

1. A (1, -1, -3), B (2, 1, -2) & C (-5, 2, -6) are the position vectors of the vertices of a triangle ABC. The length of the bisector of its internal angle at A is :

- (A) $\sqrt{10}/4$ (B) $3\sqrt{10}/4$ (C) $\sqrt{10}$ (D) none

2. Let \bar{p} is the p.v. of the orthocentre & \bar{g} is the p.v. of the centroid of the triangle ABC where circumcentre is the origin. If $\bar{p} = K\bar{g}$, then K =

- (A) 3 (B) 2 (C) 1/3 (D) 2/3

3. A vector \bar{a} has components 2p & 1 with respect to a rectangular cartesian system. The system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to the new system, \bar{a} has components p + 1 & 1 then ,

- (A) p = 0 (B) p = 1 or p = -1/3 (C) p = -1 or p = 1/3 (D) p = 1 or p = -1

4. The number of vectors of unit length perpendicular to vectors $\bar{a} = (1, 1, 0)$ & $\bar{b} = (0, 1, 1)$ is:

- (A) 1 (B) 2 (C) 3 (D) ∞

5. Four points A(+1, -1, 1) ; B(1, 3, 1) ; C(4, 3, 1) and D(4, -1, 1) taken in order are the vertices of

- (A) a parallelogram which is neither a rectangle nor a rhombus
 (B) rhombus
 (C) an isosceles trapezium
 (D) a cyclic quadrilateral.

SBG STUDY

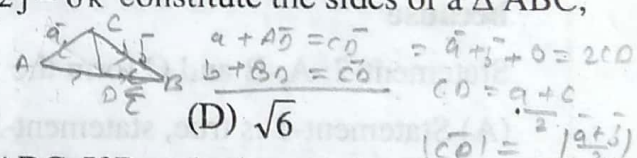
6. Let α, β & γ be distinct real numbers. The points whose position vector's are $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$; $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ and $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$



- (A) are collinear (B) form an equilateral triangle
 (C) form a scalene triangle (D) form a right angled triangle

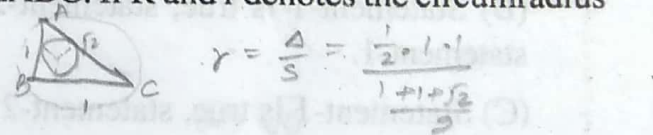
7. If the vectors $\bar{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\bar{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$ & $\bar{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ constitute the sides of a ΔABC , then the length of the median bisecting the vector \bar{c} is

- (A) $\sqrt{2}$ (B) $\sqrt{14}$ (C) $\sqrt{74}$ (D) $\sqrt{6}$



8. Let A(0, -1, 1), B(0, 0, 1), C(1, 0, 1) are the vertices of a ΔABC . If R and r denotes the circumradius

and inradius of ΔABC , then $\frac{r}{R}$ has value equal to



- (A) $\tan \frac{3\pi}{8}$ (B) $\cot \frac{3\pi}{8}$ (C) $\tan \frac{\pi}{12}$ (D) $\cot \frac{\pi}{12}$

$\bar{a}, \bar{b}, \bar{c}$ are three non-zero vectors, no two of which are collinear and the vector $\bar{a} + \bar{b}$ is collinear with \bar{c} , $\bar{b} + \bar{c}$ is collinear with \bar{a} , then $\bar{a} + \bar{b} + \bar{c}$ is equal to -

- (A) \bar{a} (B) \bar{b} (C) \bar{c} (D) none of these

10. If the three points with position vectors $(1, a, b)$; $(a, 2, b)$ and $(a, b, 3)$ are collinear in space, then the value of $a + b$ is

(A) 3

(B) 4

(C) 5

(D) none

11. Consider the following 3 lines in space

$$L_1: \vec{r} = 3\hat{i} - \hat{j} + 2\hat{k} + \lambda(2\hat{i} + 4\hat{j} - \hat{k})$$

$$L_2: \vec{r} = \hat{i} + \hat{j} - 3\hat{k} + \mu(4\hat{i} + 2\hat{j} + 4\hat{k})$$

$$L_3: \vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k} + t(2\hat{i} + \hat{j} + 2\hat{k})$$

Then which one of the following pair(s) are in the same plane.

