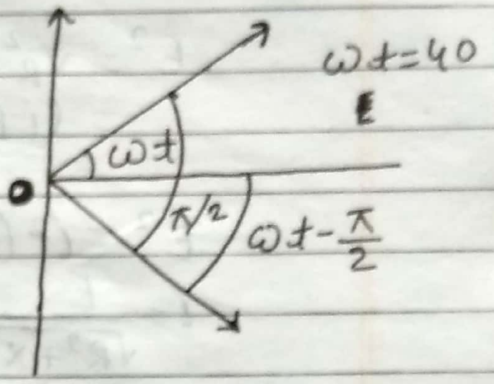
$$i = \int di = \int \frac{E_0}{L}\sin\omega t \, dt$$

$$= \frac{E_0}{L}\left[-\frac{\cos\omega t}{\omega}\right]$$

$$i = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \longrightarrow ②$$

∴ we conclude that, E leads i by an angle of $\frac{\pi}{2}$ in phase which can be shown by phasor diagram



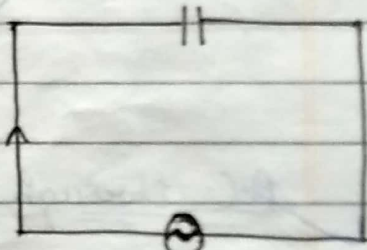


AC through pure Capacitor :

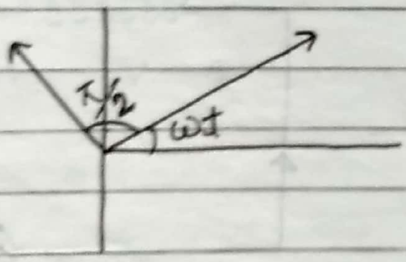
$q = CE = CE_0 \sin \omega t$

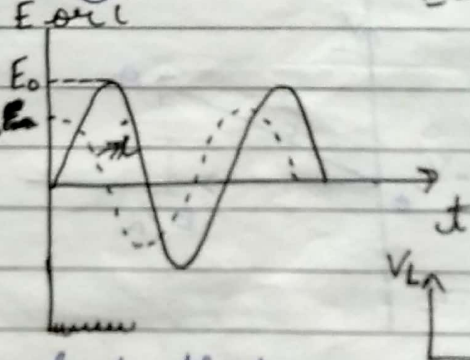
$i = \frac{dq}{dt} = CE_0 \omega \cos \omega t$

$i = \frac{E_0}{(1/\omega C)} \sin(\omega t + \pi/2)$ $\rightarrow$ ②

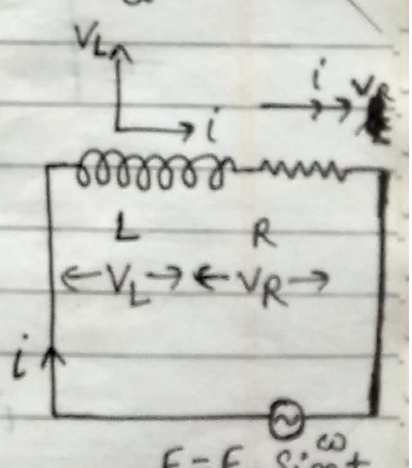


$E = E_0 \sin \omega t$ $\rightarrow$ ①





Compairing ① & ② we conclude that
E lays behind by $\frac{\pi}{2}$
or i leads ahead by $\frac{\pi}{2}$



$E = E_0 \sin \omega t$ $\rightarrow$ ①

$X_L = \omega L$ $\qquad i = \frac{E_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$

$X_C = \frac{1}{\omega C}$ $\qquad i = \frac{E_0}{1/\omega C} \sin(\omega t + \frac{\pi}{2})$

