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unit - I

Sets, Relation & function

* **Set:-** A set is a collection of unordered, distinct elements.

$A, B, C - - - - x, y$

$a, b, c, - - - - x, y$

$X = \{a, b, c, x, y\}$

$P \in A \Rightarrow P$ belongs to A

$P \notin A \Rightarrow P$ doesn't belongs to A

Q. How can we specify the sets, we

we can specify sets in two ways.

1) List the elements of sets

2) State those properties which characterised the elements of sets

Ex: $A_1 = \{2, 4, 6 - - - - \}$

$A_2 = \{x : x$ is a positive even integer, $x > 0\}$

$A_2 = \{2, 4, 6, 8 - - - \}$

Que: $E_1 = \{x : x^2 - 3x + 2 = 0\}$

$E_2 = \{1, 2\}$

Both are equal $E_1 = E_2$

Que: $E_1 = \{x : x^2 - 3x + 2 = 0\} = \{1, 2\}$

$F = \{2, 1\}$

$G = \{1, 2, 2, 1, 6/3\} \Rightarrow$ multisets

All three sets are equal, G is multisets

* Discrete maths: is the study of discrete objects.
Discrete means "distint or not connected

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* Some useful sets:
N = Sets of Natural Numbers
Z = " " Integers
Z+ = " " +ve Integer
R = Real Number — ∞ —— +ve.
continuous value
C = Set of Complex Numbers

* Universal set and Empty set:
The universal set consist of all point in the plane
For Ex: human population studies the universal set consist of all the people in the world.
this set is denoted by U.

* Empty set: It has no element.
for Ex: $E = \{x : x \text{ is a positive integer}, x^2 = 3\}$
$= \phi$

* Subsets:
$A \subset B$
↳ A is subset of B.

& $B \supseteq A$
B is super set of A

$A \not\subset B$ — A is not subset of B

$B \not\supseteq A$ — B is not superset of A

* It is not a branch of maths. It is a rather a description of a set of branch that have one common property that they are discrete not continuous

* Proper subsets:

$A \subseteq B$ & $A \neq B$

A is a proper subset of B

$C = \{1, 3, 2\}$ Defined $A \subset B$

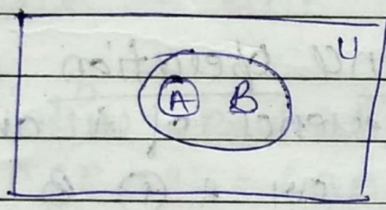
i) Set A is a proper subset of Set B and Set C

ii) Set A and Set B is subset of C but A is the proper subset of C

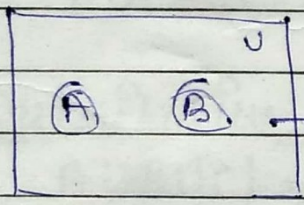
iii) Set B = Set C.

* Ven Diagram:

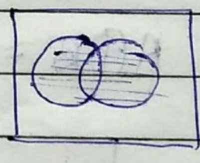
It is the pictorial Representation of the sets in which set are represented by enclosed Area.



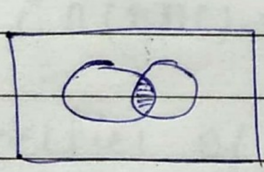
$A \subset B$



Disjoint sets



$A \cup B$



$A \cap B$

* operations on Set :

① union : $(A \cup B)$:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

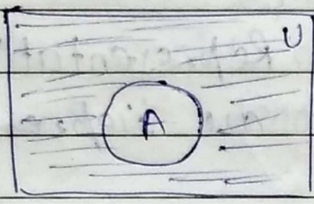
② Intersection : $(A \cap B) = \{x : x \in A \text{ and } x \in B\}$

(3) Complement operations :

let A be the any set and U is the universal set

then $\bar{A}$ or A^c

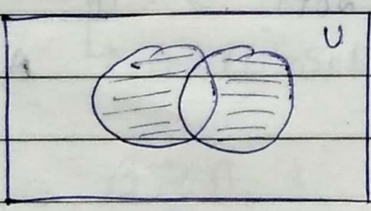
$$\bar{A} = \{x : x \in U \text{ And } x \notin A\}$$



④ Symmetric Difference operation $A \setminus B$:

The symmetric difference of A and B is denoted by $A \setminus B$ or $A \oplus B$

$$A \oplus B = (A \cup B) \setminus (A \cap B)$$



$A \oplus B$

$$A \setminus B = A \oplus B = (A \cup B) \setminus (A \cap B)$$

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Ex:- $A = \{1, 2, 3, 4, 5, 6\}$ $B = \{4, 5, 6, 7, 8, 9\}$
find symm. difference

$(A \cup B) = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$(A \cap B) = \{4, 5, 6\}$
$(A \oplus B) = \{1, 2, 3, 7, 8, 9\}$
$= (A - B) \cup (B - A)$ } Both are correct

* Algebric properties of Set :-

(1) Idempotent Law :
a) $A \cup A = A$
b) $A \cap A = A$

2) Commutative properties :
a) $A \cup B = B \cup A$
b) $A \cap B = B \cap A$

3) Associative properties :-
a) $A \cap (B \cap C) = (A \cap B) \cap C$
b) $A \cup (B \cup C) = (A \cup B) \cup C$

4) Distributive property :-
a) $A \cup (B \cup C) = (A \cup B) \wedge (A \cup C)$
b) $A \cap (B \cap C) = (A \cap B) \cup (A \cap C)$

5) Properties of Complement :-
a) $\bar{(\bar{A})} = A$
b) $A \cup \bar{A} = U$
c) $A \cap \bar{A} = \phi$
d) $\bar{\phi} = U$
e) $\bar{U} = \phi$
f) $\overline{A \cap B} = \bar{A} \cup \bar{B}$ } De-morgan's
g) $\overline{A \cup B} = \bar{A} \cap \bar{B}$ } Law.

6) Properties of universal set :
a) $A \cup U = U$
b) $A \cap U = A$

(7) Properties of empty set :
a) $A \cup \phi = A$
b) $A \cap \phi = \phi$

(8) Absorption properties :
a) $A \cup (A \cap B) = A$
b) $A \cap (A \cup B) = A$

* Cartisian Product of the sets :
Let A and B are the two sets cartisian product or cross product of these are denoted by $A \times B$

$$A \times B = \{(a, b) ; a \in A, b \in B\}$$

cartisian product of 2 same sets A and A.

$A \times A = A_2$

$A \times A \times A = A_3$

$A \times A \times A \times \ldots \times A = A_n = \{(a_1, a_2 \ldots a_i)\}$ $i = n$

Example : Let $A = \{a, b\}$
$B = \{a, c, d\}$
check $A \times B$ or which of the following statement is true
$A \times B = B \times A$
$A \times B \neq B \times A$

$(A \times B) = \{(a,a)(a,c)(a,d)(b,a)(b,c)(b,d)\}$
$(B \times A) = \{(a,a)(a,b)(c,a)(c,b)(d,b)(d,a)\}$

* finite Sets :
A Set is to be the finite set if it Contains Exactly n distinct elements where n is non-negative integer and n is said to be the 'cardanality' of the set.

this set is also known as numerator set and this Cardanality can be define as

$$\boxed{\text{Card}(A) = |A|}$$

* Inclusion - Exclusion Principle :
let A and B are the two sets then no. of elements in (AUB) or n(AUB).

