MISCELLANEOUS EXAMPLES

Short Answer : [4 Marks]

1. Two dice are thrown together. What is the probability that the sum of the numbers on the two dice is neither 9 nor 11. **[CBSE 2007, 4M]**

Sol. Let A and B be the event of getting a sum of 9 and sum of 11.

Total events $= 6 \times 6 = 36$

$A = \{\text{sum of } 9\} = \{(6, 3), (5, 4), (4, 5), (3, 6)\}$

$P(A) = \frac{4}{36}$ **[1]**

$B = \{\text{sum of } 11\} = \{(6, 5), (5, 6)\}$

$P(B) = \frac{2}{36}$ **[1]**

and $A \cap B = \phi \therefore P(A \cap B) = 0$

$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{36} + \frac{2}{36} - 0 = \frac{6}{36} = \frac{1}{6}$ **[1]**

P(neither a sum 9 nor 11) = 1 – P(A or B)

$= 1 - \frac{1}{6} = \frac{5}{6}$ **[1]**

2. Find the mean μ and variance σ^2 for the following probability distribution :

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

[CBSE 2007, 4M]

OR

Determine the binomial distribution whose mean is 20 and variance 16.

Sol. **[2]**

X	P(X)	X.P(X)	$X^2P(X)$
0	$\frac{1}{6} = \frac{5}{30}$	0	0
1	$\frac{1}{2} = \frac{15}{30}$	$\frac{15}{30}$	$\frac{15}{30}$
2	$\frac{3}{10} = \frac{9}{30}$	$\frac{18}{30}$	$\frac{36}{30}$
3	$\frac{1}{30} = \frac{1}{30}$	$\frac{3}{30}$	$\frac{9}{30}$
Total	1	$\frac{36}{30}$	$\frac{60}{30}$

(i) Mean $\mu = \Sigma X P(X) = \frac{36}{30} = 1.2$ [1]

(ii) Variance $\sigma^2 = \Sigma X^2 P(X) - [\Sigma X P(X)]^2$

$$= \frac{60}{30} - (1.2)^2 = 2 - 1.44 = 0.56$$ [1]

OR

Mean = 20

$np = 20$ ...(i)

Variance = 16 ...(ii)

$npq = 16$ [1]

$np = 20$

$npq = 16$

Solving equation (i) and (ii), we get

$20q = 16 \Rightarrow q = \frac{16}{20} \Rightarrow q = \frac{4}{5}$

$\boxed{q = \frac{16}{20}}$

$\frac{1}{q} = \frac{\cancel{20}\ 5}{\cancel{16}\ 4}$

$q = \frac{4}{5}$

$\therefore \quad p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5}$ [1]

Putting p in equation (i), we get

$n . \frac{1}{5} = 20 \Rightarrow n = 100$

$\frac{1}{2} + \frac{6}{10} + \frac{3}{30}$

$\frac{15 + 18 + 3}{30}$

$\therefore \quad n = 100, p = \frac{1}{5}, q = \frac{4}{5}$ [1]

$\therefore$ Binomial Distribution $= (q + p)^n = \left(\frac{4}{5} + \frac{1}{5}\right)^{100}$ [1]

3. 12 cards, numbered 1 to 12, are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number. **[CBSE 2008, 4M]**

Sol. Here S = {1, 2, 3, ... 12}

Total events n(s) = 12

Let E : number on the drawn card is more than 3

F : number on the card is even number

E = {4, 5, 6, 7, 8, 9, 10, 11, 12}

F = {2, 4, 6, 8, 10, 12} [1]

$E \cap F$ = {4, 6, 8, 10, 12}

$P(E) = \frac{9}{12}, P(F) = \frac{6}{12}, P(E \cap F) = \frac{5}{12}$ [1]

$P(F/E) = \frac{P(E \cap F)}{P(E)} = \frac{5/12}{9/12} = \frac{5}{9}$ [2]

4. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ? **[CBSE 2009, 4M]**

Sol. $P(\text{correct}) = \frac{1}{3}$ $\qquad P(\text{wrong}) = \frac{2}{3}$ [1]

$P(\text{4 correct}) = {}^5C_4\left(\frac{1}{3}\right)^4 \times \frac{2}{3}$ [1]

