MISCELLANEOUS EXAMPLES

Very Short Answer : [1 Mark]

1. Solve the following differential equation : $y(1-x^2)\dfrac{dy}{dx} = x(1+y^2)$. **[CBSE 2007, 1M]**

Sol. Here $y\,(1-x^2)\dfrac{dy}{dx} = x(1+y^2)$

$$\Rightarrow \frac{y}{1+y^2}dy = \frac{x}{1-x^2}dx$$

$$\Rightarrow \int\frac{y}{1+y^2}dy = \int\frac{x}{1-x^2}dx$$

$\log|1+y^2| = \dfrac{+\log(1-x^2)}{-1} + \log C$

$$\Rightarrow \int\frac{2y}{1+y^2}dy = \int\frac{2x}{1-x^2}dx$$

$$\Rightarrow \log|1+y^2| + \log|1-x^2| = \log c$$

$$\Rightarrow (1+y^2)(1-x^2) = C$$

[1]

2. Find the differential equation of the family of curves $y = a\cos(x+b)$, where a and b are arbitrary constants. **[CBSE 2007, 1M]**

Sol. Here $y = a\cos(x+b)$

Diff. w.r.t. x, we get $\dfrac{dy}{dx} = -a\sin(x+b)$

Again diff. w.r.t x, we get $\dfrac{d^2y}{dx^2} = -a\cos(x+b)$

$$\Rightarrow \frac{d^2y}{dx^2} = -y \quad \Rightarrow \frac{d^2y}{dx^2} + y = 0$$

[1]

3. Solve the following differential equation $\sec^2x\tan y\,dx + \sec^2y\tan x\,dy = 0$. **[CBSE 2007, 1M]**

Sol. Given equations is

$\sec^2x\tan y\,dx + \sec^2y\tan x\,dy = 0$

$\sec^2 y\tan x\,dy = -\sec^2 x\tan y\,dx$

$$\int\frac{\sec^2 y}{\tan y}dy = -\int\frac{\sec^2}{\tan x}dx$$

$\log|\tan y| = -\log|\tan x| + \log c$

$\log|\tan x| + \log|\tan y| = \log c$

$\log|\tan x\tan y| = \log c$

$\therefore \tan x\tan y = c$

[1]

4. Solve the following differential equation : $\tan y\,dx + \sec^2 y\tan x\,dy = 0$. **[CBSE 2007, 1M]**

Sol. $\sec^2 y \tan x\, dy = -\tan y\, dx$

$$\frac{\sec^2 y}{\tan y} dy = -\frac{dx}{\tan x}$$

$$\int \frac{\sec^2 y}{\tan y} dy = -\int \cot x\, dx$$

Put $\tan y = t \Rightarrow \sec^2 y\, dy = dt$

$$\int \frac{dt}{t} = -\log(\sin x) + \log c$$

$\log|t| = -\log|\sin x| + \log c$

$\Rightarrow \log|\tan y| + \log|\sin x| = \log c$

$\Rightarrow \log|\tan y \cdot \sin x| = \log c = \tan y \sin x = c$ [1]

5. Write the differential equation representing the family of curves $y = mx$, where m is arbitrary constant.

[CBSE 2013, 1M]

Sol. $y = mx$ ....(i) $\rightarrow m = \frac{y}{x}$

$\frac{dy}{dx} = m$ ....(ii)

from equation (i) & (ii)

$$\frac{dy}{dx} = \frac{y}{x}$$ [1]

6. Find the sum of the order and the degree of following differential equation :

$$y = x\left(\frac{dy}{dx}\right)^3 + \frac{d^2y}{dx^2}$$ **[CBSE 2015, 1M]**

Sol. Order = 2 and Degree = 1 and sum = 2 + 1 = 3. [1]

7. Find the solution of the following differential equation :

$$x\sqrt{(1+y^2)}dx + y\sqrt{(1+x^2)}dy = 0$$ **[CBSE 2015, 1M]**

Sol. $\int \frac{x}{\sqrt{1+x^2}} dx = -\int \frac{y\,dy}{\sqrt{1+y^2}}$

$$\Rightarrow \frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}} dx = -\frac{1}{2}\int \frac{2y}{\sqrt{1+y^2}} dx$$

$$\Rightarrow \frac{1}{2} \times 2\sqrt{1+x^2} = -\frac{1}{2} \times 2\sqrt{1+y^2} + C$$

$$\Rightarrow \sqrt{1+x^2} + \sqrt{1+y^2} = C$$ [1]

Short Answer : [4 Marks]

8. Solve the following differential equation: $4\frac{dy}{dx} + 8y = 5e^{-3x}$ **[CBSE 2007, 4M]**

Sol. Here $4\frac{dy}{dx}+8y=5e^{-3x}$

$\Rightarrow \frac{dy}{dx}+\frac{8}{4}y=\frac{5}{4}e^{-3x} \Rightarrow \frac{dy}{dx}+2y=\frac{5}{4}e^{-3x}$ [1]

which is linear D.E.

