ELASTICITY

If We apply any force on any body then the shape of body will change & after Removing the force body again come back to its original shape then that property of body is called Elasticity. & the force applied on body is called deforming force.

Stress = $\frac{\text{Restoring force}}{\text{Area}} = \frac{F}{A}$ * SI unit = N/m^2 * $[ML^{-1}T^{-2}]$

Strain = $\frac{\text{change in configuration (length or, volume)}}{\text{origenal configuration (length or, volume)}}$

Hook's Law → $\boxed{\text{Stress} \propto \text{Strain}}$

$\boxed{\text{Stress} = \epsilon \times \text{Strain}}$ ϵ → cofficient of Elasticity.

It is of three types—

(i) → young Modulus of Elasticity (Y) →

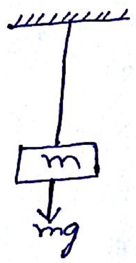
$Y = \frac{\text{Longitudnal Stress}}{\text{Longitunal Strain}}$

$\boxed{Y = \frac{F/A}{\Delta L/l}}$ A → Area of cross section

l → orginal length

$\boxed{Y = \frac{FL}{A\Delta L}}$ ΔL → change in length.

(a) → change in length of a Massless wire when 'M' Mass is hanged at Lower end →

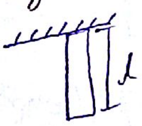


$\boxed{Y = \frac{FL}{A\Delta L}}$ r → Radius of Wire

$\boxed{\Delta L = \frac{mgl}{YA}}$

$\boxed{\Delta L = \frac{mgl}{4\pi r^2}}$

(b) → change in length due to its own weight of a uniform of mass 'm'



* $\boxed{\Delta L = \frac{mg(l/2)}{4\pi r^2}}$

$\boxed{\Delta l = \frac{mgl}{2Y\pi r^2}}$

(c) → Potential energy stored in a streached wire—

F = KΔL (like spring)

so, ~~because~~ behave like spring of force const. $\boxed{K = \frac{YA}{l}}$

pot. energy stored ⇒ $\boxed{U = \frac{1}{2}K\Delta L^2}$

$\boxed{AL = U \text{(vol.)}}$

$\boxed{U = \frac{1}{2}\left(\frac{YA}{L}\right)(\Delta L)^2}$

$$\boxed{\frac{U}{V} = \frac{1}{2} \times \text{Stress} \times \text{Strain}}$$

$$\boxed{\frac{U}{V} = \frac{1}{2}\,\frac{(\text{stress})^2}{Y}}$$

|ii| → Bulk Modulus (B) or, K (Maxwell) →

$$B = \frac{\text{volume stress}}{\text{volume strain}} = \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\boxed{B = \frac{-(\Delta P)V}{\Delta V}}$$

$$\boxed{\text{Compressibility}\,(C) = \frac{1}{B}}$$

|a| → For same $\Delta P, V$

$\boxed{B \propto \frac{1}{\Delta V}}$ $\boxed{B_{solid} > B_{liq} > B_{gas}}$

$\therefore \boxed{(\Delta V)_{solid} < (\Delta V)_{liq} < (\Delta V)_{gas}}$

|b| → There are two bulk Modulus in gases →

* Isothermal bulk modulus $= P \Rightarrow P \to$ gas
* Adiabatic ״ ״ γP $\gamma \to$ gama/γ

Isothermal

$PV = \text{const}$

$P\Delta V + V\Delta P = 0$

$P\Delta V = V\Delta P$

$-\frac{V\Delta P}{\Delta V} = P$

$\boxed{B_{isothermal} = P}$

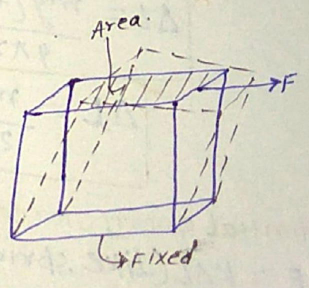
Adiabatic

$PV^{\gamma} = \text{const}$

$\gamma P = -\frac{(\Delta P)V}{\Delta V}$

$\boxed{B_{adiabatic} = \gamma P}$

|iii| → Rigidity Modulus / shear modulus (η)



$$\eta = \frac{\text{Shear stress}}{\text{shear strain}}$$

$\boxed{\eta = \frac{F/A}{\theta}}$ ($\theta \to$ shear angle)

