MISCELLANEOUS EXAMPLES

Very Short Answer : [1 Mark]

1. Find the projection of $\vec{b}+\vec{c}$ on $\vec{a}$, where $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{k}$. **[CBSE 2007, 1M]**

Sol. $\vec{b}+\vec{c}=\hat{i}+3\hat{j}+\hat{k}+\hat{i}+\hat{k}=2\hat{i}+3\hat{j}+2\hat{k}$

Projection of $\vec{b}+\vec{c}$ on $\vec{a}=\dfrac{(\vec{b}+\vec{c}).\vec{a}}{|a|}$

$$=\frac{(2\hat{i}+3\hat{j}+2\hat{k})(\hat{i}+2\hat{j}+\hat{k})}{\sqrt{(1)^2+(2)^2+(1)^2}}=\frac{2+6+2}{\sqrt{1+4+1}}=\frac{10}{\sqrt{6}}$$ **[1]**

2. If $\vec{P}(1,5,4)$ and $\vec{Q}(4,-1,-2)$, find the direction ratios of $\overrightarrow{PQ}$. **[CBSE 2008, 1M]**

Sol. $\overrightarrow{OP}=\hat{i}+5\hat{j}+4\hat{k}$,

$\overrightarrow{OQ}=4\hat{i}-\hat{j}-2\hat{k}$

$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=4\hat{i}-\hat{j}-2\hat{k}-\hat{i}-5\hat{j}-4\hat{k}=3\hat{i}-6\hat{j}-6\hat{k}$

$\therefore$ Direction ratios = 3, –6, –6. **[1]**

3. If $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-5\hat{k}$, find a unit vector in the direction of $\vec{a}-\vec{b}$. **[CBSE 2008, 1M]**

Sol. Here $\vec{a}=\hat{i}+2\hat{j}-\hat{k};\vec{b}=3\hat{i}+\hat{j}-5\hat{k}$

Let $\vec{c}=\vec{a}-\vec{b}=\hat{i}+2\hat{j}-\hat{k}-3\hat{i}-\hat{j}+5\hat{k}$

$=-2\hat{i}+\hat{j}+4\hat{k}$

$|\vec{c}|=\sqrt{4+1+16}=\sqrt{21}$

Unit vector $\hat{c}=\dfrac{\vec{c}}{|\vec{c}|}=\dfrac{-2\hat{i}+\hat{j}+4\hat{k}}{\sqrt{21}}$ **[1]**

Q. Find the value of μ and λ.
$(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+\lambda\hat{j}+\mu\hat{k})=\vec{0}$
$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 6 & 27\\ 1 & \lambda & \mu\end{vmatrix}=0\hat{i}+0\hat{j}+0\hat{k}$
equating the value of $\hat{i},\hat{j},\hat{k}$ and find μ and λ

4. If $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=2$ and $\vec{a}.\vec{b}=3$, find the angle between $\vec{a}$ and $\vec{b}$. **[CBSE 2008, 1M]**

Sol. Here $|\vec{a}|=\sqrt{3}$, $|\vec{b}|=2$ and $\vec{a}.\vec{b}=3$

$\cos\theta=\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{3}{\sqrt{3}.2}$

$\cos\theta=\dfrac{\sqrt{3}}{2}\Rightarrow 30°$ **[1]**

5. Find the value of p if $(2\hat{i}+6\hat{j}+27\hat{k})\times(\hat{i}+3\hat{j}+p\hat{k})=\vec{0}$ **[CBSE 2009, 1M]**

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & 3 & p \end{vmatrix}=0$

$\Rightarrow (6p-81)\hat{i}-(2p-27)\hat{j}+(0)\hat{k}=0$

$\Rightarrow 6p=81$

$p=\frac{81}{6}=\frac{27}{2}$. [1]

6. If $\vec{p}$ is a unit vector and $(\vec{x}-\vec{p}).(\vec{x}+\vec{p})=80$, then find $|\vec{x}|$. **[CBSE 2009, 1M]**