$P(\text{5 correct}) = \left(\frac{1}{3}\right)^5$ [1]

$\therefore P(\text{4 or 5 correct}) = {}^5C_4\left(\frac{1}{3}\right)^4 \times \frac{2}{3} + \left(\frac{1}{3}\right)^5 = \frac{11}{243}$ [1]

5. A family has 2 children. Find the probability that both are boys, if it is known that :

(i) at least one of the children is a boy,

(ii) the elder child is a boy. **[CBSE 2010, 4M]**

Sol. (i) $S = \{BB, BG, GB\}$, $E = \{BB\}$

$n(E) = 1, \quad n(S) = 3$ [1]

$P(E) = \frac{1}{3}$ [1]

(ii) $S = \{BB, BG\}$, $E = \{BB\}$

$n(E) = 1, \quad n(S) = 2$ [1]

$P(E) = \frac{1}{2}$ [1]

6. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Determine :

(i) K (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(0 < X < 3)$ **[CBSE 2011, 4M]**

OR

Find the probability of throwing at most 2 sixes in 6 throws of a single die. **[CBSE 2011, 4M]**

Sol. (i) $P(X = 0) + P(X = 1) + ... + P(X = 7) = 1$

$\Rightarrow \quad 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow \quad 10k^2 + 9k - 1 = 0 \quad \Rightarrow \quad (10k - 1)(k + 1) = 0$

$\Rightarrow (10k - 1) = 0 \quad \Rightarrow \quad k = \frac{1}{10}$ [1]

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0 + k + 2k = 3k = \frac{3}{10}$ [1]

(iii) $P(X > 6) = P(X = 7) = 7k^2 + k = \frac{7}{100} + \frac{1}{10} = \frac{17}{100}$ [1]

(iv) $P(0 < X < 3) = P(X = 1) + P(X = 2) = k + 2k = 3k = \frac{3}{10}$ [1]

OR

$26_{C_1} \times 26_{C_1}$

$\frac{26!}{1(25!} \times \frac{26!}{1(25!}$

$P(H) = P(0H) + P(1H) + P(2H)$

$$=\left(\frac{1}{2}\right)^6 + {}^6C_1\left(\frac{1}{2}\right)^6 + {}^6C_2\left(\frac{1}{2}\right)^6$$ [2]

$$=\left(\frac{1}{2}\right)^6[1+6+15] = \frac{22}{2^6} = \frac{11}{2^5}$$ [2]

7. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards: Find the mean and variance of the number of red cards **[CBSE 2012, 4M]**

Sol. Let x denotes number of red cards in a draw of two cards

$$P(x = 0) = P(\text{no. red cards}) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{26\times 25}{52\times 51} = \frac{25}{102}$$ [½]

$$P(X = 1) = P\ (\text{one red card and 1 non red card}) = \frac{{}^{26}C_1 \times {}^{26}C_1}{{}^{52}C_2} = \frac{26\times 26\times 2}{52\times 51} = \frac{26}{51}$$ [½]

$$P(X = 2) = P\ (\text{two red cards}) = \frac{{}^{26}C_2}{{}^{52}C_2} = \frac{25}{102}$$ [½]

The probability distribution of X is

X	0	1	2
P(X)	25/102	26/51	25/102

[½]

$$\text{Mean of } X = E(x) = \sum_{i=1}^{n} x_i p(x_i) = 0\times\frac{25}{102} + 1\times\frac{26}{51} + 2\times\frac{25}{102} = \frac{26}{51} + \frac{50}{102} = 1$$ [1]

$$E(x^2) = \sum_{i=1}^{n} x_i^2 p(x_i) = 0^2\times\frac{25}{102} + 1^2\times\frac{26}{51} + 2^2\times\frac{25}{102} = \frac{26}{51} + \frac{50}{51} = \frac{76}{51}$$

$$\text{Variance} = E(x^2) - [E(x)]^2 = \frac{76}{51} - 1 = \frac{25}{51}$$ [1]

8. The probabilities of two students A and B coming to the school in time are $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events, 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time. **[CBSE 2013, 4M]**