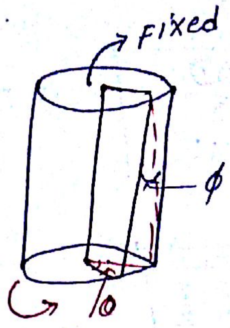
$\boxed{\eta = \frac{F}{A\theta}}$

NOTE → * Twisting of solid cyllinder.
* Angle of shear always take in Radian.

$\theta \rightarrow$ angle of twist
$r \rightarrow$ radius
$L \rightarrow$ Length
$\phi \rightarrow$ Shearangle

$AB \Rightarrow r\theta = L\phi$

**
$$\boxed{\text{Shearangle } \phi = \frac{r\theta}{L}}$$

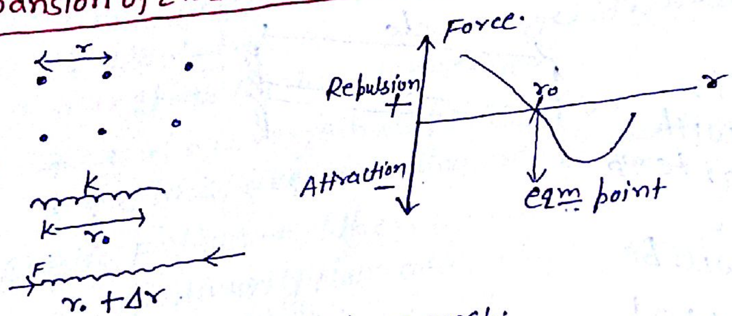


** # A Wire has length L_1 when tension is T_1 & length is found to be L_2 when tension is T_2. Find its natural length.

*
$$\boxed{L = \frac{L_1 T_2 - L_2 T_1}{T_2 - T_1}}$$

A Massless wire of length 'L' Radius 'r' is suspended vertically & pulled by force 'F' then change in length is 'L' another wire made up of same material having length '2L' & radius '2r' & pulled by force 2F then change in Length will be $\rightarrow$ $\boxed{l' = l}$ *

Explanation of Elasticity by Interatomic force $\rightarrow$



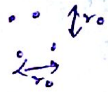
If $K \rightarrow$ Interatomic force const.

then, $F = K\Delta r$ — (I)

$\text{strain} = \frac{\Delta r}{r_0}$ — (II)

corss secction area 'A'

no. of atom $n = \frac{A}{r_0^2}$



Total Force $F_T = nF$

$= \frac{A}{r_0^2} K\Delta r$

$\frac{F_T}{A} = \frac{K\Delta r}{r_0^2}$

$\text{Stress} = \frac{K\Delta r}{r_0^2}$ — (III)

*
$$\boxed{\text{Stress} = Y \times \text{strain}}$$

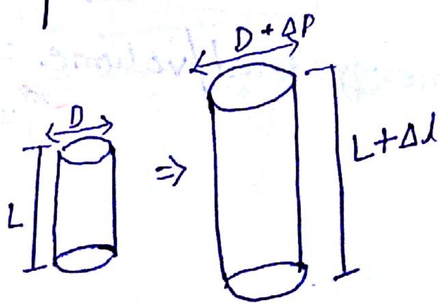
$\frac{K\Delta r}{r_0^2} = Y \times \frac{\Delta r}{r_0}$

*
$$\boxed{K = Y r_0}$$

$r_0 \rightarrow$ Interatomic distance.

