SOLVED PAPER 2016 **{OFFLINE}**

JEE MAIN

Joint Entrance Examination

INSTRUCTIONS

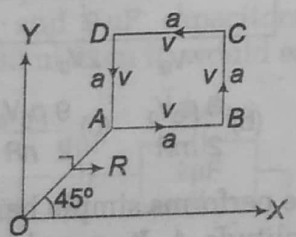
1. This test consists of 90 questions.
2. Each question is allotted 4 marks for correct response.
3. Candidates will be awarded marks as stated above in instruction no. 2 for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

Physics

1. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90s, 91s, 92s and 95s. If the minimum division in the measuring clock is 1s, then the reported mean time should be

(a) (92 ± 2)s (b) (92 ± 5)s
(c) (92 ± 1.8)s (d) (92 ± 3)s

2. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the X-Y plane as shown in the figure.



Which of the following statements is false for the angular momentum $\mathbf{L}$ about the origin?

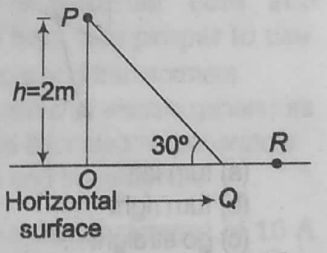
(a) $\mathbf{L} = -\frac{mv}{\sqrt{2}} R\,\hat{\mathbf{k}}$, when the particle is moving from A to B

(b) $\mathbf{L} = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{\mathbf{k}}$, when the particle is moving from B to C

(c) $\mathbf{L} = mv\left(\frac{R}{\sqrt{2}} - a\right)\hat{\mathbf{k}}$, when the particle is moving from C to D

(d) $\mathbf{L} = \frac{mv}{\sqrt{2}} R\,\hat{\mathbf{k}}$, when the particle is moving from D to A

3. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(= QR)$, are respectively close to

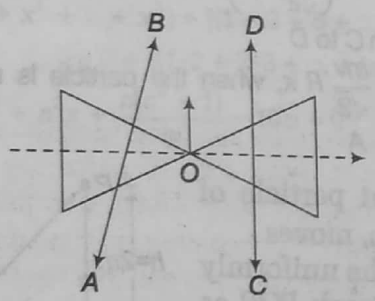


(a) 0.2 and 6.5 m
(b) 0.2 and 3.5 m
(c) 0.29 and 3.5 m
(d) 0.29 and 6.5 m

4. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% of efficiency rate. (Take, $g = 9.8 ms^{-2}$)

(a) 2.45×10^{-3} kg
(b) 6.45×10^{-3} kg
(c) 9.89×10^{-3} kg
(d) 12.89×10^{-3} kg

5. A roller is made by joining together two corners at their vertices O. It is kept on two rails AB and CD which are placed a symmetrically (see the figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see the figure). It is given a light path, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to



(a) turn left
(b) turn right
(c) go straight
(d) turn left and right alternately

6. A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of Earth $R, h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere)

(a) $\sqrt{2gR}$
(b) $\sqrt{gR}$
(c) $\sqrt{gR/2}$
(d) $\sqrt{gR}\,(\sqrt{2}-1)$

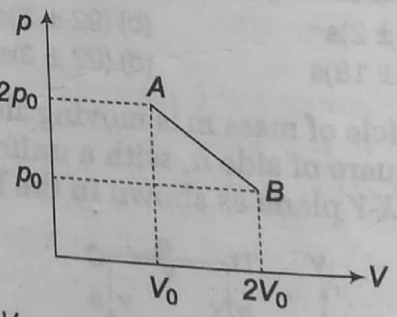
7. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s in a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coefficient of linear expansion (α) of the metal of the pendulum shaft are, respectively

(a) 25°C, $\alpha = 1.85 \times 10^{-5}$/°C
(b) 60°C, $\alpha = 1.85 \times 10^{-4}$/°C
(c) 30°C, $\alpha = 1.85 \times 10^{-3}$/°C
(d) 55°C, $\alpha = 1.85 \times 10^{-2}$/°C

8. An ideal gas undergoes a quasistatic, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure p and volume V is given by pV^n = constant, then n is given by (Here C_p and C_V are molar specific heat at constant pressure and constant volume, respectively)

(a) $n = \frac{C_p}{C_V}$ (b) $n = \frac{C - C_p}{C - C_V}$
(c) $n = \frac{C_p - C}{C - C_V}$ (d) $n = \frac{C - C_V}{C - C_p}$