* lays

A.C. through LR circuit :-

$$E^2 = V_R^2 + V_2^2$$
$$= (iR)^2 + (iX_L)^2$$

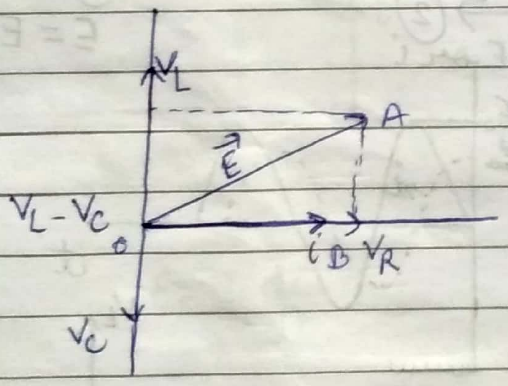
$$E^2 = i^2(R^2 + X_L^2)$$

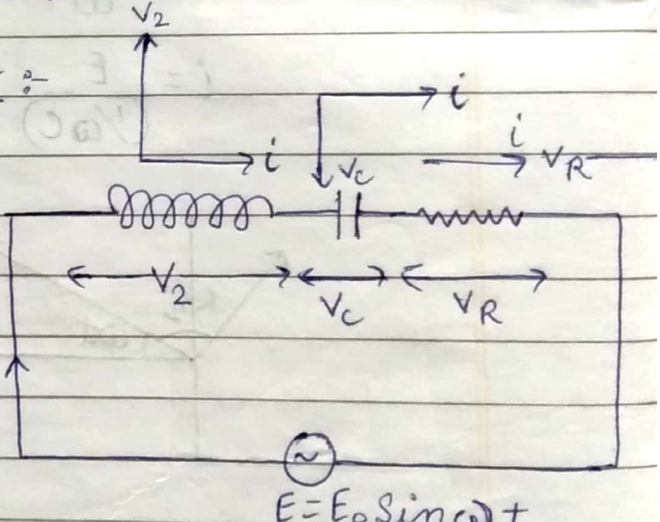
$$i = \frac{E}{\sqrt{R^2 + X_L^2}}$$

$$i = \frac{E}{Z}$$

where $Z = \sqrt{R^2 + X_L^2}$ → Impendance of LR circuit.

A.C. through LCR series circuit :-





$E = E_0 \sin \omega t$

In ΔOAB

$$E^2 = V_R^2 + (V_L - V_c)^2$$
$$= (iR)^2 + [iX_L - iX_c]^2$$
$$E^2 = i[R^2 + (X_L - X_c)^2]$$

$$i = \frac{E}{\sqrt{R^2 + (X_L - X_c)^2}}$$

$$\boxed{i = \frac{E}{Z}}$$

→ Impedance of

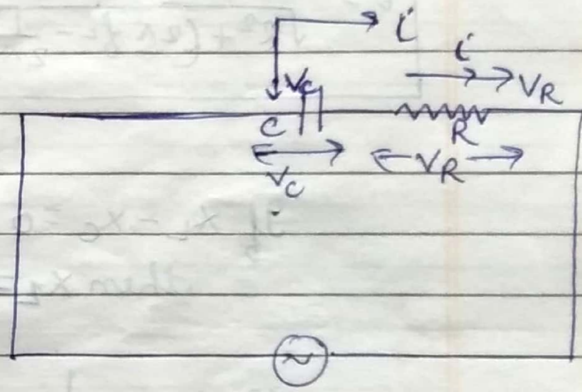
Q. To find out the phase difference between E & i?

Ans.
$$\tan\phi = \frac{BA}{OA} = \frac{V_L - V_C}{V_R} = \frac{iX_2 - iX_2}{iR} = \frac{X_L - X_C}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)}$$

∴ Where $X_L = \omega L = 2\pi f L$ = Inductive reactance
$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ = Capacitive reactance.

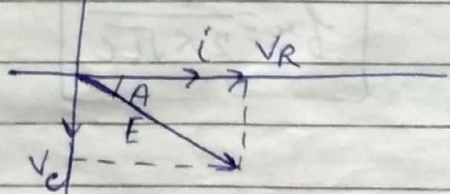
A.C through R.C circuit :-



$E = E_0 \sin\omega t$ → (1)

$$E^2 = V_C^2 + V_R^2$$
$$E^2 = (iX_C)^2 + (iR)^2$$
$$E = i\sqrt{R^2 + X_C^2}$$
$$i = \frac{E}{\sqrt{R^2 + X^2}}$$

$$i = \frac{E}{Z}$$



$$\tan\phi = \frac{V_C}{V_R} \Rightarrow \frac{iX_C}{iR} = \frac{X}{R} \Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_C}{R}\right)}$$

Ex 21. (S.L. Arora.