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

which means include n(A) & n(B) and Exclude n(A∩B)

for three finite sets A, B, and C. then,

$$\boxed{n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)}$$

we include
n(A), n(B), n(C) and n(A∩B∩C) and
Exclude n(A∩B), n(B∩C), n(C∩A)

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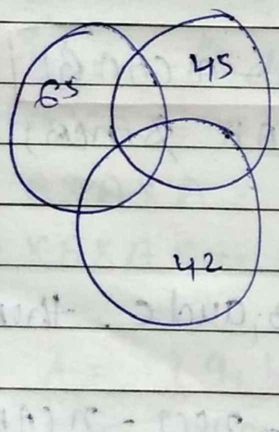
Que: Find the no. of maths students at a college taking Atleast one Language: French, german and Russian from the following data:-

65 - French
45 - German
42 - Russian
20 - French & German
25 - French & Russian
150 - German & Russian
8 = all the three Language

Soln: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 65 + 45 + 42 - 20 - 25 - 15 + 8$
$= \cancel{140} \; \cancel{-45} \quad 152 - 60 + 8$
$= 100$ Ans



$A \subseteq P(A)$
X

Let A be the finite set, then set of all possible subset of A is called Powerset of A.

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Power set :

For a given finite sets in the power set is denoted by 2^n.
where n is the no. of distinct element

So, no. of element $S = 2^{n(S)}$

$S = 2$

$\{1\ 2\ 3\}$

Power set : $\{1, 2, 3\}$

$S = \{1\}, \{2\} \{3\} \{1, 2\} \{1, 3\} \{2, 3\} \{1, 2, 3\}$
$\{\phi\}$

Let S be the finite sets then Partition of S is the Sub-division of S into non-overlaping, non-empty subsets, non empty Subsets of S.

Such That

(i) Each a_i in S belongs to one of the A_i

The sets of $\{A_i\}$ are mutually disjoints

now, non-overlapping, non-empty and all combine

$A_i = A_j$ then $A_i \cap A_j = \phi$

Ex! Let $S = \{1, 2, 3 - - - 8, 9\}$

(i) $\{1, 3, 8\} \{2, 6\} \{4, 8, 9\}$ X → is not there.

(ii) $\{1, 3, 5\} \{2, 4, 6, 8\} \{5, 7, 9\}$ X 5 two times come

(iii) $\{1, 3, 5\} \{2, 4, 6, 8\} \{7, 9\}$ ✓ All correct

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Que: In a Survey of 60 people it was found that
25 read Newsweek magazine
26 " Times "
26 " fortune "
9 both Newsweek and fortune
11 " " " times
8 " times $ fortune
3 all the three magazine

find:-
a) find the no. of people who read at least one of the three magazines.
b) filling the correct no. of elements in Each of 8 design in ven diagram.
c) find the no. of people who read Exactly one magazine

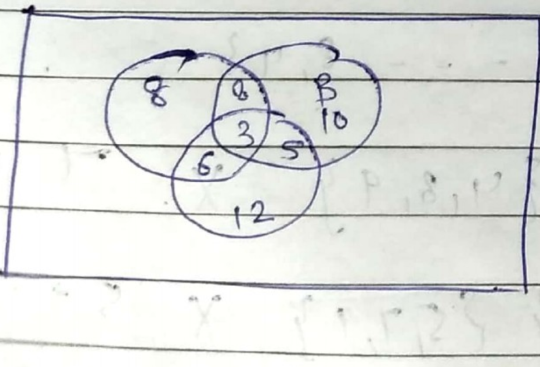
Sol^n: i) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

$= 25 + 26 + 26 - 9 - 11 - 8 + 3$

$= 77 - 28 + 3$

$= 80 - 28 = 52$ Ans

ii)



(C) $10 + 12 + 8 = 30$

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8 Reads N & T but not F
5 Reads T & F not NW
6 " N & F Not T
8 Reads only N
10 Reads " T
12 Reads " F
5 Reads all three.

Que: In a class of 80 students
50 students know English Language.
55 " " French "
46 " " German "
37 " " Eng. & F.
28 " " F and G.
25 " " E and G.
7 " " None of the Language

find out
a) How many students know all the three Lang.
b) " " " " Exactly two Lang.
c) " " " " only one Lang.

Sol^n: $(A\cup B\cup C) = 80 - 7 = 73$

$\cancel{73 = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)}$

(c) 73 =

$\cancel{73 = 50 + 55 + 46 - 37 - 28 - 25 + x}$

$n(A\cap B) + n(A\cap C) + n(B\cap C) - 3x\,(A\cap B\cap C)$

$= 37 + 28 + 25 - 3\times 12$

$90 - 36 = 54$

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* multisets : repeats sets

Ex:

{1, 1, 1, 2, 2, 3, 4} → {3.1, 2.2, 1.3, 1.4}

{1, 1, 2, 2, 3, 4}

Multisets is an unordered collection of elements where an elements occur multiple sets.

$$S = \{n_1 a_1, n_2 a_2, n_3 a_3 - - n_i a_i\}$$

$$i = 1, 2, 3 - - - n.$$

where n_1 is occurance of a_1

* Operations:

(1) union of multisets:

$A = \{1, 1, 1, 2, 2, 3\}$

$B = \{1, 1, 4, 3, 3\}$

$A \cup B = \{1, 1, 1, 2, 2, 3, 3, 4\}$

Intersection:

$(A \cap B) = \{1, 1, 3\}$

Subtraction:

$A - B = \{1, 2, 2\}$

$A + B = \{1, 1, 1, 1, 1, 2, 2, 3, 3, 3, 4\}$

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* Relations :-

Let A and B are the two sets then a Relation is a Subset of A×B.

where

(a,b) come from Cartesian Product

Suppose R is a Relation from A to B then R is a ordered pair where 1st Element of the set A and 2nd element comes from the set B.

* Domain and Range :

$Dom(R) = \{x \mid x \in A \ (x,y) \in R \ \& \ y \in B\}$

$Range(R) = \{y \mid y \in B \ for \ (x,y) \in R \ \& \ x \in A\}$

Ex :- $A = \{1, 2, 3\}$

$R = A \times A$ $\quad R \Rightarrow a < b$

$A \times A = \{(1,2)(1,3), (2,3)\}$

Domain $\{(1,2)\}$

Range $\{(2,3)\}$

Ex: R = y is square of x

$Z = \{-2, -1, 0, 1, 2\}$

R is a relation on Z $\quad R = Z \times Z$

$R = \{ \cdots (-2,4), (-1,1) \ (0,0) \ (1,1) \ (2,4) \cdots \}$

x y

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Que: Find the total no. of distinct Relation from set A to set B.

Set A, set B are finite sets

$2^{n \times n} = 2^{n^2}$ if set A and set B are same.

Power set =

If A has ~~[illegible]~~ m no. of elements and B has n no. of element.

then

$Pow(A \times B) = 2^{m \times n}$ distinct subsets

there are $2^{m.n}$ distinct relation from A to B.

