MISCELLANEOUS EXAMPLES

Very Short Answer : [1 Mark]

1. Write the direction cosines of a line equally inclined to the three coordinate axes **[CBSE 2009, 1M]**

Sol. $\ell^2 + m^2 + n^2 = 1$

given $\ell = m = n$

$3n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{3}}$

d.c's are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ and $\left(-\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$ **[1]**

2. Write the distance of the following plane from the origin : $2x - y + 2z + 1 = 0$ **[CBSE 2010, 1M]**

Sol. $d = \left|\frac{2\times0 - 1\times0 + 2\times0 + 1}{\sqrt{4+1+4}}\right| = \frac{1}{3}$ **[1]**

3. Write the vector equation of a line given by $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. **[CBSE 2011, 1M]**

Sol. $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{j}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$ **[1]**

4. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin **[CBSE 2012, 1M]**

Sol. Distance of plane $3x - 4y + 12z = 3$ from the origin

$= \left|\frac{-3}{\sqrt{9+16+144}}\right| = \frac{3}{13}$ **[1]**

5. Find the length of the perpendicular drawn from the origin to the plane $2x - 3y + 6z + 21 = 0$. **[CBSE 2013, 1M]**

Sol. $L = \left|\frac{21}{\sqrt{4+9+36}}\right|$

$\Rightarrow L = \left|\frac{21}{7}\right|$

$L = 3$ unit **[1]**

6. If the cartesian equations of a line are $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, write the vector equation for the line. **[CBSE 2014, 1M]**

Sol. Cartesian eqn of line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$

we can write it as $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

So vector eqn is $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$ where λ is constant **[1]**

7. Find the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$. **[CBSE 2015, 1M]**

Sol. $\frac{x}{1/2} = \frac{y}{1/3} = \frac{z}{-1}$ and $\frac{x}{1/6} = \frac{y}{-1} = \frac{z}{-1/4}$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{\frac{1}{12} - \frac{1}{3} + \frac{1}{4}}{\sqrt{\frac{1}{4}+\frac{1}{9}+1}\sqrt{\frac{1}{36}+1+\frac{1}{16}}} = 0$$

$$\theta = \frac{\pi}{2}$$ [1]

8. Write the sum of intercepts cut off by the plane $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})-5=0$ on the three axes.

[CBSE 2016, 1M]

Sol. $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})=5$

$2x+y-z=5$

$$\frac{x}{5/2}+\frac{y}{5}+\frac{z}{-5}=1$$ [½]

Sum of intercepts $=\frac{5}{2}+5-5=\frac{5}{2}$ [½]

Short Answer : [4 Marks]

9. Find the length and the foot of the perpendicular drawn from the point (2, –1, 5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

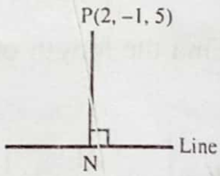
[CBSE 2008, 4M]

Sol. Given equation of line is

$$\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}=\lambda \text{ (say)} \quad \text{..... (i)}$$

Let N be the foot of perpendicular from the point P(2, –1, 5) to the line (1)

∴ coordinate of N is $(10\lambda+11, -4\lambda-2, -11\lambda-8)$ [1]



Direction ratios of PN is

$10\lambda+11-2, -4\lambda-2+1, -11\lambda-8-5$

$= 10\lambda+9, -4\lambda-1, -11\lambda-13$ [1]

and direction ratios of line (1) is 10, –4, –11 since PN ⊥ to line (1)

$10(10\lambda+9)-4(-4\lambda-1)-11(-11\lambda-13)=0 \quad [\because a_1a_2+b_1b_2+c_1c_2=0]$

$100\lambda+90+16\lambda+4+121\lambda+143=0$

$237\lambda+237=0$

$\lambda=-1$ [1]

Hence N ≡ (1,2,3)

∴ Foot of perpendicular is

$PN=\sqrt{(2-1)^2+(-1-2)^2+(5-3)^2}$

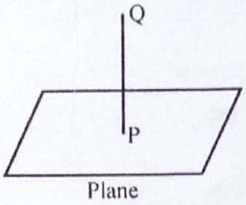
$=\sqrt{1+9+4}=\sqrt{14}$ unit [1]

10. Find the coordinates of the point where the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}$ meets the plane $x+y+4z=6$.

[CBSE 2008, 4M]

Sol. Given equation of line is

$$\frac{x+1}{2}=\frac{y+2}{3}=\frac{z+3}{4}=\lambda(\text{say})$$



$\therefore$ Any point on the line is

$P(2\lambda-1, 3\lambda-2, 4\lambda-3)$ **[1]**

Since the point P lies on the plane

$x+y+4z=6$

$\therefore$ $(2\lambda-1)+(3\lambda-2)+4(4\lambda-3)=6$ **[1]**

$2\lambda-1+3\lambda-2+16\lambda-12=6$

$21\lambda=21 \Rightarrow \lambda=1$ **[1]**

$\therefore$ Required point is

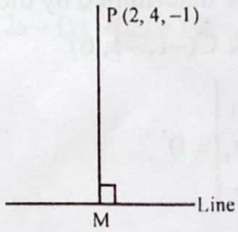
$P(2-1, 3-2, 4-3)=P(1,1,1)$ **[1]**

11. Find the equation of the perpendicular drawn from the point (2, 4, –1) to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

[CBSE 2008, 4M]

Sol. Given equation of line is

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}=\lambda(\text{say}) \quad ....(1)$$



Any point on the line (1) is

$M=(\lambda-5, 4\lambda-3, -9\lambda+6) \quad ....(2)$

dr's of line PM is

$\lambda-5-2, 4\lambda-3-4, -9\lambda+6+1$

$=\lambda-7, 4\lambda-7, -9\lambda+7$ **[1]**

Direction ratio of the given line are proportional to 1, 4, –9.

