MISCELLANEOUS EXAMPLES

LIMIT

1. $\lim\limits_{x \to 0} \frac{x}{\sqrt{1+x}} - 1.$

Sol. $\lim\limits_{x \to 0} \frac{x}{\sqrt{1+x}} - 1 = \frac{0}{\sqrt{1+0}} - 1 = 0 - 1 = -1$

2. $\lim\limits_{x \to 1} \frac{2x^2 - 3x + 1}{x^2 - 1}$

Sol. $\lim\limits_{x \to 1} \frac{2x^2 - 3x + 1}{x^2 - 1} = \lim\limits_{x \to 1} \frac{(2x-1)(x-1)}{(x+1)(x-1)} = \lim\limits_{x \to 1} \frac{2x-1}{x+1} = \frac{2 \times 1 - 1}{1+1} = \frac{1}{2}$

3. $\lim\limits_{x \to 2} \frac{x^3 + 8}{x + 2}$

Sol. $\lim\limits_{x \to 2} \frac{x^3 + 8}{x + 2} = \frac{8+8}{2+2} = \frac{16}{4} = 4$

$\boxed{\lim\limits_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}}$

4. $\lim\limits_{x \to 1} \frac{1 - x^{-1/3}}{1 - x^{-2/3}}.$

Sol. $\lim\limits_{x \to 1} \frac{1 - x^{-1/3}}{1 - x^{-2/3}} = \lim\limits_{x \to 1} \frac{-(x^{-1/3} - 1)}{-(x^{-2/3} - 1)}$

$$= \lim_{x \to 1} \frac{\left(\frac{x^{-1/3} - 1}{x - 1}\right)}{\left(\frac{x^{-2/3} - 1}{x - 1}\right)} \qquad \boxed{\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}}$$

$\frac{1}{3} \times 1 \times$

$$= \frac{-\frac{1}{3}(1)^{-\frac{1}{3}-1}}{-\frac{2}{3}(1)^{-\frac{2}{3}-1}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$$

Aliter :

$$\lim_{x \to 1} \frac{1 - x^{-1/3}}{1 - x^{-2/3}} = \lim_{x \to 1} \frac{(1 - x^{-1/3})}{(1 - x^{-1/3})(1 + x^{-1/3})}$$

$$= \lim_{x \to 1} \frac{1}{1 + x^{-1/3}} = \frac{1}{2}$$

5. $\lim\limits_{x \to 0} \frac{\sin 5x - \sin 3x}{\sin x}.$

Sol. $\lim\limits_{x\to 0}\dfrac{\sin 5x - \sin 3x}{\sin x} = \lim\limits_{x\to 0}\dfrac{5x\left(\dfrac{\sin 5x}{5x}\right) - 3x\left(\dfrac{\sin 3x}{3x}\right)}{x\left(\dfrac{\sin x}{x}\right)}$

$$\Rightarrow \frac{5\lim\limits_{x\to 0}\left(\dfrac{\sin 5x}{5x}\right) - 3\lim\limits_{x\to 0}\left(\dfrac{\sin 3x}{3x}\right)}{\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)}$$

$$= \frac{5-3}{1} = 2 \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

6. $\lim\limits_{x\to\pi/2}(\sec x - \tan x)$.

Sol. $\lim\limits_{x\to\pi/2}(\sec x - \tan x) \quad (\infty - \infty)$ form

$$= \lim_{x\to\frac{\pi}{2}}\left(\frac{1-\sin x}{\cos x}\right) \qquad \left(\frac{0}{0}\right) \text{ form}$$

$$= \lim_{h\to 0}\frac{1-\sin\left(\dfrac{\pi}{2}+h\right)}{\cos\left(\dfrac{\pi}{2}+h\right)} = \lim_{h\to 0}\frac{1-\cos h}{-\sin h} = \lim_{h\to 0}\frac{2\sin^2\dfrac{h}{2}}{-2\sin\dfrac{h}{2}\cos\dfrac{h}{2}} = -\lim_{h\to 0}\tan\frac{h}{2} = -\tan 0 = 0$$

7. $\lim\limits_{\theta\to 0}\dfrac{1-\cos\theta}{\theta^2}$

Sol. $$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2} = \lim_{\theta\to 0}\frac{2\sin^2\dfrac{\theta}{2}}{\theta^2} = \lim_{\theta\to 0}\frac{2\left(\sin\dfrac{\theta}{2}\right)^2}{4\left(\dfrac{\theta}{2}\right)^2} = \frac{1}{2}\lim_{\theta\to 0}\left(\frac{\sin\dfrac{\theta}{2}}{\dfrac{\theta}{2}}\right)^2 = \frac{1}{2}$$

8. $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{\tan^2 x}$

Sol. $\lim\limits_{x\to\pi}\dfrac{1+\cos x}{\tan^2 x} \qquad \left(\dfrac{0}{0}\right)$ form

$$= \lim_{h\to 0}\frac{1+\cos(\pi+h)}{\tan^2(\pi+h)} = \lim_{h\to 0}\frac{1-\cos h}{\tan^2 h} = \lim_{h\to 0}\frac{2\sin^2\dfrac{h}{2}}{\sin^2 h}\times\cos^2 h$$

$\dfrac{\frac{1}{2}h^2}{h^2}$

$$= \lim_{h\to 0}\frac{2\sin^2\dfrac{h}{2}}{4\sin^2\dfrac{h}{2}\cos^2\dfrac{h}{2}}\times\cos^2 h = \frac{1}{2}\,\frac{\lim\limits_{h\to 0}\cos^2 h}{\lim\limits_{h\to 0}\cos^2\dfrac{h}{2}} = \frac{1}{2}$$

9. $\lim\limits_{n\to\infty}\dfrac{(n-1)(2n+3)}{n^2}$.

