Sound : 330 m/s

26/10/17

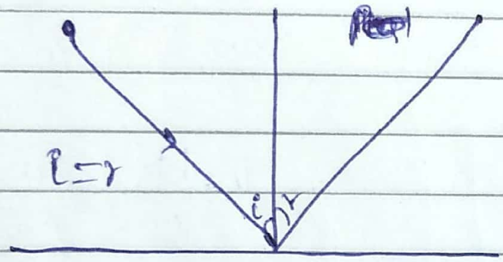
Wave optics

* Nature of Light :-

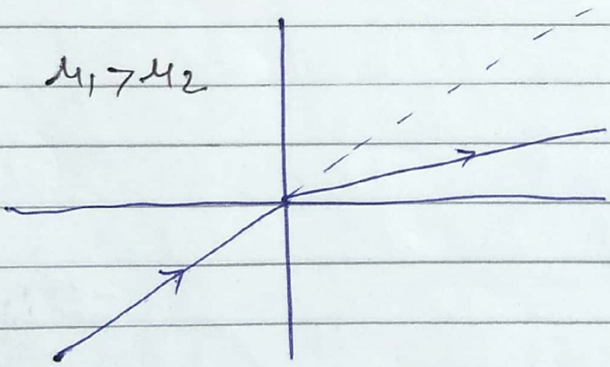
(i) Newton's Corpuscles theory :- (Particles)

Light wave consist of very small Particles called Corpuscles

In Reflection corpuscles is strick a surface perfectly elastically



In Refraction Corpuscles are more attracted by denser medium relative to rayer medium

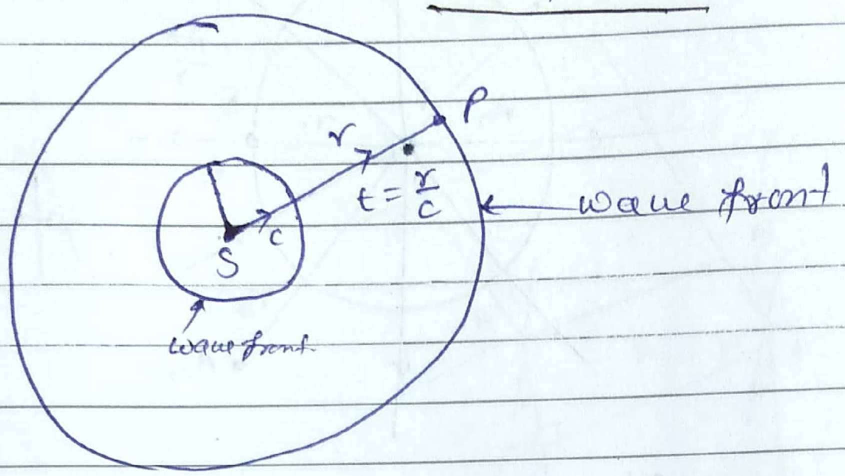


(ii) ~~Hyg~~ Huygen's Principle :-

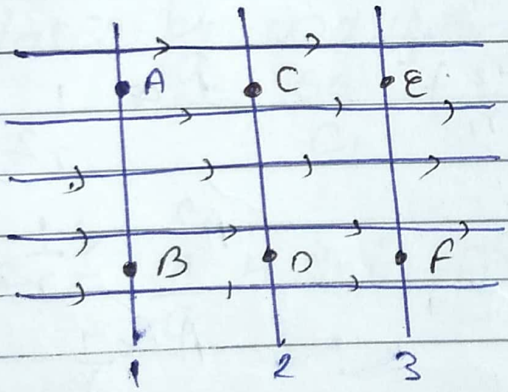
(1) there a universly Present medium called ether

(2) light wave emitting from a source these medium Particles and medium Particles are start oscillate Oscillating medium Particles becomes new source of light Produsing light wave in all directions called Secondry wave or secondry wavelets.

* A Surface at which all the Particles are vibrating in same phase called a wave front.

