# MISCELLANEOUS EXAMPLES

**Very Short Answer : [1 Mark]**

**1.** For what value of x, is following matrix singular ? **[CBSE 2008, 1M]**

$$\begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$$

**Sol.** Let $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$

$\because$ Matrix A is singular $\Rightarrow$ $|A| = 0$

$$\begin{vmatrix} 3-2x & x+1 \\ 2 & 4 \end{vmatrix} = 0$$

$(12 - 8x) - (2x + 2) = 0$

$-10x + 10 = 0$

$-10x = -10 \Rightarrow x = 1$ **[1]**

**2.** A matrix A, of order 3 × 3, has determinant value 4. Find the value of |3A|. **[CBSE 2008, 1M]**

**Sol.** Here $|A| = 4$

For 3 × 3 matrix

$|3A| = 3^3|A|$

$= 27|A| = 27 \times 4 = 108$ **[1]**

**3.** Find the value of x, if $\begin{pmatrix} 3x+y & -y \\ 2y-x & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -5 & 3 \end{pmatrix}$ **[CBSE 2009, 1M]**

**Sol.** $3x + y = 1$ & $y = -2$

$3x - 2 = 1 \Rightarrow x = 1$

$x = 1, y = -2$ **[1]**

**4.** Write the adjoint of the following matrix : $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$ **[CBSE 2010, 1M]**

**Sol.** $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

$\text{Adj}A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$ **[1]**

**5.** A is a square matrix of order 3 and |A| = 7. Write the value of |adj. A|. **[CBSE 2010, 1M]**

**Sol.** $|A| = 7$

$|\text{Adj}A| = |A|^{n-1} = (7)^2 = 49$ **[1]**

**6.** If a matrix has 5 elements, write all possible orders it can have. **[CBSE 2011, 1M]**

**Sol.** Only two possible order $1 \times 5$ or $5 \times 1$ **[1]**

**7.** If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write $A^{-1}$ in terms of A. **[CBSE 2011, 1M]**

**Sol.** $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -19 \neq 0$

$|Adj\, A| = |A|^{n-1}$

$$\text{adj}A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(A)}\,\text{adj}A = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{19}A$$ **[1]**

**8.** Find the value of x + y from the following equation :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$ **[CBSE 2012 1M]**

**Sol.** Given $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$2x + 3 = 7 \quad \Rightarrow \quad x = 2$

and $2(y - 3) + 2 = 14$

$\Rightarrow y - 3 + 1 = 7 \Rightarrow y = 9$

$x + y = 2 + 9 = 11$ **[1]**

**9.** If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$ **[CBSE 2012 1M]**

**Sol.** $B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$ **[1]**

**10.** For what value of x, is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew-symmetric matrix ? **[CBSE 2013 1M]**

**Sol.** $x = 2 \qquad [A' = -A]$ **[1]**

11. If matrix $A = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$ and $A^2 = \lambda A$, then write the value of $\lambda$. **[CBSE 2013 1M]**

**Sol.** $A^2 = \lambda A$

$A.A = \lambda A$

$\begin{bmatrix} 18 & -18 \\ -18 & 18 \end{bmatrix} = \lambda \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix}$

$18 = 3\lambda \quad \Rightarrow \quad \lambda = 6$ **[1]**

12. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an idntity matrix. **[CBSE 2014, 1M]**

**Sol.** $7A - (I + A)^3$

$7A - [(I + A)^2(I + A)] = 7A - [I^2 + A^2 + 2AI] [I + A]$

$= 7A - [I + A + 2A] [I + A]$

$= 7A - [I + 3A] [I + A]$

$= 7A - [I\,I + IA + 3AI + 3A^2]$

$= 7A - [I + A + 3A + 3A] \quad (\because A^2 = A)$

$= 7A - [I + 7A]$

$= -I$ **[1]**

13. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find the value of x + y. **[CBSE 2014, 1M]**

**Sol.** $x - y = 1 \quad$ ...(i)

$2x - y = 0 \Rightarrow y = 2x$

$x - 2x = -1 \quad$ from Eq. (i)