Sol. $\lim\limits_{n\to\infty}\dfrac{(n-1)(2n+3)}{n^2}$

$$=\lim_{n\to\infty}\frac{n\left(1-\frac{1}{n}\right).n\left(2+\frac{3}{n}\right)}{n^2}\qquad\left[\because \lim_{n\to\infty}\frac{1}{x}=0\right]$$

$$=\lim_{n\to\infty}\left(1-\frac{1}{n}\right)\left(2+\frac{3}{n}\right)=1\,.\,2=2$$

CONTINUITY

Very Short Answer : [1 Mark]

10. Discuss the continuity of the function $f(x)$ at $x=0$ if $f(x)=\begin{cases}2x-1, & x<0\\ 2x+1, & x\ge 0\end{cases}$. **[CBSE 2002, 1M]**

Sol. For the given function $f(0)=2\times 0+1=1$

$f(0^+)\ \lim\limits_{x\to0^+}f(x)=\lim\limits_{x\to0^+}(2x+1)=1$

$f(0^-)=\lim\limits_{x\to0^-}f(x)=\lim\limits_{x\to0^-}(2x-1)=-1$

Thus, $f(0)=f(0^+)\ne f(0^-)$ **[1]**

$\therefore$ $f(x)$ is not continuous at $x=0$

11. Examine the continuity of the function : $f(x)=\begin{cases}\dfrac{|\sin x|}{x}, & x\ne 0\\ 1, & x=0\end{cases}$ at $x=0$. **[CBSE 2003, 1M]**

Sol. $f(x)=\begin{cases}\dfrac{|\sin x|}{x}, & x\ne0\\ 1, & x=0\end{cases}=\begin{cases}\dfrac{\sin x}{x}, & x>0\\ 1, & x=0\\ \dfrac{-\sin x}{x}, & x<0\end{cases}$

Now $f(0)=1$

Also L.H.L. $=\lim\limits_{x\to0-}f(x)=\lim\limits_{x\to0-}-\dfrac{\sin x}{x}=-1$

Since L.H.L. $\ne f(0)$ **[1]**

f is not continuous at $x=0$

SHORT ANSWER : [4 Mark]

12. If $f(x)=\begin{cases}\dfrac{x^2-25}{x-5}, & \text{when } x\ne5\\ k, & \text{when } x=5\end{cases}$ is continuous at $x=5$, find the value of k. **[CBSE 2007, 4M]**

Sol. $\because$ f(x) is continuous at x = 5

$$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$$ **[1]**

$$\therefore \lim_{x\to 5^+} \frac{x^2-25}{x-5} = K$$ **[1]**

$\frac{x^n - a^n}{x-a}$ $na^{n-1} = \boxed{2\times 5^1 = 10}$

$$\Rightarrow \lim_{x\to 5^+}(x+5) = k \Rightarrow 5+5=k$$ **[1]**

$$\therefore k = 10$$ **[1]**

13. Find the relationship between 'a' and 'b' so that the function 'f' defined by :

$$f(x) = \begin{cases} ax+1, & \text{if} \quad x \le 3 \\ bx+3, & \text{if} \quad x > 3 \end{cases}$$ is continuous at x = 3. **[CBSE 2011, 4M]**

Sol. $f(3-h) = 3a+1$ **[1]**

& $f(3+h) = 3b+3$ **[1]**

Now, $f(3^-) = f(3^+)$

$\Rightarrow$ 3a + 1 = 3b + 3 **[1]**

$\Rightarrow$ 3a – 3b = 2

$\Rightarrow$ $a - b = \frac{2}{3}$ **[1]**

14. If the function $f(x) = \begin{cases} 3ax + b & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax - 2b & \text{if } x < 1 \end{cases}$

is continuous at x = 1, find the values of a, b. **[CBSE COMP. 2012, 4M]**

Sol. We are given that f is continuous at x = 1, therefore,

$$\underset{x\to 1^-}{\text{Lt}} f(x) = f(1) = \underset{x\to 1^+}{\text{Lt}} f(x) \Rightarrow \underset{x\to 1^-}{\text{Lt}} (5ax - 2b) = 11 = \underset{x\to 1^+}{\text{Lt}} (3ax+b)$$ **[1]**

$\Rightarrow$ 5a – 2b = 11 = 3a + b $\quad\Rightarrow\quad$ 3a + b = 11 ...(i) **[1]**

$\quad\Rightarrow\quad$ 5a – 2b = 11 ...(ii) **[1]**

Solving (i) and (ii) for a and b, we get a = 3, b = 2. **[1]**

15. Find the value of k, for which $f(x) = \begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1 \le x < 0 \\ \dfrac{2x+1}{x-1}, & \text{if } 0 \le x < 1 \end{cases}$ continuous at x = 0.