Sol. $|\vec{x}|^2-|\vec{p}|^2=80$

$|\vec{x}|^2=81 \qquad \therefore |P|=1$

$|\vec{x}|=\pm 9$ [1]

7. Write a vector of magnitude 9 units in the direction of vector : $-2\hat{i}+\hat{j}+2\hat{k}$. **[CBSE 2010, 1M]**

Sol. The required vector is

$$\frac{9(-2\hat{i}+\hat{j}+2\hat{k})}{\sqrt{4+1+4}}=3(-2\hat{i}+\hat{j}+2\hat{k})$$ [1]

8. Find λ if $\left(2\hat{i}+6\hat{j}+14\hat{k}\right)\times\left(\hat{i}-\lambda\hat{j}+7\hat{k}\right)=\vec{0}$ **[CBSE 2010, 1M]**

Sol. $(2\hat{i}+6\hat{j}+14\hat{k})\times(\hat{i}-\lambda\hat{j}+7\hat{k})=\vec{0}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 14 \\ 1 & -\lambda & 7 \end{vmatrix}=\vec{0}$

$\Rightarrow (42+14\lambda)\hat{i}+0.\hat{j}+\hat{k}(-2\lambda-6)=\vec{0}$

$\Rightarrow 42+14\lambda=0$ Also $-2\lambda-6=0$

hence $\lambda=-3$ [1]

9. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$, then what is the angle between $\vec{a}$ and $\vec{b}$? **[CBSE 2010, 1M]**

Sol. $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\cos\theta=|\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow \tan\theta=1 \Rightarrow \theta=\frac{\pi}{4}$ [1]

10. Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = \frac{2}{3}$ and $(\vec{a}\times\vec{b})$ is a unit vector. Write the angle between $\vec{a}$ and $\vec{b}$.

[CBSE 2010, 1M]

Sol. $|\vec{a}\times\vec{b}| = 1$

$\Rightarrow \ ab\sin\theta = 1 \quad \Rightarrow \quad \sqrt{3}\times\frac{2}{3}\sin\theta = 1$

$\Rightarrow \ \sin\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \theta = \frac{\pi}{3}$ **[1]**

11. Write the projection of the vector $\hat{i}-\hat{j}$ on the vector $\hat{i}+\hat{j}$. **[CBSE 2011, 1M]**

Sol. $\frac{(\hat{i}-\hat{j}).(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{1-1}{\sqrt{2}} = 0$ **[1]**

12. Write a unit vector in the direction of the vector $\vec{a} = 2\hat{i}+\hat{j}+2\hat{k}$. **[CBSE 2011, 1M]**

Sol. $\vec{a} = \frac{2\hat{i}}{3}+\frac{\hat{j}}{3}+\frac{2\hat{k}}{3}$ **[1]**

13. Write the value of $(\hat{i}\times\hat{j}).\hat{k}+\hat{i}.\hat{j}$ **[CBSE 2012, 1M]**

Sol. $(\hat{i}\times\hat{j}).\hat{k}+\hat{i}.\hat{j} = \hat{k}.\hat{k}+\hat{i}.\hat{j} = 1 + 0 = 1$

14. Find the scalar components of the vector $\overrightarrow{AB}$ with initial point A(2, 1) and terminal point B(–5, 7) **[CBSE 2012, 1M]**

Sol. Given A(2, 1) and B(–5, 7)

$\Rightarrow \overrightarrow{AB} = -7\hat{i}+6\hat{j}$, Scalar component –7, 6 **[1]**

15. Find $|\vec{x}|$, if for a unit vector $\vec{a}$, $(\vec{x}-\vec{a}).(\vec{x}+\vec{a}) = 15$. **[CBSE 2013, 1M]**

Sol. $|\vec{x}|^2 - |\vec{a}|^2 = 15$

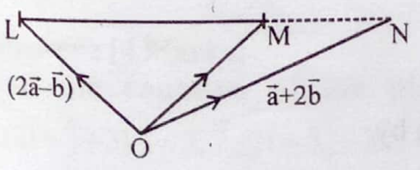
$\Rightarrow \quad |\vec{x}|^2 - 1 = 15$