On compairing by $\frac{dy}{dx}+Py=Q$

$\therefore \quad P=2,\ Q=\frac{5}{4}e^{-3x}$ [½]

Integrating factor IF $= e^{\int Pdx} = e^{\int 2dx} = e^{2x}$ [1]

$\therefore$ Required solution is

$y\,.\,IF = \int Q\ IF\ dx + c$

$\Rightarrow y\,.\,e^{2x} = \int \frac{5}{4}e^{-3x}.e^{2x}dx+c$

$\Rightarrow y\,e^{2x} = \frac{5}{4}\int e^{-x}dx + c \Rightarrow y\,e^{2x} = -\frac{5}{4}e^{-x}+c$

$\Rightarrow y\,e^{2x} = -\frac{5}{4}e^{-x}+c \Rightarrow y = -\frac{5}{4}e^{-3x}+ce^{-2x}$ [1½]

9. Solve the following differential equation : $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$. **[CBSE 2008, 4M]**

Sol. Given differential equation is

$(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$

$\frac{dy}{dx}+\frac{2x}{x^2+1}dy=\frac{\sqrt{x^2+4}}{x^2+1}$ [1]

Which is the linear differential equation in the form of $\frac{dy}{dx}+Py=Q$

Here $P=\frac{2x}{x^2+1},\ Q=\frac{\sqrt{x^2+4}}{x^2+1}$ [½]

$\therefore$ Integrating factor, IF $= e^{\int Pdx}$

I.F. $= e^{\int \frac{2x}{x^2+1}dx}$ [1]

put $x^2+1=t \Rightarrow 2x\ dx = dt$

$I = e^{\int \frac{dt}{t}} = e^{\log t} = t = x^2+1$

required solution is $y\,I.F. = \int Q.IF\,dx + c$

$$y.(x^2+1) = \int \frac{\sqrt{x^2+4}}{x^2+1}(x^2+1)dx + c$$

$$y.(x^2+1) = \int \sqrt{x^2+4}\,dx + c$$

$$\left[\because \sqrt{x^2+a^2} = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c\right]$$

$$y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + \frac{4}{2}\log\left|x+\sqrt{x^2+4}\right| + c$$

$$y(x^2+1) = \frac{x}{2}\sqrt{x^2+4} + 2\log\left|x+\sqrt{x^2+4}\right| + c$$ [1½]

10. Solve the following differential equation : $x^2\frac{dy}{dx} = y^2 + 2xy$. Given that $y = 1$, when $x = 1$.

[CBSE 2008, 4M]

Sol. Given differential equation is $x^2\frac{dy}{dx} = y^2 + 2xy$

$$\frac{dy}{dx} = \frac{y^2+2xy}{x^2}$$

This is homogeneous differential equation put $y = vx$

y|x

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$ [1]

$$v + x\frac{dv}{dx} = \frac{v^2x^2+2x.vx}{x^2} \Rightarrow v + x\frac{dv}{dx} = v^2 + 2v$$

$$\therefore \quad x\frac{dv}{dx} = v^2 + v \Rightarrow \frac{dv}{v^2+v} = \frac{dx}{x}$$ [½]

$$\therefore \quad \int\frac{dv}{v(v+1)} = \int\frac{dx}{x}$$

Let $\frac{1}{v(v+1)} = \frac{A}{v} + \frac{B}{v+1}$

$1 = A(v+1) + Bv$

Put $v = 0, -1$

$1 = A + 0 \quad \Rightarrow \quad A = 1$

$1 = 0 + B(-1) \Rightarrow \quad B = -1$ [½]

$$\therefore \quad \int\frac{dv}{v} + \int\frac{(-1)dv}{v+1} = \int\frac{dx}{x}$$

$\log|v| - \log|v+1| = \log|x| + \log c$

$$\log\left|\frac{v}{v+1}\right| = \log|cx|$$

$$\therefore \quad \frac{v}{v+1} = cx$$ [1]

putting the value of v we get

$$\frac{\frac{y}{x}}{\frac{y}{x}+1} = cx \Rightarrow \frac{y}{x+y} = cx$$

$$\therefore \quad y = cx(x+y)$$

When $x = 1, y = 1 \quad \therefore \quad c = \frac{1}{2}$

$$\therefore \quad y = \frac{1}{2}x(x+y)$$ [1]

11. Solve the following differential equation : $\frac{dy}{dx} + 2y\tan x = \sin x$ **[CBSE 2008, 4M]**

Sol. Given differential equation is

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

This is a linear differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

$\int \tan x = \log|\sec x|$

Here $P = 2\tan x$, $Q = \sin x$ [1]

$$\text{I.F.} = e^{\int P dx} = e^{\int 2\tan x\, dx} = e^{2\log\sec x} = e^{\log\sec^2 x} = \sec^2 x$$ [1]