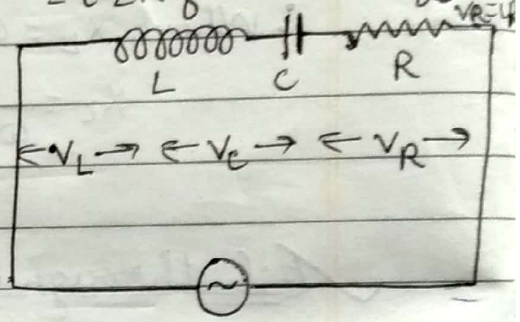
Resonance in LCR series circuit :-

$X_L \propto f$ $\qquad$ $X_C \propto \frac{1}{f}$

$V_L = i X_L = i 2\pi f L$ $\qquad$ $V_C = i \frac{1}{2\pi f C}$

$$i = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$



$$\boxed{i = \frac{E}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}}$$

$E = E_0 \sin \omega t \rightarrow (1)$

$E = E_0 \sin 2\pi f t$

$E \propto \sin f$

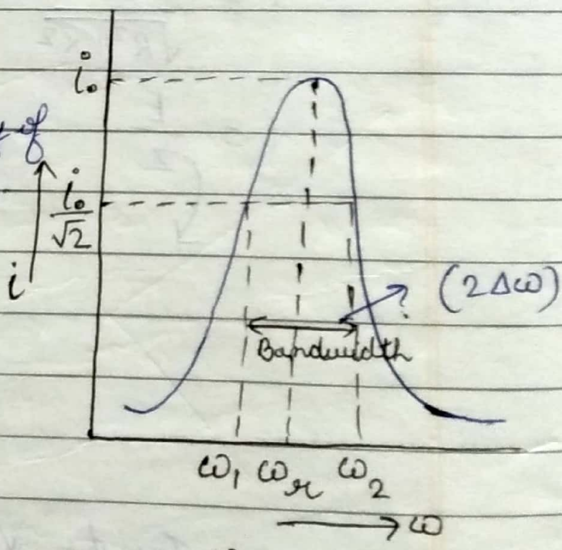
If $X_L - X_C = 0$

then $X_L = X_C$

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$ frequency of resonance.



$\omega_r = 2\pi f_r$

Quality factor (Q-factor) :-

$$Q = \frac{V_L \text{ or } V_C \text{ at resonance}}{V_R \text{ at resonance}}$$

$$= \frac{i X_L}{i R} = \frac{X_L}{R} = \frac{\omega_r L}{R}$$

$$= \frac{\frac{1}{\sqrt{LC}} \cdot L}{R} \Rightarrow \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

Example

Q43. (SL Arora) Page no. 7-26.