** # Poisson's Ratio (σ) :$\rightarrow$

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudnal strain}}$$



$$\boxed{\sigma = \frac{\frac{\Delta D}{D}}{\frac{\Delta L}{L}}}$$

*If volume is const on streching
Then, $V = \text{const}$

$$\frac{\pi D^2}{4} L = \text{const.}$$

$$D^2 L = \text{const.}$$

$$\boxed{0 < \sigma < \frac{1}{2}}$$

$$\boxed{\sigma = \frac{1}{2}}$$

MPPMT

Relation b/w Y, β, η, σ

(young, Bulk, shear, poisson)

|i| → $\boxed{Y = 3\beta(1 - 2\sigma)}$

|ii| → $\boxed{Y = 2\eta(1 + \sigma)}$

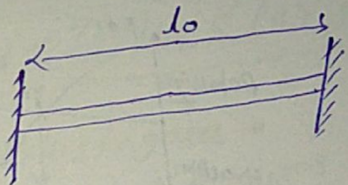
|iii| → $\boxed{\frac{9}{Y} = \frac{1}{\beta} + \frac{3}{\eta}}$

BHU

$\sigma = -1 < \sigma < 0.5$ (Theoritical limit)

$\sigma = 0.2$ to 04 (Experimental limit)

Thermal stress →



Let, a Rod of length l_0 is clamped at its end with rigid support & then temp. is increased by $\Delta\theta$

∴ Its length will be

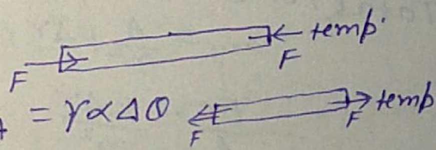
$$l = l_0(1 + \alpha\Delta\theta)$$

$$l - l_0 = l_0 \alpha \Delta\theta$$

$$\Delta l = l_0 \alpha \Delta\theta$$

$$\frac{\Delta l}{l_0} = \alpha\Delta\theta$$

$$\boxed{\text{Thermal stress} = Y\alpha\Delta\theta}$$



Let $A \rightarrow$ area of cross section

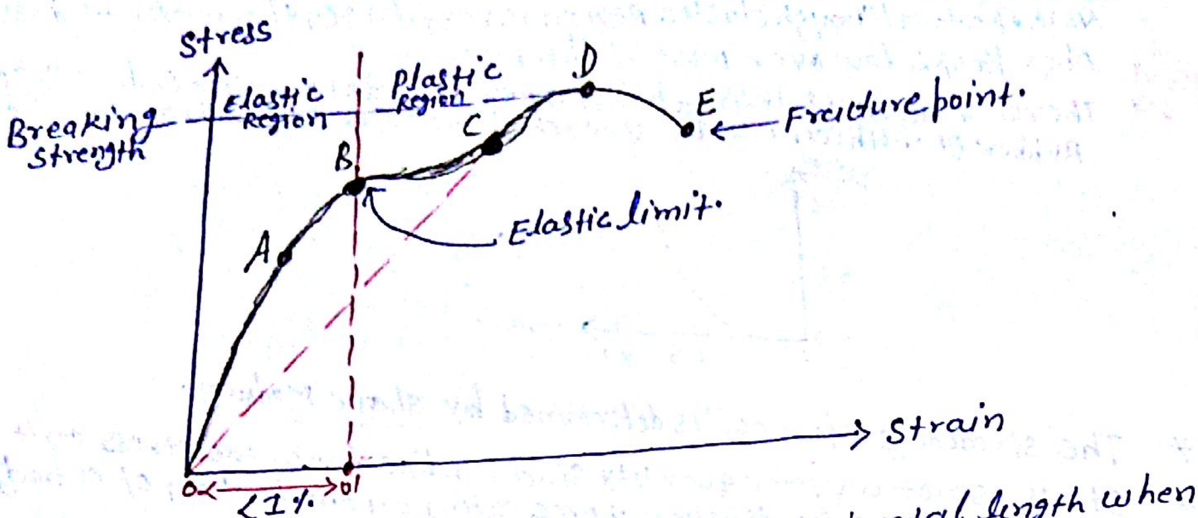
then, thermal stress $= F/A = Y\alpha\Delta\theta$

