

SBG STUDY

1. $\lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right)$ is equal to

- (A) -1 (B) 0 (C) 1 (D) D.N.E.

2. $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$ is equal to

- (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

3. $\lim_{x \rightarrow 2} \frac{\sqrt{1+\sqrt{2+x}} - \sqrt{3}}{x-2}$ is equal to

- (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) $\frac{1}{4\sqrt{3}}$ (D) $\frac{1}{8\sqrt{3}}$

4. $\lim_{x \rightarrow 1} \frac{\sqrt[n]{x} - 1}{\sqrt[m]{x} - 1}$ (m and n integers) is equal to

- (A) 0 (B) 1 (C) $\frac{m}{n}$ (D) $\frac{n}{m}$

5. If $\lim_{x \rightarrow a} \frac{2x - \sqrt{x^2 + 3a^2}}{\sqrt{x+a} - \sqrt{2a}} = \sqrt{2}$ (where $a \in \mathbb{R}^+$), then a is equal to -

- (A) $\frac{1}{3}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{3\sqrt{2}}$ (D) $\frac{1}{9}$

6. $\lim_{x \rightarrow 0} \frac{\ln(\sin 3x)}{\ln(\sin x)}$ is equal to

- (A) 0 (B) 1 (C) 2 (D) Non existent

7. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x^2} - \sqrt{1-2x}}{x+x^2}$ is equal to

- (A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) D.N.E.

8. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{7+x^3} - \sqrt{3+x^2}}{x-1}$ is equal to

- (A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $-\frac{1}{4}$ (D) $-\frac{1}{6}$

9. $\lim_{n \rightarrow \infty} \frac{(n+1)^4 - (n-1)^4}{(n+1)^4 + (n-1)^4}$ is equal to (A) -1 (B) 0 (C) 1 (D) D.N.E.

10. $\lim_{x \rightarrow \infty} \frac{(x+1)^{10} + (x+2)^{10} + \dots + (x+100)^{10}}{x^{10} + 10^{10}}$ is equal to (A) 1 (B) 100 (C) 200 (D) 10

11. $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x + 3})$ is equal to (A) $-\frac{5}{2}$ (B) $\frac{5}{2}$ (C) 0 (D) D.N.E.

12. If $\lim_{n \rightarrow \infty} (\sqrt{2n^2 + n} - \lambda \sqrt{2n^2 - n}) = \frac{1}{\sqrt{2}}$ (where λ is a real number), then- (A) $\lambda = 1$ (B) $\lambda = -1$ (C) $\lambda = \pm 1$ (D) $\lambda \in (-\infty, 1)$

13. Let $U_n = \frac{n!}{(n+2)!}$ where $n \in \mathbb{N}$. If $S_n = \sum_{n=1}^n U_n$ then $\lim_{n \rightarrow \infty} S_n$ equals (A) 2 (B) 1 (C) 1/2 (D) Non existent

14. For $n \in \mathbb{N}$, let $a_n = \sum_{k=1}^n 2k$ and $b_n = \sum_{k=1}^n (2k-1)$. Then $\lim_{n \rightarrow \infty} (\sqrt{a_n} - \sqrt{b_n})$ is equal to- (A) 1 (B) $\frac{1}{2}$ (C) 0 (D) 2

15. Let $P_n = \prod_{k=2}^n \left(1 - \frac{1}{k+1} C_2\right)$. If $\lim_{n \rightarrow \infty} P_n$ can be expressed as lowest rational in the form $\frac{a}{b}$, then value of $(a + b)$ is (A) 4 (B) 8 (C) 10 (D) 12

16. $\lim_{x \rightarrow -1} \frac{\cos 2 - \cos 2x}{x^2 - |x|}$ is equal to (A) 0 (B) $\cos 2$ (C) $2 \sin 2$ (D) $\sin 1$

17. $\lim_{x \rightarrow 0} \left(\left[\frac{-5 \sin x}{x} \right] + \left[\frac{6 \sin x}{x} \right] \right)$ (where $[.]$ denotes greatest integer function) is equal to - (A) 0 (B) -12 (C) 1 (D) 2