$\therefore$ Required solution is $\quad y.\text{IF} = \int Q.\text{IF}\, dx + c$

$$y.\sec^2 x = \int \sin x.\sec^2 x\, dx + c$$ [1]

$$y.\sec^2 x = \int \frac{\sin x}{\cos^2 x} dx + c$$

$$y\sec^2 x = \int \sec x \tan x\, dx + c$$

$$y\sec^2 x = \sec x + c$$

$$y = \frac{\sec x}{\sec^2 x} + \frac{c}{\sec^2 x}$$

$$\therefore \quad y = \cos x + c\cos^2 x$$ [1]

12. Solve the following differential equation : $\cos^2 x\frac{dy}{dx} + y = \tan x.$ **[CBSE 2008, 4M]**

Sol. $\cos^2 x \dfrac{dy}{dx} + y = \tan x$

$\dfrac{dy}{dx} + y\sec^2 x = \tan x.\sec^2 x$

$\text{I.F.} = e^{\int \sec^2 x.dx} = e^{\tan x}$ **[1]**

$y.e^{\tan x} = \int e^{\tan x}.\tan x.\sec^2 x dx$ **[1]**

(Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$)

$= \int e^t.t\,dt = te^t - \int e^t dt = te^t - e^t + C$ **[1]**

$= \tan x\, e^{\tan x} - e^{\tan x} + C$

$\Rightarrow y\, e^{\tan x} = e^{\tan x}.\tan x - e^{\tan x} + C$ **[1]**

13. Solve the following differential equation : $x\dfrac{dy}{dx} = y - x\tan\left(\dfrac{y}{x}\right)$ **[CBSE 2009, 4M]**

Homogenious

Sol. $x\dfrac{dy}{dx} = y - x\tan\left(\dfrac{y}{x}\right)$

$\dfrac{dy}{dx} = \dfrac{y}{x} - \tan\left(\dfrac{y}{x}\right)$

$\boxed{\text{Let } y = Vx}$

$\dfrac{dy}{dx} = V + x\dfrac{dV}{dx}$ **[1]**

$\cancel{V} + x\dfrac{dV}{dx} = \cancel{V} - \tan V$

$x\dfrac{dV}{dx} = -\tan V$

$\int\dfrac{dV}{\tan V} = -\int\dfrac{dx}{x} \Rightarrow -\int\dfrac{\cos V}{\sin V}dV = \int\dfrac{dx}{x}$ **[1]**

$-\ell n\,|\sin V| = \ell n\, x + C$ **[1]**

$\ell nx + \ell n\,|\sin V| + C = 0$

or $\quad \ell nx + \ell n\left|\sin\left(\dfrac{y}{x}\right)\right| + C = 0$ **[1]**

14. Solve the following differential equation : $(x^2-1)\dfrac{dy}{dx} + 2xy = \dfrac{1}{x^2-1}; |x| \neq 1$ **[CBSE 2010, 4M]**

OR

Linear

Solve the following differential equation : $\sqrt{1+x^2+y^2+x^2y^2} + xy\dfrac{dy}{dx} = 0$

Sol. $(x^2-1)\dfrac{dy}{dx} + 2xy = \dfrac{1}{x^2-1}; |x| \neq 1$

$\dfrac{dy}{dx} + \dfrac{2x}{x^2-1}y = \dfrac{1}{(x^2-1)^2}$

Let $IF = e^{\int \frac{2x}{x^2-1}dx} = e^{\ell n|x^2-1|} = (x^2-1)$ [1]

Hence the required solution be

$\Rightarrow \quad y(IF) = \int Q(IF)dx + C$ [1]

$\Rightarrow \quad y(x^2-1) = \int \frac{(x^2-1)}{(x^2-1)^2}dx + C$

$\Rightarrow \quad y(x^2-1) = \int \frac{dx}{x^2-1} + C$ [1]

$\Rightarrow \quad y(x^2-1) = \frac{1}{2}\ell n\left|\frac{x-1}{x+1}\right| + C$ [1]

OR

$\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

$\Rightarrow \quad \sqrt{(1+x^2)(1+y^2)} = -xy\frac{dy}{dx}$

$\Rightarrow \quad \frac{\sqrt{1+x^2}}{x}dx = \frac{-ydy}{\sqrt{1+y^2}}$ [1]

$\int \frac{\sqrt{1+x^2}}{x}dx = -\frac{1}{2}\int \frac{2ydy}{\sqrt{1+y^2}}$ [½]

Let $x = \tan\theta \quad \Rightarrow \quad dx = \sec^2\theta d\theta$

$\int \frac{\sec\theta \sec^2\theta d\theta}{\tan\theta} = \int \frac{\sin\theta d\theta}{\sin^2\theta\cos^2\theta} = -\int \frac{dt}{(1-t^2)t^2}$ [1]