* **Some operation on the Relation:** R S

i) $x(R \cap S)y = (xRy) \cap (xSy)$

ii) $x(R \cup S)y = (xRy) \cup (xSy)$

iii) $x(R - S)y = (xRy) - (xSy)$

eg: $R = \{(x,a), (x,b), (y,c)\}$

$S = \{(x,a)\ (x,b)\ (y,c)\}$

$R \cap S = \{(x,b)\ (y,c)\}$

$R \cup S = \{(x,a), (x,b), (y,c)\}$

$R - S = \{(x,a)\}$

* Representation of a Relation :-

let $A = \{1, 2, 3\}$

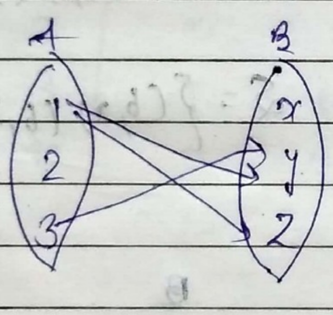
$B = \{x, y, z\}$

$R = \{(1, y), (1, z) (3, y)\}$

i) Using matrix Representation :

$$R = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \end{array}$$

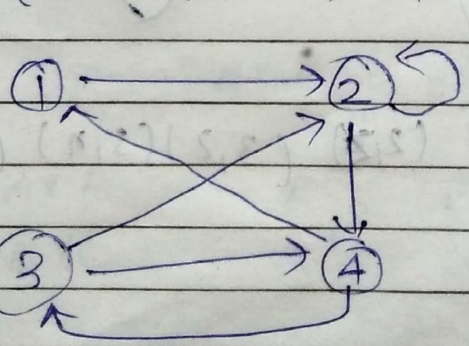
(ii) Using Arrow diagram :



* Directed Graph of a Relation :

Set $A = \{1, 2, 3, 4\}$

$A_2 = A^2 = A \times A = R = \{(1,2), (2,2), (2,4) (3,2) (3,4) (4,1) (4,3)\}$



Direction Graph.

We can represent the relation in form of graph.

* Composition of a Relation :-

A B & C

let R be the Relation $A \times B$

S " " " $B \times C$

then we can define Composite Relation $R \circ S$ from set $A \times C$

$a(R \circ S)c = \{(a,c) :$ there exist $b \in B$ & $c \in C$

$(a,b) \in R$

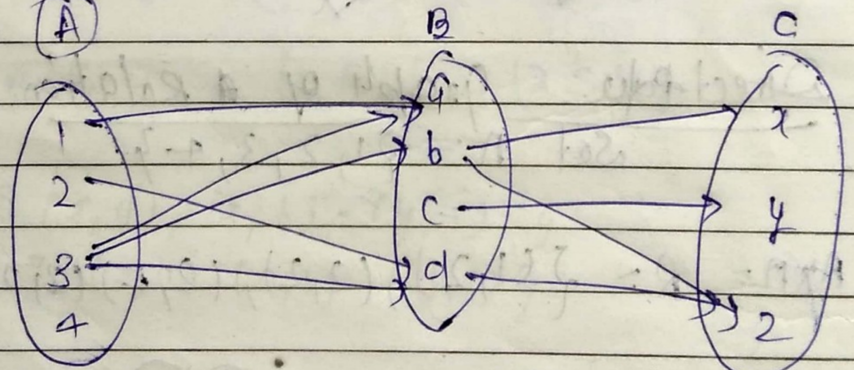
$(b,c) \in S \}$

Ex: let $A = \{1, 2, 3, 4\}$ let $R = \{(1,a)(2,d)(3,a),$
$B = \{a, b, c, d\}$ $(3,b)(3,d)\}$
$C = \{x, y, z\}$

(I)

$S = \{(b,x)(b,z)(c,y)(d,z)\}$

Find $R \circ S$?



$R \circ S = \{(2,z)(3,z)(3,x)(3,z)\}$

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* Another way of represent of Relation :

$$M_R = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}_{4\times 4} \end{array}$$

$$M_S = \begin{array}{c|c} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{4\times 3} \end{array}$$

$$= \begin{array}{c|c} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \rightarrow \text{means two times}$$

$R \circ S =$

Q. if R is a Relation $A \times B$ and S is a Relation $B \times C$ show that

$$(\underset{A\times B}{R} \circ \underset{B\times C}{S})^{-1} = S^{-1} \circ R^{-1}$$

R : $A \times B$

R	$A \times B$	R^{-1}	$B \times A$
S	$B \times C$	S^{-1}	$C \times B$

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$R \quad A \times B \qquad R^{-1} \quad B \times A$

$S \quad B \times C \qquad S^{-1} \quad C \times B$

if $x R y \quad y S z \Rightarrow$

$x(R \circ S) z$

$z(R \circ S)^{-1} x \quad - (1)$

$z S^{-1} y \,.\, y R^{-1} x \Rightarrow z (S^{-1} \circ R^{-1}) x \quad - (2)$

$(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

* Types of Relation :

(1) Identity Relation :

A Relation R on a set A is said to be Identity Relation

$I = \{ (a,a) : a \in A \}$

Ex: $A = \{1, 2, 3\}$

$R = \{ (1,1), (2,2), (3,3) \}$ – identity rel.

(2) Inverse Relation :

R on A

$(a,b) \in R$

then $(b,a) \in R^{-1}$

$R^{-1} = \{ (b,a) : (a,b) \in R \}$

Ex: $A = \{1, 2, 3\}$

$R^{-1} = \{ (2,1)\ (3,2)\ (3,1) \}$

$(R^{-1})^{-1} = R$

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(3) Reflexive Relation :
A Relation R is said to be Reflexive Relation if for $a \in R$.
$R = \{aRa : \text{for all } a \in A\}$

$= \{(1,1)\ (2,2)\ \cancel{(1,3)}\ (3,1)\ (3,2)\ (3,3)\}$
↳ Reflexive relation.

Q. Consider the following Relation where

(R) (1) $\leq$ on Z (set of integer)

(R) (2) $\subseteq$ (set inclusion) on a Collection C of set

× (3) $\perp$ on the set of Lines in the plane

(R) (4) | (divisibility) on a set of positive integer.

* Irreflexive Relation :
R A.
$(a,a) \notin R$ for every $a \in A$
$A = \{1, 2, 3\}$

$R = \{(1,2)\ (2,1)\ (1,3)\}$

* Non-Reflexive :
A relation which is neither reflexive nor irreflexive Relation

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Ex!

$A = \{1, 2, 3\}$

$R = \{(2,1)(1,1)(2,3)(3,3)\}$

(2,2) isn't coming so, it not refflexive

(1,1) (3,3) is coming, so, it is not irreflexive

So neither refflexive nor irreflexive

* Symmetric Relation!

R A

whenever $(a,b) \in R$ $(b,a) \in R$

$A = \{1, 2, 3\}$

$R = \{(1,1)(1,2)(2,1)(2,3)(3,2)\}$

* Assymetric Relation :

A Relation is said to be Assymetric if

$(a,b) \in R$ then $(b,a) \notin R$

$R = \{(1,2),(2,3),(1,3)\}$

* Anti-Symmetric :

$(a,b) \in R$ & $(b,a) \in R$ then $a = b$

$R = \{(1,1)(2,2)\}$ Symmetric Anti-Symmetric

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$R = \{(1,3)(3,1)(2,3)\}$
neither Symmetric nor Anti-symmetric.