Ans. $i = 0.49\,A$ $\quad E_{rms} = 220\,V$

$R = 400\,\Omega$ $\quad f = 50\,Hz$

$X_c = 200\,\Omega$

(i) $V_R = iR$

$V_c = iX_c = i \times 200$

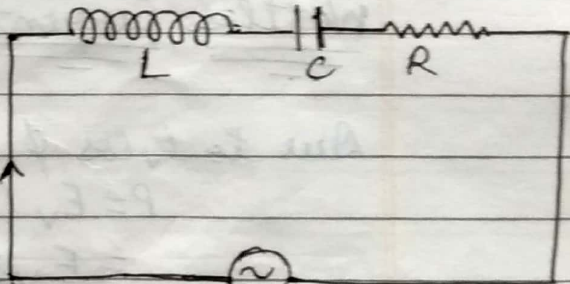
(ii) $X_L = X_c$

$2\pi f C = X_c$

$L = \frac{X_c}{2\pi f} = \frac{200 \times 7}{2 \times 22 \times 50} = 0.64\,H$

Power in AC LCR circuit:-

In one complete cycle



$\cancel{P = \frac{W}{T} = \frac{\int dW}{T} = \frac{1}{T}\int_0^T i^2 R\,dt}$

$\cancel{= \frac{1}{T}\int_0^T i_0^2 \sin^2 \omega t\, R\,dt}$

$\cancel{= \frac{1}{T} i^2 R}$

$E = E_0 \sin \omega t \longrightarrow (1)$

$P = \frac{W}{T} = \frac{\int dW}{T} = \frac{1}{T}\int_0^T E\,i\,dt$

$= \frac{1}{T}\int_0^T E_0 \sin \omega t\; i_0 \sin(\omega t + \phi)\,dt$

$= \frac{E_0 i_0}{T}\int_0^T \sin \omega t \sin(\omega t + \phi)\,dt$

$= \frac{E_0 i_0}{T}\,\frac{T}{2}\cos\phi$ ------ on evaluating the integration.

$$P = \frac{E_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos\phi$$

$$P = E_{rms} \, I_{rms} \cos\phi$$

where,

$$\cos\phi = \frac{VR}{E} = \frac{iR}{iZ} = \frac{R}{Z}$$

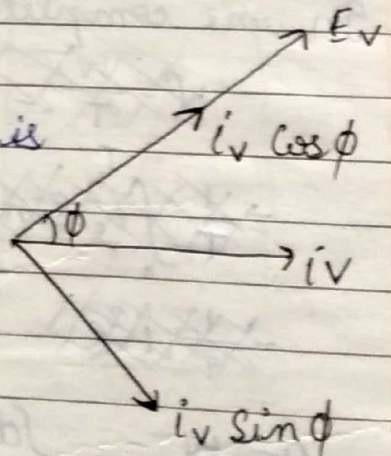
$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Is called as Power factor.

Q. What is the value of power factor at the time of resonance?

Ans. ~~$\cos\phi = 1$~~ At resonance $\phi = 0$ $\therefore \cos\phi = 1$.

Wattless Current :-



Due to $i_V \cos\phi$, power dissipated is

$$P = E_V (i_V \cos\phi) \cos 0$$
$$= E_V i_V \cos\phi \rightarrow (1)$$

Again power dissipated due to $i_V \sin\phi$

$$P = E_V (i_V \sin\phi) \cdot \cos 90^\circ$$
$$= 0.$$

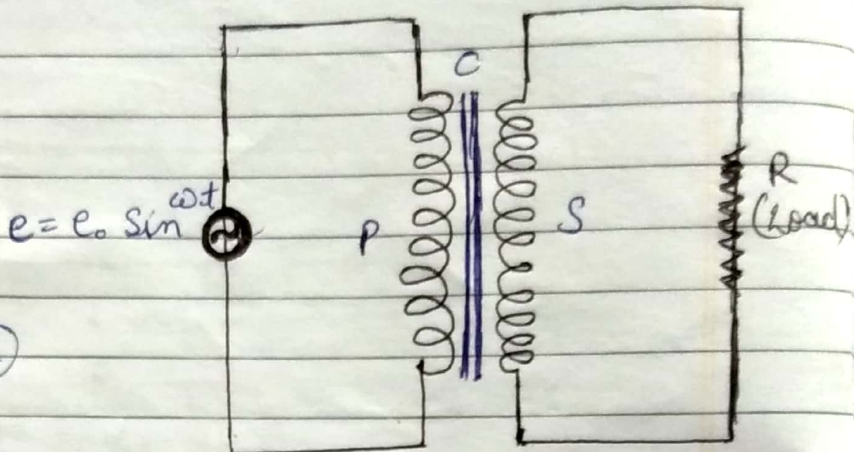
LC Oscillations :

