1. A dealer wishes to purchase a number of fans and radios. He has only Rs. 5,760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a radio Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and a radio at a profit of Rs. 18. Assuming that he can sell all the items he buys, how should he invest his money for maximum profit? Translate the problem as LPP and solve it graphically. **[CBSE 2007,09, 6M]**

Sol. Let number of fans = x

and number of radios = y

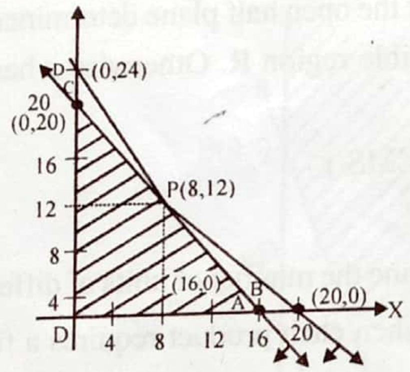
Maximum profit $z = 22x + 18y$

Subject to constraints

$x + y \leq 20$

$360x + 240y \leq 5,760$ **[2]**

Now plot the straight lines on the graph and find the corner points of feasible region.



[2]

∴ Corner points of feasible region are

A(16,0), P(8,12) and C(0,20)

Corner points	$z = 22x + 18y$
A(16,0)	$z = 352 + 0 = 352$
P(8,12)	$z = 176 + 216 = 392$
C(0,20)	$z = 0 + 360 = 360$

∴ Max profit z = Rs. 392 at x = 8, y = 12. **[2]**

2. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost, but it must satisfy the requirements of the sick person. Form the equation as LPP and solve it graphically.

[CBSE 2008, 6M]

Sol. Let food A = x unit and food B = y unit

We make the following table from the given data :

Food	Vitamins	Minerals	Calories	cost
A(x)	200	1	40	5
B(y)	100	2	40	4

Required L.P.P. is

Minimum Cost $Z = 5x + 4y$

subject to constraints

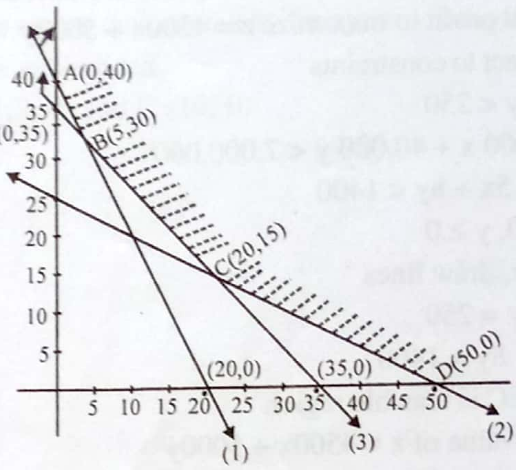
$200x + 100y \geq 4000$

$x + 2y \geq 50$

$40x + 40y \geq 1400$

$x, y \geq 0$

Now draw the graph



[2]

[2]

$\therefore$ Corner points of feasible region are

A(0, 40), B(5, 30), C(20, 15), D(50, 0)

Now evaluate Z at the corner points

corner point	$z = 5x + 4y$
A(0,40)	$Z = 0 + 160 = 160$
B(5,30)	$Z = 25 + 120 = 145$
C(20,15)	$Z = 100 + 60 = 160$
D(50,0)	$z = 250 + 0 = 250$

$\therefore$ Least cost = 145 at x = 5, y = 30 **[2]**

3. One kind of cake requires 300 g of flour and 15 g of fat, another kind of cake requires 150 g of flour and 30 g of fat. Find the maximum number of cakes which can be made from 7.5 kg of flour and 600 g of fat, assuming that there is no shortage of the other ingredients used in making the cakes. Make it as an L.P.P. and solve it graphically. **[CBSE 2010, 6M]**

Sol. Let the number of cakes of one kind be x

the number of cakes of 2^{nd} kind be y

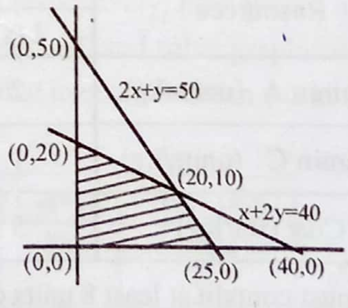
then max $Z = x + y$ (objective function)

s.t. $300 \times 10^{-3}x + 150 \times 10^{-3}y \leq 7.5$

$15 \times 10^{-3}x + 30 \times 10^{-3}y \leq 600 \times 10^{-3}$

$\Rightarrow 2x + y \leq 50$ ....(i)