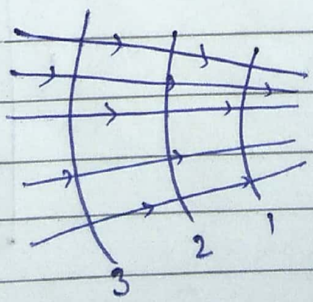


* Wave front:
1) It is the locus of all the Particles vibrating in same phase
2) It is $\perp^r$ to the direction of propagation of wave

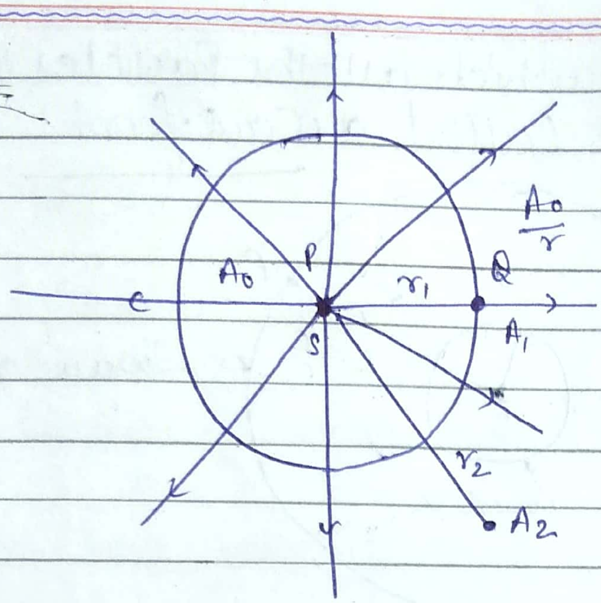


Planer

$\phi_A = \phi_B$, $\phi_C = \phi_D$ $\phi_E = \phi_F$



* **Point Source**



$I \propto A^2$

$$y = A\sin(kx - \omega t + \theta)$$

Intensity of point Q

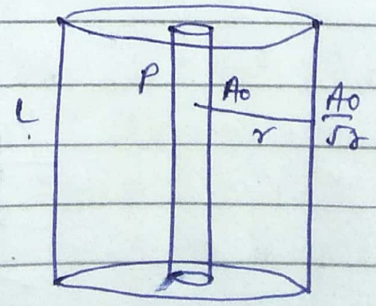
$$I = \frac{\text{Energy}}{\text{time} \times \text{Area}} = \frac{P}{4\pi r^2}$$

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 \qquad I \propto \frac{1}{r^2}$$

$$A^2 \propto \frac{1}{r^2}$$

$$\frac{A_1}{A_2} = \frac{r_2}{r_1} \qquad A \propto \frac{1}{r}$$

* **Cylinderical source** or **Extended source**
or
Line source:



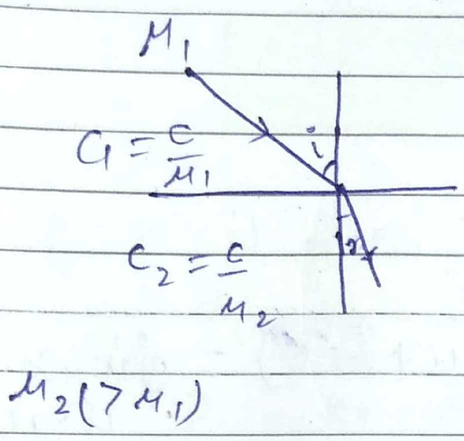
$$I = \frac{P}{2\pi r l}$$

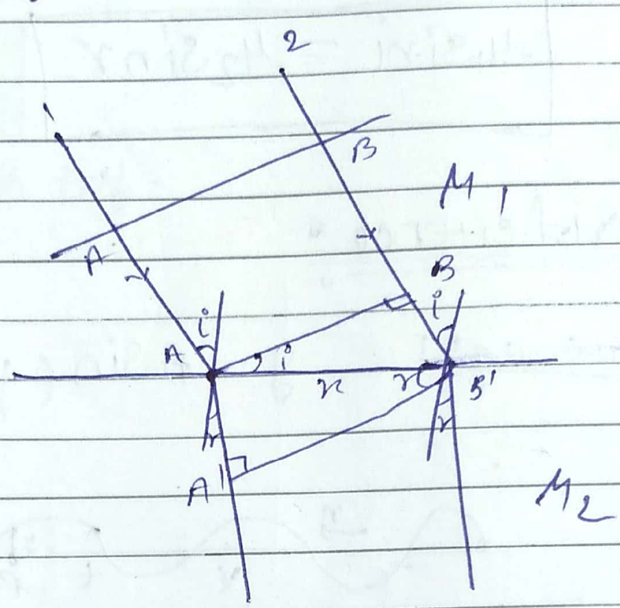
$$I \propto \frac{1}{r} \propto A^2$$

$$A \propto \frac{1}{\sqrt{r}}$$

Important for Board

* Refraction by Huygen's principle !





$\Delta ABB'$

$$\sin i = \frac{BB'}{x}$$

$$BB' = x \sin i$$

time taken by wave ~~from~~ from B to B'

$$t_1 = \frac{BB'}{c_1} = \frac{x \sin i}{\frac{c}{\mu_1}}$$

$$t_1 = \frac{x}{c} \mu \sin i \quad \text{—(1)}$$

$\Delta ABA'$ $\quad \sin r = \frac{AA'}{x}$ $\quad AA' = x \sin r$

time taken to reach from A to A'

$$t_2 = \frac{AA'}{c_2} = \frac{x \sin r}{c/\mu_2}$$

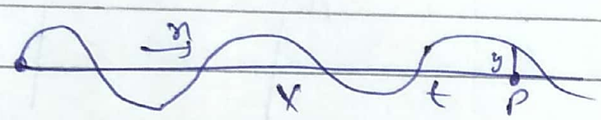
$$= \frac{x}{c} \mu_2 \sin r$$

$t_1 = t_2$

$$\boxed{\mu_1 \sin i = \mu_2 \sin r}$$

Interference:

wave eqn: $y = A\sin(kx \mp \omega t + \theta)$ – sinusoidal wave



ω – Ang. freq. $= 2\pi f = \frac{2\pi}{T}$

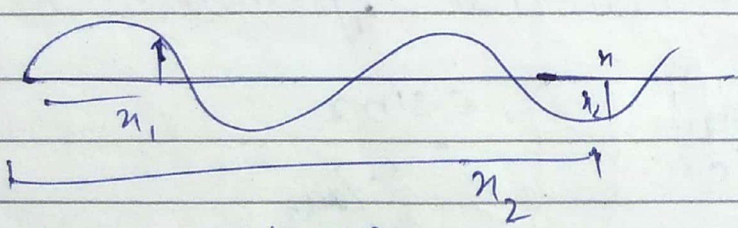
K – Propagation const. $= \frac{2\pi}{\lambda}$

Ang. wave No.

phase $= kx - \omega t + \theta$

$$\phi = \underbrace{kx - \omega t}_{\text{phase angle}} + \underbrace{\theta}_{\text{phase const.}}$$

If t is fixed



$\phi_1 = kx_1 - \omega t + \theta$

$\phi_2 = kx_2 - \omega t + \theta$

$\Delta\phi = k\Delta x$

$$\boxed{\Delta\phi = \frac{2\pi}{\lambda}\Delta x}$$

Path diff.

if x is fixed

$\phi_1 = kx - \omega t_1 + \phi$

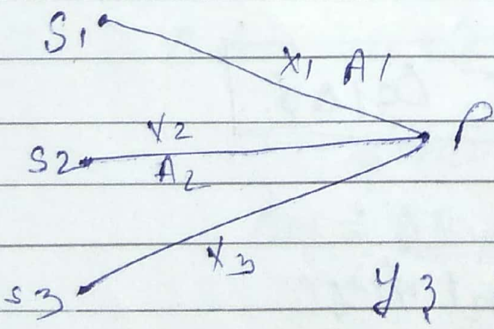
$\phi_2 = kx - \omega t + \phi$

$\Delta\phi = \text{const}$

$$\boxed{\Delta\phi = \frac{2\pi}{T}\Delta t}$$

* Superposition Principle:

Two or more waves can travel in a medium simultaneously without affecting each other. When these waves reach a medium particle simultaneously, net displacement of particle is given by sum of the displacements produced by each wave.