$\because$ line PM is perpendicular to line (1)

$\therefore$ $1(\lambda-7)+4(4\lambda-7)+(-9)(-9\lambda+7)=0$ **[1]**

$[\because a_1a_2+b_1b_2+c_1c_2=0]$

$\lambda-7+16\lambda-28+81\lambda-63=0$

$98\lambda-98=0$

$98\lambda=98 \Rightarrow \lambda=1$ **[1]**

Putting λ in eq. (2), we get

$\therefore$ Coordinate of $M=(1-5, 4-3, -9+6)=(-4, 1, -3)$

Equation of PM is

$$\frac{x-2}{-4-2}=\frac{y-4}{1-4}=\frac{z+1}{-3+1} \quad \left[\because \frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}\right]$$

$$\frac{x-2}{-6}=\frac{y-4}{-3}=\frac{z+1}{-2}$$

$$\Rightarrow \frac{x-2}{6}=\frac{y-4}{3}=\frac{z+1}{2}$$ **[1]**

12. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P(1,3,3).

[CBSE 2010, 4M]

Sol. $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}=\lambda$

Any point on the line will be

$(3\lambda-2, 2\lambda-1, 2\lambda+3)$ [1]

Given that

$$\sqrt{(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2}=5$$ [1]

$\Rightarrow 9\lambda^2-18\lambda+9+4\lambda^2-16\lambda+16+4\lambda^2=25$

$\Rightarrow 17\lambda^2-34\lambda=0$ [1]

$\Rightarrow \lambda^2-2\lambda=0 \Rightarrow \lambda=0, 2$

Hence the points are (–2, –1, 3) or (4, 3, 7) [1]

OR

Find the distance of the point P(6,5,9) from the plane determined by the points A(3,–1,2), B(5,2,4) and C(–1,–1,6).

Sol. The equation of the plane determined by the points A(3, –1, 2), B(5, 2, 4) & C(–1, –1, 6)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$ [1]

$$\Rightarrow \begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix}=0$$ [1]

$\Rightarrow 12(x-3)-(y+1)16+(z-2)12=0$

$\Rightarrow 3x-9-4y-4+3z-6=0$

$\Rightarrow 3x-4y+3z-19=0$ [1]

$$d=\frac{|3\times6-4\times5+3\times9-19|}{\sqrt{9+16+9}}=\frac{3\sqrt{34}}{17}$$ [1]

13. Find the equation of the plane passing through the point P(1,1,1) and containing the line $\vec{r}=\left(-3\hat{i}+\hat{j}+5\hat{k}\right)+\lambda\left(3\hat{i}-\hat{j}-5\hat{k}\right)$. Also, show that the plane contains the line $\vec{r}=\left(-\hat{i}+2\hat{j}+5\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}-5\hat{k}\right)$. **[CBSE 2010, 4M]**

Sol. Equation of the required plane

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a & b & c \end{vmatrix}=0$$

where $(x_1,y_1,z_1) \Rightarrow (1,1,1)$

$(x_2,y_2,z_2) \Rightarrow (-3,1,5)$

$(a,b,c) \Rightarrow (3,-1,-5)$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ -4 & 0 & 4 \\ 3 & -1 & -5 \end{vmatrix} = 0$$

$4(x-1)-8(y-1)+4(z-1)=0$

$x-2y+z=0$ is the required plane. [2]

The given line be $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(\hat{i}-2\hat{j}-5\hat{k})$

$\vec{a}=(-1, 2, 5)$, $\vec{b}=(1, -2, -5)$

Condition for the given line to lie in the plane

$x-2y+z=0$

point $(-1, 2, 5)$ satisfies the plane and $\vec{b}\cdot\hat{n}=0$

So $ax'+by'+cz'+d=0$ & $a\ell+bm+cn=0$

$-1-2\times2+5=0$ & $1\times1-2\times-2-5\times1=0$ [2]

14. Find the shortest distance between the following lines whose vector equations are :

$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$ **[CBSE 2011, 4M]**

Sol. $\vec{r}=(\hat{i}-2\hat{j}+3\hat{k})+t(-\hat{i}+\hat{j}-2\hat{k})$

$\vec{r}=(\hat{i}-\hat{j}-\hat{k})+s(\hat{i}+2\hat{j}-2\hat{k})$

we know shortest distance between lines

$\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ & $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ is

$$d=\left|\frac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$$ [1]

