Subjective Type Questions

1. **Find the domain of definition of the given functions :**

(i) $y=\sqrt{-px}\,(p>0)$ (ii) $y=\dfrac{1}{x^2+1}$ (iii) $y=\dfrac{1}{x^3-x}$ (iv) $y=\dfrac{1}{\sqrt{x^2-4x}}$

(v) $y=\sqrt{x^2-4x+3}$ (vi) $y=\dfrac{x}{\sqrt{x^2-3x+2}}$ (vii) $y=\sqrt{1-|x|}$ (viii) $y=\log_x 2$.

(ix) $y=\dfrac{1}{\log_{10}(1-x)}+\sqrt{x+2}$ (x) $y=\sqrt{x}+\sqrt[3]{\dfrac{1}{x-2}}-\log_{10}(2x-3)$

(xi) $y=\dfrac{3}{4-x^2}+\log_{10}(x^3-x)$ (xii) $y=\dfrac{1}{\sqrt{\sin x}}+\sqrt[3]{\sin x}$

(xiii) $y=\log_{10}\left(\sqrt{x-4}+\sqrt{6-x}\right)$ (xiv) $y=\log_{10}[1-\log_{10}(x^2-5x+16)]$

2. **Find the range of the following functions :**

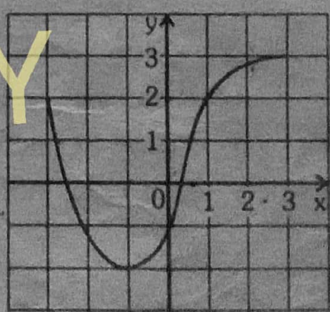
(i) $f(x)=\dfrac{x-1}{x+2}$ (ii) $f(x)=\dfrac{2}{x}$ (iii) $f(x)=\dfrac{1}{x^2-x+1}$ (iv) $f(x)=\dfrac{x^2-x+1}{x^2+x+1}$

(v) $f(x)=e^{(x-1)^2}$ (vi) $f(x)=x^3-x^2+x+1$

(vii) $f(x)=\log(x^8+x^4+x^2+1)$ (viii) $f(x)=\sin^2 x-2\sin x+4$

(ix) $f(x)=\sin(\log_2 x)$ (x) $f(x)=2^{x^2}+1$ (xi) $f(x)=\dfrac{e^{2x}-e^x+1}{e^{2x}+e^x+1}$ (xii) $f(x)=\dfrac{1}{8-3\sin x}$

3. The graph of a function f is given.



(a) State the value of $f(-1)$.

(b) For what values of x is $f(x)=2$

(c) State the domain and range of f.

(d) On what interval is f increasing ?

(e) Estimated value of $f(2)$ is -

(A) 2.2 (B) 2.8 (C) 2.5 (D) 3

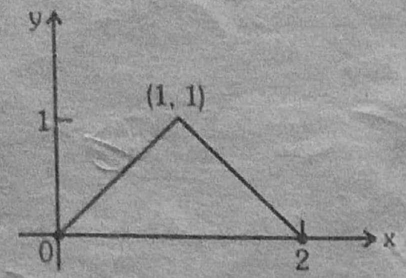
(f) Estimated value of x such that $f(x)=0$, is -

(A) –2.5 (B) 0.8 (C) –2.9 (D) 0.3

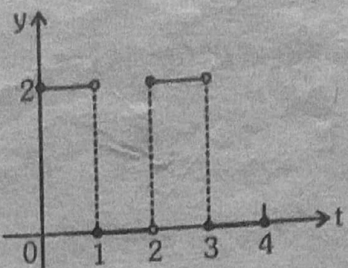
4. Graph the function $F(x)=\begin{cases}3-x, & x\le 1\\ 2x, & x>1\end{cases}$ non-uniform function

5. Find a formula for each function graphed

(a)



(b)



6. The graphs of f and g are given.

(a) State the value of $f(-4)$ and $g(3)$

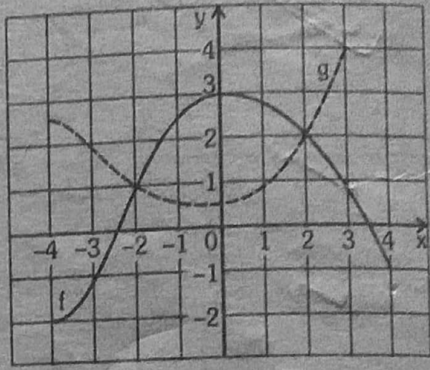
(b) For what value of x is $f(x) = g(x)$?