Transformer :-

$$e_1 = N_1 \frac{d\phi}{dt}$$

$$e_2 = \frac{N_2 d\phi}{dt}$$

$$\frac{e_2}{e_1} = \frac{N_2}{N_1} \rightarrow (1)$$



For ideal transformer

$$P_1 = P_2$$

$$e_1 i_1 = e_2 i_2$$

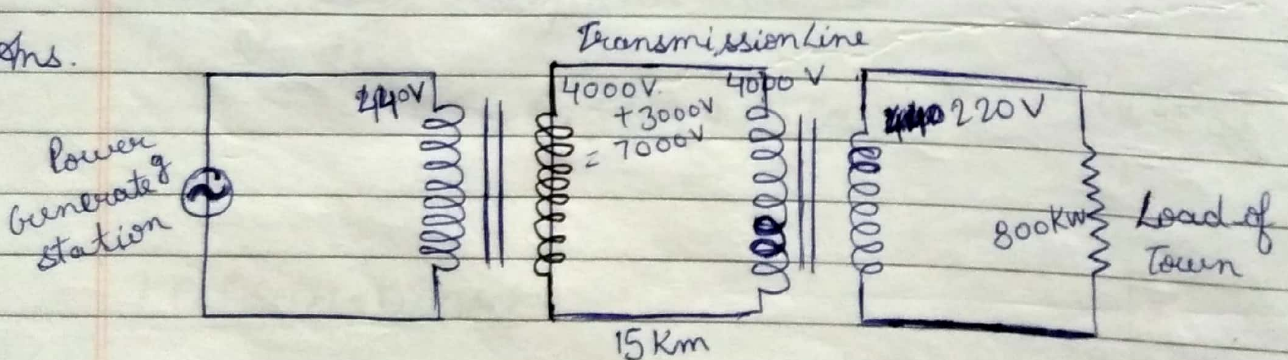
$$\frac{e_2}{e_1} = \frac{i_1}{i_2} \rightarrow (2)$$

$$\boxed{\frac{e_2}{e_1} = \frac{i_1}{i_2} = \frac{N_2}{N_1}} = K \rightarrow \text{Transformation Ratio}$$

$K > 1$ step up, $K < 1$ step down.

Q. no. 7·25 (SL. Arora) Page no. 7·73.

Ans.



Length of wire line = 30 Km

$$R = 0.5 \times 30$$
$$= 10\,\Omega$$

Current flowing through transmission line

$$i = \frac{P}{V} = \frac{800 \times 10^3}{4000}$$

$$= 200A$$

Power lost by transmission line

$$i^2R = (200)^2 \times 15\ \cancel{W} = 600kW$$

Potential drop across the transmission line $= iR$

$$= 200 \times 15$$

$$= 3000V$$

$\therefore$ Rating or characteristics of transformer at the plant $= 4000 + 3000 = 7000V$

Hence step up transformers at the plant is 440 V – 7000 V.

Q. Ex. 68. (S.L. Arora) Page no. 7.46.

Ans. $\mathcal{E}_1 = 220V$, $\mathcal{E}_2 = 22V$, $Z_2 = 220\Omega$

$$I_2 = \frac{\mathcal{E}_2}{Z_2} = \frac{22}{220} = 0.1A$$

If there are no energy losses, then Input power = Output power i.e. $\mathcal{E}_1 I_1 = \mathcal{E}_2 I_2$

$$I = \frac{\mathcal{E}_2 I_2}{\mathcal{E}_1} = \frac{22 \times 0.1}{220} = 0.01A.$$

Q. Ex 69. (S.L. Arora) Page no. 7.46.

Ans. $N_1 = 500$, $N_2 = 1000$, $E_1 = 200V$, $R_2 = 100\Omega$

$$E_2 = \frac{N_2}{N_1} E_1 = \frac{1000}{500} \times 200 = 400V$$

$$I_2 = \frac{E_2}{R_2} = \frac{4000}{1000} = 4A.$$

$$E_1 I_1 = E_2 I_2$$

$$I_1 = \frac{E_2 I_2}{E_1} = \frac{400 \times 4}{200} = 8\,A.$$

Important Questions:

Q1. What is the principle of meter bridge? How will you determine the resistance of a resistor.

Q2. What is the principle of potentiometer?

Q3. A cell is connected (EMF = 6V) across a resistance wire of length 600 cm. Assuming the internal resistance of cell to be zero. Calculate the potential gradient of the potentiometer. If a primary cell of unknown emf E is connected as shown in fig. Calculate the emf E of this test cell.

Q4. Write the working of a cyclotron with the labelled diagram. State the principle also.

Q5. With the help of a well labelled diagram explain the principle and working of a coil Galvanometer. Write two important properties of the wire used for suspending the coil. What is radial magnetic field.

Fig. of Q3.