$$\boxed{F = YA\alpha\Delta\theta}$$

Energy stored/volume $= \frac{1}{2}$ stress × strain

$$\boxed{= \frac{1}{2} Y (\alpha\Delta\theta)^2}$$

Strain - Stress curve →

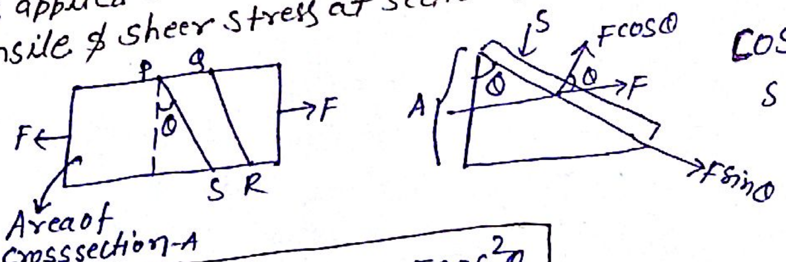


* OA → Fallows Hooks law & wire return in origental length when weight/Force is Removed.
* AB → Doesn't fallow Hooks law.
* BC → When weight removed, some permanent strain Remain.
* CD → Little extra stress cause large strain.
* D → Max stress without breaking.
** → If metal has very small plastic Region there called brittle material & having large plastic region called ductile material.

→ Elastic Fatigue → When weight on wire is applied & Removed continuosly then after some time it losses its elastic property called Elastic Fatigue.

→ Elastic After Affect → Time taken by material to regain its origenal shape when deforming force is Removed, there are some material like quartz**, Phosphor bronze** regain its origenal shape imadiately after deforming force is Removed. i.e these material has no elastic effect.

The force 'F' is applied on the face of Rectangular block as shown in fig. define the tensile & sheer stress at section PQRS.



$\cos\theta = A/S$

$S = A/\cos\theta$

$$\text{* Tensile Stress} = \frac{F\cos\theta}{A/\cos\theta} = \frac{F\cos^2\theta}{A}$$

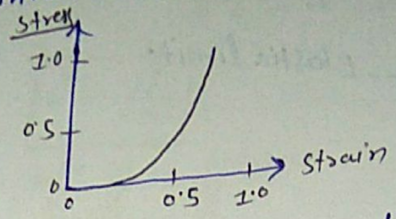
$$\text{* Sheer stress} = \frac{F\sin\theta}{A/\cos\theta} = \frac{F\sin^2\theta}{2A}$$

*** *Rubber can be pulled to several times its origenal length & still Returns to origenal shape.

* Stress & strain curve for elastic tissue of Aurta, prescent in Heart. Note that, although elastic Region is very large, the material does not obey Hooks law over most of the Region.

AIIMS Imp

* There is no well defined plastic region. substance like tissue of Aorta, Rubber etc. Which can be stretched to cause large strains called Elastomer.

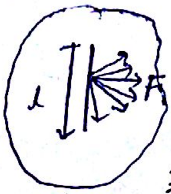


* The streaching of a coil is determined by shear Modulus.
* Stress is not a vector quantity since, unlike force, the stress can't be assignet a specific direction. Force acting on the portion of a body on a specific side of a section has definite direction.

'SURFACE TENSION'

→ It is the property of surface of liq. by wich liquid tries to minimise its surface area.

→ It surface tension (σ or, T) is force acting per unit length on a line assumed on the surface of liquid on any one side of the line.