$\Rightarrow \quad |\vec{x}|^2 = 16$

$\Rightarrow \quad |\vec{x}| = 4$ **[1]**

16. L and M are two points with position vector $2\vec{a}-\vec{b}$ and $\vec{a}+2\vec{b}$ respectively. Write the position vector of a point N which divides the line segment LM in the ratio 2 : 1 externally. **[CBSE 2013, 1M]**

Sol.



$$\overrightarrow{ON} = \frac{2\times\overrightarrow{OM} - 1\times\overrightarrow{OL}}{2-1}$$

$$\overrightarrow{ON} = \frac{2\times(\vec{a}+2\vec{b})-1\times(2\vec{a}-\vec{b})}{1}$$

$$\overrightarrow{ON} = 5\vec{b}$$

[1]

17. Find the value of 'p' for which the vectors $3\hat{i}+2\hat{j}+9\hat{k}$ and $\hat{i}-2p\hat{j}+3\hat{k}$ are parallel.

[CBSE 2014, 1M]

Sol. Two vectors are parallel if $\frac{3}{1}=\frac{2}{-2p}=\frac{9}{3}$

Then $p = -\frac{1}{3}$

[1]

18. If $\vec{a}$ and $\vec{b}$ are perpendicular vectors, $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5$, find the value of $|\vec{b}|$.

[CBSE 2014, 1M]

Sol. If $\vec{a}$ and $\vec{b}$ are perpendicular vecotrs, then $\vec{a}\,.\,\vec{b}=0$

$|\vec{a}+\vec{b}| = 13$

$|\vec{a}+\vec{b}|^2 = 169$

$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\,.\,\vec{b}=169$

$|\vec{b}|^2 = 169 - 25$

$|\vec{b}| = 12$

19. Find $\vec{a}.(\vec{b}\times\vec{c})$, if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}+2\hat{k}$. **[CBSE 2014, 1M]**

Sol. If $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=-\hat{i}+2\hat{j}+\hat{k}, \vec{c}=3\hat{i}+\hat{j}+2\hat{k}$,

Then $\vec{a}.(\vec{b}\times\vec{c}) = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$

expand along R_1 = 2[4 – 1] – 1 [–2 –3] + 3 [–1 –6]

= 6 + 5 – 21 = – 10

20. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{2}$ with y-axis and an acute angle θ with z-axis.

[CBSE 2014, 1M]

Sol. Here $l=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$ and $m=\cos\frac{\pi}{2}=0$

Therefore, $l^2+m^2+n^2=1$ gives

$$\frac{1}{2}+0+n^2=1 \Rightarrow n=\frac{1}{\sqrt{2}}$$

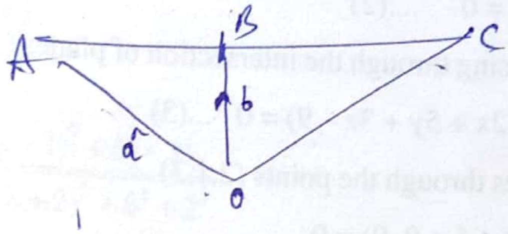
Hence, the required vector $\vec{r}=5\sqrt{2}\,(l\hat{i}+m\hat{j}+n\hat{k})$ is given by

$$\vec{r}=5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{i}+0\hat{j}+\frac{1}{\sqrt{2}}\hat{k}\right)=\vec{r}=5\hat{i}+5\hat{k}.$$

.........(1) [1]

21. In a triangle OAC, if B is the mid-point of side AC and $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, then what is $\overrightarrow{OC}$? **[CBSE 2015, 1M]**

Sol. $\overrightarrow{OB} = \dfrac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$



$\vec{b} = \dfrac{\vec{a} + \overrightarrow{OC}}{2}$

$\overrightarrow{OC} = 2\vec{b} - \vec{a}$ **[1]**

22. Find the vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. **[CBSE 2015, 1M]**

Sol. Let required vector $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{c} = \lambda\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 3 & -1 & 2 \end{vmatrix}$$