18. Let $f(x) = \left[\frac{\sin x}{x} \right] + \left[\frac{2 \sin 2x}{x} \right] + \dots + \left[\frac{10 \sin 10x}{x} \right]$ (where $[y]$ is the largest integer $\leq y$). The value of $\lim_{x \rightarrow 0} f(x)$ equals (A) 55 (B) 164 (C) 165 (D) 375

19. Let $f(x) = \frac{\sin\{x\}}{x^2 + ax + b}$. If $f(5^+)$ & $f(3^+)$ exists finitely and are not zero, then the value of $(a + b)$

is (where $\{.\}$ represents fractional part function) -

- (A) 7 (B) 10 (C) 11 (D) 20

20. $\lim_{x \rightarrow 0} \frac{|\cos(\sin(3x))| - 1}{x^2}$ equals

- (A) $-\frac{9}{2}$ (B) $-\frac{3}{2}$ (C) $\frac{3}{2}$ (D) $\frac{9}{2}$

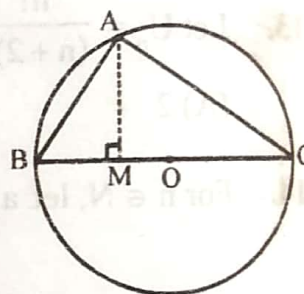
21. Let $a = \min\{x^2 + 2x + 3, x \in \mathbb{R}\}$ and $b = \lim_{\theta \rightarrow 0} \frac{1 - \cos\theta}{\theta^2}$. Then value of $\sum_{r=0}^n a^r \cdot b^{n-r}$ is :

- (A) $\frac{2^{n+1} - 1}{3 \cdot 2^n}$ (B) $\frac{2^{n+1} + 1}{3 \cdot 2^n}$ (C) $\frac{4^{n+1} - 1}{3 \cdot 2^n}$ (D) N.O.T.

22. Let BC is diameter of a circle centred at O. Point A is a variable point, moving on the circumference of circle. If $BC = 1$ unit, then

$\lim_{A \rightarrow B} \frac{BM}{(\text{Area of sector OAB})^2}$ is equal to -

- (A) 1 (B) 2
(C) 4 (D) 16



23. $\lim_{x \rightarrow \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2} \right)^x$ is equal to

- (A) 1 (B) e (C) $\frac{1}{e^2}$ (D) e^2

24. $\lim_{x \rightarrow 0} (1 + \sin x)^{\cos x}$ is equal to

- (A) 0 (B) e (C) 1 (D) $\frac{1}{e}$

25. $\lim_{x \rightarrow 0} (\cos x + a \sin bx)^{\frac{1}{x}}$ is equal to

- (A) e^a (B) e^{ab} (C) e^b (D) $e^{a/b}$

26. $\lim_{x \rightarrow 0} \left(\tan \left(\frac{\pi}{4} + x \right) \right)^{1/x}$ is equal to

- (A) e^{-2} (B) $\frac{1}{e}$ (C) e (D) e^2

27. $\lim_{n \rightarrow \infty} (4^n + 5^n)^{1/n}$ is equal to

- (A) 5 (B) 4 (C) 0 (D) D.N.E.

28. $\lim_{x \rightarrow \infty} \left(\frac{1^{1/x} + 2^{1/x} + 3^{1/x} + \dots + n^{1/x}}{n} \right)^{nx}$ is equal to

- (A) $n!$ (B) 1 (C) $\frac{1}{n!}$ (D) 0

29. If $\lim_{x \rightarrow \lambda} \left(2 - \frac{\lambda}{x} \right)^{\lambda \tan\left(\frac{\pi x}{2\lambda}\right)} = \frac{1}{e}$, then λ is equal to -

- (A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) $-\frac{2}{\pi}$

30. If $\lim_{x \rightarrow 0} (1 + ax + bx^2)^{2x} = e^3$, then

- (A) $a = \frac{3}{2}$ and $b \in \mathbb{R}$ (B) $a = \frac{3}{2}$ and $b \in \mathbb{R}^+$
 (C) $a = 0$ and $b = 1$ (D) $a = 1$ and $b = 0$