(A) only L_1L_2

(B) only L_2L_3

(C) only L_3L_1

(D) L_1L_2 and L_2L_3

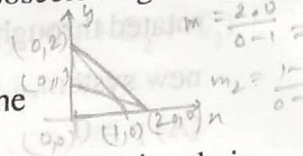
12. The acute angle between the medians drawn from the acute angles of an isosceles right angled triangle is:

(A) $\cos^{-1}(2/3)$

(B) $\cos^{-1}(3/4)$

(C) $\cos^{-1}(4/5)$

(D) none



13. The vectors $3\hat{i} - 2\hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} + 5\hat{k}$ & $2\hat{i} + \hat{j} - 4\hat{k}$ form the sides of a triangle. Then triangle is

(A) an acute angled triangle

(B) an obtuse angled triangle

(C) an equilateral triangle

(D) a right angled triangle

14. If the vectors $3\vec{p} + \vec{q}$; $5\vec{p} - 3\vec{q}$ and $2\vec{p} + \vec{q}$; $4\vec{p} - 2\vec{q}$ are pairs of mutually perpendicular vectors then $\sin(\hat{p} \hat{q})$ is

(A) $\sqrt{55}/4$

(B) $\sqrt{55}/8$

(C) $3/16$

(D) $\sqrt{247}/16$

15. Consider the points A, B and C with position vectors $(-2\hat{i} + 3\hat{j} + 5\hat{k})$, $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $7\hat{i} - \hat{k}$ respectively.

Statement-1: The vector sum, $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$

because

Statement-2: A, B and C form the vertices of a triangle.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

16. The set of values of c for which the angle between the vectors $cx\hat{i} - 6\hat{j} + 3\hat{k}$ & $x\hat{i} - 2\hat{j} + 2cx\hat{k}$ is acute for every $x \in \mathbb{R}$ is

(A) $(0, 4/3)$

(B) $[0, 4/3]$

(C) $(11/9, 4/3)$

(D) $[0, 4/3)$

17. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If \hat{n} is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $|\vec{w} \cdot \hat{n}|$ is equal to