$= \int \frac{dt}{(t^2-1)t^2} = \int \frac{dt}{t^2-1} - \int \frac{dt}{t^2} = \frac{1}{2}\ell n\left|\frac{t-1}{t+1}\right| + \frac{1}{t} + C$

$= \frac{1}{2}\ell n\left|\frac{\cos\theta-1}{\cos\theta+1}\right| + \frac{1}{\cos\theta} + C = \frac{1}{2}\ell n\left|\frac{1-\sqrt{1+x^2}}{1+\sqrt{1+x^2}}\right| + \sqrt{1+x^2} + C$ [1]

Hence, $\frac{1}{2}\ell n\left|\frac{1-\sqrt{1+x^2}}{1+\sqrt{1+x^2}}\right| + \sqrt{1+x^2} = -\sqrt{1+y^2} + C$ [½]

is the required solution.

15. Show that the differential equation $(x-y)\frac{dy}{dx} = x + 2y$, is homogeneous and solve it. **[CBSE 2010, 4M]**

Sol. Let $f(x,y) = \frac{x+2y}{x-y}$

putting $x = \lambda x$ & $y = \lambda y$

$f(\lambda x, \lambda y) = \frac{\lambda(x+2y)}{\lambda(x-y)} = \lambda^0 = f(x,y)$

$\Rightarrow$ this is a homogeneous differential equation. [½]

Let $y = xt \Rightarrow \dfrac{dy}{dx} = t + x\dfrac{dt}{dx}$ [1]

Now $t + x\dfrac{dt}{dx} = \dfrac{1+2t}{1-t}$

$x\dfrac{dt}{dx} = \dfrac{1+2t}{1-t} - t = \dfrac{1+t+t^2}{1-t}$ [1]

$\Rightarrow \quad -\int\dfrac{(t-1)dt}{t^2+t+1} = \int\dfrac{dx}{x}$

$\Rightarrow \quad -\dfrac{1}{2}\int\dfrac{(2t+1)}{t^2+t+1}dt + \dfrac{3}{2}\int\dfrac{dt}{(t^2+t+1)} = \int\dfrac{dx}{x}$ [½]

$\Rightarrow \quad -\dfrac{1}{2}\ell n|t^2+t+1| + \sqrt{3}\tan^{-1}\left(\dfrac{2\left(t+\dfrac{1}{2}\right)}{\sqrt{3}}\right)$ [½]

$= \ell n|x| + C$

$\Rightarrow \quad -\dfrac{1}{2}\ell n\left|\left(\dfrac{y}{x}\right)^2 + \dfrac{y}{x} + 1\right| + \sqrt{3}\tan^{-1}\dfrac{2}{\sqrt{3}}\left(\dfrac{y}{x}+\dfrac{1}{2}\right) = \ell n\,|x| + C$ [½]

16. Show that the following differential equation is homogeneous, and then solve it

$y\,dx + x\log\left(\dfrac{y}{x}\right)dy - 2x\,dy = 0$. **[CBSE 2010, 4M]**

Sol. $ydx + xdy[\log y - \log x - 2] = 0$

$\Rightarrow \quad \dfrac{dx}{dy} = \dfrac{x}{y}[2 + \log x - \log y]$

$\Rightarrow \quad \dfrac{dx}{dy} = \dfrac{x}{y}\left[2 + \log\left(\dfrac{x}{y}\right)\right]$ [1]

Let $f(x,y) = \dfrac{x}{y}\left[2 + \log\left(\dfrac{x}{y}\right)\right]$

Put $x = tx$, $y = ty$, we get [½]

$f(tx, ty) = \dfrac{tx}{ty}\left[2 + \log\left(\dfrac{x}{y}\right)\right] = t^0 f(x,y)$

Hence the given equation is homogeneous of the order zero.

put $x = vy \Rightarrow \dfrac{dx}{dy} = v + y\dfrac{dv}{dy}$

So $v + y\dfrac{dv}{dy} = 2v + v\log v$ [½]

$\Rightarrow \quad y\dfrac{dv}{dy} = v(1+\log v) \Rightarrow \int\dfrac{dv}{v(1+\log v)} = \int\dfrac{dy}{y}$

$\Rightarrow \quad \ell n|1+\log v| = \ell n y + \ell n c$

[1]

$$\Rightarrow \quad \ell n\left|\frac{1+\log v}{y}\right| = \ell n c \quad \Rightarrow \quad \frac{1+\log\left(\frac{x}{y}\right)}{y} = c$$

$\Rightarrow \quad 1+\log\left(\frac{x}{y}\right) = cy$ is the required solution. [1]

17. Solve the following differential equation : $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x$ **[CBSE 2010, 4M]**