Sol. Probabilities of two students A and B comming to the school in time are $P(A) = \frac{3}{7}$ and $P(B) = \frac{5}{7}$

probability of two students A and B not comming to the school in time are

$$P(A') = 1 - \frac{3}{7} = \frac{4}{7}$$

$$P(B') = 1 - \frac{5}{7} = \frac{2}{7}$$ [1]

Probability of only one of then coming to the school in time = P(A).P(B') + P(A').P(B) [½]

$$=\frac{3}{7}\times\frac{2}{7}+\frac{4}{7}\times\frac{5}{7} \Rightarrow \frac{6}{49}+\frac{20}{49}=\frac{26}{49}$$ [1½]

Value Based : Comming to school in times will lead to

(i) attending the morning prayers
(ii) Not missing the first period
} any one

[Any other individual response with suitable justification be accepted, even if there is no reference to the text]. [1]

9. An experiment succeeds thrice as often as it fails. Find the probability that in the next five trials, there will be atleast 3 successes. **[CBSE 2014, 4M]**

Sol. Let Probability of sucess P and faliure q

Given p = 3q

P + q = 1

$4q = 1 \Rightarrow q = \frac{1}{4}$ and $p = \frac{3}{4}$ [1]

$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

$$= {}^5C_3\left(\tfrac{3}{4}\right)^3\left(\tfrac{1}{4}\right)^2 + {}^5C_4\left(\tfrac{3}{4}\right)^4\left(\tfrac{1}{4}\right) + {}^5C_5\left(\tfrac{3}{4}\right)^5\left(\tfrac{1}{4}\right)^0$$ [2]

$$= \frac{10\times 27}{4^5}+\frac{5\times 81}{4^5}+\frac{243}{4^5}$$

$$= \frac{270+405+243}{1024}=\frac{918}{1024}=\frac{459}{512}$$ [1]

10. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of 5 steps, he is one step away from the starting point. **[CBSE 2015, 4M]**

OR

Suppose a girl throws a die. If she gets a 1 or 2, she tosses a coin three times and notes the number of 'tails'. If she gets 3, 4, 5 or 6, the tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die ? **[CBSE 2015, 4M]**

Sol. Since the man is one step away from starting point mean that either

(i) man has taken 3 steps forward and 2 steps backward.

(ii) man has taken 2 steps forward and 3 steps backward.

Taking, movement 1 step forward as success and 1 step backward as failure.

$\therefore$ p = Probability of success = 0.4

and q = Probability of failure = 0.6 [1]

$\therefore$ Required Probability = P {X = 3 or X = 2} = P (X = 3) + P(X = 2) $= {}^5C_3p^3q^2 + {}^5C_2p^2q^3$

$$= {}^5C_2p^2q^2(p+q) = 10\times(0.4)^2\times(0.6)^2(0.4+0.6) = 0.576 = \frac{72}{125}$$

OR

Let E_1 : '3, 4, 5 or 6 is shown on dice', and E_2 : '1 or 2 is shown on dice'

$$P(E_1) = \frac{4}{6} = \frac{2}{3} \quad \text{and} \quad P(E_2) = \frac{2}{6} = \frac{1}{3}$$ [1]

Let E : 'exactly one tail shows up'

then $P(E/E_1)$ = P (tail shows up when coin is tossed once) $= \frac{1}{2}$ [½]

and $P(E/E_2)$ = P (exactly one tail shows up when coin is tossed thrice)

$= P(\{THH, HHT, HTH\}) = \frac{3}{8}$ [½]

$\therefore$ Required probability $= P(E_1/E) = \dfrac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$ [1]

$$= \frac{\frac{1}{2}\times\frac{2}{3}}{\frac{1}{2}\times\frac{2}{3}+\frac{3}{8}\times\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \frac{8}{8+3} = \frac{8}{11}$$ [1]

11. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls where drawn from bag Y.