(A) 1

(B) 2

(C) 3

(D) 0

18. If the vector $6\hat{i} - 3\hat{j} - 6\hat{k}$ is decomposed into vectors parallel and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ then the vectors are :
- (A) $-(\hat{i} + \hat{j} + \hat{k})$ & $7\hat{i} - 2\hat{j} - 5\hat{k}$ (B) $-2(\hat{i} + \hat{j} + \hat{k})$ & $8\hat{i} - \hat{j} - 4\hat{k}$
 (C) $+2(\hat{i} + \hat{j} + \hat{k})$ & $4\hat{i} - 5\hat{j} - 8\hat{k}$ (D) none
19. Let $\vec{r} = \vec{a} + \lambda\vec{l}$ and $\vec{r} = \vec{b} + \mu\vec{m}$ be two lines in space where $\vec{a} = 5\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = -\hat{i} + 7\hat{j} + 8\hat{k}$, $\vec{l} = -4\hat{i} + \hat{j} - \hat{k}$ and $\vec{m} = 2\hat{i} - 5\hat{j} - 7\hat{k}$ then the p.v. of a point which lies on both of these lines, is
- (A) $\hat{i} + 2\hat{j} + \hat{k}$ (B) $2\hat{i} + \hat{j} + \hat{k}$
 (C) $\hat{i} + \hat{j} + 2\hat{k}$ (D) non existent as the lines are skew
20. Let A(1, 2, 3), B(0, 0, 1), C(-1, 1, 1) are the vertices of a ΔABC .
- (i) The equation of internal angle bisector through A to side BC is.
- (A) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 2\hat{j} + 3\hat{k})$ (B) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 4\hat{j} + 3\hat{k})$
 (C) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 3\hat{j} + 2\hat{k})$ (D) $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(3\hat{i} + 3\hat{j} + 4\hat{k})$
- (ii) The equation of median through C to side AB is
- (A) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(3\hat{i} - 2\hat{k})$ (B) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(3\hat{i} + 2\hat{k})$
 (C) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(-3\hat{i} + 2\hat{k})$ (D) $\vec{r} = -\hat{i} + \hat{j} + \hat{k} + p(3\hat{i} + 2\hat{j})$
- (iii) The area (ΔABC) is equal to
- (A) $\frac{9}{2}$ (B) $\frac{\sqrt{17}}{2}$ (C) $\frac{17}{2}$ (D) $\frac{7}{2}$
21. If $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then the angle between \vec{a} & \vec{b} is:
- (A) $\pi/6$ (B) $2\pi/3$ (C) $5\pi/3$ (D) $\pi/3$
22. A line passes through the point A($\hat{i} + 2\hat{j} + 3\hat{k}$) and is parallel to the vector $\vec{V}(\hat{i} + \hat{j} + \hat{k})$. The shortest distance from the origin, of the line is -
- (A) $\sqrt{2}$ (B) $\sqrt{4}$ (C) $\sqrt{5}$ (D) $\sqrt{6}$
23. Let \vec{a} , \vec{b} , \vec{c} be vectors of length 3, 4, 5 respectively. Let \vec{a} be perpendicular to $\vec{b} + \vec{c}$, \vec{b} to $\vec{c} + \vec{a}$ & \vec{c} to $\vec{a} + \vec{b}$. Then $|\vec{a} + \vec{b} + \vec{c}|$ is :
- (A) $2\sqrt{5}$ (B) $2\sqrt{2}$ (C) $10\sqrt{5}$ (D) $5\sqrt{2}$
24. The set of values of x for which the angle between the vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ acute and the angle between the vector \vec{b} and the axis of ordinates is obtuse, is
- (A) $1 < x < 2$ (B) $x > 2$ (C) $x < 1$ (D) $x < 0$