$\vec{a}_2-\vec{a}_1=\hat{j}-4\hat{k}$ [½]

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$ [1]

$=2\hat{i}-4\hat{j}-3\hat{k}$

$|b_1\times b_2|=\sqrt{29}$ [½]

$(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=-4+12=8$

$\therefore\quad d=\dfrac{8}{\sqrt{29}}$ [1]

15. Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane. **[CBSE 2012, 4M]**

Sol. The vector equation of the line through the points A(3, 4, 1) and B(5, 1, 6) is

$\vec{r}=3\hat{i}+4\hat{j}+\hat{k}+\lambda[(5-3)\hat{i}+(1-4)\hat{j}+(6-1)\hat{k}]$

$\vec{r}=(2\lambda+3)\hat{i}+(-3\lambda+4)\hat{j}+(5\lambda+1)\hat{k}$ ..... (i) [1]

Find the vector equation of the plane through the points (2, 1, –1) and (–1, 3, 4) and perpendicular to the plane x – 2y + 4z = 10. **[CBSE 2013, 4M]**

Sol. Equation of plane which passes through the point (2, 1, –1)

$a(x-2) + b(y-1) + c(z+1) = 0$ ....(i) [½]

plane (i) passes through (–1, 3, 4)

$a(-1-2) + b(3-1) + c(4+1) = 0$

$-3a + 2b + 5c = 0$ ....(ii) [½]

Required plane & given plane both are perpendicular

So $a - 2b + 4c = 0$ ....(iii) [½]

then equation (ii) & (iii)

$$\frac{a}{8+10} = \frac{-b}{-12-5} = \frac{c}{6-2}$$

$$\frac{a}{18} = \frac{b}{17} = \frac{c}{4} = \lambda$$ [1]

$a = 18\lambda,\ b = 17\lambda,\ c = 4\lambda$ put in equation (i)

$18(x-2) + 17(y-1) + 4(z+1) = 0$

$18x + 17y + 4z - 36 - 17 + 4 = 0$

$18x + 17y + 4z - 49 = 0$ [½]

$\vec{r}.(18\hat{i} + 17\hat{j} + 4\hat{k}) = 49$ [1]

Aliter :

If the points are A(2, 1, –1) and B(–1, 3, 4),

then vector normal to the plane is $\overline{AB} \times (\hat{i} - 2\hat{j} + 4\hat{k})$ [1]

i.e. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 5 \\ 1 & -2 & 4 \end{vmatrix} = 18\hat{i} + 17\hat{j} + 4\hat{k}$ [1½]

Equation of plane is

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$

$\vec{r}.(18\hat{i} + 17\hat{j} + 4\hat{k}) = 36 + 17 - 4$

$\vec{r}.(18\hat{i} + 17\hat{j} + 4\hat{k}) = 49$ [1½]

17. Find the vector and cartesian equations of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$. **[CBSE 2014, 4M]**

Sol. Let D.R. of line L which passes through the point. (2,1,3) is a_1 , a_2, a_3

Given lines are $L_1 \Rightarrow \frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

and $L_2 \Rightarrow \frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$

line $L \perp L_1$

$a_1 + 2a_2 + 3a_3 = 0$ ....(i)

and line $L \perp L_2$

$-3a_1 + 2a_2 + 5a_3 = 0$ ....(ii) **[1]**

from equation (i) and (ii)

$\frac{a_1}{10-6}=\frac{a_2}{-9-5}=\frac{a_3}{2+6} \Rightarrow \frac{a_1}{4}=\frac{a_2}{-14}=\frac{a_3}{8} \Rightarrow \frac{a_1}{2}=\frac{a_2}{-7}=\frac{a_3}{4}$ **[1]**

So cartisian equation of required line is

$\frac{x-2}{2}=\frac{y-1}{-7}=\frac{z-3}{4}$ **[1]**

and vector equation of line is

$\vec{r}=(2\hat{i}+\hat{j}+3\hat{k})+\lambda(2\hat{i}-7\hat{j}+4\hat{k})$ **[1]**

18. A line passes through (2, –1, 3) and is perpendicular to the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$. Obtain its equation is vector and Cartesian form. **[CBSE 2014, 4M]**

Sol. Line L is passing through point

$(2\hat{i}-\hat{j}+3\hat{k})$

if $L_1 \Rightarrow \vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$

$L_2 \Rightarrow \vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$

given that line L is perpendicular to L_1 and L_2

Let dr of line L = a_1, a_2, a_3

The eqn of L in vector form $\Rightarrow \vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+k(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$

k is any constant.