* Transitive Relation :-

A Relation R is said to be Transitive

if (a,b) & $(b,c) \in R$
then $(a,c) \in R$

Que!! find the no. of Reflexive Relation for a given set having n elements.
where n is set of natural numbers.

- $R = A \times A$

Q. How many Reflexive Relation are exist for a set having n elements. $n = 1,2,3 --- 10$

$2^{n(n-1)}$

let R be the Relation $R \in N \times N$

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}_{n \times n} = n^2$$

Calculate Symmetric Relation.

$\frac{n^2 - n}{2} = \frac{n(n-1)}{2}$ $\quad 2^n \times 2^{\frac{n(n-1)}{2}}$

$= 2^n \cdot \frac{(n-1)}{2}$

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* Anti-symmetry Condition

(1) find two numbers in the relation such that $(a,b) \in R$ and $(b,a) \in R$; if no such pair Exist then Relation is antisymmetric

(2) if such pair Exist and a is not opposite b then Relation is not anti-symmetry

(3) Otherwise Antisymmetric.

$R = \{(1,2)\ (1,3)\ (3,1)\ (3,3),\ (3,2)\ (1,4),\ (4,2)\ (3,4)\}$

* A Relation may be symmetric and anti-symmetric at the same time.

* Equivalence Relation :

A Relation R is said to be the eq. Relation if it satisfy the following properties

(i) R is Reflexive
(ii) R is Symmetric
(iii) R is transitive

Question: If R is a Relation define on the sets of ~~R to~~ Z.

$R \in Z \times R$

$R = \{(x,y) : x \in Z, y \in Z, (x-y)$ is divisible by $4\}$

Prove that R is equivalence Relation.

(i) Reflexive Relation:

$R = \frac{x-y}{6}$ = Yes it is Reflexive.

(2) Symmetric Relation:

Let xRy means $(x-y)$ is divisible by 6.

$\Rightarrow -(y-x)$ is also divisible by 6

$\Rightarrow yRx$.

(3) Transitive Relation:

~~xRy means $(x-y)$ is divisible by 6~~

~~$-(y-x)$ is also divisible by 6~~

then

~~xRy, yRx~~ $\Rightarrow (x, x)$

~~$(x-y) - (y-z)$~~

~~$(x-x)$ is also divisible by 6.~~

~~Hence Relation R is equivalence Relation~~

Let xRy and yRz $\in R$

$x-y$ is divisible by 6 — (i)

$y-z$ is also divisible by 6 — (ii)

add (i) and (ii)

$(x-y) + (y-z)$ is divisible by 6.

$(x-z)$ is also divisible by 6

xRz

Hence Relation R is Equivalence Relation

Dated :
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Q. Let N be the set of all Natural Numbers and R be the Relation on the set $N \times N$ define by $(a,b) R (c,d) \Rightarrow ad = bc$.

For all $(a,b), (c,d) \in N \times N$ show that R is equivalence Relation.

Sol^n: (i) Reflexive.

$a R a \quad \in R = N$

$a = a$ ~ Hence Reflexive.

(ii)

Symmetric:

~~(a,b)~~ $a R b$

then $b R a$

Q. Let N be the set of all the Natural Numbers and R be the Relation on $N \times N$ defined by

$(a,b) R (c,d) \Rightarrow ad(b+c) = bc(a+d)$

Show that R is equivalence Relation.

To Prove that Relation equivalence. this Relation is Reflexive, symmetric, and transitive

for Reflexive:

$(a,b) R (a,b) =$

$\Rightarrow ab(b+a) = ba(a+b)$

both are equal

true $\forall (a,b)$

$a,b \in N$

Dated :
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Symmetric Relation

(aRb) True (bRa)

$(a,b) \, R \, (c,d) \Rightarrow ad(b+c) = bc(a+d)$?

$(c,d) \, R \, (a,b) \Rightarrow da(c+b) = cb(d+a)$

$\Rightarrow cb(d+a) = da(c+b)$ $\forall (a,b)$

it is symmetric relation. $a,b \in N$

Transitive Relation :

aRb , $bRc \Rightarrow aRc$

let (a,b), (b,c) and $(e,f) \in N \times N$

$(a,b) \, R(c,d) \Rightarrow ad(b+c) = bc(a+d)$

$(c,d) \, R(e,f) \Rightarrow cf(d+e) = de(c+f)$

$ad(b+c) = bc(a+d)$ & $cf(d+e) = de(c+f)$

$\frac{b+c}{bc} = \frac{a+d}{ad}$ & $\frac{d+e}{de} = \frac{c+f}{cf}$

$\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ & $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ —(1) & $\frac{1}{c} - \frac{1}{d} = \frac{1}{e} - \frac{1}{f}$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f}$

$\frac{b-a}{ab} = \frac{f-e}{ef}$

$ef(b-a) = ab(f-e)$

$efb - efa = abf - abe$

$efb + abe = abf + efa$

$be(a+f) = af(b+e)$

$af(b+e) = be(a+f) \Rightarrow (a,b) \, R \, (e,f)$

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0.01$

Dated :
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Q. $R = x^2+y^2 = a^2+b^2$ (equivalence or not)
$(x,y) \; R \; (a,b)$ Prove that.

Que) Let R be a Relation on a set A Prove that
i) if R is Reflexive then R^{-1} is also Reflexive
ii) R is symmetric if and only if $R=R^{-1}$
iii) R is antisymmetric if and only if
$\boxed{R \cap R^{-1} \subseteq I}$

(i) for Reflexive $R^{-1} = (a,a)$
aRa $\cancel{R \subseteq (R^{-1})^{-1}}$ $(a,a) \in R$
then R^{-1} is $a \, R$
Then $R = R^{-1}$

(ii) R is symmetric if and only if $R = R^{-1}$
$(a,b) \in R$ then $(b,a) \in R^{-1}$
then
$(a,b) \in R \Rightarrow (b,a) \in R$

iii) $R \cap R^{-1} \subseteq I$ $(a,b) \in R$
R is antisymmetric $(b,a) \in R^{-1}$

$(a,b) \in R$, $(b,a) \in R$
then $a=b$

Let R is an antisymmetric relation
Let $(a,b) \in R \cap R^{-1}$
$(a,b) \in R$ & $(a,b) \in R^{-1}$
$(a,b) \in R$ & $(b,a) \in R$
$\boxed{a=b}$
$\boxed{R \cap R^{-1} \subseteq I}$

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* Equivalence class :

let R be an equivalence Relation on a set X. the equivalence class is denoted by [a] of each element of X. is defined as -

$$[a] = \{ b \in X : aRb \}$$

which means all the elements related to a. is called equivalence class of a and b is called Representative of equivalence class.

Ex:- let Z be the set of Integer and R be the Relation defined as Congruent modulo 5.
which means $x \equiv y \pmod 5$
$|x-y|$ is divisible by 5
Find the equivalence class of the Relation R.