31. If $f(x)$ is a polynomial of least degree, such that $\lim_{x \rightarrow 0} \left(1 + \frac{f(x) + x^2}{x^2} \right)^{1/x} = e^2$, then $f(2)$ is -

- (A) 2 (B) 8 (C) 10 (D) 12

32. Let $f(x) = \frac{\tan x}{x}$, then the value of $\lim_{x \rightarrow 0} \left([f(x)] + x^2 \right)^{\frac{1}{\{f(x)\}}}$ is equal to (where $[.]$, $\{.\}$ denotes greatest integer function and fractional part functions respectively) -

- (A) e^{-3} (B) e^3 (C) e^2 (D) non-existent

33. $\lim_{n \rightarrow \infty} \frac{e^n}{\left(1 + \frac{1}{n}\right)^{n^2}}$ equals -

- (A) 1 (B) $\frac{1}{2}$ (C) e (D) \sqrt{e}

34. If $f(x)$ is odd linear polynomial with $f(1) = 1$, then $\lim_{x \rightarrow 0} \frac{2^{f(\tan x)} - 2^{f(\sin x)}}{x^2 f(\sin x)}$ is

- (A) 1 (B) $\ln 2$ (C) $\frac{1}{2} \ln 2$ (D) $\cos 2$

35. $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$ then

- (A) $a = -5/2$ (B) $a = -3/2, b = -1/2$ (C) $a = -3/2, b = -5/2$ (D) $a = -5/2, b = -3/2$

f(x) = 2ax

1. Rationals

36. Consider following statements and identify correct options

(i) $\lim_{x \rightarrow 4} \left(\frac{2x}{x-4} - \frac{8}{x-4} \right) = \lim_{x \rightarrow 4} \frac{2x}{x-4} - \lim_{x \rightarrow 4} \frac{8}{x-4}$

(ii) $\lim_{x \rightarrow 1} \frac{x^2 + 6x - 7}{x^2 + 5x - 6} = \frac{\lim_{x \rightarrow 1} (x^2 + 6x - 7)}{\lim_{x \rightarrow 1} (x^2 + 5x - 6)}$

(iii) $\lim_{x \rightarrow 1} \frac{x-3}{x^2 + 2x - 4} = \frac{\lim_{x \rightarrow 1} (x-3)}{\lim_{x \rightarrow 1} (x^2 + 2x - 4)}$

(iv) If $\lim_{x \rightarrow 5} f(x) = 2$ and $\lim_{x \rightarrow 5} g(x) = 0$, then $\lim_{x \rightarrow 5} \frac{f(x)}{g(x)}$ does not exist.

(v) If $\lim_{x \rightarrow 5} f(x) = 0$ and $\lim_{x \rightarrow 5} g(x) = 2$, then $\lim_{x \rightarrow 5} \frac{f(x)}{g(x)}$ does not exist.

(A) Only one is true.

(B) Only two are true.

(C) Only three are false.

(D) Only two are false.

37. Which of the following limits equal to $\frac{1}{2}$

(A) $\lim_{n \rightarrow \infty} \left(\frac{1}{1.3} + \frac{1}{3.5} + \dots + \frac{1}{(2n-1)(2n+1)} \right)$

(B) $\lim_{x \rightarrow \infty} \left[\frac{3x^2}{2x+1} - \frac{(2x-1)(3x^2+x+2)}{4x^2} \right]$

(C) $\lim_{n \rightarrow \infty} \frac{1}{n^2} (1 + 2 + 3 + \dots + n)$

(D) $\lim_{n \rightarrow \infty} \frac{(n+2)! + (n+1)!}{(n+2)! - (n+1)!}$

38. Let $f(x) = \begin{cases} \sin x; & \text{where } x = \text{integer} \\ 0; & \text{otherwise} \end{cases}$; $g(x) = \begin{cases} x^2 + 1; & x \neq 0, 2 \\ 4; & x = 0 \\ 5; & x = 2 \end{cases}$, then