Sol. $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x$

$$\int dy = \int \frac{2x^2+x}{(x^2+1)(x+1)}dx$$ [1]

by partial fraction

$$\frac{2x^2+x}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{(x+1)} = \frac{(Ax+B)(x+1)+C(x^2+1)}{(x^2+1)(x+1)}$$ [1]

by comparing

$2 = A + C,\ A + B = 1,\ B + C = 0$

$A - C = 1,\ B = -C$

$A = \frac{3}{2},\ C = \frac{1}{2},\ B = -\frac{1}{2}$ [1]

Hence $\int dy = \frac{3}{2}\int\frac{x\,dx}{x^2+1} - \frac{1}{2}\int\frac{dx}{x^2+1} + \frac{1}{2}\int\frac{dx}{x+1}$

$$= \frac{3}{4}\ell n|x^2+1| - \frac{1}{2}\tan^{-1}x + \frac{1}{2}\ell n|x+1| + C$$ [1]

18. Solve the following differential equation : $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$ **[CBSE 2011, 4M]**

Sol. The given differential equation can be written as

$$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x},\ x \neq 0$$ [½]

$x\frac{dy}{dx} - y = $

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Clearly, it is homogeneous differential equation.

Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in it, we get [½]

$$\Rightarrow \quad v + x\frac{dv}{dx} = \sqrt{1+v^2} + v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \sqrt{1+v^2} \quad \Rightarrow \quad \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$ [1]

Integrating both sides, we get

$$\int\frac{1}{\sqrt{1+v^2}}dv = \int\frac{1}{x}dx$$

$\Rightarrow \quad \log|v+\sqrt{1+v^2}| = \log|x| + \log C$ [1]

$\Rightarrow \quad |v+\sqrt{1+v^2}| = |Cx|$

$\Rightarrow \quad \left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right| = |Cx| \qquad [\because v = y/x]$

$\Rightarrow \quad \left(y+\sqrt{x^2+y^2}\right)^2 = C^2x^4$ [1]

which is required solution

19. Solve the following differential equation : $x\,dy - (y + 2x^2)dx = 0$ **[CBSE 2011, 4M]**

Sol. $x\,dy - (y + 2x^2)dx = 0$

$\frac{dy}{dx} = \frac{y+2x^2}{x}$

$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = 2x$ [1]

It is linear differential equation.

$\therefore \quad \text{I.F.} = e^{\int -\frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$ [1]

Now $y\frac{1}{x} = \int 2x\frac{1}{x} + c$ [1]

$\frac{y}{x} = 2x + c$ [1]

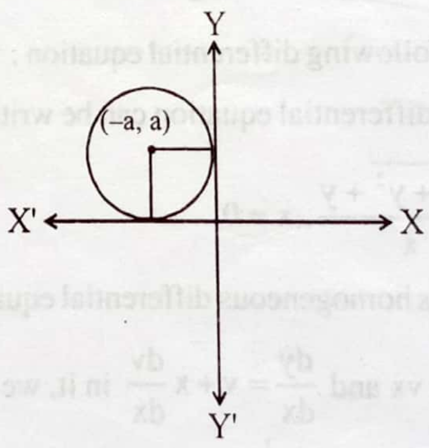
20. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes. **[CBSE 2012, 4M]**

Sol. Let C denote the family of circles in the second quadrant and touching the coordinate axes. Let (-a, a) be the coordinate of the centre of any member of this family

Equation representing the family C is

$(x + a)^2 + (y - a)^2 = a^2 \quad ....(1)$

or $\quad x^2 + y^2 + 2ax - 2ay + a^2 = 0 \quad ....(2)$ [1]



Differentiating w.r.t. x

$2x + 2y\frac{dy}{dx} + 2a - 2a\frac{dy}{dx} = 0$ [½]

or $\quad x + y\frac{dy}{dx} = a\left(\frac{dy}{dx} - 1\right)$

or $\quad a = \frac{x + y\,y'}{y' - 1} \qquad \left[\text{Let } \frac{dy}{dx} = y'\right]$ [½]

Substituting the value of a in equation (1), we get

$$\left[x + \frac{x+y\,y'}{y'-1}\right]^2 + \left[y - \frac{x+y\,y'}{y'-1}\right]^2 = \left[\frac{x+y\,y'}{y'-1}\right]^2$$ [1]

or $\quad [x\,y' - x + x + y\,y']^2 + [y\,y' - y - x - y\,y']^2 = [x + y\,y']^2$ [½]

or $\quad (x + y)^2\,y'^2 + [x + y\,]^2 = [x + y\,y']^2 \;$ or $\; (x + y)^2\,[(y')^2 + 1] = [x + y\,y']^2$ [½]

which is the differential equation representing the given family of circles.