$\boxed{T \text{ or}, \sigma = \frac{F}{l}}$ ⇒ Act ⊥r to line assumed.

*unit → N/m Imp *Dimension $= \frac{MLT^{-2}}{L} = [MT^{-2}]$*

*** → It is the property of surface of liq. & does ** not depend on length of line used.

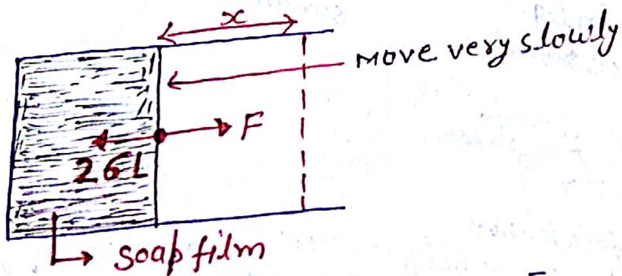
* → surface tension ↓ with rise in temp. & becomes zero at a critical temp. where interface b/w liquid & vapour disappear.

* → It depend on impurities & ↓se when impurities contaminate on the surface.

** → Generally, surface tenson ↑se with highly solible impurities like (Nacl in water) & ↓ with sparangly solible Impurities.

Work Done by surface Tension →

When surface area change



$F = 2\sigma L$

✓ $W_{net} = 0$

✓ $W_F + W_\sigma = 0$

✓ $W_F = -W_\sigma$

$W_\sigma = -Fx$

$= -2\sigma Lx$

$= -\sigma L(2x) = -\sigma \times 2(Lx)$

Work done by surface Tension $= \boxed{-\sigma(\Delta S)}$ (change in area)

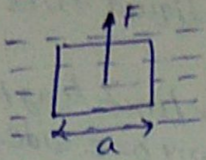
Work done on the surface is $\boxed{W = \sigma \times (\Delta S)}$

∴ Work done on the surface. ↑se the energy of surface & this work done ↑se the energy of surface.

A/F (Energy associated with the surface due to surface tension ** is also called 'surface energy') (Bcoz of this Reason liquid drops are spherical.)

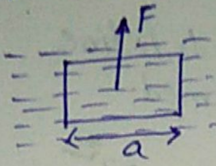
Force Required to Raise a Masses.

*|a| → square plate of side a from liquid surface.



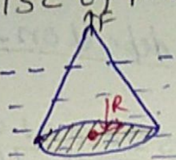
$\boxed{F = T(4L) = 4Ta}$ *

*|b| → square frame of wire side 'a'.



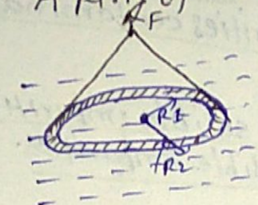
$$\boxed{\begin{aligned} F &= \text{Force on one surface} \times 4 \\ &= (T \times 2a) \times 4 \\ &= 8TA \end{aligned}}$$ *

*|c| → Disc of Radius 'R'



$$\boxed{\begin{aligned} F &= Tl \\ &= T(2\pi r) \end{aligned}}$$ *

*|d| → A Ring of inner & outer radii R_1, R_2 $(R_1 < R_2)$



$$\boxed{\begin{aligned} F &= T\cdot 2\pi R_1 + T\cdot 2\pi R_2 \\ &= 2\pi T(R_1 + R_2) \end{aligned}}$$

Free surface →

Liquid drop

1-free surface

Bubble

Bubble in liq. (one free surface)

Bubble in air.

2-free surface.

* Work done to make a liquid drop.

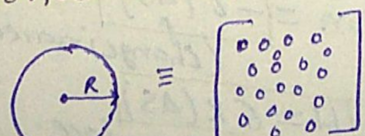
$$* \boxed{W = \sigma \times \overbrace{4\pi r^2}^{\text{area}}}$$

(σ → surface tension)

* Work done to blow a bubble in air

$$* \boxed{W = \sigma \times (4\pi r^2) \times 2 = \sigma \times 8\pi r^2}$$

* Work done to split a liquid drop.