$x + 2y \leq 40$ ....(ii)



[2]

Corner point	$z = x + y$
A(25,0)	25
B(20,10)	30
C(0,20)	20

[2]

Z is maximum at (20,10)

Hence Z max. = 30 **[2]**

4. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs. 25,000 and Rs. 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs. 70 lakhs and his profit on the desktop model is Rs. 4,500 and on the portable model is Rs. 5,000. Make an L.P.P. and solve it graphically. **[CBSE 2011, 6M]**

Sol. Let merchant stock of desktop model x unit & portable model y units.

Total profit to maximize $z = 4500x + 5000y$

subject to constraints

$x + y < 250$

$25{,}000\,x + 40{,}000\,y < 7{,}000{,}000$

$\Rightarrow \quad 5x + 8y < 1400$

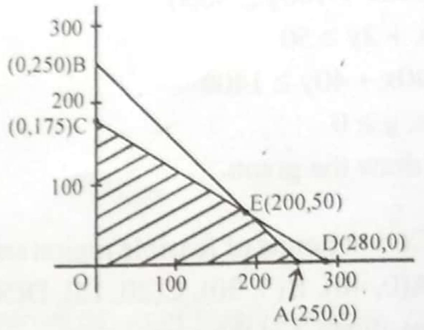
$x \geq 0, y \geq 0$

Now, draw lines

$x + y = 250$

$5x + 8y = 1400$

OAEC is feasible region



[2]

[2]

The value of $z = 4500x + 5000y$

at A $z = 4500 \times 250 + 5000 \times 0 = 1{,}125{,}000$

at E $z = 4500 \times 200 + 5000 \times 50 = 1{,}150{,}000$

at C $z = 4500 \times 0 + 5000 \times 175 = 875{,}000$

at O $z = 0$

Clearly maximum profit is 1,150,000 at E(200,50) [2]

i.e. when 200 desktop & 50 portable model is in stock

5. A dietician wishes to mix two types of foods in such a way that the vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while Food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 5 per kg to purchase Food I and ₹ 7 per kg to purchase Food II. Determine the minumum cost of such a mixture. Formulate the above as a LPP and solve it graphically. **[CBSE 2012, 6M]**

Sol. Let the mixture contain x kg of food 'I' and y kg of Food 'II'. Clearly, $x \geq 0$, $y \geq 0$, We make the following table for the given data :

Resources	Food		Requirement
	I (x)	II (y)	
Vitamin A (units/kg)	2	1	8
Vitamin C (units/kg)	1	2	10
Cost (Rs/Kg)	5	7	–

Since the mixture must contain at least 8 units of vitamin A and 10 units of vitamin C, we have the constraints :

$2x + y \geq 8$

$x + 2y \geq 10$

Total cost Z of purchasing x kg of food 'I' and y kg of Food 'II' is

$Z = 5x + 7y$

Hence, the mathematical formulation of the problem is :

Minismise $Z = 5x + 7y$

Subject to the constraints :

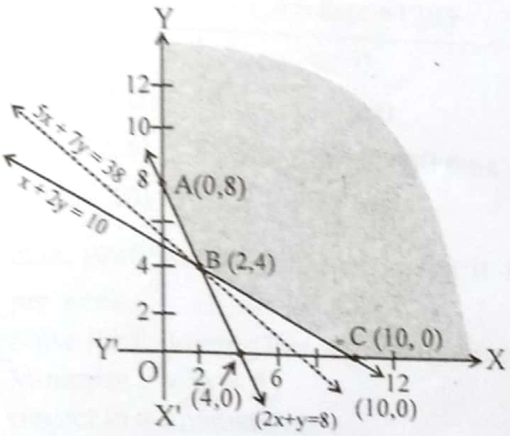
$2x + y \geq 8$ ... (2)

$x + 2y \geq 10$ ... (3)

$x, y \geq 0$ ... (4) **[2]**

Let us graph the inequalities (2) to (4). The feasible region determined by the system is shown in the fig. Here again, observe that the feasible region is unbounded.