9. n moles of an ideal gas undergoes a process A and B as shown in the figure. The maximum temperature of the gas during the process will be



(a) $\frac{9}{4}\frac{p_0V_0}{nR}$ (b) $\frac{3}{2}\frac{p_0V_0}{nR}$ (c) $\frac{9}{2}\frac{p_0V_0}{nR}$ (d) $\frac{9p_0V_0}{nR}$

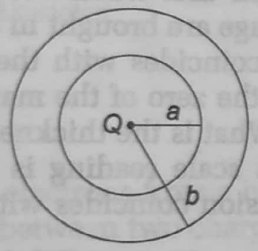
10. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is

(a) $\frac{A}{3}\sqrt{41}$
(b) $3A$
(c) $A\sqrt{3}$
(d) $\frac{7}{3}A$

11. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, $g = 10\ ms^{-2}$)

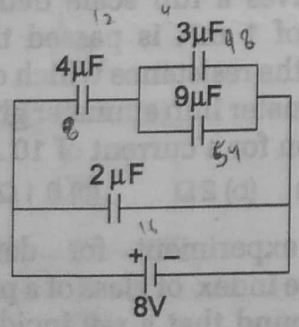
(a) $2\pi\sqrt{2}$ s (b) 2 s
(c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

12. The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is



(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$
(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

13. A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distance 30 m from it, would equal to



(a) 240 N/C
(b) 360 N/C
(c) 420 N/C
(d) 480 N/C

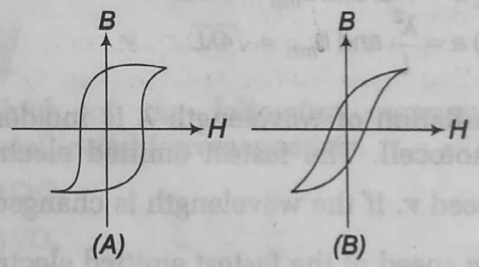
14. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by

(a) linear increase for Cu, linear increase for Si
(b) linear increase for Cu, exponential increase for Si
(c) linear increase for Cu, exponential decrease for Si
(d) linear decrease for Cu, linear decrease for Si

15. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

16. Hysteresis loops for two magnetic materials A and B are as given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use

(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

17. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to

(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

18. Arrange the following electromagnetic radiations per quantum in the order of increasing energy.

A. Blue light B. Yellow light
C. X-ray D. Radio wave

(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

19. An observer looks at a distance tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

20. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when

(a) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(b) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(c) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(d) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

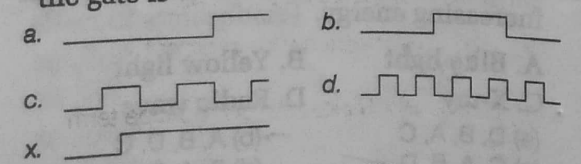
21. Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be

(a) $> v\left(\frac{4}{3}\right)^{1/2}$ (b) $< v\left(\frac{4}{3}\right)^{1/2}$
(c) $= v\left(\frac{4}{3}\right)^{1/2}$ (d) $= v\left(\frac{3}{4}\right)^{1/2}$

22. Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of A and B nuclei will be

(a) 1 : 16 (b) 4 : 1
(c) 1 : 4 (d) 5 : 4

23. If a, b, c and d are inputs to a gate and x is its output, then as per the following time graph, the gate is



(a) NOT (b) AND (c) OR (d) NAND

24. Choose the correct statement.

(a) In amplitude modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal
(b) In amplitude modulation, the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal
(c) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal
(d) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal

25. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet, if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line?

(a) 0.75 mm (b) 0.80 mm
(c) 0.70 mm (d) 0.50 mm

26. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

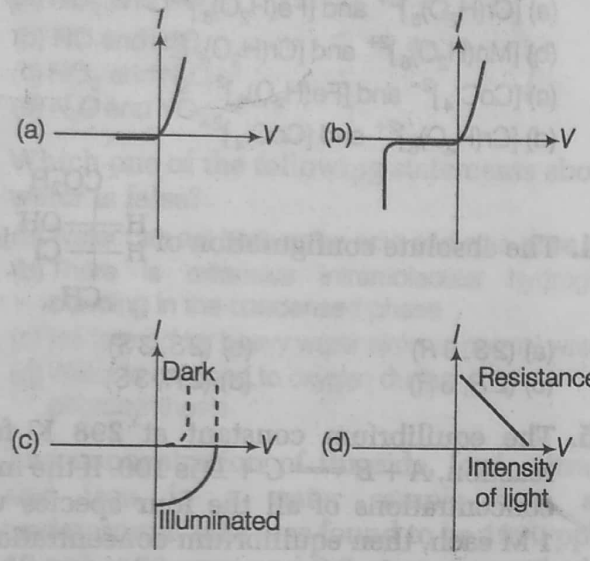
27. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

28. In an experiment for determination of refractive index of glass of a prism by i-δ plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index?