(c) Estimate the solution of the equation $f(x) = -1$.

(d) On what interval is f decreasing?

(e) State the domain and range of f.

(f) State the domain and range of g.



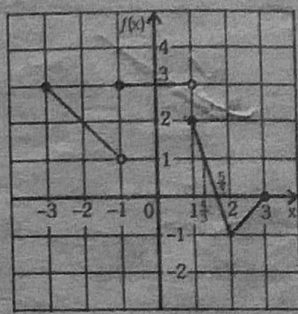
7. Solve the following inequalities using graph of f(x) :

(a) $0 \le f(x) \le 1$

(b) $-1 \le f(x) \le 2$

(c) $2 \le f(x) \le 3$

(d) $f(x) > -1$ & $f(x) < 0$



Straight Objective Type

8. If [a] denotes the greatest integer less than or equal to a and $-1 \le x < 0$, $0 \le y < 1$, $1 \le z < 2$, then

$$\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix} \text{ is equal to -}$$

(A) [x] (B) [y] (C) [z] (D) none of these

9. If [x] and {x} denotes the greatest integer function less than or equal to x and fractional part function respectively, then the number of real x, satisfying the equation $(x-2)[x] = \{x\} - 1$, is-

(A) 0 (B) 1 (C) 2 (D) infinite

10. The range of the function $f(x) = \text{sgn}\left(\dfrac{\sin^2 x + 2\sin x + 4}{\sin^2 x + 2\sin x + 3}\right)$ is (where sgn(.) denotes signum function)-

(A) {-1,0,1} (B) {-1,0} (C) {1} (D) {0,1}

11. If $2f(x) - 3f\left(\dfrac{1}{x}\right) = x^2$, x is not equal to zero, then f(2) is equal to-

(A) $-\dfrac{7}{4}$ (B) $\dfrac{5}{2}$ (C) -1 (D) none of these

12. The number of integers lying in the domain of the function $f(x)=\sqrt{\log_{0.5}\left(\frac{5-2x}{x}\right)}$ is -

(A) 3 (B) 2 (C) 1 (D) 0

13. The range of the function $f: N \to I$; $f(x)=(-1)^{x-1}$, is -

(A) $[-1, 1]$ (B) $\{-1, 1\}$ (C) $\{0, 1\}$ (D) $\{0, 1, -1\}$

14. The range of the function $f(x)=e^{-x}+e^{x}$, is -

(A) $f(x) \geq 1$ (B) $f(x) \leq 1$ (C) $f(x) \geq 2$ (D) $f(x) \leq 2$

15. If $f(x)=\frac{4^x}{4^x+2}$, then $f(x)+f(1-x)$ is equal to-

(A) 0 (B) -1 (C) 1 (D) 4

16. The range of the function $f(x)=\sqrt{4-x^2}+\sqrt{x^2-1}$ is

(A) $[\sqrt{3}, \sqrt{7}]$ (B) $[\sqrt{3}, \sqrt{5}]$ (C) $[\sqrt{2}, \sqrt{3}]$ (D) $[\sqrt{3}, \sqrt{6}]$

17. A function f has domain $[-1, 2]$ and range $[0, 1]$. The domain and range respectively of the function g defined by $g(x)=1-f(x+1)$ is

(A) $[-1, 1]$; $[-1, 0]$ (B) $[-2, 1]$; $[0, 1]$ (C) $[0, 2]$; $[-1, 0]$ (D) $[1, 3]$; $[-1, 0]$

18. For the function $f(x)=\frac{e^x+1}{e^x-1}$, if n(d) denotes the number of integers which are not in its domain and n(r) denotes the number of integers which are not in its range, then n(d) + n(r) is equal to -

(A) 2 (B) 3 (C) 4 (D) Infinite

19. If $x^4 f(x)-\sqrt{1-\sin 2\pi x}=|f(x)|-2f(x)$, then $f(-2)$ equals

(A) $\frac{1}{17}$ (B) $\frac{1}{11}$ (C) $\frac{1}{19}$ (D) 0

20. Let $f: R-\left\{\frac{-15}{2}\right\} \to R-\left\{\frac{1}{2}\right\}$ be defined by $f(x)=\frac{x+10}{2x+15}$ then $f(x)$ is-