21. Find the particular solution of the differential equation $x(x^2-1)\frac{dy}{dx} = 1$; $y = 0$ when $x = 2$

[CBSE 2012, 4M]

Sol. Given differential equation $x(x^2-1)\frac{dy}{dx} = 1$; $y = 0$ when $x = 2$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x(x^2-1)} \Rightarrow \int dy = \int \frac{1}{x(x^2-1)}dx$$

$$\Rightarrow y = \int \frac{1}{x(x^2-1)}dx \quad ......\text{(i)}$$ [½]

Let $\frac{1}{x(x^2-1)} = \frac{1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}$ [½]

When $x = 0$ then $A = -1$

When $x = -1$ then $B = \frac{1}{2}$

When $x = 1$ then $C = \frac{1}{2}$

$$\therefore \frac{1}{x(x^2-1)} = -\frac{1}{x} + \frac{1}{2}\cdot\frac{1}{x+1} + \frac{1}{2}\cdot\frac{1}{x-1}$$ [½]

So using equation (i)

$$y = \int\left(-\frac{1}{x} + \frac{1}{2}\frac{1}{x+1} + \frac{1}{2}\frac{1}{x-1}\right)dx$$

$$y = -\log x + \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) + C$$ [1]

$$y = -\log x + \frac{1}{2}[\log(x+1) + \log(x+1)] + C$$

$$y = \frac{1}{2}[-2\log x + \log(x+1) + \log(x-1)] + C$$ [½]

$$y = \frac{1}{2}[-\log x^2 + \log(x+1)(x-1)] + C$$

$$y = \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) + C$$ [½]

$$\because \; y = 0 \text{ when } x = 2 \Rightarrow C = -\frac{1}{2}\log\left(\frac{3}{4}\right)$$

$$\because \; y = \frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right) - \frac{1}{2}\log\left(\frac{3}{4}\right)$$ [½]

22. Solve the differential equation $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ **[CBSE 2014, 4M]**

Sol. $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$$

Let $P = \frac{1}{1+x^2}$ and $Q = \frac{e^{\tan^{-1}x}}{1+x^2}$ [½]

$IF = e^{\int P dx} = e^{\int \frac{1}{1+x^2}} = e^{\tan^{-1}x}$ [1]

Solution of D.E.

$$y \times e^{\tan^{-1}x} = \int \frac{e^{\tan^{-1}x}}{1+x^2} \times e^{\tan^{-1}x} dx$$ [1]

$$\Rightarrow \quad ye^{\tan^{-1}x} = \int \frac{(e^{\tan^{-1}x})^2}{1+x^2} dx$$

Put $\tan^{-1}x = t$

$$\frac{1}{1+x^2} dx = dt$$ [½]

$$\Rightarrow \quad ye^{\tan^{-1}x} = \int e^{2t} dt$$

$$= \frac{e^{2t}}{2} + C$$

$$\Rightarrow \quad ye^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + C$$ [1]

23. Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$. **[CBSE 2014, 4M]**

(1+y) + x + xy
1 + y + x(1+y)
(1+x)(1+y)

Sol. $\frac{dy}{dx} = (1+x) + y(1+x)$

or, $\frac{dy}{dx} = (1+y)(1+x)$

or, $\frac{dy}{1+y} = (1+x)dx$

$$\int \frac{dy}{1+y} = \int (1+x)dx$$

$$\log|1+y| = x + \frac{x^2}{2} + C$$ [2]

given $y = 0$ when $x = 1$

i.e., $\log|1+0| = 1 + \frac{1}{2} + C$

$$\Rightarrow \quad C = -\frac{3}{2}$$ [1]

$\therefore$ The particular solution is

$$\log|1+y| = \frac{x^2}{2} + x - \frac{3}{2}$$

or the answer can expressed as

$$\log|1+y| = \frac{x^2 + 2x - 3}{2}$$

or $1 + y = e^{(x^2+2x-3)/2}$

or, $y = e^{(x^2+2x-3)/2} - 1$ **[1]**

24. Find the particular solution of the differential equation $x(1 + y^2)dx - y(1 + x^2)dy = 0$, given that $y = 1$ when $x = 0$. **[CBSE 2014, 4M]**

Sol. $x(1 + y^2)dx - y(1 + x^2)dy = 0$

$\Rightarrow x(1 + y^2)dx = y(1 + x^2)dy$

$\Rightarrow \int \frac{2y}{1+y^2}.dy \Rightarrow \int \frac{2x}{1+x^2}dx$

By property

$= \log(1 + y^2) = \log(1 + x^2) + \log C \quad \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$ ....(1) **[2]**

when $x = 0$ given that $y = 1$

$\log c = \log 2$ **[1]**

put in equation (1)

$\Rightarrow \log(1 + y^2) = \log(1 + x^2) + \log 2$

$\Rightarrow (1 + y^2) = 2(1 + x^2)$

$\Rightarrow 2x^2 - y^2 + 1 = 0$ **[1]**

25. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that $y = 0$ when $x = 0$. **[CBSE 2014, 4M]**