L_1 is perpendicular to L $a_1a_2+b_1b_2+c_1c_2=0$

$2a_1 - 2a_2 + a_3 = 0$ ... (1) **[1]**

and also

$L \perp L_2$

so, $a_1 + 2a_2 + 2a_3 = 0$ ....(2) **[1]**

from equation (1) and (2)

$2a_1 - 2a_2 + a_3 = 0$

$a_1 + 2a_2 + 2a_3 = 0$

$$\frac{a_1}{-4-2}=\frac{a_2}{1-4}=\frac{a_3}{4+2}$$

$$\frac{a_1}{-6}=\frac{a_2}{-3}=\frac{a_3}{6}$$

$$\frac{a_1}{-2}=\frac{a_2}{-1}=\frac{a_3}{2}$$

So direction ratio of line L is (–2, –1, 2) **[1]**

Hance vectors equation of a line $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+k(-2\hat{i}-\hat{j}+2\hat{k})$ and cartesian equation of a line is

$$\frac{x-2}{-2}=\frac{y+1}{-1}=\frac{z-3}{2}$$ **[1]**

19. Find the value of p, so that the lines $l_1 : \frac{1-x}{3}=\frac{7y-14}{p}=\frac{z-3}{2}$ and $l_2 : \frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other. Also find the equations of line passing through a point (3,2 –4) and parallel to line l_1. **[CBSE 2014, 4M]**

Sol. Given lines written as

$$\ell_1 \Rightarrow \frac{x-1}{-3}=\frac{y-2}{p/7}=\frac{z-3}{2}$$

$$\ell_2 \Rightarrow \frac{x-1}{\frac{-3p}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}$$ **[1]**

ℓ_1 and ℓ_2 due perpendicular to each other

So $a_1a_2+b_1b_2+c_1c_2=0$

$$\Rightarrow (-3)\times\left(\frac{-3p}{7}\right)+\left(\frac{p}{7}\right)\times 1+2\times(-5)=0$$

$\Rightarrow p=7$ **[2]**

Now equation of a line passing through a point (3, 2, –4) and parallel the line ℓ_1

$$\frac{x-3}{-3}=\frac{y-2}{1}=\frac{z+4}{2}$$ **[1]**

20. Find the shortest distance between the following lines :

$$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$$

$$\vec{r}=(2\hat{i}+4\hat{j}+5\hat{k})+\mu(4\hat{i}+6\hat{j}+8\hat{k})$$

[CBSE 2015, 4M]

OR

Find the equation of the plane passing through the line of intersection of the planes $2x+y-z=3$ and $5x-3y+4z+9=0$ and is parallel to the line $\frac{x-1}{2}=\frac{y-3}{4}=\frac{5-z}{-5}$. **[CBSE 2015, 4M]**

Sol. $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

$\vec{r}=(2\hat{i}+4\hat{j}+5\hat{k})+2\mu(2\hat{i}+3\hat{j}+4\hat{k})$

Let $\vec{a}_1=\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ **[1]**

$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ and $|\vec{b}| = \sqrt{4+9+16} = \sqrt{29}$ **[1]**

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 2 & 2 \end{vmatrix}$$

$= \hat{i}(6-8) - \hat{j}(4-4) + \hat{k}(4-3)$

$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = -2\hat{i} - 0\hat{j} + \hat{k}$ **[1]**

$|\vec{b} \times (\vec{a}_2 - \vec{a}_1)| = \sqrt{4+1} = \sqrt{5}$

$$D = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \frac{\sqrt{5}}{\sqrt{29}}$$ **[1]**

OR

Let $P_1 = 2x + y - z - 3 = 0$

$P_2 = 5x - 3y + 4z + 9 = 0$

Required plane is $P_1 + \lambda P_2 = 0$

$\Rightarrow (2x + y - z - 3) + \lambda(5x - 3y + 4z + 9) = 0$

$\Rightarrow (2 + 5\lambda)x + (1 - 3\lambda)y + (-1 + 4\lambda)z - 3 + 9\lambda = 0$ ....(1) **[1½]**

plane (1) parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

$\therefore$ Normal of plane (1) is

perpendicular to the line $(a_1a_2 + b_1b_2 + c_1c_2) = 0$

$\Rightarrow (2 + 5\lambda) \times 2 + (1 - 3\lambda) \times 4 + (-1 + 4\lambda) \times 5 = 0$

$\Rightarrow \lambda = -\frac{1}{6}$, put is equation (1) **[1]**

$$\left(2 - \frac{5}{6}\right)x + \left(1 + \frac{1}{2}\right)y + \left(-1 - \frac{2}{3}\right)z - 3 - \frac{3}{2} = 0$$

$\Rightarrow \frac{7}{6}x + \frac{3}{2}y - \frac{5}{3}z - \frac{9}{2} = 0$

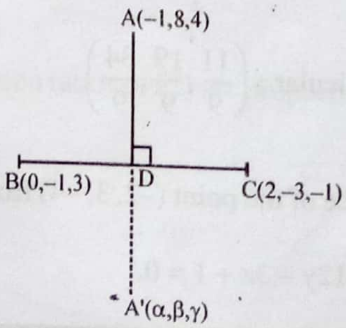
$\Rightarrow 7x + 9y - 10z - 27 = 0$ **[1½]**

21. Find the coordinates of the foot of perpendicular drawn from the point A(–1, 8, 4) to the line joining the points B(0, –1, 3) and C(2, –3, –1). Hence find the image of the point A in the line BC.

