1. Let $f(x) = [\tan^2 x]$, (where [.] denotes greatest integer function). Then -

(A) $\lim_{x\to 0} f(x)$ does not exist
(B) $f(x)$ is continuous at $x = 0$.
(C) $f(x)$ is not differentiable at $x = 0$
(D) $f'(0) = 1$

2. The number of points where $f(x) = [\sin x + \cos x]$ (where [] denotes the greatest integer function), $x \in (0,2\pi)$ is not continuous is -

(A) 3
(B) 4
(C) 5
(D) 6

3. If 6,8 and 12 are l^{th}, m^{th} and n^{th} terms of an A.P. and $f(x) = nx^2 + 2lx - 2m$, then the equation $f(x) = 0$ has -

(A) a root between 0 and 1
(B) both roots imaginary.
(C) both roots negative.
(D) both roots greater than 1.

4. Let f be differentiable at $x = 0$ and $f'(0) = 1$. Then $\lim_{h\to 0}\dfrac{f(h)-f(-2h)}{h} =$

(A) 3
(B) 2
(C) 1
(D) –1

SBG STUDY

5. Let $g(x) = \begin{cases} 3x^2 - 4\sqrt{x} + 1 & \text{for } x < 1 \\ ax + b & \text{for } x \geq 1 \end{cases}$.

If g(x) is continuous and differentiable for all numbers in its domain then -

(A) a = b = 4
(B) a = b = – 4
(C) a = 4 and b = – 4
(D) a = – 4 and b = 4

6. If $f(x)\, f(y) + 2 = f(x) + f(y) + f(xy)$ and $f(1) = 2$, $f'(1) = 2$ then sgn $f(x)$ is equal to (where sgn denotes signum function) -

(A) 0
(B) 1
(C) –1
(D) 4

7. The function $g(x) = \begin{cases} x + b, & x < 0 \\ \cos x, & x \geq 0 \end{cases}$ can be made differentiable at x = 0 -

(A) if b is equal to zero
(B) if b is not equal to zero
(C) if b takes any real value
(D) for no value of b

8. Which one of the following functions is continuous everywhere in its domain but has atleast one point where it is not differentiable ?

(A) $f(x) = x^{1/3}$
(B) $f(x) = \dfrac{|x|}{x}$
(C) $f(x) = e^{-x}$
(D) $f(x) = \tan x$

9. If the right hand derivative of $f(x) = [x] \tan \pi x$ at x = 7 is $k\pi$, then k is equal to ([y] denotes greatest integer $\leq$ y)

(A) 6
(B) 7
(C) –7
(D) 49

10. Let $f : R \to R$ be a continuous onto function satisfying $f(x) + f(-x) = 0, \forall x \in R$. If $f(-3) = 2$ and $f(5) = 4$ in $[-5,5]$, then the equation $f(x) = 0$ has -

(A) exactly three real roots
(B) exactly two real roots
(C) atleast five real roots
(D) atleast three real roots

11. Let $f(x) = \begin{cases} \lim\limits_{n\to\infty} \dfrac{ax(x-1)\left(\cot\frac{\pi x}{4}\right)^n + (px^2+2)}{\left(\cot\frac{\pi x}{4}\right)^n + 1}, & x \in (0,1)\cup(1,2) \\ 0 \quad , & x = 1 \end{cases}$

If $f(x)$ is differentiable for all $x \in (0,2)$ then $(a^2 + p^2)$ equals -

(A) 18
(B) 20
(C) 22
(D) 24

12. If $2x + 3|y| = 4y$, then y as a function of x i.e. $y = f(x)$, is -

(A) discontinuous at one point
(B) non differentiable at one point
(C) discontinuous & non differentiable at same point
(D) continuous & differentiable everywhere

13. If $f(x) = (x^5 + 1)\,|x^2 - 4x - 5| + \sin|x| + \cos(|x - 1|)$, then $f(x)$ is not differentiable at -

(A) 2 points
(B) 3 points
(C) 4 points
(D) zero points

14. Let $f(x) = \begin{cases} x^3 + 2x^2 & x \in Q \\ -x^3 + 2x^2 + ax & x \notin Q \end{cases}$, then the integral value of 'a' so that $f(x)$ is differentiable at x = 1, is

(A) 2
(B) 6
(C) 7
(D) not possible

15. Let R be the set of real numbers and $f : R \to R$, be a differentiable function such that $|f(x) - f(y)| \le |x - y|^3 \ \forall\, x, y \in R$. If $f(10) = 100$, then the value of $f(20)$ is equal to -

(A) 0
(B) 10
(C) 20
(D) 100

16. For what triplets of real numbers (a, b, c) with $a \neq 0$ the function

$f(x) = \begin{cases} x & x \le 1 \\ ax^2 + bx + c & \text{otherwise} \end{cases}$ is differentiable for all real x ?

(A) $\{(a, 1 - 2a, a) \mid a \in R, a \neq 0\}$
(B) $\{(a, 1 - 2a, c) \mid a, c \in R, a \neq 0\}$
(C) $\{(a, b, c) \mid a, b, c \in R, a + b + c = 1\}$
(D) $\{(a, 1 - 2a, 0) \mid a \in R, a \neq 0\}$

17. Number of points of non-differentiability of the function $g(x) = [x^2]\{\cos^2 4x\} + \{x^2\}[\cos^2 4x] + x^2\sin^2 4x + [x^2][\cos^2 4x] + \{x^2\}\{\cos^2 4x\}$ in $(-50, 50)$ where $[x]$ and $\{x\}$ denotes the greatest integer function and fractional part function of x respectively, is equal to :-

(A) 98
(B) 99
(C) 100
(D) 0