[CBSE 2016, 4M]

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

Sol. Let E_1 = event of drawing bag X

E_2 = event of drawing bag Y

E = event of drawing one white and one black ball.

$$P(E_1) = P(E_2) = \frac{1}{2}$$

$$P\left(\frac{E}{E_1}\right) = \left(\frac{4}{6}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right) = \frac{16}{30}$$

$$P\left(\frac{E}{E_2}\right) = \left(\frac{3}{6}\right)\left(\frac{3}{5}\right) + \left(\frac{3}{6}\right)\left(\frac{3}{5}\right) = \frac{18}{30}$$ [1½]

By Bayes theorem

$$P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\times P(E/E_2)}{P(E_1)\times P(E/E_1) + P(E_2)\times P(E/E_2)} = \frac{\frac{1}{2}\times\frac{18}{30}}{\left(\frac{1}{2}\times\frac{16}{30}\right)+\left(\frac{1}{2}\times\frac{18}{30}\right)} = \frac{9}{17}$$ [2]

OR

Let A_i and B_i be the events of throwing 10 by A and B.

$$P(A_i) = P(B_i) = \frac{3}{36} = \frac{1}{12}$$

$$P(\bar{A}_i) = P(\bar{B}_i) = \frac{11}{12} \qquad [1\tfrac{1}{2}]$$

$$P(\text{win}A) = P(A) + P(\bar{A})P(\bar{B})P(A)$$

$$= \frac{1}{12} + \frac{1}{12} \times \left(\frac{11}{12}\right)^2 + \frac{1}{12} \times \left(\frac{11}{12}\right)^4 + \ldots.. \qquad [1]$$

$$= \frac{1}{12}\left[1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \ldots..\right] = \frac{1}{12}\left[\frac{1}{1-\frac{11^2}{12^2}}\right]$$

$$P(\text{win}A) = \frac{1}{12} \times \left[\frac{12^2}{144-121}\right] = \frac{12}{23} \qquad [1]$$

$$P(\text{win}B) = 1 - \frac{12}{23} = \frac{11}{23} \qquad [\tfrac{1}{2}]$$

Long Answer : [6 Marks]

12. An urn contains 4 red and 7 blue balls. Two balls are drawn at random with replacement. Find the probability of getting : (i) 2 red balls (ii) 2 blue balls (iii) one red and one blue ball.

[CBSE 2007, 6M]

Sol. Red balls = 4

Blue balls = 7

Total balls = 4 + 7 = 11

(i) $P(\text{2 Red balls}) = P(B).P(B) = \frac{4}{11}.\frac{4}{11} = \frac{16}{121}$ = $\frac{^4C_2}{^{11}C_2}$ = 4x3 [2]

(ii) $P(\text{2 Blue balls}) = P(RR) = \frac{7}{11}.\frac{7}{11} = \frac{49}{121}$ [2]

(iii) P(one red and one blue ball)

$= P(R).P(B) + P(B).P(R)$

$$= \frac{4}{11}.\frac{7}{11} + \frac{7}{11}.\frac{4}{11} = \frac{28}{121} + \frac{28}{121} = \frac{56}{121} \qquad [2]$$

13. There are two bags I and II. Bag I contains 2 white and 4 red balls and bag II contains 5 white and 3 red balls one ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II. **[CBSE 2007, 6M]**

Sol. Bag I | Bag II

White balls = 2 | White balls = 5

red balls = 4 | red balls = 3

Total balls = 2 + 4 = 6 | Total = 5 + 3 = 8

Let E_1 and E_2 be the events to select bag I and respectively and let E be the event to draw red ball.

$$\therefore \quad P(E_1) = \frac{1}{2},\ P(E_2) = \frac{1}{2} \qquad [1]$$

$$P(P/E_1) = \frac{4}{6} = \frac{2}{3};\quad P(E/E_2) = \frac{3}{8} \qquad [1]$$

P(that the red ball was drawn from Second bag)

$= P(E_2/E)$

$$= \frac{P(E_2).P(E/E_2)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)} \qquad [2]$$

$$= \frac{\frac{1}{2}.\frac{3}{8}}{\frac{1}{2}.\frac{2}{3} + \frac{1}{2}.\frac{3}{8}} = \frac{\frac{3}{8}}{\frac{2}{3} + \frac{3}{8}} = \frac{\frac{3}{8}}{\frac{16+9}{24}} = \frac{3}{8} \times \frac{24}{25} = \frac{9}{25} \qquad [2]$$

$$\frac{\frac{3}{16}}{\frac{2}{6} + \frac{3}{24}} = \frac{\frac{3}{16}}{\frac{8+3}{24}} \qquad \frac{3}{16} \times \frac{24}{11} = \frac{9}{22}$$