[CBSE 2016, 4M]

Sol. Equation of BC

$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = \lambda$ **[1]**



Let foot of $\perp$ is

$D(2\lambda, -2\lambda - 1, -4\lambda + 3)$

D. ratio of line AD$(2\lambda + 1, -2\lambda - 9, -4\lambda - 1)$ **[1]**

D. ratio of line BC (2, –2, –4)

AD $\perp$ BC

$\therefore 2(2\lambda + 1) - 2(-2\lambda - 9) - 4(-4\lambda - 1) = 0$

$24\lambda + 24 = 0$

$\lambda = -1$

D(–2, 1, 7) [1]

Let A'(α, β, γ) is a image of A.

$\therefore \quad \frac{-1+\alpha}{2} = -2; \ \frac{8+\beta}{2} = 1; \ \frac{4+\gamma}{2} = 7$

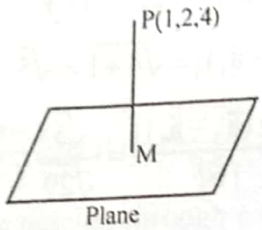
$\alpha = -3; \beta = -6, \gamma = 10, \ A'(-3, -6, 10)$ [1]

Long Answer : [6 Marks]

22. From the point P(1, 2, 4), perpendicular is drawn on the plane $2x + y - 2z + 3 = 0$. Find the equation, the length and the coordinates of the foot of the perpendicular. **[CBSE 2008, 6M]**

Sol. Given equation of plane is

$2x + y - 2z + 3 = 0 \qquad .....(1)$



dr's of the plane are 2, 1, –2

∴ dr's of line normal to plane are 2, 1, –2

Equation of line PM is

$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = \lambda(\text{say})$ [1]

$\left[\because \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}\right]$

∴ Coordinate of M = $(2\lambda + 1, \lambda + 2, -2\lambda + 4)$

Since point M lies on the plane (1)

$\therefore \quad 2(2\lambda + 1) + (\lambda + 2) - 2(-2\lambda + 4) + 3 = 0$

$4\lambda + 2 + \lambda + 2 + 4\lambda - 8 + 3 = 0$ [1]

$9\lambda - 1 = 0 \quad \Rightarrow \quad \lambda = \frac{1}{9}$

∴ Foot of perpendicular $= \left(\frac{2}{9}+1, \frac{2}{9}+2, \frac{-2}{9}+4\right) = \left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

Length of perpendicular from (1, 2, 4)

$PM = \left|\frac{2(1)+2-2(4)+3}{\sqrt{4+1+4}}\right| = \left|\frac{2+2-8+3}{\sqrt{9}}\right| = \frac{1}{3}$ unit [2]

Hence, equation of line : $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2}$ [1]

Length of perpendicular $= \frac{1}{3}$ unit

foot of perpendicular $= \left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$ [1]

23. Find the distance of the point (–2, 3, –4) from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$. **[CBSE 2008, 6M]**

Sol. Let the given point be P(–2, 3, –4) and equation of given line be

$$\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}=\lambda(\text{say})$$

$\therefore$ Any point on the line be

$$Q\left(3\lambda-2,\frac{4\lambda-3}{2},\frac{5\lambda-4}{3}\right)$$ [1]

dr's of the line PQ is

$$3\lambda-2+2,\ \frac{4\lambda-3}{2}-3,\ \frac{5\lambda-4}{3}+4$$

$$=3\lambda,\ \frac{4\lambda-3-6}{2},\ \frac{5\lambda-4+12}{3}$$

$$=3\lambda,\ \frac{4\lambda-9}{2},\ \frac{5\lambda+8}{3}$$ [1]

Since line PQ is parallel to the plane

$4x + 12y - 3z + 1 = 0$

$$\therefore\ 4(3\lambda)+12\left(\frac{4\lambda-9}{2}\right)-3\left(\frac{5\lambda+8}{3}\right)=0$$

$[\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$

$12\lambda + 24\lambda - 54 - 5\lambda - 8 = 0$ [1]

$31\lambda = 62 \Rightarrow \lambda = 2$ [1]

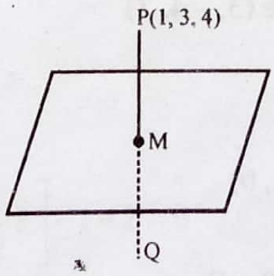
$$\therefore\ \text{The coordinate of } Q=\left(6-2,\frac{8-3}{2},\frac{10-4}{3}\right)=\left(4,\frac{5}{2},2\right)$$

$$\therefore\ \text{Length PQ is } \sqrt{(4+2)^2+\left(\frac{5}{2}-3\right)^2+(2+4)^2}$$ [1]

$$=\sqrt{36+\frac{1}{4}+36}=\sqrt{72+\frac{1}{4}}=\sqrt{\frac{289}{4}}=\frac{17}{2}$$ [1]

24. Find the coordinates of the image of the point (1, 3, 4) in the plane $2x-y+z+3=0$. **[CBSE 2008, 6M]**

Sol. Let Q be the image of the point P(1, 3, 4) in the plane $2x - y + z + 3 = 0$ .....(1)



Then PQ is normal to the plane. Therefore, direction ratios of PQ are proportional to the dr's of plane 2, –1, 1. [1]

