# EXERCISE (O-1)

SBG STUDY

1. If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ equals

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$ (C) $g(x)\,g(\pi)$ (D) $[g(x)/g(\pi)]$

2. $\int_0^1 \frac{\tan^{-1}x}{x}dx =$

$x = \tan\theta$

(A) $\int_0^{\pi/4} \frac{\sin x}{x}dx$ (B) $\int_0^{\pi/2} \frac{x}{\sin x}dx$ (C) $\frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x}dx$ (D) $\frac{1}{2}\int_0^{\pi/4} \frac{x}{\sin x}dx$

3. The value of the defined integral $\int_0^{\pi/2} (\sin x + \cos x).\sqrt{\frac{e^x}{\sin x}}dx$ equals

(A) $2\sqrt{e^{\pi/2}}$ (B) $\sqrt{e^{\pi/2}}$ (C) $2\sqrt{e^{\pi/2}}.\cos 1$ (D) $\frac{1}{2}e^{\pi/4}$

4. Variable x and y are related by equation $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$. The value of $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{y}{\sqrt{1+y^2}}$ (B) y (C) $\frac{2y}{\sqrt{1+y^2}}$ (D) 4y

5. If $\int_0^x f(t)\,dt = x + \int_x^1 t^2 \cdot f(t)dt + \frac{\pi}{4} - 1$, then the value of the integral $\int_{-1}^1 f(x)\,dx$ is equal to

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) $\pi$

6. If $I = \int_0^{\pi/2} \ell n(\sin x)dx$ then $\int_{-\pi/4}^{\pi/4} \ell n(\sin x + \cos x)dx$

(A) $\frac{I}{2}$ (B) $\frac{I}{4}$ (C) $\frac{I}{\sqrt{2}}$ (D) I

7. If $f(x) = x\sin x^2$; $g(x) = x\cos x^2$ for $x \in [-1, 2]$

$A = \int_{-1}^2 f(x)\,dx$ ; $B = \int_{-1}^2 g(x)\,dx$, then

(A) $A > 0$ ; $B < 0$ (B) $A < 0$ ; $B > 0$ (C) $A > 0$ ; $B > 0$ (D) $A < 0$ ; $B < 0$

8. The value of the definite integral $\int_0^{\pi/2} \sin x \sin 2x \sin 3x\,dx$ is equal to :

(A) $\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{6}$

9. Value of the definite integral $\int_{-1/2}^{1/2} (\sin^{-1}(3x - 4x^3) - \cos^{-1}(4x^3 - 3x))\,dx$

(A) 0 (B) $-\frac{\pi}{2}$ (C) $\frac{7\pi}{2}$ (D) $\frac{\pi}{2}$

10. $\lim_{x\to\infty}\left(x^3\int_{-1/x}^{1/x}\frac{\ln(1+t^2)}{1+e^t}dt\right)$ equals

(A) 1/3 (B) 2/3 (C) 1 (D) 0

11. If $\frac{C_0}{1}+\frac{C_1}{2}+\frac{C_2}{3}=0$, where $C_0,C_1,C_2$ are all real, the equation $C_2x^2+C_1x+C_0=0$ has :

(A) atleast one root is (0,1)

(B) one root is (1,2) & other is (3,4)

(C) one root is (–1,1) & the other is (–5,–2)

(D) both roots imaginary

12. The value of the definite integral $\int_1^e\left((x+1)e^x.\ell nx\right)dx$ is -

(A) e (B) $e^{e+1}$ (C) $e^e(e-1)$ (D) $e^e(e-1)+e$

13. $\lim_{n\to\infty}\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+.....+\frac{1}{\sqrt{n}\sqrt{4n}}$ is equal to -

(A) 2 (B) 4 (C) $2\left(\sqrt{2}-1\right)$ (D) $2\sqrt{2}-1$

14. The value of the definite integral $\int_{19}^{37}\left(\{x\}^2+3(\sin 2\pi x)\right)dx$ where { x } denotes the fractional part function.

(A) 0 (B) 6 (C) 9 (D) can't be determined

15. $\lim_{x\to 0}\frac{\int_0^x \sin t^2 dt}{x(1-\cos x)}$ equals -

(A) $\frac{1}{3}$ (B) 2 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$

16. If g (x) is the inverse of f (x) and f (x) has domain x ∈ [1, 5], where f (1) = 2 and f (5) = 10 then the values of $\int_1^5 f(x)dx+\int_2^{10}g(y)dy$ equals -

(A) 48 (B) 64 (C) 71 (D) 52

17. The value of the definite integral $\int_2^4\left(x(3-x)(4+x)(6-x)(10-x)+\sin x\right)dx$ equals-

(A) cos 2 + cos 4 (B) cos 2 – cos 4 (C) sin 2 + sin 4 (D) sin 2 – sin 4

18. The value of $\int_{-1}^{1}\frac{dx}{\sqrt{|x|}}$ is -

(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) undefined

19. $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1} dx = a\ln 2 + b$ then -

(A) a = 2; b = 1 (B) a = 2; b = 0 (C) a = 3; b = –2 (D) a = 4; b = –1

20. The true solution set of the inequality, $\sqrt{5x-6-x^2} + \left(\frac{\pi}{2}\int_0^x dz\right) > x\int_0^{\pi} \sin^2 x dx$ is :

(A) R (B) (1,6) (C) (–6,1) (D) (2,3)

21. Let $I_1 = \int_0^{\pi/2} e^{-x^2}\sin(x)dx; I_2 = \int_0^{\pi/2} e^{-x^2}dx; I_3 = \int_0^{\pi/2} e^{-x^2}(1+x)dx$

and consider the statements

I $I_1 < I_2$ II $I_2 < I_3$ III $I_1 = I_3$

Which of the following is(are) true ?

(A) I only (B) II only

(C) Neither I nor II nor III (D) Both I and II

22. Let $u = \int_0^1 \frac{\ln(x+1)}{x^2+1} dx$ and $v = \int_0^{\pi/2} \ln(\sin 2x)dx$ then -

(A) u = 4v (B) 4u + v = 0 (C) u + 4v = 0 (D) 2u + v = 0

23. $\int_{\frac{1}{2}}^{3\frac{1}{2}} \left\{\frac{1}{2}(|x-3|+|1-x|-4)\right\} dx$ equals-

(A) $-\frac{3}{2}$ (B) $\frac{9}{8}$ (C) $\frac{1}{4}$ (D) $\frac{3}{2}$

Where {.} denotes the fraction part function.

24. Suppose that F(x) is an antiderivative of $f(x) = \frac{\sin x}{x}$, x > 0 then $\int_1^3 \frac{\sin 2x}{x}$ can be expressed as -

(A) F(6) – F(2) (B) $\frac{1}{2}$(F(6) – F(2)) (C) $\frac{1}{2}$(F(3) – F(1)) (D) 2(F(6) – F(2))

25. Let $f$ be a positive function. Let $I_1 = \int_{1-k}^{k} x f(x(1-x))dx$ ; $I_2 = \int_{1-k}^{k} f(x(1-x))dx$ , where 2k – 1 > 0.

Then $\frac{I_2}{I_1}$ is -

(A) k (B) 1/2 (C) 1 (D) 2

26. $\lim_{h\to 0} \frac{\int_a^{x+h} \ln^2 t\,dt - \int_a^{x} \ln^2 t\,dt}{h} =$

(A) 0 (B) $\ln^2 x$ (C) $\frac{2\ln x}{x}$ (D) does not exist