$\vec{c} = \lambda(\hat{i} - 11\hat{j} - 7\hat{k})$ ....(1)

$|\vec{c}| = \lambda\sqrt{1 + 121 + 49}$

$\sqrt{171} = \lambda\sqrt{171}$

$\lambda = 1$ put in equation (1)

$\vec{c} = (\hat{i} - 11\hat{j} - 7\hat{k})$ **[1]**

23. If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$. **[CBSE 2016, 1M]**

Sol. Unit vector $= \dfrac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \dfrac{6\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{36 + 9 + 4}} = \dfrac{6\hat{i} - 3\hat{j} + 2\hat{k}}{7}$ **[1]**

24. Find λ and μ if $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = \vec{0}$. **[CBSE 2016, 1M]**

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

$\hat{i}(3\mu + 9\lambda) - \hat{j}(\mu - 27) + \hat{k}(-\lambda - 9) = 0$

$\mu - 27 = 0$ and $-\lambda - 9 = 0$

$\mu = 27$ $\lambda = -9$ **[½ + ½]**

Short Answer : [4 Marks]

25. Find the equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i} + \hat{j} + 3\hat{k}) = 7$, $\vec{r}.(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and the point (2, 1, 3). **[CBSE 2007, 4M]**

OR

Find the equation of line, which is parallel to $2\hat{i} - \hat{j} + 3\hat{k}$ and which passes through the point (5, –2, 4).

Sol. Given equations of plane in cartesian form is

$2x + y + 3z - 7 = 0$ ....(1)

$2x + 5y + 3z - 9 = 0$ ....(2)

Equation of plane passing through the intersection of planes (1) and (2) is

$(2x + y + 3z - 7) + \lambda(2x + 5y + 3z - 9) = 0$ ...(3)

$\because$ The plane (3) passes through the points (2,1,3)

$\therefore\ (4 + 1 + 9 - 7) + \lambda(4 + 5 + 9 - 9) = 0$

$7 + 9\lambda = 0 \Rightarrow \lambda = \frac{-7}{9}$

Putting the value of λ in equation (3), we get

$(2x + y + 3z - 7) - \frac{7}{9}(2x + 5y + 3z - 9) = 0$

$18x + 9y + 27z - 63 - 14x - 35y - 21z + 63 = 0$

$2x - 13y + 3z = 0$

Equation of plane in vector form is

$\vec{r}.(2\hat{i} - 13\hat{j} + 3\hat{k}) = 0$

OR

The equation of the line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\therefore \vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

Equation of line in cartesian form is

$\frac{x-5}{2} = \frac{y+2}{-1} = \frac{z-4}{3} = \lambda$

26. If vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are such that $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 3, |\vec{b}| = 5$ and $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$. **[CBSE 2008, 4M]**

Sol. $\because\ \vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a} + \vec{b} = -\vec{c}$

$(\vec{a} + \vec{b}) . (\vec{a} + \vec{b}) = (-\vec{c}) . (-\vec{c})$ [1]

$\vec{a} . \vec{a} + \vec{a} . \vec{b} + \vec{b} . \vec{a} + \vec{b} . \vec{b} = \vec{c} . \vec{c}$

$\vec{a} . \vec{a} + 2\vec{a} . \vec{b} + \vec{b} . \vec{b} = \vec{c} . \vec{c}$ $[\because \vec{a} . \vec{b} = \vec{b} . \vec{a}]$

$|\vec{a}|^2 + 2|\vec{a}||\vec{b}|\cos\theta + |\vec{b}|^2 = |\vec{c}|^2$ [1]

$(3)^2 + 2(3)(5)\cos\theta + (5)^2 = (7)^2$

$9 + 30\cos\theta + 25 = 49$ [1]

$30\cos\theta = 15$

$\cos\theta = \frac{15}{30} = \frac{1}{2} = \cos 60°$

$\theta = 60°$. [1]

27. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ. **[CBSE 2009, 4M]**

Sol. $\vec{S} = (2\hat{i} + 4\hat{j} - 5\hat{k}) + (\lambda\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{S} = \hat{i}(2+\lambda) + 6\hat{j} - 2\hat{k}$

Unit vector of $\vec{S} = \hat{S} = \dfrac{\vec{S}}{|S|} = \dfrac{(\lambda+2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(\lambda+2)^2 + 6^2 + 2^2}}$ [1½]

Given : $(\hat{i} + \hat{j} + \hat{k}).\hat{S} = 1$

$\therefore \dfrac{(\lambda+2)+6-2}{\sqrt{(\lambda+2)^2 + 40}} = 1$ [1]

or $(\lambda + 6) = \sqrt{(\lambda+2)^2 + 40}$ or Squaring both side

$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

$\Rightarrow 8\lambda = 8 \Rightarrow \lambda = 1$ [1½]

28. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$. **[CBSE 2010, 4M]**

OR

Sol. Let the required vector be $\vec{r}$

$\vec{r} = \lambda(2\vec{a} - \vec{b} + 3\vec{c})$ [1]

$= \lambda[2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}]$

$= \lambda[\hat{i} - 2\hat{j} + 2\hat{k}]$ [½]

As $|\vec{r}| = 6$ [½]

$\Rightarrow \sqrt{\lambda^2(1+4+4)} = 6 \Rightarrow |\lambda| \times 3 = 6$

$\Rightarrow \lambda = \pm 2$ [1]

Hence, the required vector be $\pm 2(\hat{i} - 2\hat{j} + 2\hat{k})$ [1]

29. Using vectors, find the area of the triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5). **[CBSE 2011, 4M]**

Sol. A(1, 1, 2) B(2, 3, 5) C(1, 5, 5)

$\Delta = \left|\dfrac{1}{2}|\overrightarrow{AB} \times \overrightarrow{BC}|\right|$ [1½]

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{BC} = -\hat{i} + 2\hat{j}$

$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 0 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 4\hat{k}$ [1½]

$\Delta = \dfrac{1}{2}\sqrt{61}$ [1]

30. Let $\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$, $\vec{b} = 3\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c} = 18$ **[CBSE 2010, 2012, 4M]**

Sol. Given $\vec{a} = \hat{i}+4\hat{j}+2\hat{k}$, $\vec{b} = 3\hat{i}-2\hat{j}+7\hat{k}$, $\vec{c} = 2\hat{i}-\hat{j}+4\hat{k}$ and $\vec{p}.\vec{c} = 18$

The vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ must be parallel to $\vec{a}\times\vec{b}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$= \hat{i}(28+4) - \hat{j}(7-6) + \hat{k}(-2-12) = 32\hat{i}-\hat{j}-14\hat{k}$ [1]

since $\vec{p}$ is $\perp$ to both $\vec{a}$ and $\vec{b}$

i.e., $\vec{p} \parallel |\vec{a}\times\vec{b}|$ [1]

Let $\vec{p} = \lambda(\vec{a}\times\vec{b}) = \lambda(32\hat{i}-\hat{j}-14\hat{k})$

$\vec{p}.\vec{c} = 18 \Rightarrow \lambda(32\hat{i}-\hat{j}-14\hat{k}).(2\hat{i}-\hat{j}+4\hat{k}) = 18$ [½]

then $2(32\lambda) + (-1)(-\lambda) + 4(-14\lambda) = 18$

If $64\lambda + \lambda - 56\lambda = 18$

$9\lambda = 18 \Rightarrow \lambda = 2$ [½]

$\vec{p} = 2(32\hat{i}-\hat{j}-14\hat{k})$ [1]

31. If $\vec{a} = \hat{i}-\hat{j}+7\hat{k}$ and $\vec{b} = 5\hat{i}-\hat{j}+\lambda k$ then find the value of λ so that $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular vectors. **[CBSE 2013, 4M]**

Sol. $\vec{a} = \hat{i}-\hat{j}+7\hat{k}$ & $\vec{b} = 5\hat{i}-\hat{j}+\lambda k$