~~So, only~~

Sol^n:

$[0] = \{ \ldots -15, -10, -5, 0, 5, 10, 15, \ldots \}$

$[1] = \{ \ldots -14, -9, -4, 1, 6, 11, 16, \ldots \}$ all are partitions

$[2] = \{ \ldots, -13, -8, -3, 2, 7, 12, 17, \ldots \}$

$[3] = \{ \ldots, -12, -7, -2, 3, 8, 13, 18, \ldots \}$

$[4] = \{ \ldots, -11, -6, -1, 4, 9, 14, 19, \ldots \}$

~~[5] = {..., -15, -10, -5, 0, 5, 10, 15, 20, ...}~~

$[0] \cup [1] \cup [2] \cup [3] \cup [4] = Z$

- If we have two equivalence class then these two classes are equal or disjoint. And these are called Partition and collection of all equivalence classes are called quotient set

Q. let $A = \{1, 2, \ldots 9\}$ and R be the Relation defined on $A \times A$ given by
$(a,b) R (c,d) \Rightarrow a+d = b+c$
i) Prove that R is equivalence Relation.
ii) find $[2,5]$ or equivalence classes of $[2,5]$

Sol^n: (i) $A = \{1,2,3,4,5,6,7,8,9\}$ $R \subseteq A \times A$

~~for~~ Reflexive Relation.

$(a,b) R (a,b) \Rightarrow a+b = b+a$

Hence it is equal so, satisfy ~~equivalence~~ relation.

(ii) ~~for~~ Symmetric Relation

$(a,b) R (c,d) \Rightarrow a+d = b+c$

$b+c = a+d$

$(c,d) R (a,b)$ $\Rightarrow c+b = d+a$

so, it is Symmetric Relation

(iii) Transitive Relation

Dated :

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$(2,5)\ R\ (c,d) \Rightarrow a+d = b+c$

$[2] = \{2, 4, 6, 8\}, \cancel{2},$

$[5] = \{5\}$

$(2,5)\ R\ (c,d) \Rightarrow$

$(2,5)\ R\ (1,4)$

$(2,5)\ R\ (2,5)$

$(2,5)\ R\ (3,6)$

$(2,5)\ R\ (4,7)$

$(2,5)\ R\ (5,8)$

$(2,5)\ R\ (6,9)$

Q Let R be the equivalent Relation defined on the set X

$X = \{1, 2, 3, 4, 5, 6\}$ is given by

$R = \{(1,1)\ (1,5), (2,3)\ (2,6), (2,2)\ (3,2)\ (3,3)\ (3,6), (4,4)$
$(5,1)\ (5,5), (6,2)\ (6,3), (6,6)\}$

Find the Partitions of X. $[a] = b \ \forall\ aRb$
$b \in X.$

$[1] = \{1, 5\}$

$[2] = \{3, 2, 6\}$

$\cancel{[3] = \{2, 3, 6\}}$

$[4] = \{4\}$

$= \{[1,5]\ (2,3,6)\ (4)\}$

Dated :
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Que:- let $A = R \times R$ where R is the set of Real No.s defined on a

$$(a,b) \, R \, (c,d) \Rightarrow a^2+b^2 = c^2+d^2$$

~~find~~ (i) Verify that (a,R) is an equivalence relation
(ii) find the equivalence class of the Relation R

Equivalence class:- denoted by $[x]$, first equivalance.

Q. $A = [1,2,3,4,5]$

$R = \{(1,1)(2,2)(3,3)(4,4)(5,5)(1,2)(2,1)(4,5)(5,4)\}$

$[1] = \{1,2\}$
$[2] = \{2,1\}$ or $\{1,2\}$
$[3] = \{3\}$
$[4] = \{4,5\}$
$[5] = \{4,5\}$

called Partitions
3 Partition

All Partition of Relation is equal to A.

$$P_1 \cup P_2 \cup P_3 \cdots P_n = A$$
$$P_1 \cap P_2 \cap P_3 \cdots P_n = \phi$$

Q P_1 P_2
$\{1,2,3\}$ $\{4,5\}$

find Relation

P_1 P_2
$\{1,2,3\}$ $\{4,5\}$
$\times$ $\times$
$\{1,2,3\}$ $\{4,5\}$

* Closure Relation :

The Reflexive closer of an Relation R defined on non-Empty set A is the smallest Reflexive Relation that contain R as a ~~subject~~ subset

For Ex. let R be a Relation

Que: Closure of Relations :
1. Reflexive closure
2. Symmetric "
3. Transitive "

(1) Reflexive closure : (R_1)

$$R_1 = R \cup \Delta$$

$$\Delta = \{ (a,a) : a \in A \}$$

For Ex: $A = \{1, 2, 3\}$

$R = (1,1), (2,1), (1,2), (3,3) \}$

find the Reflexive closure

$$R_1 = \{(1,1), (2,1), (1,2) (3,3)\} \cup \{(1,1), (2,2) (3,3)\}$$

$$= \{(1,1) (2,1), (1,2), (3,3), (2,2)\}$$

Que: find Reflexive closure Relation R

$R = \{ (a,b) : a < b \}$ where $a, b \in Z^+$

find the Reflexive closure

$R_1 = \{ (a,b) : a \leq b \}$ if a and b is equal.

Dated:
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(2) Symmetric closure:
S.C of a Relation R is the smallest symmetric Relation which contain R as a subset.

Symmetric closure of R = $R \cup R^{-1}$

Q. Let Relation R = {(1,2), (1,1) (1,4), (3,4), (2,2)}

A = {1, 2, 3, 4}
Find symmetric closure.

$R = R \cup R^{-1}$
= {(1,2), (1,1) (1,4) (3,4) (2,2) (2,1) (4,1) (4,3)}

(3) Transitive closure:
Que: Let A = {1, 2, 3}
R = {(1,1), (1,3), (2,2), (3,1), (3,2)}

Find Transitive closure
Step-① first write the matrix of the Relation R and suppose it is denoted by M_R.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}_{3\times3}$$

Step-2 Compute different power of M_R using the Boolean matrix multiplication.

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Stop the computation when M_R^n is equal to any of $M_R, M_R^2, M_R^3 \ldots, M_R^{n-1}$

$$M_R^2 = M_R \cdot M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_R^3 = M_R^2 \cdot M_R = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Step-3 Select distinct M_R^n from the computed list of matrices.

for Ex: M_R, M_R^2, M_R^3 are distinct in this case

St. 4 Joint the matrices selected in the step 3 using Boolean OR on the element of matrices. it is denoted by M_R^∞

$$M_R^\infty = M_R \vee M_R^2$$

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$$M_R^{\infty} = M_R \vee M_R^2$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Step-5 Convert M_R^{∞} into Transitive closure R^* or R^{∞}

$= \{ (1,1)\ (1,2)\ (1,3)\ (2,2)\ (3,1)\ (3,2)\ (3,3) \}$.

Que: Let $R = \{(1,2)\ (2,3)\ (3,1)\}$

$A = \{1, 2, 3\}$

Find Reflexive closure, symmetric closure, Transitive closure.