[CBSE 2013, 4M]

Sol. f(x) is continuous at x = 0

So RHL = LHL = f(0) [1]

$$f(0) = \frac{0+1}{0-1} = -1$$ [½]

$$\text{RHL} = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{\sqrt{1+kh}-\sqrt{1-kh}}{h}$$

$$= \lim_{h\to 0} \frac{(1+kh)-(1-kh)}{h(\sqrt{1+kh}+\sqrt{1-kh})} = \lim_{h\to 0} \frac{2kh}{h(\sqrt{1+kh}+\sqrt{1-kh})}$$

$$\Rightarrow \frac{2k}{2} = k$$ [2]

Hence f(0) = RHL ⇒ k = –1 [½]

16. Discuss the continuity and differentiability of the function f(x) = |x| + |x – 1| in the interval (–1, 2).

[CBSE 2015, 4M]

Sol. $f(x) = |x| + |x-1|$

$$f(x) = \begin{cases} 1-2x & -1 < x \le 0 \\ 1 & 0 < x \le 1 \\ 2x-1 & 1 < x < 2 \end{cases}$$

The possible point of discontinuity and non differentiability are x = 0 and x = 1

(i) We have to check cont. at x = 0

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1 = 1$$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -2x+1 = 1$$

$f(0) = 1$

So RHL = LHL = $f(0)$

function is cont. at x = 0 [1]

(ii) We have to check cont. at x = 1

$$\text{RHL} = \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x-1 = 1$$

$$\text{LHL} = \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} 1 = 1$$

$f(1) = 1$

RHL = LHL = $f(1)$

So function is continuous at x = 1 [1]

(iii) We have to check differentiability at x = 0

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{1-1}{h} = 0$$

$$\text{LHD} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$\Rightarrow \quad \lim_{h\to 0}\frac{-2(-h)+1-1}{-h} = -2$$

RHL $\neq$ LHD

So $f(x)$ is not differentiable at $x = 0$. **[1]**

(iv) We have to check diff. at x = 1

$$\text{RHD} = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h\to 0}\frac{[2(1+h)-1]-1}{h} = 2$$

$$\text{LHD} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$= \lim_{h\to 0}\frac{1-1}{-h} = 0$$

RHD $\neq$ LHD

So $f(x)$ is not diff. at x = 1 **[1]**

17. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x} &, x<0 \\ 2 &, x=0 \\ \dfrac{\sqrt{1+bx}-1}{x} &, x>0 \end{cases}$ is continuous at $x = 0$, then find the values of a and b.

[CBSE 2016, 4M]

Sol. $f(x)$ is continuous at $x = 0$.

So RHL = LHL = $f(0)$

$$\text{RHL} = \lim_{x\to 0^+}\frac{\sqrt{1+bx}-1}{x}$$

$$= \lim_{x\to 0^+}\frac{\sqrt{1+bx}-1}{x}\times\frac{\sqrt{1+bx}+1}{\sqrt{1+bx}+1}$$

$$= \lim_{x\to 0^+}\frac{(1+bx)-1}{x[\sqrt{1+bx}+1]} = \frac{b}{2}$$ **[1½]**

$$\text{LHL} = \lim_{x\to 0^-}\frac{\sin(a+1)x}{x}+\frac{2\sin x}{x}$$

$$\Rightarrow \quad \lim_{x\to 0^-}(a+1)\frac{\sin(a+1)x}{\sin(a+1)x}+2\lim_{x\to 0^-}\frac{\sin x}{x}$$

$(a+1)+2 = a+3$ **[1½]**

$f(0) = 2$

$$\therefore \quad \frac{b}{2} = a+3 = 2$$ **[½]**

$a = -1$ and $b = 4$ **[½]**

DIFFERENTIABILITY & DIFFERENTIATION

Very Short Answer : [1 Mark]

18. If $y=\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}$, show that $\frac{dy}{dx}+\sec^2\left(\frac{\pi}{4}-x\right)=0$. **[CBSE 2002, 1M]**

Sol. $y=\sqrt{\frac{1-\sin 2x}{1+\sin 2x}}=\sqrt{\frac{(\cos x-\sin x)^2}{(\cos x+\sin x)^2}}$

$$=\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{1-\tan x}{1+\tan x}=\tan\left(\frac{\pi}{4}-x\right)$$

Diff. w.r.t. x

$$\frac{dy}{dx}=\sec^2\left(\frac{\pi}{4}-x\right).(-1)$$

$$\Rightarrow \frac{dy}{dx}+\sec^2\left(\frac{\pi}{4}-x\right)=0$$ [1]

19. Differentiate $\tan^{-1}\left[\frac{1-\cos x}{\sin x}\right]$ w.r.t. x. **[CBSE 2002, 1M]**

Sol. Let $y=\tan^{-1}\left[\frac{1-\cos x}{\sin x}\right]$

$$=\tan^{-1}\left[\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right]=\tan^{-1}\left(\tan\frac{x}{2}\right)=\frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}$$ [1]

Short Answer : [4 Marks]

20. If $y=\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$, find $\frac{dy}{dx}$. **[CBSE 2006, 4M]**

Sol. Here, $y=\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$

Put : $x^2=\cos\theta$

$$\therefore y=\tan^{-1}\left[\frac{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}\right]$$ [1]

$$= \tan^{-1}\left[\frac{\sqrt{2}\cos\frac{\theta}{2}-\sqrt{2}\sin\frac{\theta}{2}}{\sqrt{2}\cos\frac{\theta}{2}+\sqrt{2}\sin\frac{\theta}{2}}\right]$$

$$= \tan^{-1}\left[\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right]$$ [1½]

$$= \frac{\pi}{4}-\frac{\theta}{2} = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x^2$$