(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

29. Identify the semiconductor devices whose characteristics are as given below, in the order (a),(b),(c),(d).



(a) Simple diode, Zener diode, Solar cell, Light dependent resistance
(b) Zener diode, Simple diode, Light dependent resistance, Solar cell
(c) Solar cell, Light dependent resistance, Zener diode, Simple diode
(d) Zener diode, Solar cell, Simple diode, Light dependent resistance

30. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is

(a) $\frac{1}{\alpha}=\frac{1}{\beta}+1$
(b) $\alpha=\frac{\beta}{1-\beta}$
(c) $\alpha=\frac{\beta}{1+\beta}$
(d) $\alpha=\frac{\beta^2}{1+\beta^2}$

Chemistry

1. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of h/λ (where, λ is wavelength associated with electron wave) is given by

(a) $2meV$ (b) $\sqrt{mev}$
(c) $\sqrt{2mev}$ (d) mev

2. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields

I. $C_2H_5CH_2\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-OCH_3$

II. $C_2H_5CH_2\underset{\displaystyle CH_3}{\overset{}{\overset{|}{C}}}=CH_2$

III. $C_2H_5CH=\underset{\displaystyle CH_3}{\overset{}{\overset{|}{C}}}-CH_3$

(a) Both I and III (b) Only III
(c) Both I and II (d) All of these

3. Which of the following compounds is metallic and ferromagnetic?

(a) CrO_2
(b) VO_2
(c) MnO_2
(d) TiO_2

4. Which of the following statements about low density polythene is false?

(a) It is a poor conductor of electricity
(b) Its synthesis required dioxygen or a peroxide initiator as a catalyst
(c) It is used in the manufacture of buckets, dustbins etc
(d) Its synthesis requires high pressure

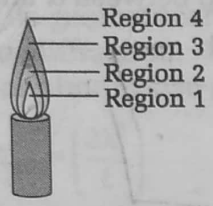
5. For a linear plot of $\log(x/m)$ *versus* $\log p$ in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

(a) $1/n$ appears as the intercept
(b) Only $1/n$ appears as the slope
(c) $\log\left(\frac{1}{n}\right)$ appears as the intercept
(d) Both k and $1/n$ appear in the slope term

6. The heats of combustion of carbon and carbon monoxide are -393.5 and -283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is

(a) 676.5 (b) –676.5
(c) –110.5 (d) 110.5

7. The hottest region of Bunsen flame shown in the figure given below is



(a) region 2 (b) region 3 (c) region 4 (d) region 1

8. Which of the following is an anionic detergent?

(a) Sodium lauryl sulphate
(b) Cetyltrimethyl ammonium bromide
(c) Glyceryl oleate
(d) Sodium stearate

9. 18 g of glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapour pressure of water (in torr) for this aqueous solution is

(a) 76.0 (b) 752.4 (c) 759.0 (d) 7.6

10. The distillation technique most suited for separating glycerol from spent lye in the soap industry is

(a) fractional distillation
(b) steam distillation
(c) distillation under reduced pressure
(d) simple distillation

11. The species in which the N-atom is in a state of *sp* hybridisation is

(a) NO_2^- (b) NO_3^- (c) NO_2 (d) NO_2^+

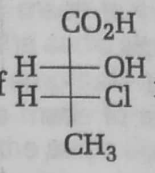
12. Decomposition of H_2O_2 follows a first order reaction. In 50 min, the concentration of H_2O_2 decreases from 0.5 to 0.125 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be

(a) 6.93×10^{-4} mol min^{-1} (b) 2.66 L min^{-1} at STP
(c) 1.34×10^{-2} mol min^{-1} (d) 6.93×10^{-2} mol min^{-1}

13. The pair having the same magnetic moment is
[at. no. Cr = 24, Mn = 25, Fe = 26 and Co = 27]

(a) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$
(b) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$
(c) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$
(d) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