Let us evaluate Z at the corner points A (0, 8), B(2, 4) and C (10, 0)



[2]

Corner Point	$Z = 5x + 7y$
(0, 8)	56
(2, 4)	38 ← Minimum
(10, 0)	50

[2]

Thus, the minimum value of Z is 38 attained at the point (2, 4). Hence, the optimal mixing strategy for the dietician would be to mix 2 kg of Food 'I' and 4 kg of Food 'II', and with this strategy, the minium cost of the mixture will be Rs 38.

6. A manufacturer considers that men and women workers are equally efficient and so he pays them at the same rate. He has 30 and 17 units of workers (male and female) and capital respectively, which he uses to produce two types of goods A and B. To produce one unit of A, 2 workers and 3 units of capital are required while 3 workers and 1 unit of capital is required to produce one unit of B. If A and B are priced at ₹ 100 and ₹ 120 per unit respectively, how should he use his resources to maximise the total revenue ? From the above as an LPP and solve graphically.

Do you agree with this view of the manufacturer that men and women workers are equally efficient and so should be paid at the same rate ? **[CBSE 2013, 6M]**

Sol.

Type of goods	Quantity of goods	Workers	Capital	Prices (Rs.)
A	x	2x	3x	100x
B	y	3y	1y	120y

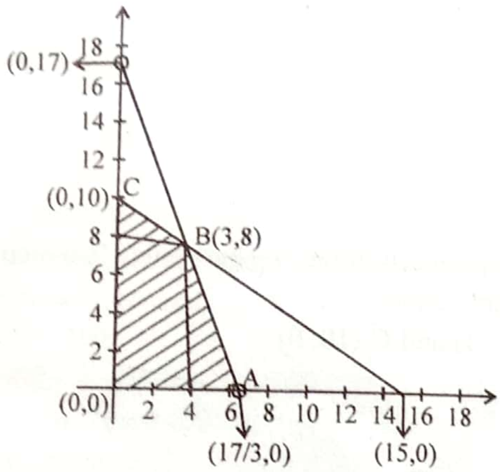
Mathematical form of L.P.P. is

(max) $z = 100x + 120y$

subject to $2x + 3y \leq 30$

$3x + y \leq 17$

$x \geq 0; y \geq 0$ **[2]**



[1½]

corner point	z = 100x + 120y
0(0, 0)	0
A(17/3, 0)	566.10
B(3, 8)	1260
C(0, 10)	1200

[½]

For maximise the total revenue

Produced type of goods A and B respectively 3, 8 [1]

Yes men & women are equally efficient.

[Any other individual response with suitable justification be accepted, even if there is no reference to the text]. [1]

7. A manufacturing company makes two types of teaching aids A and B of Mathematics for class XII. Each type of A requires 9 labour hours of fabricating and 1 labour hour for finishing. Each type of B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available per week are 180 and 30 respectively. The company makes a profit of ₹ 80 on each piece of type A and ₹ 120 on each piece of type B. How many pieces of type A and Type B should be manufactured per week to get a maximum profit ? Make it as an LPP and solve graphically. What is the maximum profit per week? **[CBSE 2014, 6M]**

Sol.

Type	Let Quantity	Fabricating (hrs)	Finishing (hrs)	Profit (Rs.)
A	x	9	1	80
B	y	12	3	120
		≤ 180	≤ 30	

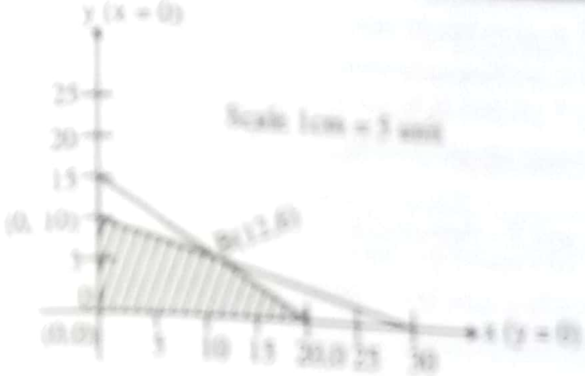
Mathematical form of L.P.P. is

Objective $= 80x + 120y$

Subject to : $9x + 12y \le 180$

$x + 3y \le 30$

$x \ge 0 \; ; \; y \ge 0$ [2]



[2]

Corner point	Z = 80x + 120y
O (0,0)	0
A (20, 0)	1600
B (12,6)	960 + 720 = 1680 max.
C (0,10)	1200

max. profit is Rs. 1680 per week is 12 piece of type A. and 6 piece of type B manufactured per week.