$-x = -1 \Rightarrow x = 1$ & $y = 2$

then $x + y = 1 + 2 = 3$ **[1]**

14. If $\begin{vmatrix} 3x & 7 \\ -2 & 4 \end{vmatrix} = \begin{vmatrix} 8 & 7 \\ 6 & 4 \end{vmatrix}$, find the value of x. **[CBSE 2014, 1M]**

**Sol.** $12x + 14 = 32 - 42$

$12x = -24$

$x = -2$ **[1]**

15. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of Det($A^n$). **[CBSE 2015, 1M]**

**Sol.** $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$|A^n| = \cos^2 n\theta + \sin^2 n\theta = 1$ **[1]**

**16.** If $A=\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}$, find α satisfying $0<\alpha<\frac{\pi}{2}$ when $A+A^T=\sqrt{2}I_2$; where $A^T$ is transpose of A. **[CBSE 2016, 1M]**

**Sol.** $A+A^T=\sqrt{2}I_2$

$\Rightarrow \begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}+\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}=\sqrt{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ **[½]**

$\Rightarrow \begin{pmatrix}2\cos\alpha & 0\\ 0 & 2\cos\alpha\end{pmatrix}=\begin{pmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{pmatrix}$

$\because\ 2\cos\alpha=\sqrt{2}$

$\cos\alpha=\frac{1}{\sqrt{2}}$

$\alpha=\frac{\pi}{4}\in\left(0,\frac{\pi}{2}\right)$ **[½]**

**Short Answer : [4 Marks]**

**17.** If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=0$. Hence find $A^{-1}$. **[CBSE 2007, 4M]**

**Sol.** $A^2=A\cdot A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$

$A^2-5A+7I=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}+\begin{bmatrix}-15 & -5\\ 5 & -10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=O$ **[2]**

Now $A^2-5A+7I=0$

$\Rightarrow AA-5A+7I=0$

$\Rightarrow AAA^{-1}-5AA^{-1}+7IA^{-1}=0$

$\Rightarrow AI-5I+7A^{-1}=0$

$\Rightarrow 7A^{-1}=5I-A \Rightarrow A^{-1}=\frac{1}{7}(5I-A)$

$\Rightarrow A^{-1}=\frac{1}{7}\left(5\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}-\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}\right)=\frac{1}{7}\begin{pmatrix}2 & -1\\ 1 & 3\end{pmatrix}$ **[2]**

**18.** If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$, find the value of λ so that $A^2=\lambda A-2I$. Hence find $A^{-1}$. **[CBSE 2007, 4M]**

**Sol.** $A^2=A.A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}$

$\because\ A^2=\lambda A-2I$

$$\begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}=\lambda\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}-2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & -2\\4 & -2\end{bmatrix}=\begin{bmatrix}3\lambda-2 & -2\lambda\\4\lambda & -2\lambda-2\end{bmatrix}$$

$\therefore \quad 4\lambda = 4 \Rightarrow \lambda = 1$

$\therefore \quad A^2 = 1.A - 2I$

$A^2 = A - 2I$

$AA.A^{-1} = A.A^{-1} - 2IA^{-1}$ [$\because$ $AA^{-1} = I$, $IA = A$]

$AI = I - 2A^{-1}$

$2A^{-1} = I - AI \Rightarrow 2A^{-1} = I - A$

$$2A^{-1}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}-\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}=\begin{bmatrix}-2 & 2\\-4 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{2}\begin{bmatrix}-2 & 2\\-4 & 3\end{bmatrix}$$

**19.** Express the following matrix as the sum of a symmetric and skew symmetric matrix, and verify yo

result : $\begin{pmatrix}3 & -2 & -4\\3 & -2 & -5\\-1 & 1 & 2\end{pmatrix}$ **[CBSE 2010, 4M**