25. If a vector \vec{a} of magnitude 50 is collinear with vector $\vec{b} = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with positive z-axis then :
- (A) $\vec{a} = 4\vec{b}$ (B) $\vec{a} = -4\vec{b}$ (C) $\vec{b} = 4\vec{a}$ (D) none
26. A, B, C & D are four points in a plane with pv's $\vec{a}, \vec{b}, \vec{c}$ & \vec{d} respectively such that $(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) = (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) = 0$. Then for the triangle ABC, D is its
- (A) incentre (B) circumcentre (C) orthocentre (D) centroid
27. \vec{a} and \vec{b} are unit vectors inclined to each other at an angle $\alpha, \alpha \in (0, \pi)$ and $|\vec{a} + \vec{b}| < 1$. Then $\alpha \in$
- (A) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$ (B) $\left(\frac{2\pi}{3}, \pi\right)$ (C) $\left(0, \frac{\pi}{3}\right)$ (D) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
28. Image of the point P with position vector $7\hat{i} - \hat{j} + 2\hat{k}$ in the line whose vector equation is, $\vec{r} = 9\hat{i} + 5\hat{j} + 5\hat{k} + \lambda(\hat{i} + 3\hat{j} + 5\hat{k})$ has the position vector
- (A) $(-9, 5, 2)$ (B) $(9, 5, -2)$ (C) $(9, -5, -2)$ (D) none
29. Let $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{a} + \hat{b} + \hat{c}$ is also a unit vector. If pairwise angles between $\hat{a}, \hat{b}, \hat{c}$ are θ_1, θ_2 and θ_3 respectively then $\cos \theta_1 + \cos \theta_2 + \cos \theta_3$ equals
- (A) 3 (B) -3 (C) 1 (D) -1
30. A tangent is drawn to the curve $y = \frac{8}{x^2}$ at a point A (x_1, y_1) , where $x_1 = 2$. The tangent cuts the x-axis at point B. Then the scalar product of the vectors \vec{AB} & \vec{OB} is
- (A) 3 (B) -3 (C) 6 (D) -6
31. Cosine of an angle between the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ if $|\vec{a}| = 2, |\vec{b}| = 1$ and $\vec{a} \wedge \vec{b} = 60^\circ$ is
- (A) $\sqrt{3/7}$ (B) $9/\sqrt{21}$ (C) $3/\sqrt{7}$ (D) none
32. An arc AC of a circle subtends a right angle at the centre O. The point B divides the arc in the ratio 1 : 2. If $\vec{OA} = \vec{a}$ & $\vec{OB} = \vec{b}$, then the vector \vec{OC} in terms of \vec{a} & \vec{b} , is
- (A) $\sqrt{3}\vec{a} - 2\vec{b}$ (B) $-\sqrt{3}\vec{a} + 2\vec{b}$ (C) $2\vec{a} - \sqrt{3}\vec{b}$ (D) $-2\vec{a} + \sqrt{3}\vec{b}$
33. Given three vectors \vec{a}, \vec{b} & \vec{c} each two of which are non collinear. Further if $(\vec{a} + \vec{b})$ is collinear with $\vec{c}, (\vec{b} + \vec{c})$ is collinear with \vec{a} & $|\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{2}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$:
- (A) is 3 (B) is -3 (C) is 0 (D) cannot be evaluated
34. The vector equations of two lines L_1 and L_2 are respectively
- $\vec{r} = 17\hat{i} - 9\hat{j} + 9\hat{k} + \lambda(3\hat{i} + \hat{j} + 5\hat{k})$ and $\vec{r} = 15\hat{i} - 8\hat{j} - \hat{k} + \mu(4\hat{i} + 3\hat{j})$
- I L_1 and L_2 are skew lines
- II $(11, -11, -1)$ is the point of intersection of L_1 and L_2
- III $(-11, 11, 1)$ is the point of intersection of L_1 and L_2
- IV $\cos^{-1}(3/\sqrt{35})$ is the acute angle between L_1 and L_2
- then, which of the following is true?
- (A) II and IV (B) I and IV (C) IV only (D) III and IV

35. For two particular vectors \vec{A} and \vec{B} it is known that $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$. What must be true about the two vectors?

(A) At least one of the two vectors must be the zero vector.

(B) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$ is true for any two vectors.

(C) One of the two vectors is a scalar multiple of the other vector.

(D) The two vectors must be perpendicular to each other.

36. For some non zero vector \vec{V} , if the sum of \vec{V} and the vector obtained from \vec{V} by rotating it by an angle 2α equals to the vector obtained from \vec{V} by rotating it by α then the value of α , is

(A) $2n\pi \pm \frac{\pi}{3}$

(B) $n\pi \pm \frac{\pi}{3}$

(C) $2n\pi \pm \frac{2\pi}{3}$

(D) $n\pi \pm \frac{2\pi}{3}$

where n is an integer.

37. Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}|=1, |\vec{v}|=2, |\vec{w}|=3$. If the projection of \vec{v} along \vec{u} is equal to that of \vec{w} along \vec{u} and vectors \vec{v}, \vec{w} are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals

(A) 2

(B) $\sqrt{7}$

(C) $\sqrt{14}$

(D) 14

38. If \vec{a} and \vec{b} are non zero, non collinear, and the linear combination

$(2x - y)\vec{a} + 4\vec{b} = 5\vec{a} + (x - 2y)\vec{b}$ holds for real x and y then x + y has the value equal to

(A) -3

(B) 1

(C) 17

(D) 3

39. Given an equilateral triangle ABC with side length equal to 'a'. Let M and N be two points respectively

on the side AB and AC such that $\vec{AN} = K\vec{AC}$ and $\vec{AM} = \frac{\vec{AB}}{3}$. If \vec{BN} and \vec{CM} are orthogonal then the value of K is equal to