[2]

8. Solve the following linear programming problem graphically.

Minimise $z = 3x + 5y$

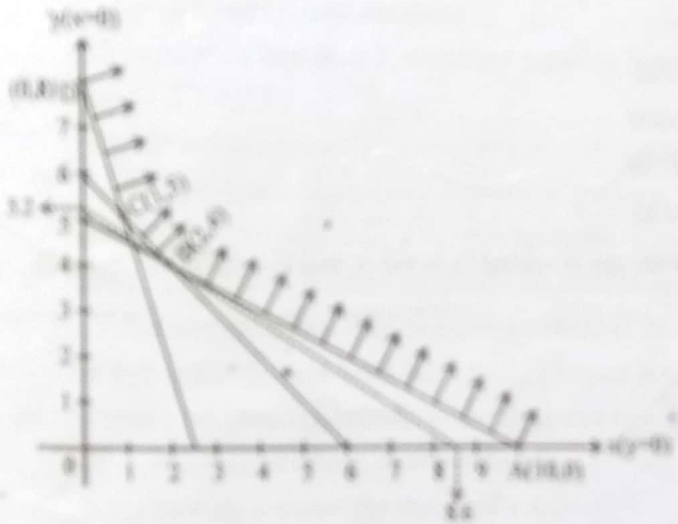
subject to the constraints

$$x + 2y \geq 10$$
$$x + y \geq 6$$
$$3x + y \geq 8$$
$$x, y \geq 0.$$

[CBSE 2015, 6M]

Sol.

[3]



coner point	z = 3x + 5y
A(10,0)	30
B(2,4)	6 + 20 = 26
C(1,5)	3 + 25 = 28
D(0,8)	40

[2]

$3x + 5y < 26$ has no common point in feasible region

Hence min. value of $z = 26$ at B(2, 4)

[1]

9. A retired person wants to invest an amount of ₹ 50,000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹ 20,000 in bond 'A' and at least ₹ 10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns. **[CBSE 2016, 6M]**

Sol. Let x and y be the invested amount in bond A and B respectively

$$z = \frac{10x}{100} + \frac{9y}{100}$$

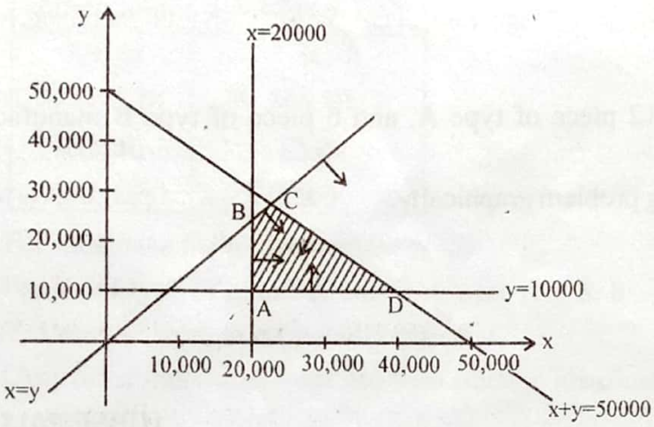
(Max) $z = .1x + .09y$ [1]

Sub to $x \geq 20{,}000$ $x + y \leq 50{,}000$

$y \geq 10{,}000$

$x \geq y$

$x \geq 0, y \geq 0$ [2]



[2]

C.point	$z = .1x + .09y$
A(20,000,10,000)	$2000 + 900 = 2900$
B(20,000,20,000)	$2000 + 1800 = 3800$
C(25,000,25,000)	$2500 + 2250 = 4750$
D(40,000,10,000)	$4000 + 900 = 4900$

Max. return is Rs. 4900, when Rs. 40000 are invested in bond A and Rs. 10000 in bond B. [1]

NCERT IMPORTANT QUESTIONS	
Examples	6, 9, 10
Exercise # 12.1	4
Exercise # 12.2	3, 8
Miscellaneous Examples	5, 8, 10