14. The absolute configuration of



is

(a) (2*S*, 3*R*) (b) (2*S*, 3*S*)
(c) (2*R*, 3*R*) (d) (2*R*, 3*S*)

15. The equilibrium constant at 298 K for a reaction, $A + B \rightleftharpoons C + D$ is 100. If the initial concentrations of all the four species were 1 M each, then equilibrium concentration of *D* (in mol L^{-1}) will be

(a) 0.818 (b) 1.818 (c) 1.182 (d) 0.182

16. Which one of the following ores is best concentrated by froth floatation method?

(a) Siderite (b) Galena
(c) Malachite (d) Magnetite

17. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion, the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is

(a) C_3H_8 (b) C_4H_8 (c) C_4H_{10} (d) C_3H_6

18. The pair in which phosphorus atoms have a formal oxidation state of +3 is

(a) pyrophosphorous and hypophosphoric acids
(b) orthophosphorous and hypophosphoric acids
(c) pyrophosphorous and pyrophosphoric acids
(d) orthophosphorous and pyrophosphorous acids

19. Which one of the following complexes shows optical isomerism?

(a) *cis* $[Co(en)_2Cl_2]Cl$
(b) *trans* $[Co(en)_2Cl_2]Cl$
(c) $[Co(NH_3)_4Cl_2]Cl$
(d) $[Co(NH_3)_3Cl_3]$
(en = ethylenediamine)

20. The reaction of zinc with dilute and concentrated nitric acid, respectively, produce
(a) NO_2 and NO
(b) NO and N_2O
(c) NO_2 and N_2O
(d) N_2O and NO_2

21. Which one of the following statements about water is false?
(a) Water can act both as an acid and as a base
(b) There is extensive intramolecular hydrogen bonding in the condensed phase
(c) Ice formed by heavy water sinks in normal water
(d) Water is oxidised to oxygen during photosynthesis

22. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of
(a) lead
(b) nitrate
(c) iron
(d) fluoride

23. The main oxides formed on combustion of Li, Na and K in excess of air respectively are
(a) LiO_2, Na_2O_2 and K_2O
(b) Li_2O_2, Na_2O_2 and KO_2
(c) Li_2O, Na_2O_2 and KO_2
(d) Li_2O, Na_2O and KO_2

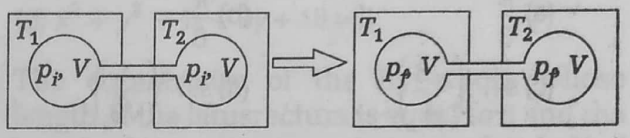
24. Thiol group is present in
(a) cystine
(b) cysteine
(c) methionine
(d) cytosine

25. Galvanisation is applying a coating of
(a) Cr (b) Cu
(c) Zn (d) Pb

26. Which of the following atoms has the highest first ionisation energy?
(a) Na
(b) K
(c) Sc
(d) Rb

27. In the Hofmann-bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are
(a) four moles of NaOH and two moles of Br_2
(b) two moles of NaOH and two moles of Br_2
(c) four moles of NaOH and one mole of Br_2
(d) one mole of NaOH and one mole of Br_2

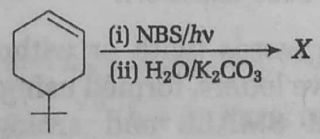
28. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is

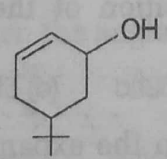


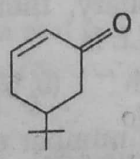
(a) $2p_i\left(\frac{T_1}{T_1+T_2}\right)$ (b) $2p_i\left(\frac{T_2}{T_1+T_2}\right)$
(c) $2p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$ (d) $p_i\left(\frac{T_1T_2}{T_1+T_2}\right)$

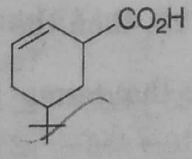
29. The reaction of propene with HOCl ($Cl_2 + H_2O$) proceeds through the intermediate
(a) $CH_3—\overset{+}{C}H—CH_2—Cl$
(b) $CH_3—CH(OH)—\overset{+}{C}H_2$
(c) $CH_3—CHCl—\overset{+}{C}H_2$
(d) $CH_3—\overset{+}{C}H—CH_2—OH$

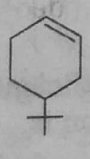
30. The product of the reaction given below is