$\vec{a}+\vec{b} = 6\hat{i}-2\hat{j}+(7+\lambda)\hat{k}$ [½]

$\vec{a}-\vec{b} = -4\hat{i}-0\hat{j}+(7-\lambda)\hat{k}$ [½]

$(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$ are perpendicular

So $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 0$ [1½]

$\Rightarrow [6\hat{i}-2\hat{j}+(7+\lambda)\hat{k}].[-4\hat{i}+0\hat{j}+(7-\lambda)\hat{k}] = 0$

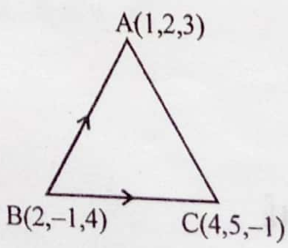
$\Rightarrow -24+0+49-\lambda^2 = 0 \Rightarrow \lambda^2 = 25$ [1]

$\lambda = \pm 5$

32. Using vectors, find the area of the triangle ABC, whose vertices are A(1, 2, 3), B(2, –1, 4) and C(4, 5,–1). **[CBSE 2013, 4**

Sol. Side $\overrightarrow{BA} = -\hat{i}+3\hat{j}-\hat{k}$

& $\overrightarrow{BC} = 2\hat{i}+6\hat{j}-5\hat{k}$



area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{BA}\times\overrightarrow{BC}\right|$

$$\overrightarrow{BA}\times\overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 2 & 6 & -5 \end{vmatrix}$$

$= \hat{i}(-15+6) - \hat{j}(5+2) + \hat{k}(-6-6) = -9\hat{i} - 7\hat{j} - 12\hat{k}$

$|\overrightarrow{BA} \times \overrightarrow{BC}| = \sqrt{81+49+144}$

$\Rightarrow \quad \sqrt{274}$

Now area of ΔABC is $= \frac{1}{2}\sqrt{274}$ [1]

33. Show that the four points A, B, C and D with position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$ respectively are coplanar. **[CBSE 2014, 4M]**

OR

The scalar product of the vector $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i}+4\hat{j}-6\hat{k}$ and $\vec{c} = \lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

Sol. $\overrightarrow{AB} = (-\hat{j}-\hat{k}) - (4\hat{i}+5\hat{j}+\hat{k}) = -4\hat{i}-6\hat{j}-2\hat{k}$

$\overrightarrow{AC} = (3\hat{i}+9\hat{j}+4\hat{k}) - (4\hat{i}+5\hat{j}+\hat{k}) = -\hat{i}+4\hat{j}+3\hat{k}$

$\overrightarrow{AD} = (-4\hat{i}+4\hat{j}+4\hat{k}) - (4\hat{i}+5\hat{j}+\hat{k}) = -8\hat{i}-\hat{j}+3\hat{k}$ [1½]

four points A,B,C,D with position vectors are coplanar if

$$[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$ [1½]

$\Rightarrow -4(12+3) + 6(-3+24) - 2(1+32)$

$= -60 + 126 - 66 = 0$ [1]

OR

Let $\vec{S} = \vec{b}+\vec{c}$

$\vec{S} = (2\hat{i}+4\hat{j}-5\hat{k}) + (\lambda\hat{i}+2\hat{j}+3\hat{k})$

$\vec{S} = \hat{i}(2+\lambda) + 6\hat{j} - 2\hat{k}$

Unit vector of $\vec{S} = \hat{S} = \frac{\vec{S}}{|\vec{S}|} = \frac{(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(\lambda+2)^2+6^2+2^2}}$ ....(i) [1]

Given : $(\hat{i}+\hat{j}+\hat{k}).\hat{S} = 1$

$\therefore \quad \frac{(\lambda+2)+6-2}{\sqrt{(\lambda+2)^2+40}} = 1$ [1]

or $(\lambda+6) = \sqrt{(\lambda+2)^2+40}$ or Squaring both side

$\lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

$\Rightarrow \quad 8\lambda = 8 \quad \Rightarrow \lambda = 1$ [1]