Diff. w.r.t. x

$$\frac{dy}{dx} = 0-\frac{1}{2}.\frac{d}{dx}(\cos^{-1}x^2) = -\frac{1}{2}.\frac{-1}{\sqrt{1-(x^2)^2}}.\frac{d}{dx}(x^2)$$

$$= \frac{1}{2}.\frac{1}{\sqrt{1-x^4}}.2x = \frac{x}{\sqrt{1-x^4}}$$ Cos 1 m 2 [1½]

21. If $y = 3e^{2x} + 2e^{3x}$, prove that $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$. **[CBSE 2007, 4M]**

Sol. Here $y = 3e^{2x} + 2e^{3x}$

$$\frac{dy}{dx} = 3e^{2x}.2+2e^{3x}.3$$ [½]

$$\frac{dy}{dx} = 6e^{2x}+6e^{3x}$$ [½]

Now, $\frac{d^2y}{dx^2} = 6e^{2x}.2+6e^{3x}.3$ [½]

$$\frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x}$$ [½]

$$\text{LHS} = \frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y$$

$= (12e^{2x} + 18e^{3x}) - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x})$ [1]

$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x}$

$= 30e^{2x} + 30e^{3x} - 30e^{2x} - 30e^{3x} = 0$ [1]

22. If $y=\sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$, find $\frac{dy}{dx}$. **[CBSE 2008, 4M]**

Doubt

Sol. $y=\sin^{-1}\left[\frac{5x+12\sqrt{1-x^2}}{13}\right]$ [½]

Let $x = \sin\theta \Rightarrow \theta = \sin^{-1}x$

$$\therefore\ y=\sin^{-1}\left[\frac{5\sin\theta+12\sqrt{1-\sin^2\theta}}{13}\right]$$

$$\Rightarrow \quad y = \sin^{-1}\left[\frac{5\sin\theta + 12\cos\theta}{13}\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\sin\theta + \frac{12}{13}\cos\theta\right]$$ [½]

Let $\cos\alpha = \frac{5}{13} \Rightarrow \alpha = \cos^{-1}\frac{5}{13}$

$\therefore \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$ [1]

$\therefore \quad y = \sin^{-1}[\cos\alpha\sin\theta + \sin\alpha\cos\theta]$

$y = \sin^{-1}(\sin(\alpha+\theta))$ [1]

$y = \alpha + \theta$

$y = \cos^{-1}\frac{5}{13} + \sin^{-1}x$

Differentiate w.r.t. x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$ [1]

23. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = a\sin\theta$, find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$. **[CBSE 2008, 4M]**

Q. $x = a(\cos t + \log\tan\frac{t}{2})$ $y = a\sin t$

Find $\left(\frac{d^2y}{dx^2}\right)_{\pi/4}$

$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}\right)$ — ①

$\frac{dy}{dx} = a\cos t$ ②

Sol. $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{1}{\tan\frac{\theta}{2}}.\sec^2\frac{\theta}{2}.\frac{1}{2}\right]$$ [½]

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}.\frac{1}{\cos^2\frac{\theta}{2}}.\frac{1}{2}\right]$$ [½]

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$$ [½]

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{1}{\sin\theta}\right]$$ [½]

$$\frac{dx}{d\theta} = a\left[\frac{-\sin^2\theta + 1}{\sin\theta}\right]$$

$$\frac{dx}{d\theta} = a\frac{\cos^2\theta}{\sin\theta}$$ [½]

Now $y = a\sin\theta$

$$\frac{dy}{d\theta} = a\cos\theta$$ [½]

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta}{\frac{a\cos^2\theta}{\sin\theta}}$$

$$\frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$ [½]

$$\therefore \left(\frac{dy}{dx}\right)_{at\ \theta=\frac{\pi}{4}} = \tan\frac{\pi}{4} = 1$$ [½]

24. If $\sin y = x\sin(a+y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **[CBSE 2009, 4M]**

Sol. $\sin y = x \sin a \cos y + x \cos a \sin y$

or $\cos y\frac{dy}{dx} = \sin a \cos y - x \sin a \sin y \frac{dy}{dx} + \cos a \sin y + x \cos a \cos y \frac{dy}{dx}$ [1]

Sina 1st level

cont.

or $\frac{dy}{dx}[\cos y + x(\sin a \sin y - \cos a \cos y)] = \sin a \cos y + \cos a \sin y$ [1]

or $\frac{dy}{dx}[\cos y - x\cos(a+y)] = \sin(a+y)$ [½]

or $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\cos y \sin(a+y) - \sin y \cos(a+y)}$ [1]

$= \frac{\sin^2(a+y)}{\sin a}$ $\left[x = \frac{\sin y}{\sin(a+y)}\right]$ [½]

25. If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$. **[CBSE 2009, 4M]**

Sol. $(\cos x)^y = (\sin y)^x$

taking log on both side.

or $y\log(\cos x) = x\log(\sin y)$ [½]

or $y\left(\frac{1}{\cos x}\right)(-\sin x) + \log(\cos x).\frac{dy}{dx}$

$= x\frac{1}{\sin y}\cos y\frac{dy}{dx} + \log(\sin y)$ [1]

or $-y\tan x + \frac{dy}{dx}\log\cos x$

$= \frac{dy}{dx}.(x\cot y) + \log(\sin y)$ [½]

or $\frac{dy}{dx}(\log(\cos x) - x\cot y)$

$= y\tan x + \log(\sin y)$ [1]

$$\frac{dy}{dx} = \frac{y\tan x + \log(\sin y)}{\log\cos x - x\cot y}$$ [1]