**Sol.** Let $A=\begin{bmatrix}3 & -2 & -4\\3 & -2 & -5\\-1 & 1 & 2\end{bmatrix}$

$$A=\frac{1}{2}[A+A']+\frac{1}{2}[A-A']$$ [1

$$A'=\begin{bmatrix}3 & 3 & -1\\-2 & -2 & 1\\-4 & -5 & 2\end{bmatrix}$$

$$A+A'=\begin{bmatrix}6 & 1 & -5\\1 & -4 & -4\\-5 & -4 & 4\end{bmatrix} \;\&\; A-A'=\begin{bmatrix}0 & -5 & -3\\5 & 0 & -6\\3 & 6 & 0\end{bmatrix}$$ [1]

$$\text{Hence } P=\frac{1}{2}\begin{bmatrix}6 & 1 & -5\\1 & -4 & -4\\-5 & -4 & 4\end{bmatrix}=\begin{bmatrix}3 & \frac{1}{2} & -\frac{5}{2}\\\frac{1}{2} & -2 & -2\\-\frac{5}{2} & -2 & 2\end{bmatrix}$$ [1]

$$Q=\frac{1}{2}\begin{bmatrix}0 & -5 & -3\\ 5 & 0 & -6\\ 3 & 6 & 0\end{bmatrix}=\begin{bmatrix}0 & -\frac{5}{2} & -\frac{3}{2}\\ \frac{5}{2} & 0 & -3\\ \frac{3}{2} & 3 & 0\end{bmatrix}$$ [1]

$\because$ P' = P(symmetric) & Q' = –Q(skew symmetric)

A = P + Q Hence proved.

**20.** For the following matrices A and B, verify that (AB)' = B'A'.

$$A=\begin{pmatrix}1\\ -4\\ 3\end{pmatrix},\ B=(-1\quad 2\quad 1)$$ **[CBSE 2010, 4M]**

**Sol.** $$AB=\begin{pmatrix}1\\ -4\\ 3\end{pmatrix}_{3\times1}(-1\quad 2\quad 1)_{1\times3}=\begin{bmatrix}-1 & 2 & 1\\ 4 & -8 & -4\\ -3 & 6 & 3\end{bmatrix}$$

$$(AB)'=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$$ [2]

$$B'=\begin{pmatrix}-1\\ 2\\ 1\end{pmatrix},\ A'=(1\quad -4\quad 3)$$

$$B'A'=\begin{pmatrix}-1\\ 2\\ 1\end{pmatrix}(1\quad -4\quad 3)=\begin{bmatrix}-1 & 4 & -3\\ 2 & -8 & 6\\ 1 & -4 & 3\end{bmatrix}$$ [2]

**21.** There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. Then recommended daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using matrices. Using matrix multiplication, calculate the total requirement of calories and proteins for each of the 2 families. What awareness can you create among people about the balanced diet from this question ? **[CBSE 2015, 4M]**

**Sol.** The numbers of the two families can be represented by the 2 × 3 matrix.

$$F=\begin{matrix} & M & W & C\\ A & 4 & 6 & 2\\ B & 2 & 2 & 4\end{matrix}$$ [1]

and the recommended daily allowance of calories and protein for each member can be represented by 3 × 2 matrix

$$R=\begin{matrix} & \text{Calories} & \text{Proteins}\\ M & 2400 & 45\\ W & 1900 & 55\\ C & 1800 & 33\end{matrix}$$ [1]

The total requirement of calories and proteins for each of the two families is given by the matrix multiplication :

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} = \begin{matrix} A \\ B \end{matrix} \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$$ [2]

**Value Based :** Balanced diet gives best health.

**22.** Using elementary row operations (transformations), find the inverse of the following matrix :

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{pmatrix}$$

**[CBSE 2015, 4M]**

**OR**

If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, then calculate AC, BC and (A + B)C. Also verify that (A + B)C = AC + BC.