'n' identical drops (each of radius 'r')

$$\rho \times \frac{4}{3}\pi R^3 = \rho \times \left(\frac{4}{3}\pi r^3\right)^n \Rightarrow \boxed{R = n^{1/3} r} *$$

$$* \boxed{W = (n^{1/3} - 1)\, \sigma \times 4\pi R^2} *$$

✓ If this process is consider as adiabatic then temp. of system will fall so,

$$\boxed{\Delta\theta = \frac{3T}{\rho s J}\left(\frac{1}{r} - \frac{1}{R}\right)}$$

AMU 2016

When, $\boxed{\begin{array}{l}\text{Bigger drop} \longrightarrow \text{smaller drops} \\ \text{surface} \uparrow \\ \text{Surface energy} \uparrow \\ \text{temp. of system} \downarrow\end{array}}$

*

In this case,

$$\boxed{\frac{\text{Initial surface energy}}{\text{Final surface energy}} = \frac{1}{N^{1/3}}}$$

* Work done to ↑ Radius of a liq. drop from R_1 to R_2.

$$\boxed{W = 4\pi T\left(R_2^2 - R_1^2\right)}$$

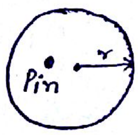
* Work done to ↑ radius of a soap bubble from R_1 to R_2.

$$\boxed{W = 8\pi T\left(R_2^2 - R_1^2\right)}$$

* Two identical liq. drop of Radius r_1 & r_2 combine to form a single spherical drop. In Isothermal condition then Ratio of New drop.

*$\boxed{r = \sqrt{r_1^2 + r_2^2}}$

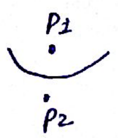
Excess Pressure → The extra pressure inside a drop or bubble.



•Pout

**

$$\boxed{P = P_{in} - P_{out} = \begin{cases} \frac{2\sigma}{r} \text{ ; For one free surface.} \\ \frac{4\sigma}{r} \text{ ; For two free surface.} \end{cases}}$$

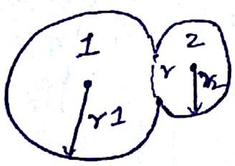


$P_1 > P_2$

$P_1 > P_2$

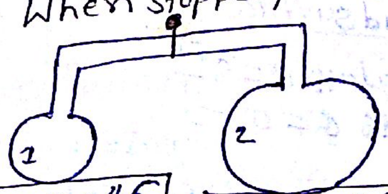
$P_1 - P_2 = \frac{2\sigma}{r}$

** Two soap/air bubble when made in contact to form double bubble.



*$\boxed{r = \frac{r_1 r_2}{r_1 - r_2}}$

AIIMS ** When stopper/knob is opened how size of ① & ② will change.

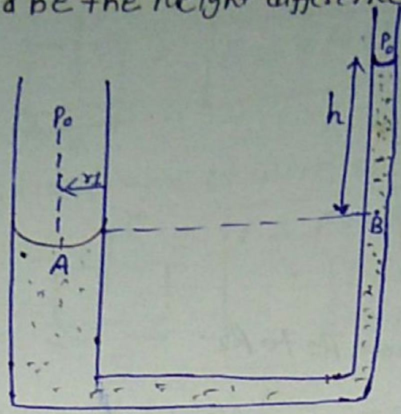


$\boxed{P_1 = P_0 + \frac{4\sigma}{r_1}}$ $\boxed{P_2 = P_0 + \frac{4\sigma}{r_2}}$

as, *$\boxed{\begin{array}{l} r_1 < r_2 \\ P_1 > P_2 \end{array}}$ *$\boxed{\Delta P \propto \frac{1}{R}}$

Hence, When stopper is opened smaller bubble reduces its size while bigger bubble expand its size.