(A) one-one but not onto (B) many one but not-onto

(C) one-one and onto (D) many one and onto

21. $f: R \to R$ $f(x)=\frac{2x^2-5x+3}{8x^2+9x+11}$, then f is -

(A) one-one onto (B) many-one onto (C) one-one into (D) many one into

22. If $f: R \to R$ & $f(x)=\frac{\sin([x]\pi)}{x^2+2x+3}+2x-1+\sqrt{x(x-1)+\frac{1}{4}}$ (where [x] denotes integral part of x), then f(x) is -

(A) one-one but not onto (B) one-one & onto

(C) onto but not one-one (D) neither one-one nor onto

23. Which of the following function is surjective but not injective

(A) $f : R \to R$ $f(x) = x^4 + 2x^3 - x^2 + 1$ (B) $f : R \to R$ $f(x) = x^3 + x + 1$

(C) $f : R \to R^+$ $f(x) = \sqrt{1+x^2}$ (D) $f : R \to R$ $f(x) = x^3 + 2x^2 - x + 1$

24. If $f(x) = x|x|$ then $f^{-1}(x)$ equals-

(A) $\sqrt{|x|}$ (B) $(\text{sgn } x).\sqrt{|x|}$ (C) $-\sqrt{|x|}$ (D) Does not exist

(where sgn(x) denotes signum function of x)

25. If $f : (-\infty,3] \to [7,\infty)$; $f(x) = x^2 - 6x + 16$, then which of the following is true -

(A) $f^{-1}(x) = 3 + \sqrt{x-7}$ (B) $f^{-1}(x) = 3 - \sqrt{x-7}$

(C) $f^{-1}(x) = \dfrac{1}{x^2 - 6x + 16}$ (D) f is many-one

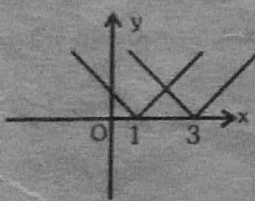
26. $f : R \to R$ such that $f(x) = \ell n\left(x + \sqrt{x^2+1}\right)$. Another function g(x) is defined such that $gof(x) = x\ \forall\ x \in R$. Then g(2) is -

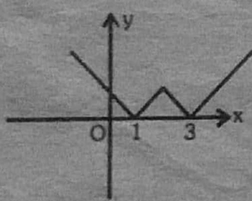
(A) $\dfrac{e^2 + e^{-2}}{2}$ (B) e^2 (C) $\dfrac{e^2 - e^{-2}}{2}$ (D) e^{-2}

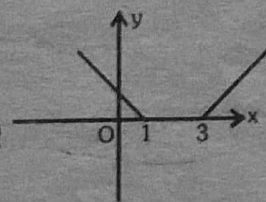
27. Let $P(x) = kx^3 + 2k^2x^2 + k^3$. The sum of all real numbers k for which $(x - 2)$ is a factor of P(x), is

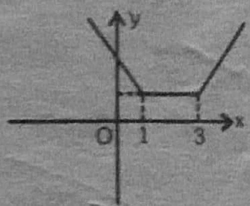
(A) 4 (B) 8 (C) –4 (D) –8

28. Which of the following is the graph of $y = |x - 1| + |x - 3|$?

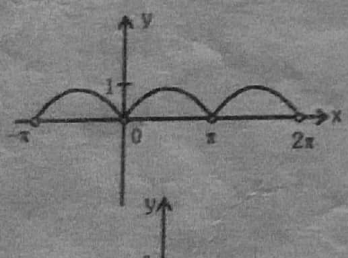
(A) 

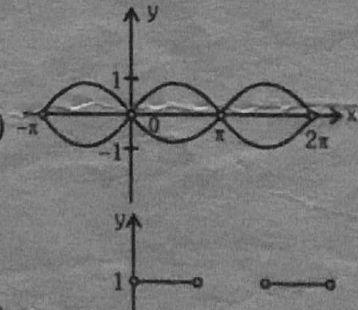
(B) 

(C) 

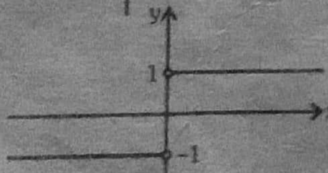
(D) 

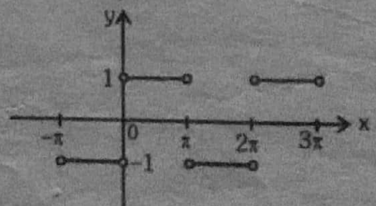
29. Which of the following is the graph of $y = \dfrac{|\sin x|}{\sin x}$?