$y_1 = A_1 \sin(kx_1 - \omega t)$ $I_1 \propto A_1^2$

$y_2 = A_2 \sin(kx_2 - \omega t)$ ϕ_2

$I_2 \propto A_2^2$

$y_3 = A_3 \sin(kx_3 - \omega t)$ $I_3 \propto A_3^2$

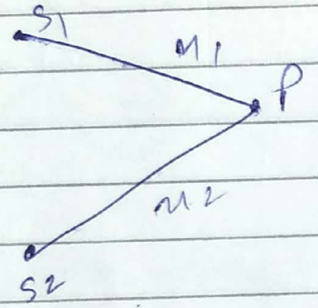
$y = y_1 + y_2 + y_3$

$I \propto A^2$

Interference :

It is the phenomina of redistribution of energy in a medium due to super position of two or more waves

At some pt amplitude and Intensity are maximum called Constractive Interfarence. and where Intensity and Amplitud are minimum called distereshive Interference.



$$\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\phi}}$$

$$y_1 = A_1 \sin(kx_1 - \omega t) \qquad I_1 \propto A_1^2$$

$$y_2 = A_2 \sin(kx_2 - \omega t) \qquad I_2 \propto A_2^2$$

Phase diff $\Delta\phi = k\Delta x$ (Path diff.)

$I \propto A^2$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\phi$$

$$\frac{I}{\alpha} = \frac{I_1}{\alpha} + \frac{I_2}{\alpha} + \frac{2\sqrt{I_1 I_2}}{\alpha} \cos\Delta\phi$$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\Delta\phi.}$$

(i) for Constractive Intelference :
or
maxima or Bright fringe

A & I = max

$\cos\Delta\phi$ = max

$\cos\Delta\phi = 1$

$\Delta\phi = 2n\pi$

$\Delta\phi = 0$ – zero order max

$\Delta\phi = 2\pi$ – Ist order max

$\Delta\phi = 4\pi$ – 2nd

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = 2n\pi$

$\Delta x = n\lambda$

$\Delta x = \lambda$ – Ist $\qquad |x_1 - x_2| = n\lambda$

$A_{max} = A_1 + A_2$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

(2) for Distestive Interfarence : or minima or Dark frinze :

$\Delta\phi$ I = min

$\cos\Delta\phi$ = min = –1

$\phi = (2n \pm 1)\pi$

$\Delta\phi = \pi, 3\pi, 5\pi, 7\pi$

Istorder $\quad$ second order

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = (2n \pm 1)\pi$

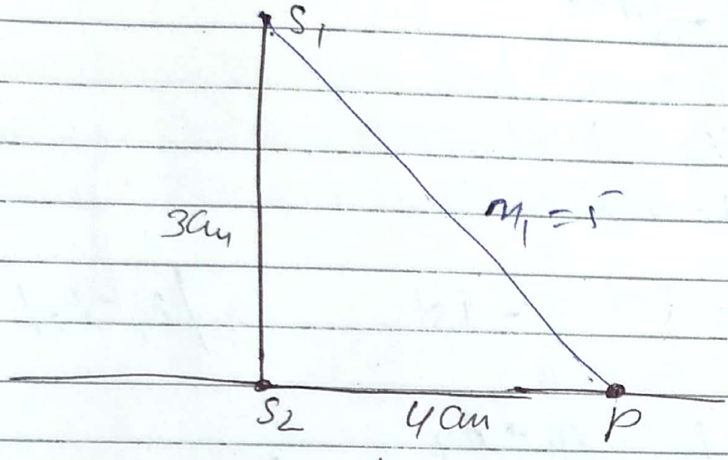
$\Delta x = (2n \pm 1)\frac{\lambda}{2}$