26. If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}-y=0$ **[CBSE 2009, 4M]**

Sol. $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

$$\frac{dy}{dx}=\frac{1}{1-x^2}+\frac{x\sin^{-1}x}{(1-x^2)^{3/2}}$$ [1]

$$=\frac{1}{1-x^2}+\frac{yx}{1-x^2}$$ [½]

or $(1-x^2)\dfrac{dy}{dx}=1+yx$ [½]

or $\dfrac{d^2y}{dx^2}(1-x^2)+\dfrac{dy}{dx}(-2x)=x\dfrac{dy}{dx}+y$ [1½]

or $\dfrac{d^2y}{dx^2}(1-x^2)-3x\dfrac{dy}{dx}-y=0$ [½]

27. If $y=\cos^{-1}\left(\dfrac{3x+4\sqrt{1-x^2}}{5}\right)$, find $\dfrac{dy}{dx}$. **[CBSE 2010, 4M]**

Sol. $y=\cos^{-1}\left(\dfrac{3x+4\sqrt{1-x^2}}{5}\right)$

Let $x=\cos\theta$; $\theta=\cos^{-1}x$ [½]

$y=\cos^{-1}\left[\dfrac{3}{5}\cos\theta+\dfrac{4}{5}\sin\theta\right]$ [½]

Let $\cos\phi=\dfrac{3}{5}$ and $\sin\phi=\dfrac{4}{5}$

$y=\cos^{-1}[\cos(\theta-\phi)]$ [1]

$y=\theta-\phi=\cos^{-1}x-\cos^{-1}\dfrac{3}{5}$ [1]

$\Rightarrow \dfrac{dy}{dx}=-\dfrac{1}{\sqrt{1-x^2}}$ [1]

28. If $y=\operatorname{cosec}^{-1}x,\ x>1$, then show that $x(x^2-1)\dfrac{d^2y}{dx^2}+(2x^2-1)\dfrac{dy}{dx}=0$ **[CBSE 2010, 4M]**

Sol. $y=\operatorname{cosec}^{-1}x$

putting $x=\operatorname{cosec}\theta$.

$\Rightarrow y=\theta.$

$\Rightarrow \dfrac{dy}{d\theta}=1$ [½]

$\dfrac{dx}{d\theta}=-\operatorname{cosec}\theta\cot\theta$

$$\frac{dy}{dx} = -\frac{1}{\operatorname{cosec}\theta\cot\theta} = -\frac{\sin^2\theta}{\cos\theta}$$ [½]

$$\frac{d^2y}{dx^2} = \frac{-(\cos\theta 2\sin\theta\cos\theta + \sin^3\theta)}{\cos^2\theta} \times \frac{-1}{\operatorname{cosec}\theta\cot\theta}$$ [1]

$$\frac{d^2y}{dx^2} = \frac{\sin\theta(1+\cos^2\theta)\sin^2\theta}{\cos^2\theta\times\cos\theta} = \frac{\sin^3\theta(1+\cos^2\theta)}{\cos^3\theta}$$ [½]

$$\text{L.H.S.} = \operatorname{cosec}\theta\,(\cot^2\theta) \times \frac{\sin^3\theta(1+\cos^2\theta)}{\cos^3\theta} + (2\operatorname{cosec}^2\theta - 1)\left(-\frac{\sin^2\theta}{\cos\theta}\right)$$ [½]

$$= \frac{(1+\cos^2\theta)}{\cos\theta} - \frac{(1+\cos^2\theta)}{\cos\theta} = 0 \quad \text{H.P.}$$ [1]

29. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$ **[CBSE 2011, 4M]**

Sol. We have, $x = \tan\left(\frac{1}{a}\log y\right)$

$$\Rightarrow \tan^{-1}x = \frac{1}{a}\log y \Rightarrow a\tan^{-1}x = \log y$$ [1]

Differentiating w.r.t. x, we get

$$\frac{a}{1+x^2} = \frac{1}{y}\frac{dy}{dx} \Rightarrow (1+x^2)\frac{dy}{dx} = ay$$ [1]

Differentiating w.r.t. x

$$(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = a\frac{dy}{dx}$$ [1]

$$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$$ [1]

30. If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$. **[CBSE 2011, 4M]** 2013

Sol. We have, $x^y = e^{x-y}$

$$\Rightarrow e^{y\log x} = e^{x-y} \Rightarrow y\log x = x - y$$ [1]

$$\Rightarrow y\log x + y = x \Rightarrow y(1+\log x) = x$$ [½]

$$\Rightarrow y = \frac{x}{1+\log x}$$ [½]

On differentiating both sides w.r.to x, we get

$$\frac{dy}{dx}=\frac{(1+\log x).1-x.\left(0+\frac{1}{x}\right)}{(1+\log x)^2}$$ [1]

$$=\frac{\log x}{(1+\log x)^2}$$

$$=\frac{\log x}{[\log(ex)]^2}$$ [1]