Hence unit vector along $\vec{S} = \vec{b}+\vec{c} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}}$ (put $\lambda = 1$ in equation (i))

$\hat{S} = \frac{1}{7}[3\hat{i}+6\hat{j}-2\hat{k}]$ [1]

34. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 27$. **[CBSE 2015, 4M]**

Sol. Given $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{c}.\vec{d} = 27$

The vector which is perpendicular to both $\vec{a}$ and $\vec{b}$ must be parallel to $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$= \hat{i}(28 + 4) - \hat{j}(7 - 6) + \hat{k}(-2 - 12) = 32\hat{i} - \hat{j} - 14\hat{k}$ **[1]**

since $\vec{d}$ is $\perp$ to both $\vec{a}$ and $\vec{b}$

i.e., $\vec{d} \parallel (\vec{a} \times \vec{b})$ **[1]**

Let $\vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda(32\hat{i} - \hat{j} - 14\hat{k})$

$\vec{c}.\vec{d} = 27 \Rightarrow \lambda(32\hat{i} - \hat{j} - 14\hat{k}).(2\hat{i} - \hat{j} + 4\hat{k}) = 27$ **[½]**

then $2(32\lambda) + (-1)(-\lambda) + 4(-14\lambda) = 27$

If $64\lambda + \lambda - 56\lambda = 27$

$9\lambda = 27 \Rightarrow \lambda = 3$ **[½]**

$\vec{d} = 3(32\hat{i} - \hat{j} - 14\hat{k})$ **[1]**

35. Show that the four points A(4, 5, 1), B(0, –1, –1), C(3, 9, 4) and D(–4, 4, 4) are coplanar. **[CBSE 2016, 4M]**

Sol. A(4, 5, 1), B(0, –1, –1), C(3, 9, 4), D(–4, 4, 4) are coplanar if $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

$\therefore \quad \overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ **[1½]**

$$[\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}] = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$ **[1½]**

$= -4(12 + 3) + 6(-3 + 24) - 2(1 + 32)$

$= -60 + 126 - 66 = 0$ (which is true hence points are coplanar) **[1]**

Long Answer : [6 Marks]

36. Find the value of λ which makes the vectors $\vec{a}, \vec{b}, \vec{c}$ coplanar where $\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $b = -\hat{i} + 4\hat{j} + 3\hat{k}$ and $\vec{c} = -8\hat{i} - \hat{j} + \lambda\hat{k}$. **[CBSE 2007, 6M]**

Sol. The vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if

$\vec{a}.(\vec{b} \times \vec{c}) = 0$

$$\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & \lambda \end{vmatrix} = 0$$

$-4(4\lambda+3)+6(-\lambda+24)-2(1+32)=0$

$-16\lambda-12-6\lambda+144-66=0$

$-22\lambda=-66 \Rightarrow \lambda=3$

37. Find the shortest distance between the lines ℓ_1 and ℓ_2 whose vector equations are $\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$. **[CBSE 2007, 6M]**

Sol. On compairing by

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$\therefore \quad \vec{a}_1=\hat{i}+\hat{j}, \vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$$

$$\vec{a}_2=2\hat{i}+\hat{j}-\hat{k}, \vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$$

$$a_2-a_1=2\hat{i}+\hat{j}-\hat{k}-\hat{i}-\hat{j}=\hat{i}-\hat{k}$$

$$\vec{b}_2\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix}$$

$$=\hat{i}(-2+5)-\hat{j}(4-3)+\hat{k}(-10+3)$$

$$=3\hat{i}-\hat{j}-7\hat{k}$$

$$\therefore \quad |\vec{b}_1\times\vec{b}_2|=\sqrt{9+1+49}=\sqrt{59}$$

Therefore, shortest distance between the given lines is

$$SD=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right|$$

$$=\left|\frac{(3\hat{i}-\hat{j}-7\hat{k}).(i-k)}{\sqrt{59}}\right|=\left|\frac{3-0+7}{\sqrt{59}}\right|=\frac{10}{\sqrt{59}}$$