(i) Reflexive closure:-

$R_1 = R \cup \Delta$

$\Delta = \{(a,a) : a \in A\}$

$R_1 = \{(1,2)\ (2,3)\ (3,1),\ (1,1)\ (2,2),\ (3,3)\}$

(ii) Symmetric closure :

$= \{(1,2)(2,3)(3,1)(2,1)(3,2)(1,3)\}$

(iii) Transitive closure :

$$M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_R^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_R^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_R^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_R = M_R^4$$

$$M_R^\infty = M_R \vee \cancel{M_R^4} \; M_R^2 \vee M_R^3$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R^* = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

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Function :

let A and B are two sets, then the rule or correspondance which associate each element of A to a unique elements of set B is called a function

x A
y B

$y = f(x)$

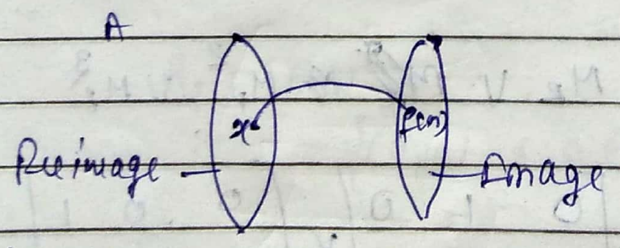
Let x be the general element of set A and y be the general element of set B the $y = f(x)$ be a function

* What is Domain, codomain and Range & Preimage :

Domain : Element of Set A is called Domain

codomain : " " " B " " Codomain

Range : set of all F-images of all elements of Set A is called image set or Range of the function and it is denoted by



* Preimage : It is the value of $f(x)$ in set A.

Q. Let $A = \{-2, -1, 0, 1, 2\}$
and B is the set of integer. $\forall x \in A$ and $f(x) \in B$
$f(x) = x^2$

Sol^n

domain $(f) = \{-2, 1, 0, 1, 2\}$
co-domain $(f) = \{Z\}$
Range $(f) = \{0, 1, 4\}$

$\boxed{\text{Range} \subseteq \text{co-domain}}$

Q. Let $A = \{1, 2, 3, 4\}$
$B = \{1, 2, 3\}$
$f(x) = \{(1,2)\ (2,3)\ (3,3)\ (4,2)\}$

domain $(f) = \{1, 2, 3, 4\}$
Range$(f) = \{2, 3\}$
co-domain $= \{1, 2, 3\}$
Preimage $= \{\cancel{1}, 2, 3, \cancel{4}\}$

* Type of function:
1. One-one function: A function f from $A \to B$ is said to be one-one function (injective) if distinct element of set A have distinct images in set B.
it means that for every $b \in B$
there exist at least
one $a \in A$

$y = |x|$ $f(x) = \pm x$.

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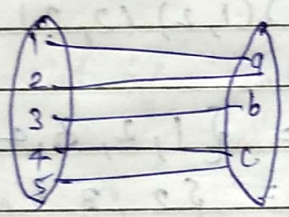
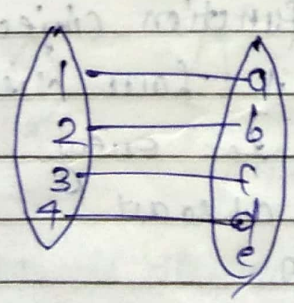
(2) many one f^n:
A f^n $f: A \to B$ is called many one if atleast one Element of Co-domain or Set B as two or more Pre images in set A

(3) onto function: (Surjective):
A f^n $f: A \to B$ is said to be onto function if there is no element in set B which is not an image of some element of A i.e every element of Set B appears as a image of at least one element of set A.



Into f^n:
A f^n $f: A \to B$ is called an into f^n. if there is atleast one element of set B which as no Preimage in set A.



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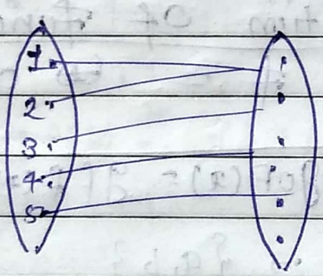
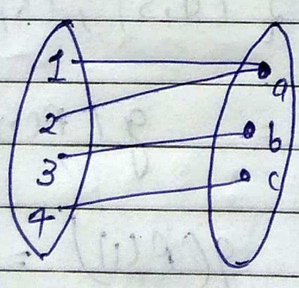
(5) one-one into function :-
it is a function which is Both one-one and into.
i.e different Points in A are join to different point of Set B and there are some points in B which are not connected in A.

(6) one-one onto function (Bijective) :
It is a f^n which is both one-one and onto
i.e different points in A are joined to the diff. points of B and no point in B is left vacent

(7) many one into function :-
A function many one into function is both Many one and into



(8) many one onto function :
Satisfy many one and onto



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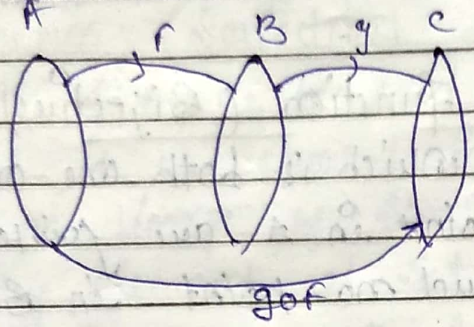
Que set $A = \{1, 2, 3\}$
$B = \{x, y, z\}$

Composition of function :-

Let $f : A \to B$ and $g : B \to C$



then the Composition of function of f and g can be define from set A to set C. and it is denoted by gof.

For a Composition of function for ~~domo~~

$\boxed{\text{codomain of } f = \text{domain of } g}$

$gof(x) = g(f(x))$

Que let $f : \{1, 2, 3\} \to \{a, b\}$
$g : \{a, b\} \to \{5, 6, 7\}$
defined by $f = \{(1, a), (2, a) (3, b)\}$
$g = \{(a, 5), (b, 7)\}$

$(gof)(x) = g(f(x))$

$(gof)(1) = g(f(1)) = g(a) = 5$

$gof(2) = g(f(2)) = g(a) = 5$

$gof(3) = g(f(3)) = g(b) = 7$

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$(gof)(a) = \{(1,5), (2,5), (3,7)\}.$

Q. Let $f : R \to R$ and $g : R \to R$ given by

$f(x) = x^2$

$g(x) = 2x+1$

Check weather $gof(x) = fog(x)$

Sol:

Not equal

$gof(x) \quad g(f(x)) = g(x^2)$

$= 2x^2+1$

$fog(x) \Rightarrow f(g(x)) = f(2x+1)$

$= (2x+1)^2$

$= \cancel{2x}$

$4x^2+1+4x$

$gof(x) \neq fog(x)$

Q $f, g, h : R \to R$

$f(x) = x^3 - 4x \quad g(x) = \dfrac{1}{x^2+1}, \quad h(x) = x^4$

Find i) $\cancel{f(g(h(x)))} = (fogoh)(x)$

ii) $(gog)(x)$

iii) $(hogof)(x)$

iv) $(goh)(x)$

(i) $f(g(x^4)) = f\left(\frac{1}{x^4+1}\right) = \left(\frac{1}{x^4+1}\right)^3 - 4\left(\frac{1}{x^4+1}\right)$

$= (x^4+1)^{-3} - 4(x^4+1)^{-1}$

(ii) $(gog)(x) = g\left(\frac{1}{x^2+1}\right) = \dfrac{1}{(x^2+1)^{-2}+1} = ((x^2+1)^{-2}+1)^{-1}$