(A) $\lim_{x \rightarrow 0} g(f(x)) = 4$

(B) $\lim_{x \rightarrow 0} f(g(x)) = 0$

(C) $\lim_{x \rightarrow 1} f(g(x)) = 0$

(D) $\lim_{x \rightarrow 1} g(f(x)) = 5$

39. If $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$, then

(A) $\lim_{x \rightarrow 0} f(x) = 0$

(B) $\lim_{x \rightarrow 0} f(x)$ does not exist

(C) $\lim_{x \rightarrow 2} f(x) = 4$

(D) $\lim_{x \rightarrow 2} f(x)$ does not exist

40. Let $f(\beta) = \lim_{\alpha \rightarrow \beta} \frac{\sin^2 \alpha - \sin^2 \beta}{\alpha^2 - \beta^2}$, then $f\left(\frac{\pi}{4}\right)$ is greater than-

(A) $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin 2x}$

(B) $\lim_{x \rightarrow \pi/2} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

(C) $\lim_{x \rightarrow \infty} (\cos \sqrt{x+1} - \cos \sqrt{x})$

(D) $\lim_{x \rightarrow a} \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}}$ where $a > 0$

41. If $\frac{\sin x + ae^x + be^{-x} + c \ln(1+x)}{x^3}$ has a finite limit L as $x \rightarrow 0$, then

- (A) $a = -\frac{1}{2}$ (B) $b = \frac{1}{2}$ (C) $c = 0$ (D) $L = -\frac{1}{3}$

42. Let $\ell = \lim_{x \rightarrow \infty} \frac{a^x - a^{-x}}{a^x + a^{-x}}$ ($a > 0$), then

- (A) $\ell = 1 \forall a > 0$ (B) $\ell = -1 \forall a \in (0, 1)$ (C) $\ell = 0$, if $a = 1$ (D) $\ell = 1 \forall a > 1$

[MATCH THE COLUMN TYPE]

43. For the function $g(t)$ whose graph is given, match the entries of column-I to column-II

Column-I

Column-II

(A) $\lim_{t \rightarrow 0^+} g(t) + \lim_{t \rightarrow 2^-} g(t)$

(P) $\lim_{t \rightarrow 2^+} g(t)$

(B) $\lim_{t \rightarrow 0^-} g(t) + g(2)$

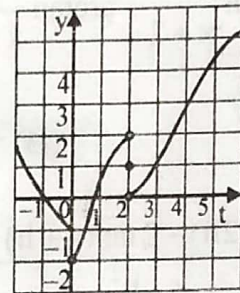
(Q) does not exist

(C) $\lim_{t \rightarrow 0} g(t)$

(R) 0

(D) $\lim_{t \rightarrow 2} g(t)$

(S) $\lim_{t \rightarrow 4} g(t)$



44. **Column-I**

Column-II

(A) $\lim_{n \rightarrow \infty} n \sin\left(\frac{\pi}{4n}\right) \cos\left(\frac{\pi}{4n}\right)$ is equal to

(P) 0

(B) $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$ is equal to

(Q) $\frac{1}{2}$

(C) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)$ is equal to

(R) $\frac{\pi}{4}$

(D) $\lim_{x \rightarrow \pi/2} \frac{1 + \cos 2x}{(\pi - 2x)^2}$ is equal to

(S) $\frac{\pi}{180}$

45. **Column-I**

Column-II

(A) $\lim_{x \rightarrow \infty} \frac{a^x}{a^x + 1}$ ($a > 0$) can be equal to

(P) $\lim_{x \rightarrow \infty} x(e^{1/x} - 1)$

(B) $\lim_{x \rightarrow 2} \frac{\sin(e^{x-2} - 1)}{\log(x-1)}$ is equal to

(Q) $\lim_{x \rightarrow 0} \frac{a^x + b^x + c^x - 3}{x}$ ($a, b, c > 0$ & $abc = 1$)

(C) $\lim_{x \rightarrow e} \frac{(\ln x - 1)e}{x - e}$ is equal to

(R) $\lim_{x \rightarrow 0} \frac{e^{4x} - e^{3x}}{x}$

(D) $\lim_{x \rightarrow 0} \frac{x(5^x - 1)}{(1 - \cos x)4 \ln 5}$ is equal to

(S) $\frac{1}{2}$

(T) 0