**[CBSE 2015, 4M]**

**Sol.** Let $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

We know that A = IA

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ [½]

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -9 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$ [½]

$R_1 \to R_1 - 2R_2$ and $R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$ [1]

$R_2 \to R_2 - 2R_3$ and $R_1 \to R_1 + R_3$

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}3&-2&1\\-9&6&-2\\5&-3&1\end{bmatrix}A$$ [1]

Hence $A^{-1}=\begin{bmatrix}3&-2&1\\-9&6&-2\\5&-3&1\end{bmatrix}$ [½]

**OR**

$$AC=\begin{bmatrix}0&6&7\\-6&0&8\\7&-8&0\end{bmatrix}\begin{bmatrix}2\\-2\\3\end{bmatrix}$$

$$AC\Rightarrow\begin{bmatrix}0-12+21\\-12+0+24\\14+16+0\end{bmatrix}\Rightarrow\begin{bmatrix}9\\12\\30\end{bmatrix}$$ .........(i) [1]

$$BC=\begin{bmatrix}0&1&1\\1&0&2\\1&2&0\end{bmatrix}\begin{bmatrix}2\\-2\\3\end{bmatrix}$$

$$BC\Rightarrow\begin{bmatrix}0-2+3\\2+0+6\\2-4+0\end{bmatrix}\Rightarrow\begin{bmatrix}1\\8\\-2\end{bmatrix}$$ ..........(ii) [1]

$$(A+B)C=\left\{\begin{bmatrix}0&6&7\\-6&0&8\\7&-8&0\end{bmatrix}+\begin{bmatrix}0&1&1\\1&0&2\\1&2&0\end{bmatrix}\right\}\begin{bmatrix}2\\-2\\3\end{bmatrix}$$

$$=\begin{bmatrix}0&7&8\\-5&0&10\\8&-6&0\end{bmatrix}\begin{bmatrix}2\\-2\\3\end{bmatrix}$$

$$=\begin{bmatrix}0-14+24\\-10+0+30\\16+12+0\end{bmatrix}=\begin{bmatrix}10\\20\\28\end{bmatrix}$$ .........(iii) [1]

Now LHS $=(A+B)C=\begin{bmatrix}10\\20\\28\end{bmatrix}$ from equation (iii)

$$RHS = AC + BC = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$ from equation (i) and (ii)

$$= \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$

Hence (A +B) . C = AC + BC [1]

**23.** A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only ₹ 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

**[CBSE 2016, 4M]**

**Sol.** Let x and y be the charges of one Eng. and one Hindi page respectively.

$10x + 3y = 145$

$3x + 10y = 180$

matrix form, $\begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 145 \\ 180 \end{bmatrix}$ [1½]

Let A X = B

$X = A^{-1}.B$

$$A^{-1} = \frac{adjA}{|A|}$$

$$|A| = 91 \text{ and } adjA = \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}$$

$$X = \frac{1}{91}\begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix}\begin{bmatrix} 145 \\ 180 \end{bmatrix}$$

$$X = \frac{1}{91}\begin{bmatrix} 910 \\ 1365 \end{bmatrix}$$

$$X = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$ [1½]

x = Rs. 10, y = Rs. 15

Typist charged from poor student = 2 × 5 = Rs.10

actual price is 15 × 5 = Rs. 75

The poor boy was charged = 75 – 10 = Rs. 65

always helps poor pepule. [1]

## Long Answer : [6 Marks]

**24.** Using matrix method solve the following system of linear equations : **[CBSE 2005, 07, 2011 6M]**

$x + 2y + z = 7$

$x + 3z = 11$

$2x - 3y = 1$

**Sol.** We can write the given eqs. as

$AX = B$ .....(1)

Where $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ; $B = \begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$

$= 1(0 + 9) - 2(0 - 6) + 1(-3 - 0)$

$= 9 + 12 - 3 = 18 \neq 0$

$\Rightarrow A^{-1}$ exists and it is given by **[1]**

$A^{-1} = \frac{1}{|A|}$ adj. $A = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}$ **[2]**

From eq. (1)

$X = A^{-1} B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 9 & -3 & 6 \\ 6 & -2 & -2 \\ -3 & 7 & -2 \end{bmatrix}\begin{bmatrix} 7 \\ 11 \\ 1 \end{bmatrix} = \frac{1}{18}\begin{bmatrix} 36 \\ 18 \\ 54 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ **[2]**