(A) 

(B) 

Sin x Function

(C) 

(D) 

30. Period of function $f(x) = \min\{\sin x, |x|\} + \dfrac{x}{\pi} - \left[\dfrac{x}{\pi}\right]$ (where [.] denotes greatest integer function) is -

(A) $\pi/2$ (B) π (C) 2π (D) 4π

Multiple Correct Answer

31. Which of the following function(s) have the same domain and range ?

(A) $f(x) = \sqrt{1-x^2}$ (B) $g(x) = \dfrac{1}{x}$ (C) $h(x) = \sqrt{x}$ (D) $l(x) = \sqrt{4-x}$

32. Let $f(x) = x^2 + 3x + 2$, then number of solutions to -

(A) $f(|x|) = 2$ is 1 (B) $f(|x|) = 2$ is 3 (C) $|f(x)| = 0.125$ is 4 (D) $|f(|x|)| = 0.125$ is 8

33. Which of the following pair(s) of function have same graphs?

(A) $f(x) = \frac{\sec x}{\cos x} - \frac{\tan x}{\cot x}$, $g(x) = \frac{\cos x}{\sec x} + \frac{\sin x}{\operatorname{cosec} x}$

(B) $f(x) = \operatorname{sgn}(x^2 - 4x + 5)$, $g(x) = \operatorname{sgn}\left(\cos^2 x + \sin^2\left(x + \frac{\pi}{3}\right)\right)$ where sgn denotes signum function.

(C) $f(x) = e^{\ln(x^2+3x+3)}$, $g(x) = x^2 + 3x + 3$

(D) $f(x) = \frac{\sin x}{\sec x} + \frac{\cos x}{\operatorname{cosec} x}$, $g(x) = \frac{2\cos^2 x}{\cot x}$

34. If a function is defined by an implicit equation $2^{|x|+|y|} + 2^{|x|-|y|} = 2$, then -

(A) Domain of function is singleton

(B) Range of function is singleton

(C) graph of the function intersects the line $y = x$

(D) maximum value of function is 2

35. For each real x, let $f(x) = \max\{x, x^2, x^3, x^4\}$, then $f(x)$ is -

(A) x^4 for $x \leq -1$ (B) x^2 for $-1 < x \leq 0$ (C) $f\left(\frac{1}{2}\right) = \frac{1}{2}$ (D) $f\left(\frac{1}{2}\right) = \frac{1}{4}$

36. Let $f(x) = \sin^6 x + \cos^6 x$, then -

(A) $f(x) \in [0, 1]\ \forall\ x \in R$

(B) $f(x) = 0$ has no solution

(C) $f(x) \in \left[\frac{1}{4}, 1\right] \forall x \in R$

(D) $f(x)$ is an injective function

37. Let $f(x) = \begin{cases} x^2 - 3x + 4 & ; & x < 3 \\ x + 7 & ; & x \geq 3 \end{cases}$ and $g(x) = \begin{cases} x + 6 & ; & x < 4 \\ x^2 + x + 2 & ; & x \geq 4 \end{cases}$, then which of the following is/are true -

(A) $(f + g)(1) = 9$ (B) $(f - g)(3.5) = 1$ (C) $(f\,g)(0) = 24$ (D) $\left(\frac{f}{g}\right)(5) = \frac{8}{3}$

Matrix Match Type

38. Match the functions given in column-I correctly with mappings given in column-II.

Column-I	Column-II
(A) $f : \left[-\frac{1}{2}, \frac{1}{2}\right] \rightarrow \left[\frac{4}{7}, \frac{4}{3}\right]$	(P) Injective mapping
$f(x) = \frac{1}{x^2 + x + 1}$	(Q) Non-injective mapping
(B) $f : [-2, 2] \rightarrow [-1, 1]$	(R) Surjective mapping
$f(x) = \sin x$	(S) Non-surjective mapping
(C) $f : R - I \rightarrow R$	(T) Bijective mapping
$f(x) = \ell n\{x\}$, (where {.} represents fractional part function)	
(D) $f : (-\infty, 0] \rightarrow [1, \infty)$, $f(x) = (1 + \sqrt{-x}) + (\sqrt{-x} - x)$	