(a) 

(b) 

(c) 

(d) 

Mathematics

1. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$,

$x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S

(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements

2. A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

3. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is

(a) 3 (b) – 4
(c) 6 (d) 5

4. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,\text{adj}\,A = AA^T$, then $5a + b$ is equal to

(a) – 1 (b) 5 (c) 4 (d) 13

5. The system of linear equations $x + \lambda y - z = 0$; $\lambda x - y - z = 0$; $x + y - \lambda z = 0$ has a non-trivial solution for

(a) infinitely many values of λ
(b) exactly one value of λ
(c) exactly two values of λ
(d) exactly three values of λ

6. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary, then the position of the word SMALL is

(a) 46th (b) 59th (c) 52nd (d) 58th

7. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion is

(a) 64 (b) 2187 (c) 243 (d) 729

8. If the 2nd, 5th and 9th terms of a non-constant AP are in GP, then the common ratio of this GP is

(a) $\frac{8}{5}$ (b) $\frac{4}{3}$
(c) 1 (d) $\frac{7}{4}$

9. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots$, is $\frac{16}{5}m$, then m is equal to

(a) 102 (b) 101 (c) 100 (d) 99

10. Let $p = \lim\limits_{x \to 0^+} (1 + \tan^2\sqrt{x})^{1/2x}$, then $\log p$ is equal to

(a) 2 (b) 1
(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

11. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

(a) g is not differentiable at $x = 0$
(b) $g'(0) = \cos(\log 2)$
(c) $g'(0) = -\cos(\log 2)$
(d) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

12. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1 + \sin x}{1 - \sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$.

A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point

(a) (0, 0) (b) $\left(0, \frac{2\pi}{3}\right)$
(c) $\left(\frac{\pi}{6}, 0\right)$ (d) $\left(\frac{\pi}{4}, 0\right)$

13. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then

(a) $2x = (\pi + 4)r$ (b) $(4 - \pi)x = \pi r$
(c) $x = 2r$ (d) $2x = r$

14. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to

(a) $\frac{-x^5}{(x^5+x^3+1)^2}+C$ (b) $\frac{x^{10}}{2(x^5+x^3+1)^2}+C$

(c) $\frac{x^5}{2(x^5+x^3+1)^2}+C$ (d) $\frac{-x^{10}}{2(x^5+x^3+1)^2}+C$

where, C is an arbitrary constant.

15. $\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right)^{1/n}$ is equal to

(a) $\frac{18}{e^4}$ (b) $\frac{27}{e^2}$

(c) $\frac{9}{e^2}$ (d) $3\log 3-2$

16. The area (in sq units) of the region $\{(x,y): y^2\geq 2x$ and $x^2+y^2\leq 4x,\ x\geq 0,\ y\geq 0\}$ is

(a) $\pi-\frac{4}{3}$ (b) $\pi-\frac{8}{3}$

(c) $\pi-\frac{4\sqrt{2}}{3}$ (d) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$

17. If a curve $y=f(x)$ passes through the point $(1,-1)$ and satisfies the differential equation, $y(1+xy)dx=xdy$, then $f\left(-\frac{1}{2}\right)$ is equal to

(a) $-\frac{2}{5}$ (b) $-\frac{4}{5}$

(c) $\frac{2}{5}$ (d) $\frac{4}{5}$

18. Two sides of a rhombus are along the lines, $x-y+1=0$ and $7x-y-5=0$. If its diagonals intersect at $(-1,-2)$, then which one of the following is a vertex of this rhombus?

(a) $(-3,-9)$ (b) $(-3,-8)$

(c) $\left(\frac{1}{3},-\frac{8}{3}\right)$ (d) $\left(-\frac{10}{3},-\frac{7}{3}\right)$

19. The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the X-axis, lie on

(a) a circle

(b) an ellipse which is not a circle

(c) a hyperbola

(d) a parabola

20. If one of the diameters of the circle, given by the equation, $x^2+y^2-4x+6y-12=0$, is a chord of a circle S, whose centre is at $(-3,2)$, then the radius of S is

(a) $5\sqrt{2}$ (b) $5\sqrt{3}$

(c) 5 (d) 10

21. Let P be the point on the parabola, $y^2=8x$, which is at a minimum distance from the centre C of the circle, $x^2+(y+6)^2=1$. Then, the equation of the circle, passing through C and having its centre at P is