14. In a bulb factory, machines. A, B and C manufacture 60%, 30% and 10% bulbs respectively. 1%, 2% and 3% of the bulb produced respectively by A, B and C are found to the defective. A bulb is picked up at random from the total production and found to be defective. Find the probability that this bulb was produced by the machine A. **[CBSE 2008, 6M]**

Sol. Here $P(A) = 60\% = \frac{60}{100} = \frac{6}{10}$

$P(B) = 30\% = \frac{30}{100} = \frac{3}{10}$,

$P(C) = 10\% = \frac{10}{100} = \frac{1}{10}$ [2]

$$\frac{\frac{3}{16}}{\frac{2}{6} + \frac{3}{24}} \qquad \frac{3}{11} \qquad \frac{8+3}{24} \qquad \frac{3}{16} \times \frac{24}{11} \qquad \frac{9}{22}$$

Let E be the event of defective bulb

$$\therefore \quad P(E/A) = 1\% = \frac{1}{100},\ P(E/B) = 2\% = \frac{2}{100} \qquad [1]$$

$$P(E/C) = 3\% = \frac{3}{100}$$

Required probability = P(A/E)

$$= \frac{P(A)P(E/A)}{P(A)P(E/A) + P(B)P(E/B) + P(C)P(E/C)} \qquad [1]$$

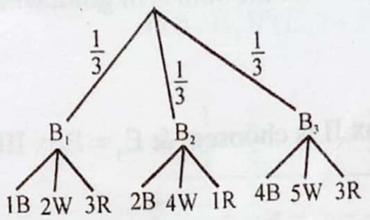
$$= \frac{\frac{6}{10} \times \frac{1}{100}}{\frac{6}{10} \times \frac{1}{100} + \frac{3}{10} \times \frac{2}{100} + \frac{1}{10} \times \frac{3}{100}} = \frac{6}{6+6+3} = \frac{6}{15} = \frac{2}{5} \qquad [2]$$

15. Coloured balls are distributed in three bags as shown in the following table :

Bag	Colour of the ball		
	Black	**White**	**Red**
I	1	2	3
II	2	4	1
III	4	5	3

A bag is selected at random and them two balls are randomly drawn from the selected bag. They happen to be black and red. What is the probability that they came from bag I ? **[CBSE 2009, 6M]**

Sol. Probability of selecting each bag $= \frac{1}{3}$ **[1]**



Probability that they came from bag 1 (from Baye's Theorem)

$$= \frac{\frac{1}{3}\times\frac{{}^3C_1\times{}^1C_1}{{}^6C_2}}{\frac{1}{3}\left[\frac{{}^3C_1\times{}^1C_1}{{}^6C_2}+\frac{{}^2C_1\times{}^1C_1}{{}^7C_2}+\frac{{}^4C_1\times{}^3C_1}{{}^{12}C_2}\right]}$$ **[2]**

$$= \frac{\frac{3}{15}}{\frac{3}{15}+\frac{2}{21}+\frac{12}{66}}$$ **[2]**

$$= \frac{21\times 22}{(462+220+420)} = \frac{21\times 22}{1102} = \frac{462}{1102} = \frac{231}{551}$$ **[1]**

16. A bag contains 4 balls. Two balls are drawn at random, and are found to be white. What is the probability that all balls are white ? **[CBSE 2010, 6M]**

Sol. E : 2 balls drawn at random found white.

A_0 : 0 white ball

A_1 : 1 white ball

A_2 : 2 white balls

A_3 : 3 white balls

A_4 : 4 white balls

$P(A_i) = 1/5$ **[1]**

$$P\left(\frac{A_4}{E}\right)=\frac{P\left(\frac{E}{A_4}\right).P(A_4)}{\sum_{i=0}^{4}P\left(\frac{E}{A_i}\right).P(A_i)}$$

$P\left[\frac{E}{A_4}\right]$

[2]

$$=\frac{1}{\frac{^3C_2}{^4C_2}+\frac{^2C_2}{^4C_2}+\frac{^4C_2}{^4C_2}}=\frac{1}{\frac{1}{2}+\frac{1}{6}+1}=\frac{3}{5}$$ [3]

17. Given three identical boxes I, II and III each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold? **[CBSE 2011, 6M]**

Sol. Consider the following events :

E_1 = Box I is choosen, E_2 = Box II is choosen & E_3 = Box III is choosen.