31. If $x=\sqrt{a^{\sin^{-1}t}}$, $y=\sqrt{a^{\cos^{-1}t}}$, show that $\frac{dy}{dx}=-\frac{y}{x}$ **[CBSE 2012, 4M]**

Sol. Given $x=\sqrt{a^{\sin^{-1}t}}$, $y=\sqrt{a^{\cos^{-1}t}}$

i.e., $x=a^{\frac{1}{2}\sin^{-1}t}$ and $y=a^{\frac{1}{2}\cos^{-1}t}$

$$\therefore\quad \frac{dx}{dt}=a^{\frac{1}{2}\sin^{-1}t}\log a\frac{d}{dt}\left(\frac{1}{2}\sin^{-1}t\right)$$

$$=a^{\frac{1}{2}\sin^{-1}t}\log a\left(\frac{1}{2\sqrt{1-t^2}}\right)=\frac{a^{\frac{1}{2}\sin^{-1}t}\log a}{2\sqrt{1-t^2}}\quad\text{and}\quad\frac{dy}{dt}=a^{\frac{1}{2}\cos^{-1}t}\log a\frac{d}{dt}\left(\frac{1}{2}\cos^{-1}t\right)$$ [1]

$$=a^{\frac{1}{2}\cos^{-1}t}\log a\left(\frac{-1}{2\sqrt{1-t^2}}\right)=\frac{-a^{\frac{1}{2}\cos^{-1}t}\log a}{2\sqrt{1-t^2}}$$ [1]

$$\Rightarrow\quad\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{-a^{\frac{1}{2}\cos^{-1}t}}{a^{\frac{1}{2}\sin^{-1}t}}$$ [1]

$$=-a^{\frac{1}{2}(\cos^{-1}t-\sin^{-1}t)}=\frac{\sqrt{a^{\cos^{-1}t}}}{\sqrt{a^{\sin^{-1}t}}}=\frac{-y}{x}$$ [1]

32. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$ with respect to x **[CBSE 2004, 2012, 4M]**

Sol. Let $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$. Putting $x=\tan\theta$, then

$$y=\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)=\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$ [1]

$$\Rightarrow\quad y=\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$ [1]

$\Rightarrow\ y = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\theta = \frac{1}{2}\tan^{-1} x$ [1]

$\therefore\ \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{1+x^2}\right)$ [1]

33. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, $0 < t < \frac{\pi}{2}$, find $\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$.

[CBSE 2012, 4M]

Sol. $x = a(\cos t + t\sin t)$, $y = (\sin t - t\cos t)$ $0 < t < \pi/2$

$\frac{dx}{dt} = a[-\sin t + \sin t + t\cos t]$

$\frac{dx}{dt} = a\ (t\cos t)$ ....(i) [½]

$\frac{d^2x}{dt^2} = a[\cos t - t\sin t]$ [½]

and $y = a(\sin t - t\cos t)$

$\frac{dy}{dt} = a\ (\cos t - \cos t + t\sin t)$

$\frac{dy}{dt} = a\, t\sin t$ ....(ii) [½]

$\frac{d^2y}{dt^2} = a(\sin t + t\cos t)$ [½]

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{at\sin t}{at\cos t} = \tan t$ Using (i) and (ii) [1]

$\frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{(dx/dt)} \Rightarrow \frac{d^2y}{dx^2} = \sec^2 t \times \frac{1}{at\cos t} = \frac{1}{at\cos^3 t}$ [1]

34. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. **[CBSE 2013, 4M]**

Sol. $x = a\cos^3\theta$; $y = a\sin^3\theta$

$\frac{dx}{d\theta} = 3\cos^2\theta(-\sin\theta)$ ....(i) [½]

$\frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$ ....(ii) [½]

from (ii) / (i)

$$\frac{dy}{dx}=\frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}=-\tan\theta$$ [1]

$$\frac{d^2y}{dx^2}=-\sec^2\theta\times\frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2}=-\sec^2\theta\times\frac{1}{-3a\cos^2\theta\sin\theta}$$

$$\left(\frac{d^2y}{dx^2}\right)=\frac{1}{3a.\cos^4\theta.\sin\theta}$$ [1]

$$\left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{6}}=\frac{1}{3a\times\left(\frac{\sqrt{3}}{2}\right)^4\times\frac{1}{2}}=\frac{32}{27a}$$ [1]

35. If $y^x = e^{y-x}$, then prove that $\frac{dy}{dx}=\frac{(1+\log y)^2}{\log y}$. **[CBSE 2013, 4M]**

2011

Sol. $y^x = e^{y-x}$

$x\log y = y - x$

$x(\log y + 1) = y$

$$x=\frac{y}{1+\log y}$$ [1]

$$\frac{dx}{dy}=\frac{(1+\log y).1-y\times\frac{1}{y}}{(1+\log y)^2}$$ [1½]

$$\Rightarrow\frac{dx}{dy}=\frac{\log y}{(1+\log y)^2}$$

$$\Rightarrow\frac{dy}{dx}=\frac{(1+\log y)^2}{(\log y)}$$ [1½]

36. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$ **[CBSE 2013, 4M]**

Sol. Let $y=\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$

$$y=\sin^{-1}\left(\frac{2.(6)^x}{1+(6^x)^2}\right)$$ $= \sin\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\sin(\sin 2\theta)$

$\theta = \tan^{-1} 6^x$

put $6^x = \tan\theta$; $\theta = \tan^{-1}6^x$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2(\tan^{-1}6^x)$ [2]

$$\frac{dy}{dx}=\frac{2}{1+(36)^x}\times6^x\log_e 6$$ [2]

37. If $x \sin(a+y) + \sin a \cos(a+y) = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ **[CBSE 2013, 4M]**

Sol. $x \sin(a+y) + \sin a \cos(a+y) = 0$

$$x = -\frac{\sin a \cos(a+y)}{\sin(a+y)} = -\sin a \cot(a+y)$$ [1]

$$\frac{dx}{dy} = [\sin a \operatorname{cosec}^2(a+y)]$$ [1]

$$\frac{dx}{dy} = \frac{\sin a}{\sin^2(a+y)}$$ [1]