$\therefore$ Equation of line PQ is

$$\frac{x-1}{2}=\frac{y-3}{-1}=\frac{z-4}{1}=\lambda(\text{say})$$ [1]

$\therefore$ The coordinate of $Q = (2\lambda + 1, -\lambda + 3, \lambda + 4)$

Since M is mid-point of line PQ

$\therefore$ The coordinate of

$$M = \left(\frac{2\lambda+1+1}{2}, \frac{-\lambda+3+3}{2}, \frac{\lambda+4+4}{2}\right) = \left(\frac{2\lambda+2}{2}, \frac{-\lambda+6}{2}, \frac{\lambda+8}{2}\right)$$ [1]

Since M lies on the plane (1)

$$\therefore \quad 2\left(\frac{2\lambda+2}{2}\right) - \left(\frac{-\lambda+6}{2}\right) + \left(\frac{\lambda+8}{2}\right) + 3 = 0$$ [1]

$$\frac{4\lambda+4+\lambda-6+\lambda+8+6}{2} = 0$$ [1]

$6\lambda + 12 = 0 \Rightarrow \lambda = -2$

$\therefore$ Coordinate of $Q = (-4 + 1, 2 + 3, -2 + 4)$ [1]

$= (-3, 5, 2)$

25. Find the equation of the plane determined by the point A(3, –1, 2), B(5, 2, 4) and C(–1, –1, 6). Also find the distance of the point P(6, 5, 9) from the plane. **[CBSE 2009, 6M]**

Sol. A (3, –1, 2); B (5, 2, 4); C (–1, –1, 6)

$\overrightarrow{AB} = 2\vec{i} + 3\vec{j} + 2\vec{k}$

$\overrightarrow{AC} = -4\vec{i} + 0\vec{j} + 4\vec{k}$ [1]

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ = normal vector of plane.

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 12\hat{i} - (8+8)\hat{j} + (12)\hat{k}$$

$\vec{n} = 12\hat{i} - 16\hat{j} + 12\hat{k}$ [1]

$\Rightarrow$ dr's of the normal to the plane are (3, – 4, 3)

$\Rightarrow$ Equation of plane is given by

$3x - 4y + 3z = d$

this plane passes through (–1, –1, 6)

$\Rightarrow$ d = 19

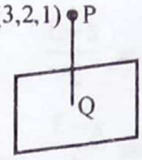
Equation of plane is $3x - 4y + 3z - 19 = 0$ [2]

$$\text{distance d from } (6, 5, 9) = \left|\frac{3.6 - 4.5 + 3.9 - 19}{\sqrt{3^2 + 4^2 + 3^2}}\right|$$

$$= \left|\frac{18 - 20 + 27 - 19}{\sqrt{34}}\right| = \frac{6}{\sqrt{34}}$$ [2]

26. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(3,2,1) from the plane $2x - y + z + 1 = 0$. Find also the image of the point in the plane. **[CBSE 2010, 6M]**

Sol. Let Q be the foot of perpendicular



$\Rightarrow$ equation of PQ is $\frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = \lambda$

coordinates of Q is $3 + 2\lambda, 2 - \lambda, 1 + \lambda$ **[1]**

point Q also lie in the given plane.

$\Rightarrow 2(3 + 2\lambda) - (2 - \lambda) + 1 + \lambda + 1 = 0$

$\Rightarrow 6 + 4\lambda - 2 + \lambda + 1 + \lambda + 1 = 0$

$\Rightarrow 6\lambda = -6 \Rightarrow \lambda = -1$ **[1]**

Point Q is (1,3,0).

Let the image of the point P in the plane be P'.

Image of point P is (–1,4,–1) **[1]**

$PQ = \sqrt{(1-3)^2 + (3-2)^2 + (0-1)^2}$

$PQ = \sqrt{4+1+1} = \sqrt{6}$ **[1]**

27. Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis. **[CBSE 2011, 6M]**

Sol. Equation of plane passing through intersection of

planes $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$

is $\left[\vec{r}.(\hat{i} + \hat{j} + \hat{k}) - 1\right] + \lambda\left[\vec{r}.(2\hat{i} + 3\hat{j} - \hat{k}) + 4\right] = 0$ **[1½]**

$\vec{r}.\left[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1-\lambda)\hat{k}\right] - 1 + 4\lambda = 0$ ....(1)

If plane (1) is parallel to x-axis then

$1 + 2\lambda = 0$

$\lambda = -\frac{1}{2}$ **[1]**

$\therefore$ required plane is $\vec{r}.\left[-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right] - 1 - 2 = 0$ **[1½]**

$\Rightarrow \vec{r}.\left[-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right] - 1 - 2 = 0$

$\Rightarrow \vec{r}.\left[-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k}\right] = 3 \Rightarrow -\frac{1}{2}y + \frac{3}{2}z = 3$

$\Rightarrow -y + 3z = 6$ **[2]**

28. If the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines. **[CBSE 2012, 6M]**

Sol. Given lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ ..... (i)

$\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ ..... (ii)

$\because$ Line (i) $\perp$ line (ii)