(A) $\frac{1}{5}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

40. If \vec{p} & \vec{s} are not perpendicular to each other and $\vec{r} \times \vec{p} = \vec{q} \times \vec{p}$ & $\vec{r} \cdot \vec{s} = 0$, then $\vec{r} =$

(A) $\vec{p} \cdot \vec{s}$

(B) $\vec{q} + \left(\frac{\vec{q} \cdot \vec{p}}{\vec{p} \cdot \vec{s}} \right) \vec{p}$

(C) $\vec{q} - \left(\frac{\vec{q} \cdot \vec{s}}{\vec{p} \cdot \vec{s}} \right) \vec{p}$

(D) $\vec{q} + \mu \vec{p}$ for all scalars μ

41. If \vec{u} and \vec{v} are two vectors such that $|\vec{u}|=3; |\vec{v}|=2$ and $|\vec{u} \times \vec{v}|=6$ then the correct statement is

(A) $\vec{u} \wedge \vec{v} \in (0, 90^\circ)$

(B) $\vec{u} \wedge \vec{v} \in (90^\circ, 180^\circ)$

(C) $\vec{u} \wedge \vec{v} = 90^\circ$

(D) $(\vec{u} \times \vec{v}) \times \vec{u} = 6\vec{v}$

42. Given a parallelogram OACB. The lengths of the vectors \vec{OA}, \vec{OB} & \vec{AB} are a, b & c respectively.

The scalar product of the vectors \vec{OC} & \vec{OB} is :

(A) $\frac{a^2 - 3b^2 + c^2}{2}$

(B) $\frac{3a^2 + b^2 - c^2}{2}$

(C) $\frac{3a^2 - b^2 + c^2}{2}$

(D) $\frac{a^2 + 3b^2 - c^2}{2}$

43. Vectors \vec{a} & \vec{b} make an angle $\theta = \frac{2\pi}{3}$. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ then $\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\}^2 =$

- (A) 225 (B) 250 (C) 275 (D) 300

44. If the vector product of a constant vector \vec{OA} with a variable vector \vec{OB} in a fixed plane OAB be a constant vector, then locus of B is :

- (A) a straight line perpendicular to \vec{OA} (B) a circle with centre O radius equal to $|\vec{OA}|$
 (C) a straight line parallel to \vec{OA} (D) none of these

45. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $|\vec{a} \times \vec{b} \cdot \vec{c}| = |\vec{a}| |\vec{b}| |\vec{c}|$ holds if and only if ;

- (A) $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$ (B) $\vec{c} \cdot \vec{a} = 0, \vec{a} \cdot \vec{b} = 0$
 (C) $\vec{a} \cdot \vec{c} = 0, \vec{b} \cdot \vec{c} = 0$ (D) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

46. The vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ & $\vec{c} = 3\hat{i} + \hat{j} + 4\hat{k}$ are so placed that the end point of one vector is the starting point of the next vector. Then the vectors are -

- (A) not coplanar (B) coplanar but cannot form a triangle
 (C) coplanar but can form a triangle (D) coplanar & can form a right angled triangle

47. Given the vectors

$$\vec{u} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{v} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{w} = \hat{i} - \hat{k}$$

If the volume of the parallelepiped having $c\vec{u}$, \vec{v} and $c\vec{w}$ as concurrent edges, is 8 then 'c' can be equal to

- (A) ± 2 (B) 4 (C) 8 (D) can not be determined

48. Given $\vec{a} = x\hat{i} + y\hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j}$; $(\vec{a} \wedge \vec{b}) \cdot \vec{c} = \pi/2$, $\vec{a} \cdot \vec{c} = 4$ then

- (A) $[\vec{a} \vec{b} \vec{c}]^2 = |\vec{a}|$ (B) $[\vec{a} \vec{b} \vec{c}] = |\vec{a}|$ (C) $[\vec{a} \vec{b} \vec{c}] = 0$ (D) $[\vec{a} \vec{b} \vec{c}] = |\vec{a}|^2$

49. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$; $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$; $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that \vec{c} is a unit vector perpendicular to both \vec{a} & \vec{b} . If the angle between \vec{a} & \vec{b} is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 =$$

$$(|\vec{a}| |\vec{b}| |\vec{c}| \sin 30^\circ)^2 = (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2) \cdot \frac{1}{4}$$

- (A) 0
 (B) 1
 (C) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$
 (D) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

50. For three vectors $\vec{u}, \vec{v}, \vec{w}$ which of the following expressions is not equal to any of the remaining three?