(a) $x^2+y^2-4x+8y+12=0$

(b) $x^2+y^2-x+4y-12=0$

(c) $x^2+y^2-\frac{x}{4}+2y-24=0$

(d) $x^2+y^2-4x+9y+18=0$

22. The eccentricity of the hyperbola whose length of the latusrectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\sqrt{3}$

23. The distance of the point $(1,-5,9)$ from the plane $x-y+z=5$ measured along the line $x=y=z$ is

(a) $3\sqrt{10}$ (b) $10\sqrt{3}$ (c) $\frac{10}{\sqrt{3}}$ (d) $\frac{20}{3}$

24. If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane $lx+my-z=9$, then l^2+m^2 is equal to

(a) 26 (b) 18

(c) 5 (d) 2

25. Let $\hat{a}$, $\hat{b}$ and $\hat{c}$ be three unit vectors such that $\hat{a}\times(\hat{b}\times\hat{c})=\frac{\sqrt{3}}{2}(\hat{b}+\hat{c})$. If $\hat{b}$ is not parallel to $\hat{c}$, then the angle between $\hat{a}$ and $\hat{b}$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

26. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?

(a) $3a^2-26a+55=0$ (b) $3a^2-32a+84=0$

(c) $3a^2-34a+91=0$ (d) $3a^2-23a+44=0$

27. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four, E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is not true?

(a) E_1 and E_2 are independent
(b) E_2 and E_3 are independent
(c) E_1 and E_3 are independent
(d) E_1, E_2 and E_3 are independent

28. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is

(a) 3 (b) 5
(c) 7 (d) 9

29. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 min from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then, the time taken (in minutes) by him, from B to reach the pillar, is

(a) 6 (b) 10
(c) 20 (d) 5

30. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to

(a) $\sim p \wedge q$ (b) $p \wedge q$
(c) $p \vee q$ (d) $p \vee \sim q$

ANSWER WITH EXPLANATION

Physics

1. *(a)* Arithmetic mean time of a oscillating simple pendulum

$$= \frac{\Sigma x_i}{N} = \frac{90 + 91 + 92 + 95}{4} = 92\,\text{s}$$

Mean deviation of a simple pendulum

$$= \frac{\Sigma |\bar{x} - x_i|}{N} = \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Given, minimum division in the measuring clock, i.e. simple pendulum = 1 s. Thus, the reported mean time of a oscillating simple pendulum = (92 ± 2) s

2. *(b, d)* For a particle of mass m is moving along the side of a square of side a. Such that

Angular momentum L about the origin

$$= \mathbf{L} = \mathbf{r} \times \mathbf{p} = r_p \sin\theta\, \hat{\mathbf{n}} \text{ or } \mathbf{L} = \mathrm{r}\,(\mathrm{p})\,\hat{\mathbf{n}}$$

When a particle is moving from D to A,

$$\mathbf{L} = \frac{R}{\sqrt{2}} mv(-\hat{\mathbf{k}})$$

A particle is moving from A to B,

$$\mathbf{L} = \frac{R}{\sqrt{2}} mv(-\hat{\mathbf{k}})$$

and it moves from C to D,

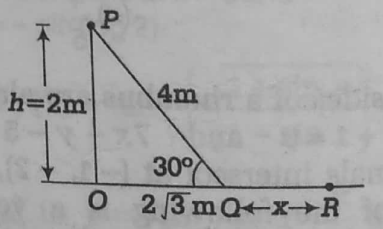
$$\mathbf{L} = \left(\frac{R}{\sqrt{2}} + a\right) mv\,(\hat{\mathbf{k}})$$

For B to C, we have

$$\mathbf{L} = \left(\frac{R}{\sqrt{2}} + a\right) mv\,(\hat{\mathbf{k}})$$

Hence, options (b) and (d) are incorrect.

3. *(c)* Energy lost over path $PQ = \mu mg\cos\theta \times 4$



Energy lost over path $QR = \mu mgx$

i.e $\mu mg\cos 30° \times 4 = \mu mgx$ $(\because \theta = 3$

$$x = 2\sqrt{3} = 3.45\,\text{m}$$

From Q to R, energy loss is half of the total ener loss.

i.e. $$\mu mgx = \frac{1}{2} \times mgh$$

$$\Rightarrow \quad \mu = 0.29$$

The values of the coefficient of friction μ and distance $x(= QR)$ are 0.29 and 3.5 m.