** There is U-tube having their arms of Radius r_1 & r_2 ($r_1 > r_2$) then what Would be the height difference of Hg. In both arm angle of contact 0°.

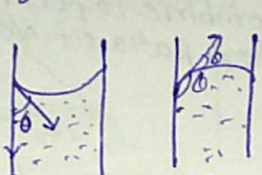


* $$\boxed{h = \frac{2T}{\rho g}\left(\frac{r_1 - r_2}{r_1 r_2}\right)}$$

Angle of contact (θ) →

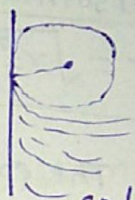
θ = At meniscus, angle b/w tangent of liq. surface in contact & tangent on solid surface inside liquid.

F_A → Adhesive force b/w solid & liquid molecule.
F_C → cohesive force b/w liq. molecule.



θ = acute angle
cohesive force (C.F) <
*Adhesive force (AF)
(Wetting liq)

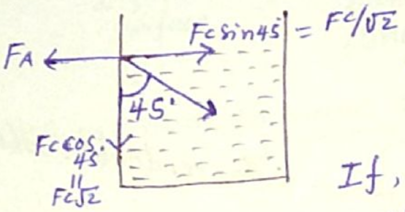
θ = obtuse
$\boxed{C.F > A.F}$
*Non-Wetting liq



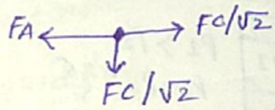
here $\boxed{\theta = 0°}$
liq = water

*Solid ⇒ clean glass

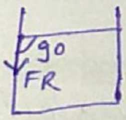
|i| → If $F_A = F_C/\sqrt{2}$



⇒



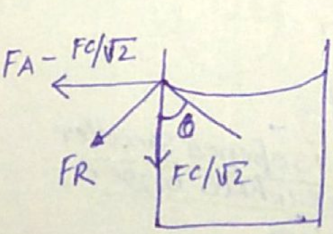
If, $F_A = F_C/\sqrt{2}$
then, $F_R = \frac{F_C}{\sqrt{2}}$ = downward



* $\boxed{\text{Angle of contact} = 90°}$

Imp * Eg → [Water in silver glass.]

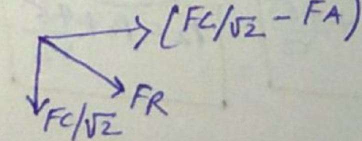
|ii| → If $F_A > F_C/\sqrt{2}$



$\theta < 90°$ (Acute angle)
such liq wet the solid surface.

Imp
*For normal water glass $\theta = 8°$ **
*For pure water glass $\theta = 0°$

|iii| → If $F_A < F_C/\sqrt{2}$
then $\theta > 90°$ (obtuse angle)
From such liq surface is not wet. Eg → *glass Hg
* $\boxed{\theta = 135°}$



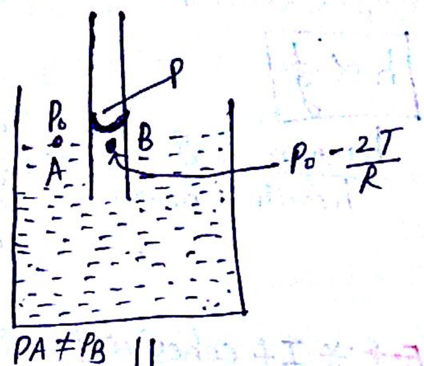
Rise of liquid level in a cappillary tube →

$P_A = P_B$

$P_o = P_o - \frac{2T}{R} + h\rho g$

$\frac{2T}{R} = h\rho g$

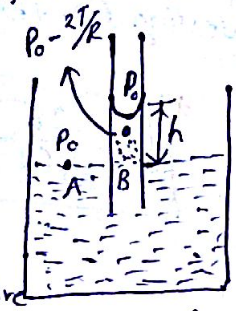
* $\boxed{h = \frac{2T}{R\rho g}}$



Meniscus.