$\Rightarrow x = 2, y = 1, z = 3.$ **[1]**

**25.** Using elementary transformations, find the inverse of the following matrix : $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

**[CBSE 2008, 6M]**

**Sol.** Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

In order to use elementary row transformation, we may write A = IA

$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ **[1]**

Applying $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$

[1½]

Applying $R_2 \to R_2 - R_3$, $R_1 \to R_1 - 3R_3$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & -3 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

[1½]

Applying $R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A$$

[1]

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

[1]

**26.** Using matrices, solve the following system of equations : **[CBSE 2009, 6M]**

$x + y + z = 6$

$x + 2z = 7$

$3x + y + z = 12$

**OR**

Obtain the inverse of the following matrix using elementary operations : $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

**Sol.** Given equation can be written as $AX = B \Rightarrow X = A^{-1}B$

where $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 2 \end{vmatrix} = 4 \neq 0 \quad \Rightarrow A^{-1} \text{ exist.}$$

[1]

$$A^{-1} = \frac{1}{(A)} \text{adj } A = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

[2]

$$X = A^{-1}B = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix}$$

[2]

$x = 3, y = 1, z = 2$

[1]

**OR**

$A = IA$

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ [1]

$C_1 \to C_1 + 2C_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A$$ [1]

$C_3 \to C_3 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{bmatrix} A$$ [1]

$C_1 \to C_1 - C_3, \quad C_2 \to C_2 - C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & -3 & 3 \end{bmatrix} A$$ [1]

$C_3 \to C_3 - 2C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 3 \\ 0 & 1 & -2 \\ -1 & -3 & 9 \end{bmatrix} A$$ [1]

$C_1 \to C_1 + C_3, \; C_2 \to C_2 - C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ -8 & -12 & 9 \end{bmatrix} A$$ [1]

**27.** Find the inverse of the following matrix using elementary operations : $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

**[CBSE 2010, 6M]**

**Sol.** $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

$AI = A$ [1]

$$A\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}1&2&-2\\-1&3&0\\0&-2&1\end{bmatrix}$$ [1]

$C_2 \to C_2 + C_3$

$$A\begin{bmatrix}1&0&0\\0&1&0\\0&1&1\end{bmatrix}=\begin{bmatrix}1&0&-2\\-1&3&0\\0&-1&1\end{bmatrix}$$

$C_3 \to C_3 + 2C_1$ [1½]

$$A\begin{bmatrix}1&0&2\\0&1&0\\0&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&3&-2\\0&-1&1\end{bmatrix}$$

$C_2 \to C_2 + C_3$ [1]

$$A\begin{bmatrix}1&2&2\\0&1&0\\0&2&1\end{bmatrix}=\begin{bmatrix}1&0&0\\-1&1&-2\\0&0&1\end{bmatrix}$$

$C_1 \to C_1 + C_2$, $C_3 \to C_3 + 2C_2$

$$A\begin{bmatrix}3&2&6\\1&1&2\\2&2&5\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Hence $A^{-1}=\begin{bmatrix}3&2&6\\1&1&2\\2&2&5\end{bmatrix}$ [1½]

**28.** Using matrices, solve the following system of equation :

$2x + 3y + 3z = 5,\ x - 2y + z = -4,\ 3x - y - 2z = 3$ **[CBSE 2012 6M]**

**Sol.** Given system of equations $2x + 3y + 3z = 5$, $x - 2y + z = -4$ and $3x - y - 2z = 3$

The given system can be written as AX = B,

where $A=\begin{vmatrix}2&3&3\\1&-2&1\\3&-1&-2\end{vmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$

and $B=\begin{bmatrix}5\\-4\\3\end{bmatrix}$ [1]

Here $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$

$= 2(4+1) - 3(-2-3) + 3(-1+6)$

$= 10 + 15 + 15 = 40 \neq 0$ [1]

therefore, the given system is consistent and has a unique solution given by

$X = A^{-1}B = \left\{\frac{1}{|A|}(\text{adj}A)\right\}B,\quad \text{adj } A = \begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}^T = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$ [2]