$\Rightarrow -3k-2k+10=0$

$-5k=-10$

$k=2$ [2]

Equation of planes containing the given lines

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ -3 & 2k & 2 \\ k & 1 & 5 \end{vmatrix}=0$$ [1]

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$ [2]

$-22(x-1)+19(y-2)+5(z-3)=0$

$\Rightarrow 22x-19y-5z+31=0$ [2]

29. Find the equation of plane passing through the line of intersection of the plane $\vec{r}.(\hat{i}+3\hat{j})-6=0$ and $\vec{r}.(3\hat{i}-\hat{j}-4\hat{k})=0$, whose perpendicular distance from origin is unity. **[CBSE 2013, 6M]**

OR

Find the vector equation of the line passing through the point (1, 2, 3) and parallel to the plane $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5$ and $\vec{r}.(3\hat{i}+\hat{j}+\hat{k})=6$.

Sol. Required plane is $P_1+\lambda P_2=0$

$\left[\vec{r}.(\hat{i}+3\hat{j})\right]-6+\lambda[\vec{r}.(3\hat{i}-\hat{j}-4\hat{k})]=0$ [1]

$\vec{r}.[(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}+(0-4\lambda)\hat{k}]=6$ ....(i) [1]

Perpendicular distance from origin is units

$$\left|\frac{\vec{a}.\vec{n}-p}{|\vec{n}|}\right|=1$$

$$\left|\frac{(0\hat{i}+0\hat{j}+0\hat{k}).(1+3\lambda)\hat{i}+(3-\lambda)\hat{j}+(0-4\lambda)\hat{k}-6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(0-4\lambda)^2}}\right|=1$$ [1]

$\Rightarrow\ 36=1+9\lambda^2+6\lambda+9+\lambda^2-6\lambda+16\lambda^2$

$\Rightarrow\ 36-10=26\lambda^2 \Rightarrow\ 26\lambda^2=26$

$\lambda=\pm1$ [1]

equation of plane at $\lambda = 1$ (put in equation (i))

$\vec{r}.[4\hat{i}+2\hat{j}-4\hat{k}]-6=0$ [1]

equation of plane at $\lambda = -1$ (put in equation (i)

$\vec{r}.[-2\hat{i}+4\hat{j}+4\hat{k}]-6=0$ [1]

OR

Equation of the line passing through the point (1, 2, 3) is

$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$ ......(i) [1]

equation (i) is parallel to the given planes

So $a - b + 2c = 0$ ....(ii) [1]

$3a + b + c = 0$ ....(iii) [1]

from equation (ii) & (iii)

$$\frac{a}{-1-2}=\frac{b}{6-1}=\frac{c}{1+3}$$

$$\frac{a}{-3}=\frac{b}{5}=\frac{c}{4}$$ [1]

Hence the required line is $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(-3\hat{i}+5\hat{j}+4\hat{k})$ [2]

Aliter :

Equation of the line passing through the point (1, 2, 3) is

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda\vec{b}$ ......(i)

$\vec{b}=\vec{n}_1\times\vec{n}_2=(\hat{i}-\hat{j}+2\hat{k})\times(3\hat{i}+\hat{j}+\hat{k})$ [2]

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 3 & 1 & 1\end{vmatrix}=-3\hat{i}+5\hat{j}+4\hat{k}$$ [2]

$\vec{b}=-3\hat{i}+5\hat{j}+4\hat{k}$ (put in equation (i))

Hence the required line is $\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(-3\hat{i}+5\hat{j}+4\hat{k})$ [2]

30. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin. **[CBSE 2014, 6M]**

OR

Find the distance of the point (2, 12, 5) from the point of intersection of the line $\vec{r}=2\hat{i}-4\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ and the plane $\vec{r}.(\hat{i}-2\hat{j}+\hat{k})=0$.

Sol. Eqn of given planes are

$P_1 \Rightarrow x + y + z - 1 = 0$

$P_2 \Rightarrow 2x + 3y + 4z - 5 = 0$

Eqn of plane through the line of intersection of planes P_1, P_2 is

$P_1 + \lambda P_2 = 0$

$(x + y + z - 1) + \lambda (2x + 3y + 4z - 5) = 0$ .... (1) [2]

$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 5\lambda) = 0$

given that plane represented by eqn (1) is perpendicular to plane

$x - y + z = 0$

so we use formula $a_1a_2 + b_1b_2 + c_1c_2 = 0$

so $(1 + 2\lambda).1 + (1 + 3\lambda).(-1) + (1 + 4\lambda).1 = 0$

$1 + 2\lambda - 1 - 3\lambda + 1 + 4\lambda = 0$

$3\lambda + 1 = 0$

$\lambda = \frac{-1}{3}$ [2]

Put $\lambda = -\frac{1}{3}$ in eqn (1) so we get

$$\left(1-\frac{2}{3}\right)x + (1-1)y + \left(1-\frac{4}{3}\right)z + \frac{2}{3} = 0$$

$$\frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$$

$x - z + 2 = 0$ [1]

Distance of the plane from the origin $= \left|\frac{2}{\sqrt{1^2+1^2}}\right| = \frac{2}{\sqrt{2}} = \sqrt{2}$ [1]