Sol. $\log\left(\frac{dy}{dx}\right) = 3x + 4y$

$\Rightarrow \frac{dy}{dx} = e^{3x+4y}$

$\Rightarrow \frac{dy}{dx} = e^{3x}.e^{4y}$

$\Rightarrow \int e^{-4y}\,dy = \int e^{3x}\,dx$

$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$ ...(i) **[2]**

Given that $y = 0$ when $x = 0$

$C = \frac{-1}{4} - \frac{1}{3} = \frac{-7}{12}$ putting Eq. (i) **[1]**

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

$3e^{-4y} + 4e^{3x} = 7$ **[1]**

26. Find the particular solution of the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$ given that $x = 0$ when $y = 1$. **[CBSE 2016, 4M]**

Sol. $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$

$\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}}$

Given D.E. is homogenous D.E.

put x = vy

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$ [1]

$$v + y.\frac{dv}{dy} = \frac{2vye^{vy/y} - y}{2ye^{vy/y}}$$

$$v + y.\frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}}$$

$$y\frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}} - v$$

$$y\frac{dv}{dy} = \frac{2ve^{v} - 1 - 2ve^{v}}{2e^{v}}$$

$$y\frac{dv}{dy} = \frac{-1}{2e^{v}}$$

$$2\int e^{v}.dv = -\int\frac{dy}{y}$$ [1]

$2e^{v} = -\log y + C$

$2e^{x/y} = \log y + C$ .....(1) [1]

when x = 0, y = 1

1 = 0 + c, c = 2

$e^{x/y} = \log y + 2$ [1]

27. Find the particular solution of differential equation : $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$ given that y = 1 when x = 0.

[CBSE 2016, 4M]

Sol. $\frac{dy}{dx} = -\frac{x + y\cos x}{1 + \sin x}$

$$\frac{dy}{dx} + \left(\frac{\cos x}{1 + \sin x}\right)y = -\frac{x}{1 + \sin x}$$ [1]

Given D.E. is identical with

Linear D.E. $\frac{dy}{dx} + Py = Q$

where $P = \frac{\cos x}{1 + \sin x}$ and $Q = -\frac{x}{1 + \sin x}$

$\text{I.F.} = e^{\int P dx}$

$= e^{\int \frac{\cos x}{1 + \sin x} dx}$

$= e^{\log(1 + \sin x)} = 1 + \sin x.$ [1]

solution is $y \times \text{I.F.} = \int (Q \times \text{I.F.})dx$

$$\Rightarrow \quad y(1+\sin x) = \int -\frac{x}{1+\sin x} \times (1+\sin x)dx$$

$$\Rightarrow \quad y(1+\sin x) = -\frac{x^2}{2} + C \quad .....(1)$$ **[1]**

When $x = 0, y = 1$

$\Rightarrow \quad c = 1$

$$y(1+\sin x) = -\frac{x^2}{2} + 1$$ **[1]**

Long Answer : [6 Marks]

28. Find the particular solution of the differential equation $\frac{dx}{dy} + x\cot y = 2y + y^2 \cot y, \ (y \neq 0)$, given that $x = 0$ when $y = \frac{\pi}{2}$. **[CBSE 2013, 6M]**

Sol. $\frac{dx}{dy} + x\cot y = 2y + y^2 \cot y$

$\int \cot x = \log(\sin x)$?

form of $\frac{dx}{dy} + Px = Q$

$\sqrt{2x}$

where $P = \cot y, \ \ Q = 2y + y^2 \cot y$ **[½ + ½]**

$$I.F = e^{\int P.dy} = e^{\int \cot y dy} = e^{\log \sin y} = \sin y$$ **[1]**

Solution of D.E. is

$$x \times IF = \int (Q \times IF).dy$$

$$x \sin y = \int (2y + y^2 \cot y)\sin y.dy$$ **[1]**

$$= \int (2y \sin y + y^2 \cos y)dy$$ **[½]**

$$= \int 2y \sin y dy + y^2 \sin y - \int 2y \sin y dy$$ **[1]**

$x\sin x = y^2 \sin y + C \quad ....(i)$ **[½]**

at $x = 0, \ y = \frac{\pi}{2}, \ C = -\frac{\pi^2}{4}$ Put in equation (i) **[½]**

$$x\sin x = y^2 \sin y - \frac{\pi^2}{4}$$ **[½]**

29. Find the particular solution of the differential equation $(\tan^{-1}y - x)dy = (1+y^2)dx$, given that $x = 0$ when $y = 0$. **[CBSE 2013, 6M]**

Sol. $$\frac{dx}{dy} = \frac{\tan^{-1}y - x}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$$

$$\frac{dx}{dy} + Px = Q$$

where $P = \frac{1}{1+y^2}$ & $Q = \frac{\tan^{-1} y}{1+y^2}$ [½ + ½]

$$\text{I.F.} = e^{\int P.dy} = e^{\int \frac{1}{1+y^2}.dy} = e^{\tan^{-1} y}$$ [1]