$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$

$A_{min} = |A_1 - A_2|$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$

Ex



monochromatic source.

find Possible values of wavelength so that there is maxima at Point P

$\frac{4 \times 3}{4 \times 4} = \frac{12}{16}$

$\frac{3}{4} = \frac{max}{min}$ $\frac{12}{4}$

$x_2 = 4$

Path diff

$\Delta x = 1cm$

$1 = n\lambda$

$\lambda = \frac{1}{n}$

Q. ~~Interference~~ Interference pattern ratio of maxⁿ to minⁿ intensity is $\frac{42}{16} = \frac{I_{max}}{I_{min}}$

find the Ratio of Amplitude of Interference wave

$\frac{11}{3}$ ~~$\frac{4}{16}$~~

$$\frac{I_{max}}{I_{min}} = \frac{49}{16}$$

$$\left(\frac{A_{max}}{A_{min}}\right)^2 = \frac{49}{16}$$

$$\frac{A_{max}}{A_{min}} = \frac{7}{4}$$

$$\frac{A_1 + A_2}{A_1 - A_2} = \frac{7}{4}$$

* Condition for good Interference:

(1) Amplitude and Intesity of Intesphesic waves must be nearly equal

$A_1 = 100$

$A_2 = 2$

$A_1 = 100$

$A_2 = 98$

$A_1 = 100$

$A_2 = 100$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\phi}$$

$$= \cancel{A_0\sqrt{2(1+\cos}}$$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\Delta\phi$$

$$A_1 = A_2 = A_0$$

$$I_1 = I_2 = I_0$$

$$= A_0\sqrt{2(1+\cos\Delta\phi)}$$

$$= A_0\sqrt{2 \times 2\cos^2\frac{\Delta\phi}{2}}$$

$$\boxed{A = 2A_0\cos\frac{\Delta\phi}{2}}$$

$$\boxed{I = 4I_0\cos^2\frac{\Delta\phi}{2}}$$

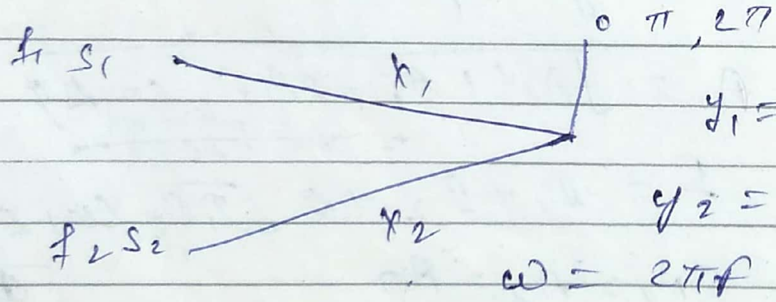
(2) frequency of both the wave must be equal

(3) Phase diff b/w the waves must be independent of time

(4) Sources producing wave must be ~~totally~~ coherent

* Coherent Sources: All the sources producing waves of same frequency called coherent source

Phase difference b/w the coherent source does not change with time or Independent of time