- (A) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (B) $(\vec{v} \times \vec{w}) \cdot \vec{u}$ (C) $\vec{v} \cdot (\vec{u} \times \vec{w})$ (D) $(\vec{u} \times \vec{v}) \cdot \vec{w}$

51. Let $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ & $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors, $\hat{i} - 2\hat{j} + \hat{k}$, $3\hat{i} + 2\hat{j} - \hat{k}$ & \vec{c} are coplanar then $\frac{\alpha}{\beta}$ is
- (A) 1 (B) 2 (C) 3 (D) -3
52. A rigid body rotates with constant angular velocity ω about the line whose vector equation is, $\vec{r} = \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$. The speed of the particle at the instant it passes through the point with p.v. $2\hat{i} + 3\hat{j} + 5\hat{k}$ is:

- (A) $\omega\sqrt{2}$ (B) 2ω (C) $\omega/\sqrt{2}$ (D) none

53. Given 3 vectors $\vec{V}_1 = a\hat{i} + b\hat{j} + c\hat{k}$; $\vec{V}_2 = b\hat{i} + c\hat{j} + a\hat{k}$; $\vec{V}_3 = c\hat{i} + a\hat{j} + b\hat{k}$

In which one of the following conditions \vec{V}_1 , \vec{V}_2 and \vec{V}_3 are linearly independent?

- (A) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$
 (B) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
 (C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$
 (D) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

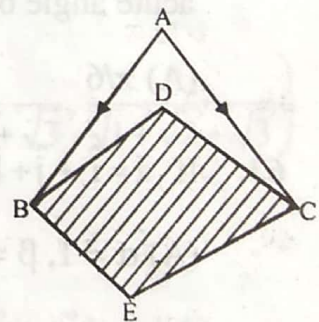
$$(a+b+c) [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$

54. Given unit vectors \vec{m} , \vec{n} & \vec{p} such that angle between \vec{m} & $\vec{n} =$ angle between \vec{p} and $(\vec{m} \times \vec{n}) = \pi/6$, then $[\vec{n} \vec{p} \vec{m}] =$

- (A) $\sqrt{3}/4$ (B) $3/4$ (C) $1/4$ (D) none

55. Let $\vec{AB} = 3\hat{i} - \hat{j}$, $\vec{AC} = 2\hat{i} + 3\hat{j}$ and $\vec{DE} = 4\hat{i} - 2\hat{j}$. The area of the shaded region in the adjacent figure, is-

- (A) 5 (B) 6
 (C) 7 (D) 8



56. The altitude of a parallelepiped whose three coterminous edges are the vectors, $\vec{A} = \hat{i} + \hat{j} + \hat{k}$; $\vec{B} = 2\hat{i} + 4\hat{j} - \hat{k}$ & $\vec{C} = \hat{i} + \hat{j} + 3\hat{k}$ with \vec{A} and \vec{B} as the sides of the base of the parallelepiped, is
- (A) $2/\sqrt{19}$ (B) $4/\sqrt{19}$ (C) $2\sqrt{38}/19$ (D) none

57. Consider ΔABC with $A \equiv (\vec{a})$; $B \equiv (\vec{b})$ & $C \equiv (\vec{c})$. If $\vec{b} \cdot (\vec{a} + \vec{c}) = \vec{b} \cdot \vec{b} + \vec{a} \cdot \vec{c}$; $|\vec{b} - \vec{a}| = 3$; $|\vec{c} - \vec{b}| = 4$ then the angle between the medians \vec{AM} & \vec{BD} is

- (A) $\pi - \cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$ (B) $\pi - \cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$
 (C) $\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$ (D) $\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$