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$ [1]

38. If $y = Pe^{ax} + Qe^{bx}$, show that $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$ **[CBSE 2014, 4M]**

Sol. $y = Pe^{ax} + Qe^{bx}$ ...(i)

$$\frac{dy}{dx} = aPe^{ax} + bQe^{bx}$$ [½]

$$\frac{dy}{dx} = a[y - Qe^{bx}] + bQe^{bx} \quad \text{From eq. (i)}$$ [1]

$$\frac{dy}{dx} = ay - aQe^{bx} + bQe^{bx}$$

$$\frac{dy}{dx} = ay - (a-b)Qe^{bx} \quad ..(ii)$$ [1]

$$\frac{d^2y}{dx^2} = a\frac{dy}{dx} - (a-b)Qbe^{bx}$$

$$\frac{d^2y}{dx^2} = \left[a\frac{dy}{dx} + b\frac{dy}{dx} - aby\right] \quad \text{From eq. (ii)}$$ [1]

$$\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = 0$$ [½]

39. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^{\theta}(\sin\theta - \cos\theta)$ and $y = ae^{\theta}(\sin\theta + \cos\theta)$. **[CBSE 2014, 4M]**

Sol. $y = ae^{\theta}(\sin\theta + \cos\theta)$

$x = ae^{\theta}(\sin\theta - \cos\theta)$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \quad \text{(Applying parametric defferentiation)} \quad ....(1)$$

Now, $\frac{dy}{d\theta} = ae^{\theta}(\cos\theta - \sin\theta) + ae^{\theta}(\sin\theta + \cos\theta)$

$$= 2ae^{\theta}(\cos\theta)$$ [1]

$$\frac{dx}{d\theta} = ae^{\theta}(\cos\theta + \sin\theta) + ae^{\theta}(\sin\theta - \cos\theta)$$

$$= 2ae^{\theta}(\sin\theta)$$ [1]

Substituting the value of $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ in (1)

$$\frac{dy}{dx} = \frac{2ae^{\theta}\cos\theta}{2ae^{\theta}\sin\theta} = \cot\theta$$ [1]

Now $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

$$[\cot\theta]_{\theta=\pi/4} = \cot\frac{\pi}{4} = 1$$ [1]

40. If $x = a\sin 2t\,(1+\cos 2t)$ and $y = b\cos 2t\,(1-\cos 2t)$, show that at $t = \frac{\pi}{4}, \left(\frac{dy}{dx}\right) = \frac{b}{a}$. **[CBSE 2014, 4M]**

Sol. $x = a\sin2t(1+\cos2t)$

$$x = a\sin2t + \frac{a}{2}2\sin 2t\cos 2t$$

$$\frac{dx}{dt} = 2a\cos 2t + \frac{4a}{2}\cos 4t$$

$$\frac{dx}{dt} = 2a(\cos 2t + \cos 4t) \quad \text{..........(i)}$$ [1]

Now $y = b\cos2t(1-\cos2t)$

$$y = b\cos2t - b\cos^2 2t$$

$$y = b\cos2t - b\left(\frac{1+\cos 4t}{2}\right) \qquad \because \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$y = b\cos 2t - \frac{b}{2} - \frac{b}{2}\cos 4t$$

$$\Rightarrow \frac{dy}{dt} = -2b\sin 2t + \frac{4b}{2}\sin 4t$$

$$\Rightarrow \frac{dy}{dt} = 2b(\sin 4t - \sin 2t) \quad \text{......(ii)}$$ [1]

By chain rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2b(\sin 4t - \sin 2t)}{2a(\cos 2t + \cos 4t)}$$ [1]

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \frac{b}{a}\left[\frac{\sin\pi - \sin\frac{\pi}{2}}{\cos\frac{\pi}{2} + \cos\pi}\right]$$

$$\left(\frac{dy}{dx}\right)_{\frac{\pi}{4}} = \frac{b}{a}$$ [1]

41. If $x = \cos t\,(3 - 2\cos^2 t)$ and $y = \sin t(3 - 2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. **[CBSE 2014, 4M]**

Sol. $x = 3\cos t - 2\cos^3 t$

$$\frac{dx}{dt} = -3\sin t + 2 \times 3\cos^2 t \sin t$$
$$= 3\sin t\,(2\cos^2 t - 1)$$
$$= 3\sin t.\cos 2t \qquad ...(i) \qquad [1]$$

$y = \sin t(3 - 2\sin^2 t)$

$$\frac{dy}{dt} = 3\sin t - 2\sin^3 t$$
$$= 3\cos t - 2 \times 3\sin^2 t \cos t$$
$$= 3\cos t(1 - 2\sin^2 t)$$
$$= 3\cos t.\cos 2t \qquad ...(ii) \qquad [1]$$

By chain rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\cos t.\cos 2t}{3\sin t.\cos 2t} = \cot t \qquad [1]$$

$$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \cot\frac{\pi}{4} = 1 \qquad [1]$$

42. If $x = a(\cos 2t + 2t \sin 2t)$ and $y = a(\sin 2t - 2t \cos 2t)$, then find the $\frac{d^2y}{dx^2}$. **[CBSE 2015, 4M]**