A = The coin drawn is of gold.

We have, $P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$ [1]

Box 2

$P(A/E_1)$ = Probability of drawing a gold coin from box I.

$\Rightarrow \quad P(A/E_1)=\frac{2}{2}=1$ [1]

$P(A/E_2)$ = Probability of drawing a gold coin from box II

$\Rightarrow \quad P(A/E_2)=0$ [1]

$P(A/E_3)$ = Probability of drawing a gold coin from box III.

$\Rightarrow \quad P(A/E_3)=\frac{1}{2}$ [1]

Now, Probability that the other coin in the box is of gold

= Probability that gold coin is drawn from the box = $P(E_1/A)$

$$=\frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)+P(E_2)P(A/E_3)}$$ [1]

$$=\frac{\frac{1}{3}\times 1}{\frac{1}{3}\times 1+\frac{1}{3}\times 0+\frac{1}{3}\times\frac{1}{2}}=\frac{2}{3}$$ [1]

18. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin 3 times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2, 3 or 4 with the die. **[CBSE 2012, 6M]**

Sol. Let E_1 : '1, 2, 3 or 4 is shown on dice', and E_2 : '5 or 6 is shown on dice', then E_1 and E_2 are mutually exclusive and exhaustive. Moreover,

$$P(E_1) = \frac{4}{6} = \frac{2}{3} \quad \text{and} \quad P(E_2) = \frac{2}{6} = \frac{1}{3}$$ **[1+1]**

Let E : 'exactly one head shows up'

then $P(E/E_1)$ = P (head shows up when coin is tossed once) $= \frac{1}{2}$ **[1]**

and $P(E/E_2)$ = P (exactly one head shows up when coin is tosed thrice)

$= P(\{HTT, THT, TTH\}) = \frac{3}{8}$ **[1]**

$\therefore$ Required probability $= P(E_1/E) = \dfrac{P(E/E_1)P(E_1)}{P(E/E_1)P(E_1) + P(E/E_2)P(E_2)}$ **[1]**

$$= \frac{\frac{1}{2}\times\frac{2}{3}}{\frac{1}{2}\times\frac{2}{3}+\frac{3}{8}\times\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{8}} = \frac{8}{8+3} = \frac{8}{11}$$ **[1]**

19. In a hockey match, both teams A and B scored same number of goals up to the end of the game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, find their respective probabilities of winning the match and state whether the decision of the referee was fair or not. **[CBSE 2013, 6M]**

Sol. Let E = captain A gets a six

F = captain B gets a six

$P(E) = \frac{1}{6} \qquad P(F) = \frac{1}{6}$

$P(\bar{E}) = \frac{5}{6} \qquad P(\bar{F}) = \frac{5}{6}$ **[1]**

A wins if the throws a six in 1st or 3rd or 5th throw .... **[1]**

Wining probability in first through $= P(E) = \frac{1}{6}$

third through $= P(E')\,P(F')\,P(E) = \left(\frac{5}{6}\right)^2 \times \frac{1}{6}$

fifth throw $= P(E')\,P(F')P(E')P(F')P(E) = \left(\frac{5}{6}\right)^4 \times \frac{1}{6}$

Hence probability of wining of A

$$P(A) = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \ldots\ldots$$ **[1]**

$$P(A) = \frac{1/6}{1-(5/6)^2} = \frac{6}{11}$$ **[1]**

Probability of wining of P(B) = $1 - P(A) = 1 - \frac{6}{11} = \frac{5}{11}$ [1]

Value Based : The decision of the referece is not fair because wining probability of captain A is much than B. [1]

20. There are three coins. One is a two-heated coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-heated coin ?

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution. **[CBSE 2014, 6M]**

Sol. Let E_1 = Two headed Coin

E_2 = Biased coin that cones up heads (75%)

E_3 = Baised coin that Comes up tails (40 %)

E = Heads comes up

$$P(E_1) = P(E_2) = P(E_3) = 1/3$$

$$P\left(E/E_1\right) = 1 ; P\left(E/E_2\right) = 3/4 ; P\left(E/E_3\right) = 1 - \frac{2}{5} = \frac{3}{5}$$ [2]