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(iii) $(h o g o f)(x)$

$h(g(x^3 - 4x)) = h\left(\frac{1}{(x^3-4x)+1}\right)$

$= \left(\frac{1}{(x^3-4x)+1}\right)^4 = ((x^3+4x)+1)^{-4}$

(iv) $g o h(x) = g(x^4) = \frac{1}{(x^4)^2+1} = \frac{1}{x^8+1}$

Q. Let f, g, $f: A \to B$

$g: B \to C$

$h: C \to D$

Show that ~~hg~~ $h o(g o f)(x) = (h o g) o f$

$h o(g o f)$: $g o f: A \to C$, $h: C \to D$

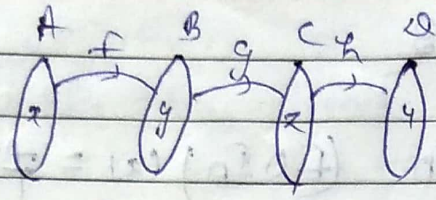
$= ~~\cdot~~ A \to D$

$h o (g o f): A \to D$

$(h o g) o f$: $h o g: B \to D$ $f: A \to B$

$(h o g) o f : A \to D$

⟹



x, y, z

$x \in A, \; y \in B, \; z \in C, \; 4 \in D$

$f(x) = y, \; g(y) = z, \; h(z) = 4$

$(g \circ f) : A \to C \qquad h : C \to D$

$h \circ (g \circ f) : A \to D$

Now

$(h \circ (g \circ f))(x) = h(g(f(x))) = h(g(y)) = h(z) = 4 \quad \text{— (1)}$

$((h \circ g) \circ f)(x) = t \circ f(x) = t(f(x)) = t(y) = h \circ g(y)$

let t

$= h(g(y)) = h(z) = 4 \quad \text{— (2)}$

$eq^n \; (1) = eq^n \; (2)$

So, composition of function is associated

Ident!

Composition of any function with the identity function is function itself.

$f \circ I_A = I_B \circ f = f$

Let f be the function $f : A \to B$ and I be the identity function which is defined $A \to A$

~~$f \circ I$~~ =?

$I_A : A \to A$

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$f \circ I_A : A \to B$

let $x \in A$ then $(f \circ I_A)(x) = f(I_A(x)) = f(x)$
$= f$
— (1)

Now $f : A \to B$ and $I_B : B \to B$

$I_B \circ f : A \to B$

$(I_B \circ f)(x) = I_B(f(x)) = y = f(x) = f$ (2)

Que Composition of any function

Que: function $f(x) = 2x+3$, $f : R \to R$ $\forall x \in R$

for injective

let x_1, x_2 are defined on R

$x_1 \neq x_2$

$f(x_1) = 2x_1 + 3$ $\quad f(x_2) = 2x_2 + 3$

$2x_1 + 3 \quad 2x_2 + 3$

$x_1 \neq x_2$

$f(x_1) \neq f(x_2)$

f^n is injective

let $f(x) = 2x + 3 = y$

$y = 2x + 3 \quad \forall x \in R$

then $2x = y - 3$

$x = \frac{y-3}{2}$

$\boxed{f(x) = \frac{x-3}{2}}$ $\forall x \in R$

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onto

For every element $y = f(x) \in B$ there exist $x \in A$

Such that $f(x) = y$

Note ~~y=~~ Let $y = 2x + 3$ be any element in the codomain of function x.

then $x = \left(\frac{y-3}{2}\right)$ be the preimage of y in the domain of $f(x)$

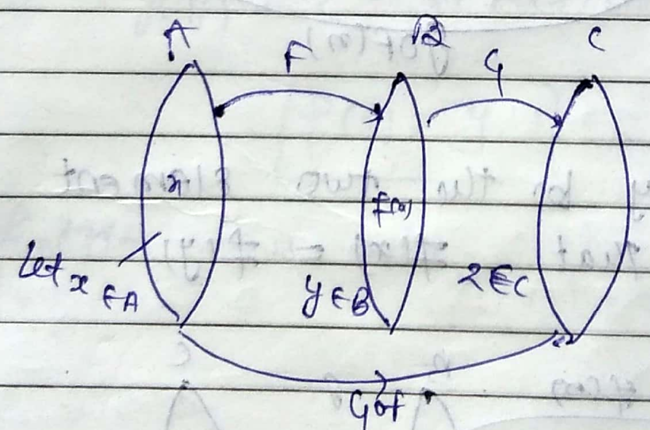
Such that

$$f(x) = f\left(\frac{y-3}{2}\right)$$

$$= 2\left(\frac{y-3}{2}\right) + 3 = y$$

Que Let f^n ~~$f: A \to B$, $g: B \to C$~~

(i) $g \circ f: A \to C$ is onto $\Rightarrow g: B \to C$ is onto



gof = g(f(x))
g(y) = z

for every $z \in C$ their exist $g(y) \in B$ Such that $g(y) = z$

$g(f(x))$

$$g \circ f(x) = z = g(y)$$

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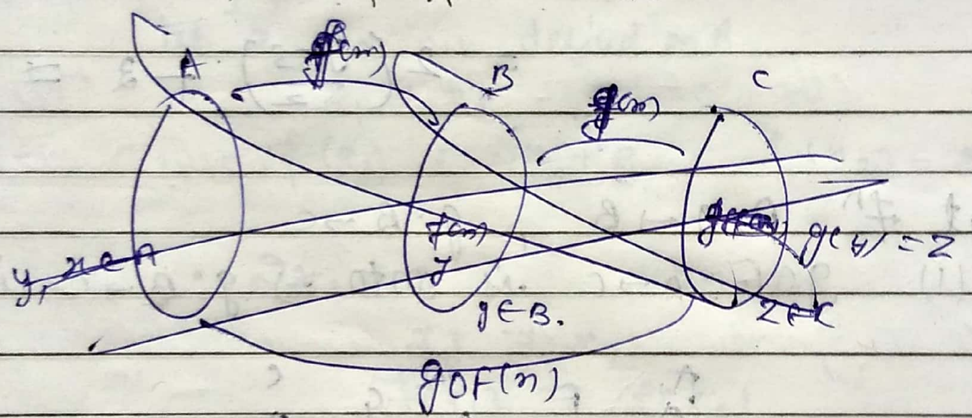
Since gof is onto function then for every $z \in C$ their exist a $x \in A$ such that

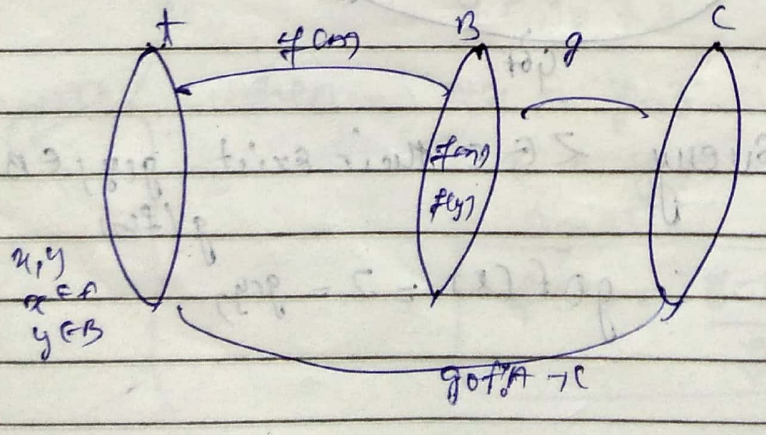
$$gof(x) = z\ g(f(x))$$
$$= g(y)$$
$$= z$$