Solution of given Q.E. is

$$x \times \text{I.F.} = \int (Q \times \text{I.F.}).dy$$ [1]

$$\Rightarrow x \times e^{\tan^{-1} y} = \int \frac{\tan^{-1} y}{1+y^2} \times e^{\tan^{-1} y}.dy$$ [½]

Put $\tan^{-1} y = t$

$$\frac{1}{1+y^2}.dy = dt$$ [½]

$$\Rightarrow xe^{\tan^{-1} y} = \int t.e^t dt$$

$$= t.e^t - \int e^t.dt$$ [1]

$$xe^{\tan^{-1} y} = t.e^t - e^t + C$$

$$xe^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + C \quad ......(1)$$

at $x = 0$, $y = 0$

$C = 1$ put inequation (1)

$$xe^{\tan^{-1} y} = e^{\tan^{-1} y}(\tan^{-1} y - 1) + 1$$ [1]

30. Show that the differential equation $(x - y)\frac{dy}{dx} = x + 2y$ is homogeneous and solve it also.

[CBSE 2015, 6M]

OR

Find the differential equation of the family of curves $(x - h)^2 + (y - k)^2 = r^2$, where h and k are arbitrary constants. **[CBSE 2015, 6M]**

Sol. $(x - y)\frac{dy}{dx} = x + 2y$

I-अ

$$\frac{dy}{dx} = \frac{x + 2y}{x - y}$$

Let $f(x, y) = \frac{x + 2y}{x - y}$

put $x = \lambda x$ and $y = \lambda y$

$$f(\lambda x, \lambda y) = \frac{\lambda x + 2\lambda y}{\lambda x - \lambda y}$$

$$f(\lambda x, \lambda y) = \lambda^{o}\left(\frac{x+2y}{x-y}\right)$$

$$f(\lambda x, \lambda y) = \lambda^{o} f(x, y)$$ [1]

Hence D.E. is zero degree homogenous D.E.

Now

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\text{Put} \quad \left.\begin{aligned} y &= vx \\ \frac{dy}{dx} &= v + \frac{xdv}{dx} \end{aligned}\right\}$$ [1]

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx}$$

$$v + x\frac{dv}{dx} = \frac{1+2v}{1-v}$$

$$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1+v+v^2}{1-v}$$ [1]

$$\Rightarrow \quad \int \frac{1-v}{1+v+v^2} dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int \left[\frac{(2v+1)-3}{v^2+v+1}\right] dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\int \frac{2v+1}{v^2+v+1} dv - \frac{3}{2}\int \frac{1}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dv = -\int \frac{dx}{x}$$ [1]

$$= \frac{1}{2}\log(v^2+v+1) - \frac{3}{2} \times \frac{2}{\sqrt{3}} \tan^{-1}\frac{2v+1}{\sqrt{3}} = -\log x + C$$ [1]

$$= \frac{1}{2}\log\left(\frac{y^2}{x^2} + \frac{y}{x} + 1\right) - \sqrt{3}\tan^{-1}\left(\frac{2y/x+1}{\sqrt{3}}\right) = -\log x + C$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{y^2+xy+x^2}{x^2}\right) - \sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = -\log x + C$$

$$\Rightarrow \quad \log(x^2+xy+y^2)^{1/2} - \sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right) = C$$ [1]

OR

$(x-h)^2 + (y-k)^2 = r^2$ ....(i)

Differentiating (i) w.r.t. x, we get

$$2(x-h) + 2(y-k)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (x-h) + (y-k)\frac{dy}{dx} = 0 \quad \text{.........(ii)}$$

[1]

Differentiating (ii) w.r.t. x, we get

[1]

$$1 + (y-k)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \quad \text{.........(iii)}$$

From (iii), we get

[1]

$$y - k = -\frac{1 + (dy/dx)^2}{d^2y/dx^2}$$

Putting the value of $(y-k)$ is (ii), we obtain

[1]

$$x - h = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}\frac{dy}{dx}}{d^2y/dx^2}$$

Substituting the values of $(x-h)$ and $(y-k)$ in (i), we get

[1]

$$\frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^2\left(\frac{dy}{dx}\right)^2}{(d^2y/dx^2)^2} + \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^2}{(d^2y/dx^2)^2} = r^2$$

[1]

$$\Rightarrow \quad \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$$

This the required differential equation.

Completed

1M

1000,000

NCERT IMPORTANT QUESTIONS	
Examples	5, 7, 16, 20, 28
Exercise # 9.1	1, 12
Exercise # 9.2	6
Exercise # 9.4	4, 10
Exercise # 9.5	6, 8, 11, 13
Exercise # 9.6	5, 10, 15
Miscellaneous Exercise	6, 9, 11, 14