$X = \frac{1}{40}\begin{bmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{bmatrix}^T \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix}$ [1]

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow x = 1, y = 2, z = -1.$ [1]

29. The management committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers so keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method, find the number of awardees of each category. Apart from these values, namely, honesty, cooperation and supervision, suggest one more value of which the management of the colony must include for awards. **[CBSE 2013 6M]**

**Sol.** x = awarded members for honesty

y = awarded members for helping (cooperation)

z = awarded members for supervision.

Sum of all the awarders is 12

so $x + y + z = 12$ .........(i)

Three times the sum of awardess for y and z.

added to two times the x is 33

$3(y + z) + 2x = 33$

$2x + 3y + 3z = 33$ .........(ii)

The sum of number of for x and z is twice the y

$x + z = 2y$

$x - 2y + z = 0$ ........(iii)

(for any two correct) [1½]

above all three equation can be written as matrix form

$$\begin{bmatrix}1 & 1 & 1\\2 & 3 & 3\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}12\\33\\0\end{bmatrix}$$

Let A X = B

$$X = A^{-1}.B \Rightarrow \frac{\text{adj}A}{|A|}.B$$

$$|A| = 1(3+6) - 1(2-3) + 1(-4-3)$$
$$= 9 + 1 - 7 = 3$$

$$\text{adj}A = [C_{ij}]^T$$

$$=\begin{bmatrix}9 & 1 & -7\\-3 & 0 & 3\\0 & -1 & 1\end{bmatrix}^T=\begin{bmatrix}9 & -3 & 0\\1 & 0 & -1\\-7 & 3 & 1\end{bmatrix}$$ [½]

[½]

$$[X]=\frac{1}{3}\begin{bmatrix}9 & -3 & 0\\1 & 0 & -1\\-7 & 3 & 1\end{bmatrix}\begin{bmatrix}12\\33\\0\end{bmatrix}=\frac{1}{3}\begin{bmatrix}108 & -99 & +0\\12 & +0 & +0\\-84 & +99 & +0\end{bmatrix}=\frac{1}{3}\begin{bmatrix}9\\12\\15\end{bmatrix}$$ [1]

$$\begin{bmatrix}x\\y\\z\end{bmatrix}\Rightarrow\begin{bmatrix}3\\4\\5\end{bmatrix} \Rightarrow x = 3, y = 4, z = 5$$ [1]

Value Based : Those who keep their surrounding clean

[Any other individual response with suitable justification be accepted, even if there is no reference to the text]. [1]

**30.** Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs. x each, Rs. y each and Rs. z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs. 1,600. school B wants to spend Rs. 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs. 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award. **[CBSE 2014 6M]**

**Sol.** According to statement

$$3x + 2y + z = 1600$$
$$4x + y + 3z = 2300$$
$$x + y + z = 900$$ [1½]

We can represent the given equation in matrix multiplication as

$$\begin{bmatrix}3 & 2 & 1\\4 & 1 & 3\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1600\\2300\\900\end{bmatrix}$$

Let $AX = B$

$X = A^{-1}.B$

$\because\ A^{-1} = \dfrac{\text{adj}A}{|A|}$

$|A| = 3(-2) - 2(1) + 1(3) = -5$ [1]

$\text{adj}A = [c_{ij}]^T$

$$\Rightarrow \quad \text{adj}A = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$ [1½]

$$\therefore \quad X = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}\begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$ [1]

Value Based : Award can also be given for punctuality.

[Any other individual response with suitable justification be accepted, even if there is no reference to the text]. [1]

| NCERT IMPORTANT QUESTIONS | |
|---|---|
| Example | 3, 10, 18, 22, 24 |
| Exercise # 3.1 | 5 (i), 9, 10 |
| Exercise # 3.2 | 2 (i), 10, 15, 18 |
| Exercise # 3.3 | 9, 11 |
| Miscellaneous Exercise | 7, 13 |