EXERCISE (O-1)

Straight Objective Type

1. The domain of the function $\sin^{-1}\left(\log_2\left(\frac{x}{3}\right)\right)$ is-

(A) $\left[\frac{1}{2},3\right]$ (B) $\left[\frac{1}{2},3\right]$ (C) $\left[\frac{3}{2},6\right]$ (D) $\left[\frac{1}{2},2\right]$

2. Domain of the function $f(x)=\log_e\cos^{-1}\{\sqrt{x}\}$ is, where $\{.\}$ represents fractional part function -

(A) $x \in R$ (B) $x \in [0,\infty)$ (C) $x \in (0,\infty)$ (D) $x \in R-\{x \mid x \in I\}$

3. $\tan^{-1}\left(1-x^2-\frac{1}{x^2}\right)+\sin^{-1}\left(x^2+\frac{1}{x^2}-1\right)$ (where $x \neq 0$) is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) π

4. The value of $\tan^2(\sec^{-1}3)+\cot^2(\operatorname{cosec}^{-1}4)$ is -

(A) 9 (B) 16 (C) 25 (D) 23

5. If $x>0$ $\cos^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2}-\cos^{-1}\left(\frac{16}{x}\right)$ then x is -

(A) 12 (B) 16 (C) 20 (D) 320

6. If $2 \le a < 3$, then the value of $\cos^{-1}\cos[a]+\operatorname{cosec}^{-1}\operatorname{cosec}[a]+\cot^{-1}\cot[a]$, (where [.] denotes greatest integer less than equal to x) is equal to

(A) $2-\pi$ (B) $2+\pi$ (C) π (D) 6

7. If $\cos^{-1}(2x^2-1)=2\pi-2\cos^{-1}x$, then -

(A) $x \in [-1,0]$ (B) $x \in [0,1]$ (C) $x \in \left[0,\frac{1}{\sqrt{2}}\right]$ (D) $x \in \left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$

8. Number of integral ordered pairs (a,b) for which $\sin^{-1}(1+b+b^2+....\infty)+\cos^{-1}\left(a-\frac{a^2}{3}+\frac{a^3}{9}-.....\infty\right)=\frac{\pi}{2}$ is-

(A) 0 (B) 4 (C) 9 (D) Infinitely many

9. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\tan^{-1}\frac{2r+1}{r^4+2r^3+r^2+1}$ is equal to -

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{8}$

Multiple Correct Answer Type

10. If $\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\cos\left(2\tan^{-1}\left(\frac{1}{3}\right)\right)=\frac{p}{q}$, where p & q are relatively prime then digit at units place of $(p-q)^{2k+1}$, $k \in N$, can be -

(A) 1 (B) 3 (C) 7 (D) 9

11. Consider the function $f(x)=e^x$ and $g(x)=\sin^{-1}x$, then which of the following is/are necessarily true.

(A) Domain of gof = Domain of f (B) Range of $gof \subset$ Range of g

(C) Domain of gof is $(-\infty, 0]$ (D) Range of gof is $\left[-\frac{\pi}{2}, 0\right)$

12. Let $f(x)=\sin^{-1}(\tan x)+\cos^{-1}(\cot x)$ then

(A) $f(x)=\frac{\pi}{2}$ wherever defined (B) domain of $f(x)$ is $x=n\pi\pm\frac{\pi}{4}$, $n \in I$

(C) period of $f(x)$ is $\frac{\pi}{2}$ (D) $f(x)$ is many one function

13. Let $f(x)=e^{x^3-x^2+x}$ be an invertible function such that $f^{-1}=g$, then -

(A) $g(e)=0$ (B) Domain of 'g' is R^+

(C) Range of 'g' is R (D) $f(g(e))=e$

14. Value of $3\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)+\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ is greater than

(A) $\frac{\pi}{2}$ (B) $\frac{2\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{6}$

15. Which of the following is/are correct ?

(A) $\cot^{-1}(x)=\tan^{-1}\left(\frac{1}{x}\right) \forall\ x \in R-\{0\}$

(B) If $f: R \to R$ such that $f(x)=\text{sgn}(e^x)$ then $f(x)$ is an into function.

(C) If $f: R^+ \to R$ such that $f(x)=\sin x+x$ then $f(x)$ is an odd function.

(D) If $f: R \to R$ such that $f(x)=\frac{e^x}{e^{[x]}}$ then $f(x)$ is a periodic function .

(where [.] represents greatest integer function)