R → Radius of curvature of

$P_A \neq P_B$



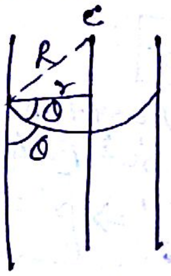
Let, r → radius of curvature

θ → angle of contact.

$\cos\theta = r/R$

$R = \frac{r}{\cos\theta}$

$\therefore h = \frac{2T}{\frac{r}{\cos\theta}\rho g}$



* $\boxed{h = \frac{2T\cos\theta}{r\rho g}}$

|i| → For liquid gas →

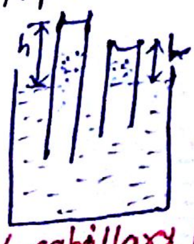
θ, T, ρ → const.

$\therefore \boxed{h \propto \frac{1}{r}}$ ⟹ Zurin Law (BHU)

* $\boxed{h_1 r_1 = h_2 r_2}$

|ii| → If capillary has insufficient length then water can't come out from top but radius of meniscus at top does not change.

* $\boxed{Lr = Lr^{-1}}$



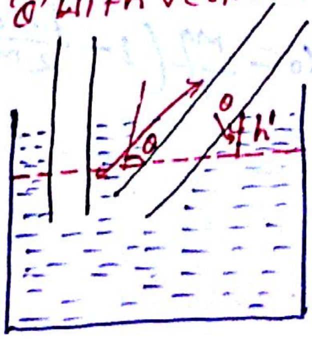
|iii| → If capillary is inclined at angle 'θ' with vertical.

Let, L = length of liq. column in inclined tube

then, $h' = h$

$\boxed{L\cos\theta = h}$

* $\boxed{L = \frac{h}{\cos\theta}}$ $\boxed{L > h}$



$\cos\theta = \frac{h'}{L}$

$L = \frac{h'}{\cos\theta}$

** → Height raised in capillary tube is found to be 'h' on earth surface. If all exp. is taken at Moon then height Rise become.

Imp

$g_{moon} = \frac{g_{earth}}{6}$

$$\boxed{h \propto \frac{1}{g}}$$

$$\frac{h_{Moon}}{h_{earth}} = \frac{g_e}{h_{moon}} = \frac{g_e}{\frac{g_e}{6}} \therefore \boxed{h_m = 6 h_e}$$

NOTE → * If cohesive force (C.F) are less than adhesive force (A.F) i.e $\theta \Rightarrow$ acute or, Liq is wetting liq. then the level of liq. in capillary tube will rise.

$$* \boxed{h = \frac{2\sigma\cos\theta}{\rho g R}} \qquad * \boxed{h = \frac{2\sigma}{r\rho g} = \frac{2\sigma}{\rho g r}}$$

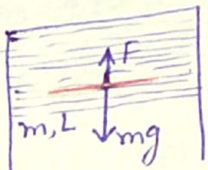
* In any capillary

$$\boxed{hr = \text{const.}}$$

i.e If capillary tube is of insufficient length of liq. will not over flow but Radius of meniscous ↑se.

$$hr = h'r' \Rightarrow \boxed{r' = \frac{hr}{h'}}$$

Imp
* If needle is in equilibrium find surface tension of soap film.



$F = mg$

$$\sigma \times 2L = mg \Rightarrow \boxed{\sigma = \frac{mg}{2L}}$$

Imp
* If capillary & beker system is taken at the artificial satellite or, Freely falling lift then height rise in capillary is equal to its Full length.

A uniform cyllinder of length 'L' & Mass 'M' having cross-section area Area 'A' suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liq. of density 'σ' at eqm position. The extension X_0 of the spring when it is in eqm →

$$* \boxed{X_0 = \frac{Mg}{K}\left(1 - \frac{LA\sigma}{2M}\right)}$$