for every $z \in C$
there exist $f(x) \in B$
such that
$$g(f(x)) = g(y) = z$$

Q $gof: A \to C$ one one
f^n $f: A \to B$ is one-one



Let, x, y be the two element such that $f(x) = f(y)$



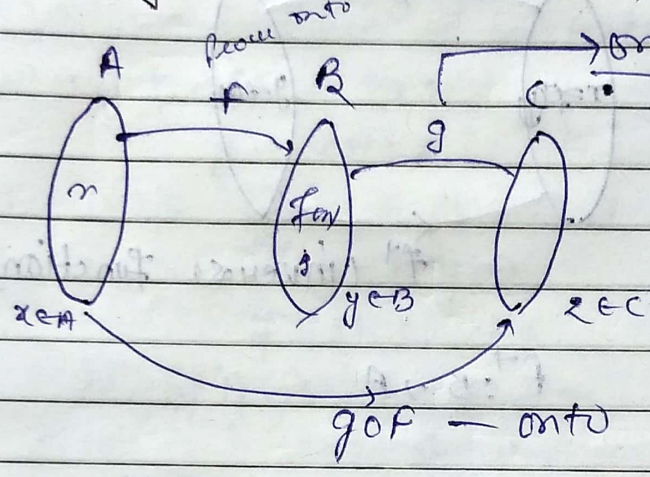
$f(x) = f(y)$

$g(f(x)) = g(f(y))$

$gof(x) = gof(y)$

$\boxed{x = y}$

Q. f^n $gof: A \to C$ is onto and $g: B \to C$ is one $\Rightarrow$ $f: A \to B$ is one-one, onto



$gof(x) = z$ Let $y = f(x)$

$g(f(x)) = z$ $g(y) = z$

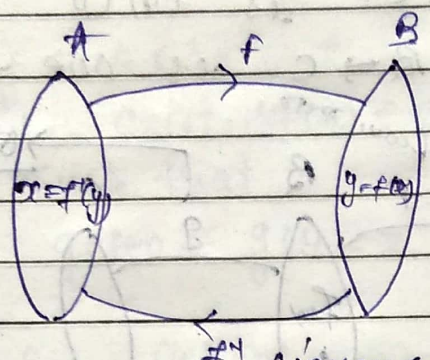
$g(y) = z = g(f(x))$

$\boxed{f(x) = y}$

function f is onto

* Inverse function :-

Let $f: A \to B$ be an one-one onto (bijective) function. Then then $f^{-1}: B \to A$ is called Inverse function



f^{-1} (inverse function).

$f^{-1}: B \to A$

* Necessary Condition for finding the inverse
- one-one onto (bijective)

Que Suppose that function $f: R \to R$ is Bijective function

$f(x) = 3x - 1$

find $f^{-1}(5)$

Let $f(x) = y$ $\qquad$ $y = 3x - 1$

$f^{-1}(y) = x$ $\qquad$ $3x = \frac{y+1}{3}$

$f^{-1}(y) = \frac{y+1}{3}$

$f^{-1}(x) = \frac{x+1}{3}$

$= \frac{5+1}{3} = 2$

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Que: Let f and g : R → R given by $f(x) = x+2$

$g(x) = \frac{1}{x^2+1}$

Compute $f^{-1}(g(x))$

$g(x) = \frac{1}{x^2+1}$, $f(x) = x+2$

$f^{-1}\left(\frac{1}{x^2+1}\right)$

$y = f(x) = x+2$

$y = x+2$

$x = y-2$

$f^{-1}(y) = y-2$

$f^{-1}(x) = x-2$

$f^{-1}(x) = \frac{1}{(x-2)^2+1} = \frac{1}{x^2+4-2x+1}$

$= \frac{1}{x^2-2x+5}$

$= \frac{1}{x^2+1} - 2$

$= \frac{1-2x^2-2}{x^2+1} = \frac{-2x^2-1}{x^2+1}$

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Que if function $f: X \to Y$ and A and B are the two subsets of X and Y. then Prove that

(i) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$

(ii) $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$

(i) L.H.S

Let x be an arbitrary elem of $f^{-1}(A \cup B)$

$x \in f^{-1}(A \cup B)$

$f(x) \in A \cup B$

$f(x) \in A$ OR $f(x) \in B$

$x \in f^{-1}(A)$ OR $x \in f^{-1}(B)$

$x = f^{-1}(A) \cup f^{-1}(B)$ —— ①

$f^{-1}(A \cup B) \subseteq f^{-1}(A) \cup f^{-1}(B)$

RHS

Let x be any arbitarary

$x \in (f^{-1}(A) \cup f^{-1}(B))$

$x \in f^{-1}(A)$ OR $x \in f^{-1}(B)$

$f(x) \in A$ OR $f(x) = B$

$f(x) \in A \cup B$

$x \in f^{-1}(A \cup B)$

$x \subseteq f^{-1}(A \cup B)$ ②

$f^{-1}(A) \cup f^{-1}(B) \subseteq f^{-1}(A \cup B)$ —— ②

from eq^n ① and ②

$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$

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(ii) L.H.S

Let x be an arbitrary of $f^{-1}(A \cap B)$

$x \in f^{-1}(A \cap B)$

$f(x) \in A \cap B$

$f(x) \in A$ and $f(x) \in B$

$x \in f^{-1}(A)$ and $x \in f^{-1}(B)$

$x \in f^{-1}(A) \cap f^{-1}(B)$

$f^{-1}(A \cap B) \subseteq f^{-1}(A) \cap f^{-1}(B)$ — (1)

RHS

Let x be any arbitrary

$x \in f^{-1}(A) \cap f^{-1}(B)$

$x \in f^{-1}(A)$ and $x \in f^{-1}(B)$

$f(x) \in A$ and $f(x) = B$

$\cancel{f(x) \in f^{-1}(A \cap B)}$

$f(x) \in A \cap B$

$x \in f^{-1}(A \cap B)$

$x \subseteq f^{-1}(A \cap B)$

$f^{-1}(A) \cap f^{-1}(B) \subseteq f^{-1}(A \cap B)$ — (2)

from eqn (1) and (2)

$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$

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Que If $f: A \to B$ & $g: B \to C$ are two one-one onto function then prove that

i) $gof: A \to C$ is one-one onto

ii) gof is invertable i.e $\bar{gof} = f^{-1} o g^{-1} : C \to A$

Sol^n: $f: A \to B$ $g: B \to C$

Let $x_1, x_2 \in A$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

~~$f(x_1) \in A$~~ ~~$f(x_2) \in B$~~

$g(f(x_1)) \neq g(f(x_2))$

$(gof)(x_1) \neq (gof)(x_2)$

$x_1, x_2 \in A$; $x_1 \neq x_2$

$(gof)(x_1) \neq (gof)(x_2)$

(gof) is one-one.

Onto: Let $z \in C$ be an arbitery element of set C

If g is onto function then for every $z \in C$ there exist $y \in B$ such that $g(y) = z$

If f is onto function for every $y \in B$, there exist x such that $f(x) = y$

$$z = g(y) = g(f(x)) = (gof)(x)$$

$$z = (gof)(x)$$

for every $z \in C$ there exist a x belongs to A such that $(g o f)(x) = z$