1. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$, is

(A) 0 (B) 1 (C) 2 (D) more than 2

2. Number of straight lines which satisfy the differential equation $\frac{dy}{dx} + x\left(\frac{dy}{dx}\right)^2 - y = 0$ is:

(A) 1 (B) 2 (C) 3 (D) 4

3. The value of the constant 'm' and 'c' for which $y = mx + c$ is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$.

(A) is $m = -1; c = 3/4$ (B) is $m = 1; c = -3/4$ (C) no such real m, c (D) is $m = 1; c = 3/4$

4. Consider the two statements

Statement-1: $y = \sin kt$ satisfies the differential equation $y'' + 9y = 0$.

Statement-2: $y = e^{kt}$ satisfy the differential equation $y'' + y' - 6y = 0$

The value of k for which both the statements are correct is

(A) -3 (B) 0 (C) 2 (D) 3

5. If $y = \frac{x}{\ln|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$ then the function $\phi\left(\frac{x}{y}\right)$ is :

(A) $\frac{x^2}{y^2}$ (B) $-\frac{x^2}{y^2}$ (C) $\frac{y^2}{x^2}$ (D) $-\frac{y^2}{x^2}$

6. The differential equation corresponding to the family of curves $y = e^x(ax + b)$ is

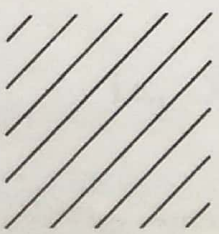
(A) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - y = 0$ (B) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$ (C) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ (D) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = 0$

7. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole.

If t is measured in minutes and $k = \frac{1}{15}$ then the time to drain the tank if the water is 4 meter deep to start with is

(A) 30 min (B) 45 min (C) 60 min (D) 80 min

8. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ is a family of curves which looks most like which of the following?

(A) 

(B) 

(C) 

(D) 

9. Spherical rain drop evaporates at a rate proportional to its surface area. The differential equation corresponding to the rate of change of the radius of the rain drop if the constant of proportionality is $K > 0$, is

(A) $\frac{dr}{dt} + K = 0$ (B) $\frac{dr}{dt} - K = 0$ (C) $\frac{dr}{dt} = Kr$ (D) none

10. The x-intercept of the tangent to a curve is equal to the ordinate of the point of contact. The equation of the curve through the point (1, 1) is

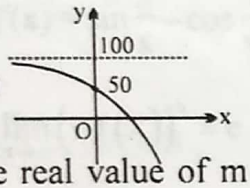
(A) $ye^{\frac{x}{y}} = e$ (B) $xe^{\frac{x}{y}} = e$ (C) $xe^{\frac{y}{x}} = e$ (D) $ye^{\frac{y}{x}} = e$

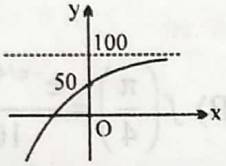
11. A function $f(x)$ satisfying $\int_0^1 f(tx)\,dt = nf(x)$, where $x > 0$, is

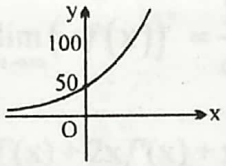
(A) $f(x) = c \cdot x^{\frac{1-n}{n}}$ (B) $f(x) = c \cdot x^{\frac{n}{n-1}}$ (C) $f(x) = c \cdot x^{\frac{1}{n}}$ (D) $f(x) = c \cdot x^{(1-n)}$

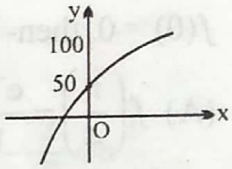
12. Which one of the following curves represents the solution of the initial value problem

$Dy = 100 - y$, where $y(0) = 50$

(A) 

(B) 

(C) 

(D) 

13. The real value of m for which the substitution, $y = u^m$ will transform the differential equation, $2x^4y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is :

(A) $m = 0$ (B) $m = 1$ (C) $m = 3/2$ (D) no value of m

14. A curve C passes through origin and has the property that at each point (x, y) on it the normal line at that point passes through (1, 0). The equation of a common tangent to the curve C and the parabola $y^2 = 4x$ is

(A) $x = 0$ (B) $y = 0$ (C) $y = x + 1$ (D) $x + y + 1 = 0$

15. A function $y = f(x)$ satisfies $(x+1).f'(x) - 2(x^2+x)f(x) = \frac{e^{x^2}}{(x+1)}$, $\forall\ x > -1$

If $f(0) = 5$, then $f(x)$ is

(A) $\left(\frac{3x+5}{x+1}\right).e^{x^2}$ (B) $\left(\frac{6x+5}{x+1}\right).e^{x^2}$ (C) $\left(\frac{6x+5}{(x+1)^2}\right).e^{x^2}$ (D) $\left(\frac{5-6x}{x+1}\right).e^{x^2}$

16. The equation to the orthogonal trajectories of the system of parabolas $y = ax^2$ is

(A) $\frac{x^2}{2} + y^2 = c$ (B) $x^2 + \frac{y^2}{2} = c$ (C) $\frac{x^2}{2} - y^2 = c$ (D) $x^2 - \frac{y^2}{2} = c$

17. If $\int_a^x t\,y(t)dt = x^2 + y(x)$ then y as a function of x is

(A) $y = 2 - (2 + a^2)e^{\frac{x^2-a^2}{2}}$ (B) $y = 1 - (2 + a^2)e^{\frac{x^2-a^2}{2}}$

(C) $y = 2 - (1 + a^2)e^{\frac{x^2-a^2}{2}}$ (D) none