Sol. $x = a(\cos 2t + 2t\sin 2t)$

$$\frac{dx}{dt} = a[-2\sin 2t + 2(2t\cos 2t + \sin 2t)]$$

$$\frac{dx}{dt} = 4at\cos 2t \qquad ....(1) \qquad [1]$$

$y = a(\sin 2t - 2t\cos 2t)$

$$\frac{dy}{dt} = a\left[2\cos 2t - 2(-2t\sin 2t + \cos 2t)\right]$$

$$\frac{dy}{dt} = 4at\sin 2t \qquad ....(2) \qquad [1]$$

By chain rule

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at\sin 2t}{4at\cos 2t}$$

$$\frac{dy}{dx} = \tan 2t \qquad [1]$$

$$\frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = 2\sec^2 2t \times \frac{1}{4at\cos 2t}$$

$$\frac{d^2y}{dx^2} = \frac{1}{2at} \times \sec^3 2t \qquad [1]$$

43. If $(ax+b)e^{y/x} = x$, then show that $x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2$ **[CBSE 2015, 4M]**

Sol. $(ax+b)e^{\frac{y}{x}} = x$

$$e^{y/x} = \frac{x}{ax+b} \qquad \text{.......(1)}$$

$$\frac{y}{x} = [\log x - \log(ax+b)] \qquad [½]$$

differentiate w.r.t. x

Bony

$$\frac{x.\frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{a}{ax+b} \qquad [1]$$

$$\frac{x.\frac{dy}{dx} - y}{x^2} = \frac{ax+b-ax}{x(ax+b)} \qquad [½]$$

$$x\frac{dy}{dx} - y = \frac{bx}{(ax+b)}$$

from equation (1)

$$x\frac{dy}{dx} - y = be^{y/x} \qquad \text{........(2)} \qquad [½]$$

differentiate w.r.t. x

$$x.\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{dy}{dx} = \frac{be^{y/x}\left(x.\frac{dy}{dx} - y\right)}{x^2}$$

Completed Zuel ...

from equation (2)

$$x^3\left(\frac{d^2y}{dx^2}\right) = \left(x\frac{dy}{dx} - y\right)^2 \qquad [1]$$

44. If $x\cos(a+y) = \cos y$, then prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. **[CBSE 2016, 4M]**

Hence show that $\sin a\frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$.

OR

Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

Sol. $x\cos(a+y) = \cos y$

$$x = \frac{\cos y}{\cos(a+y)}$$

$$\Rightarrow \frac{dx}{dy}=\frac{-\cos(a+y)\sin y+\cos y\sin(a+y)}{\cos^2(a+y)}$$

$$\Rightarrow \frac{dx}{dy}=\frac{\sin a}{\cos^2(a+y)} \Rightarrow \frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$$ [2]

Hence $\frac{d^2y}{dx^2}=\frac{-2\cos(a+y)\sin(a+y)}{\sin a}\cdot\frac{dy}{dx}$

$$\frac{d^2y}{dx^2}=\frac{-\sin 2(a+y)}{\sin a}\cdot\frac{dy}{dx}$$ [1½]

$$\sin a\frac{d^2y}{dx^2}+\sin 2(a+y)\frac{dy}{dx}=0$$ [½]

OR

$$y=\sin^{-1}\left[\frac{6x-4\sqrt{1-4x^2}}{5}\right]$$

$$y=\sin^{-1}\left[\frac{3}{5}.2x-\frac{4}{5}\sqrt{1-(2x)^2}\right]$$

put $2x=\sin\theta$ and $\frac{3}{5}=\cos\phi$

$\theta=\sin^{-1}2x$ $\qquad \frac{4}{5}=\sin\phi$ [1]

$$y=\sin^{-1}\left[\sin\theta\cos\phi-\cos\theta\sin\phi\right]$$ [1]

$$y=\sin^{-1}[\sin(\theta-\phi)]$$

$$y=\theta-\phi$$

$$y=\sin^{-1}2x-\sin^{-1}\frac{4}{5}$$ [1]

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-4x^2}}.2-0$$

$$\frac{dy}{dx}=\frac{2}{\sqrt{1-4x^2}}$$ [1]

Q. if $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$
P. that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

Sol^n: substitution method
put $x=\sin\theta,\ y=\sin\phi$

$\sqrt{1-\sin^2\theta}+\sqrt{1-\sin^2\phi}=a(\sin\theta-\sin\phi)$

$\frac{\cos\theta+\cos\phi}{\sin\theta-\sin\phi}=a$

$\frac{2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)}{2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)}=a$

$\cot\frac{\theta-\phi}{2}=a$

$\frac{\theta-\phi}{2}=\cot^{-1}a$

$\theta-\phi=2\cot^{-1}a$

$\sin^{-1}x-\sin^{-1}y=2\cot^{-1}x$

differentiate $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{-2}{1+x^2}$

$\frac{1}{\sqrt{1-y^2}}\left(\frac{1}{2}\right)\frac{dy}{dx}=$

$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

Completed

NCERT IMPORTANT QUESTIONS	
Examples	37, 41, 45 (iii)
Exercise # 5.1	24, 26, 30
Exercise # 5.2	10
Exercise # 5.3	7, 10, 15
Exercise # 5.4	7, 10
Exercise # 5.5	9, 12, 14, 15
Exercise # 5.6	7, 11
Exercise # 5.7	13, 14, 16, 17
Miscellaneous Exercise	6, 15, 16, 17, 23