OR

General points on the line:

$x = 2 + 3\lambda,\ y = -4 + 4\lambda,\ z = 2 + 2\lambda$

The equation of the plane :

$\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$

The point of intersection of the line and the plane :

$\left[(2+3\lambda)\hat{i} + (-4+4\lambda)\hat{j} + (2+2\lambda)\hat{k}\right].(\hat{i} - 2\hat{j} + \hat{k}) = 0$ [2]

$(2+3\lambda).1 + (-4+4\lambda)(-2) + (2+2\lambda).1 = 0$

$12 - 3\lambda = 0$ or, $\lambda = 4$ [1]

$\therefore$ the point of intersection is :

$(2 + 3(4), -4 + 4(4), 2 + 2(4)) = (14, 12, 10)$ [1]

Distance of this point from (2, 12, 5) is

$= \sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2}$

$= \sqrt{12^2 + 5^2}$

= 13 Ans. [2]

31. Find the equation of a plane passing through the point P(6, 5, 9) and parallel to the plane determined by the points A(3, –1, 2), B(5, 2, 4) and C(–1, –1, 6). Also find the distance of this plane from the point A.

[CBSE 2015, 6M]

Sol. Let equation of a plane which passes through (6, 5, 9)

is $a(x-6)+b(y-5)+c(z-9)=0$ .....(1)

Equation of a plane which passes through A(3, –1, 2), B(5, 2, 4), C(–1, –1, 6)

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix}=0$$

$\Rightarrow \quad (x-3)\times 12-(y+1)\times 16+(z-2)\times 12=0$

$\Rightarrow \quad 12x-16y+12z-76=0$ ......(2) **[2]**

Plane (1) and plane (2) is parallel.

So required plane is $12(x-6)-16(y-5)+12(z-9)=0$

$12x-16y+12z-100=0$ ......(3) **[2]**

distance of this plane (3), from the point A(3, –1, 2)

$$D=\left|\frac{(12\times 3)-(16\times -1)+(12\times 2)-100}{\sqrt{144+256+144}}\right|$$

$$D=\left|\frac{-24}{\sqrt{544}}\right|=\frac{24}{\sqrt{544}}=\frac{24}{\sqrt{16\times 34}}=\frac{6}{\sqrt{34}}$$ **[2]**

Aliter :

Normal vector of plane which passes through, three points

$\vec{n}=\overline{AB}\times\overline{AC}$

$=(2\hat{i}+3\hat{j}+2\hat{k})\times(-4\hat{i}+0\hat{j}+4\hat{k})$

$$=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix}$$

$=12\hat{i}-16\hat{j}+12\hat{k}$ **[2]**

Required plane is parallel to the given plane

So $\vec{r}.\vec{n}=\vec{a}.\vec{n}$

$\vec{r}.(12\hat{i}-16\hat{j}+12\hat{k})=100$ ......(1) **[2]**

Distance of this plane (1) from the point (3, –1, 2)

$$D=\left|\frac{(3\hat{i}-\hat{j}+2\hat{k}).(12\hat{i}-16\hat{j}+12)-100}{\sqrt{144+256+144}}\right|$$

$$D=\frac{24}{\sqrt{544}}$$

$$\Rightarrow \frac{24}{\sqrt{16\times 34}}=\frac{24}{4\sqrt{34}}=\frac{6}{\sqrt{34}}$$ **[2]**

32. Find the equation of the plane which contains the line the intersection of the planes $\vec{r}.(\hat{i}-2\hat{j}+3\hat{k})-4=0$ and $\vec{r}.(-2\hat{i}+\hat{j}+\hat{k})+5=0$ and whose intercept on x-axis is equal to that of on y-axis.

[CBSE 2016, 6M]

Sol. Required plane $P_1+\lambda P_2=0$

$$\left[\vec{r}.(\hat{i}-2\hat{j}+3\hat{k})-4\right]+\lambda\left[\vec{r}.(-2\hat{i}+\hat{j}+\hat{k})+5\right]=0$$

$$\Rightarrow \quad \vec{r}.\left[(1-2\lambda)\hat{i}+(-2+\lambda)\hat{j}+(3+\lambda)\hat{k}\right]=4-5\lambda \qquad \text{........(1)}$$ [2]

x-axis intercept = y-axis intercept

$$\frac{1-2\lambda}{4-5\lambda}=\frac{-2+\lambda}{4-5\lambda}$$ [1]

$1-2\lambda=-2+\lambda$

$3\lambda=3$

$\lambda=1$ put in equation (1) [rejecting $\lambda=4/5$] [1]

$\vec{r}.(-\hat{i}-\hat{j}+4\hat{k})=-1$

$\vec{r}.(-\hat{i}-\hat{j}+4\hat{k})+1=0$ [2]

NCERT IMPORTANT QUESTIONS

Examples	11, 21, 22, 24, 25
Exercise # 11.1	4, 5
Exercise # 11.2	6, 12, 15, 17
Exercise # 11.3	9
Miscellaneous Exercise	6, 7, 10, 13, 14, 15, 17, 18, 19, 20