By Baye's Theorom : -

$$P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \times P\left(E/E_1\right)}{P(E_1) \times P\left(E/E_1\right) + P(E_2) \times P\left(E/E_2\right) + P(E_3) \times P\left(E/E_3\right)}$$ [2]

By Baye's Theorem

$$= \frac{1/3 \times 1}{\left(1/3 \times 1\right) + \left(1/3 \times 3/4\right) + \left(1/3 \times 3/5\right)} = \frac{20}{47}$$ [2]

OR

S = {1, 2, 3, 4, 5, 6}

X denote the larger of the two numbers

if X = 2 then favourable cases are $\{(1,2)(2,1)\}$

if X = 3 then favourable cases are $\{(1,3)(2,3)(3,1)(3,2)\}$

if X = 4 then favourable cases are $\{(1,4)(2,4)(3,4)(4,1)(4,2)(4,3)\}$

if X = 5 then favourable cases are $\{(1,5)(2,5)(3,5)(4,5)(5,1)(5,2)(5,3)(5,4)\}$

if X = 6 then favourable cases are $\{(1,6)(2,6)(3,6)(4,6)(5,6)(6,1)(6,2)(6,3)(6,4)(6,5)\}$ [2]

X	P(X)	X.P(X)
2	$\frac{2}{30}$	$\frac{4}{30}$
3	$\frac{4}{30}$	$\frac{12}{30}$
4	$\frac{6}{30}$	$\frac{24}{30}$
5	$\frac{8}{30}$	$\frac{40}{30}$
6	$\frac{10}{30}$	$\frac{60}{30}$

[2]

Mean = $\Sigma X.\ P(X)$

$$= \frac{4+12+24+40+60}{30} = \frac{140}{30} = \frac{14}{3}$$ [2]

21. An urn contains 5 red and 2 black balls. Two balls are randomly drawn, without replacement. Let X represent the number of black balls drawn. What are the possible values of X ? Is X a random variable ? If yes, find the mean and variance of X. **[CBSE 2015, 6M]**

Sol. X = number of black balls drawn

X = 0, 1, 2 [1]

x	P(x)	x.P(x)	x^2.P(x)
0	$\frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{20}{42}$	0	0
1	$\frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$

[2]

$\because \quad \Sigma p(x) = 1$

So x is a random variable [1]

$$\text{mean} = \Sigma x.p(x) = 0 + \frac{10}{21} + \frac{2}{21} = \frac{12}{21} = \frac{4}{7}$$ [1]

$$\text{Var}(x) = \Sigma x^2.\ p(x) - \left[\sum xP(x)\right]^2$$

$$= 0 + \frac{10}{21} + \frac{4}{21} - \left(\frac{4}{7}\right)^2 = \frac{14}{21} - \frac{16}{49} = \frac{50}{147}$$ [1]

22. Three numbers are selected at random (without replacement) from first six positive integers. Let X denotes the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution. **[CBSE 2016, 6M]**

Sol. $S = \{1, 2, 3, 4, 5, 6\}$ [2]

$X = \{\text{largest of the three numbers}\} = \{3, 4, 5, 6\}$

X	Out comes	P(x)	x.P(x)	x^2.P(x)
3	(3,1,2)	1/20	3/20	9/20
4	(4,1,2),(4,2,3),(4,1,3)	3/20	12/20	48/20
5	(5,1,2),(5,2,3),(5,3,4),(5,1,3),(5,1,4),(5,2,4)	6/20	30/20	150/20
6	(6,1,2),(6,2,3),(6,3,4),(6,4,5),(6,1,3),(6,1,4),(6,1,5),(6,2,4),(6,2,5),(6,3,5)	10/20	60/20	360/20

[2]

Mean $E(x) = \Sigma x.P(x)$

3 4 5 6

$= \frac{105}{20} = \frac{21}{4}$ [1]

$Var.(X) = \Sigma x^2.P(x) - [E(x)]^2$

$= \frac{567}{20} - \left(\frac{105}{20}\right)^2 = \frac{63}{80}$ [1]

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10/20

30/20

50

30

Amanul

Zero