$y_1 = A_1 \sin(kx_1 - \omega t)$

$y_2 = A_2 \sin(kx_2 - \omega t)$

$\omega = 2\pi f$

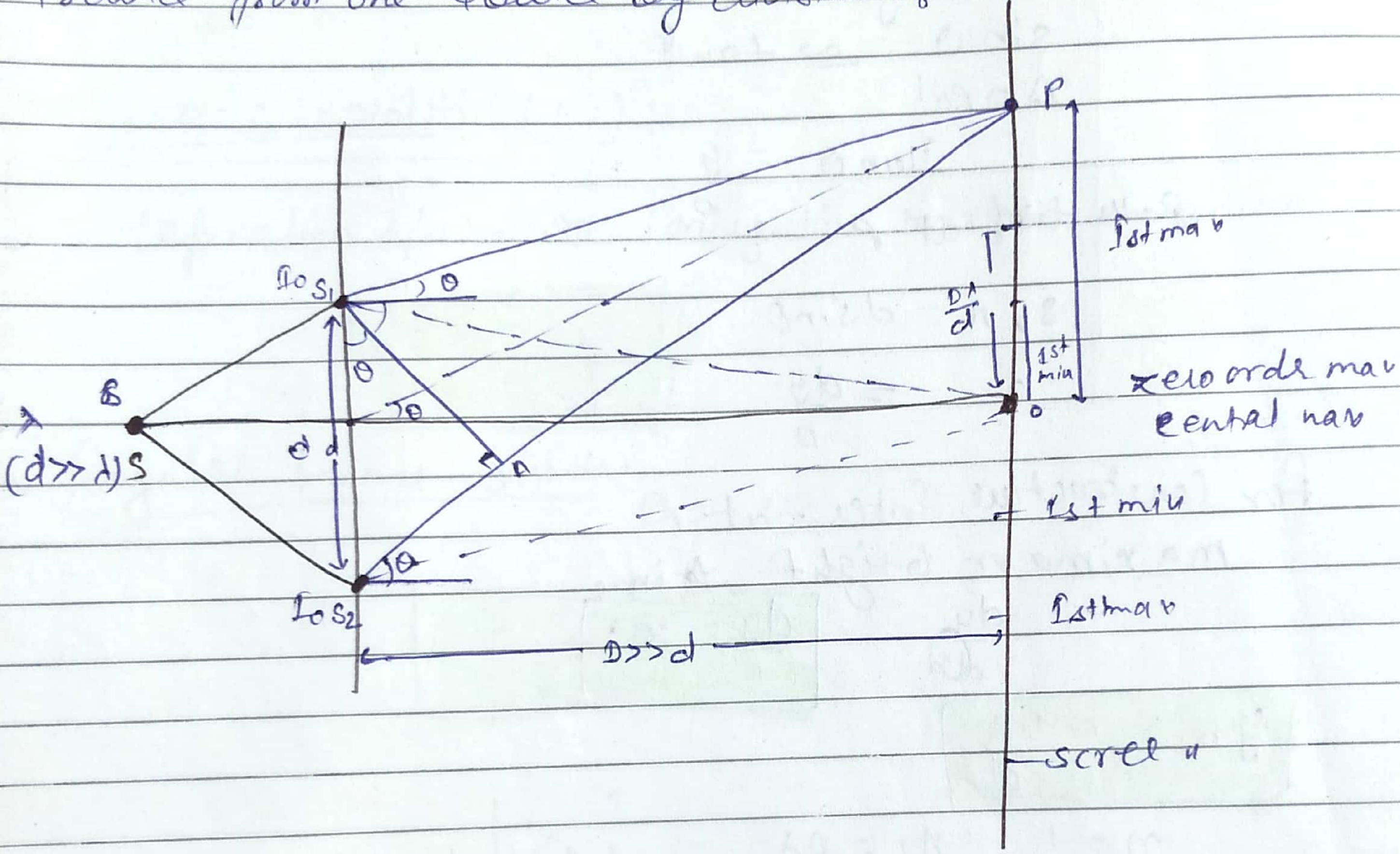
Due to coherent source interference pattern does not change with time

Two Independent source can never be coherent

To produce coherent source two or more than two sources are produce from one source

* Youngs double slets Exp.: YDSE:

This exp. Profe wave nature of light. two sources are Produce from one source by division of wave front



at O

path diff = 0

phase diff = $\frac{2\pi}{\lambda}$ path diff

$= 0$

$I = 4 I_0 \cos^2 \frac{\Delta\phi}{2}$

at Point P   path diff = $S_2P - S_1P$

$= S_2A$

$\Delta S_1 S_2 A$

$\sin\theta = \frac{S_2 A}{d}$

Path diff at P

$S_2A = d\sin\theta$

$\sin\theta$ $D >> d$

θ – very small

$\sin\theta \simeq \tan\theta$

$\Delta OPO'$

$\tan\theta = \frac{y}{D}$

Path diff at P

$S_2A = d\sin\theta$

$= \frac{dy}{D}$

for Constructive Inter. at P
maxima or bright fringe

$\frac{dy}{D}$ $\boxed{\frac{dy}{D} = n\lambda}$

$$\boxed{y_n = \frac{nD\lambda}{d}}$$

$n = 1$ $y_1 = \frac{D\lambda}{d}$ – Ist order

$n = 2$ $y_2 = \frac{2D\lambda}{d}$ – 2nd order

$n = 3$ $y_3 = \frac{3D\lambda}{d}$

for Distruction Int. minima or dark fringe

$$\boxed{\frac{dy}{D} = (2n-1)\frac{\lambda}{2}}$$

$y_n = (2n-1)\frac{D\lambda}{2d}$

$y_1 = \frac{D\lambda}{2d}$ Ist order min

$$y_2 = \frac{3D\lambda}{2d}$$

$$y_3 = \frac{5D\lambda}{2d}$$

* **Frinze width (ω):-**

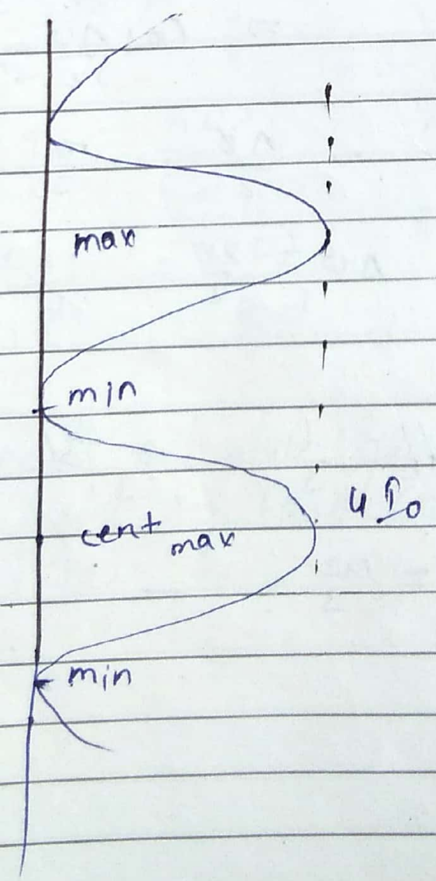
Sepration b/w two Conjugative maxima or minima.

$$\boxed{\omega = \frac{D\lambda}{d}}$$

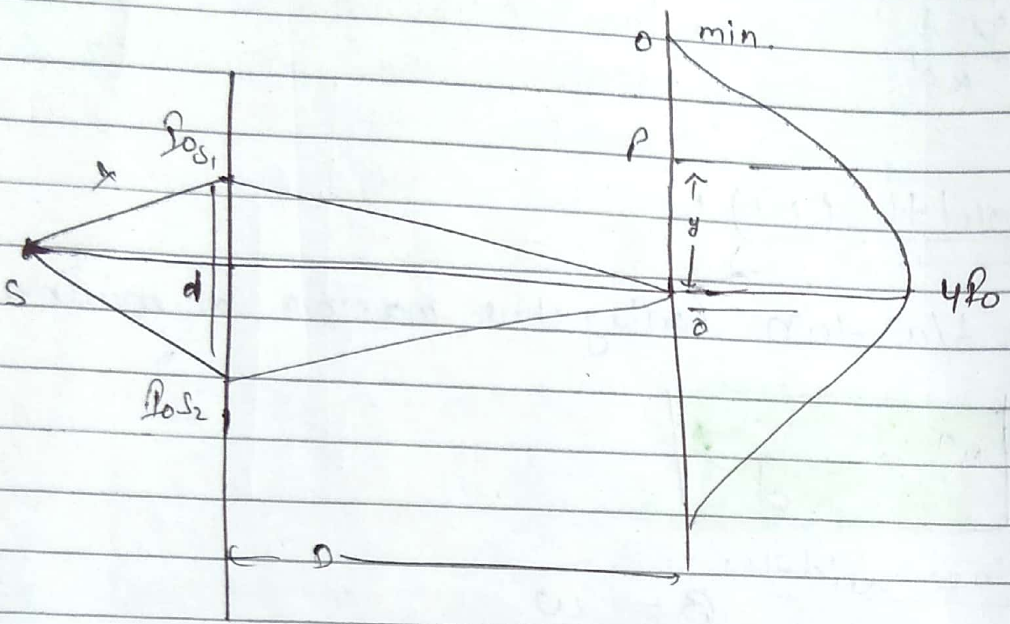
* Angular frinze width:

$$\beta = \frac{\omega}{D}$$

$$\boxed{\beta = \frac{\lambda}{d}}$$



Que: ~~find y~~ find y so that Intensity at P is 25% of Intensity at central maxima



$$I = 4 I_0 \cos^2 \frac{\Delta\phi}{2}$$

$$I_0 = 4 I_0 \cos^2 \frac{\Delta\phi}{2}$$

$$\cos^2 \frac{\Delta\phi}{2} = \frac{1}{4} \Rightarrow \cos \frac{\Delta\phi}{2} = \pm \frac{1}{2}$$

$$\cos \frac{\Delta\phi}{2} = \frac{1}{2} \Rightarrow \frac{\Delta\phi}{2} = \frac{\pi}{3}$$

$$\Delta\phi = \frac{2\pi}{3}$$

$$\frac{2\pi}{\lambda} \text{ Path} = \frac{2\pi}{3}$$

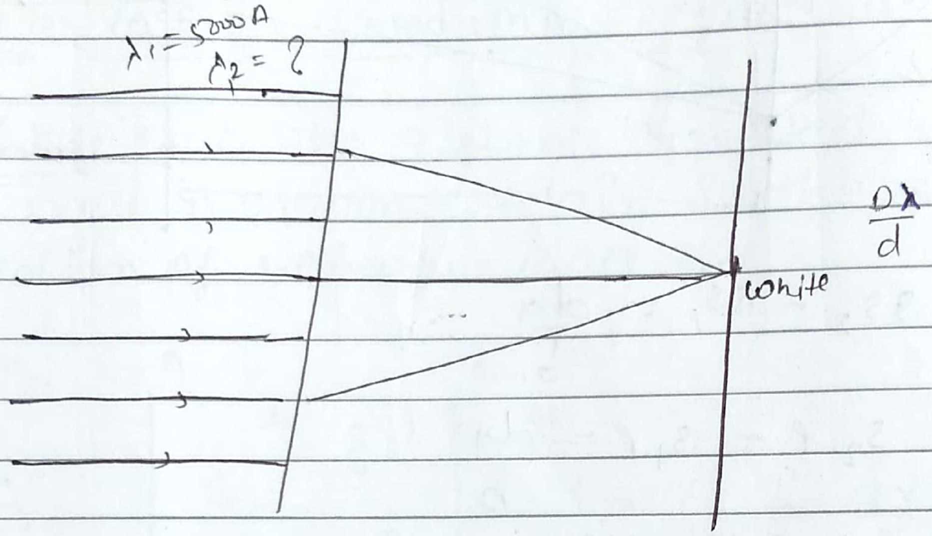
$$\text{Path diff} = \frac{\lambda}{3} \Rightarrow \frac{dy}{D} = \frac{\lambda}{3}$$

$$y = \frac{D\lambda}{3d} = \frac{\omega}{3}$$

Case-1 If monochromatic light is not used

If white light is used in YDSE instead of monochromatic light then central maxima is white and rest of the screen is coloured.

Ex:-



find λ_2 if 3rd Maxima of λ_1 consided to the 3rd min of λ_2

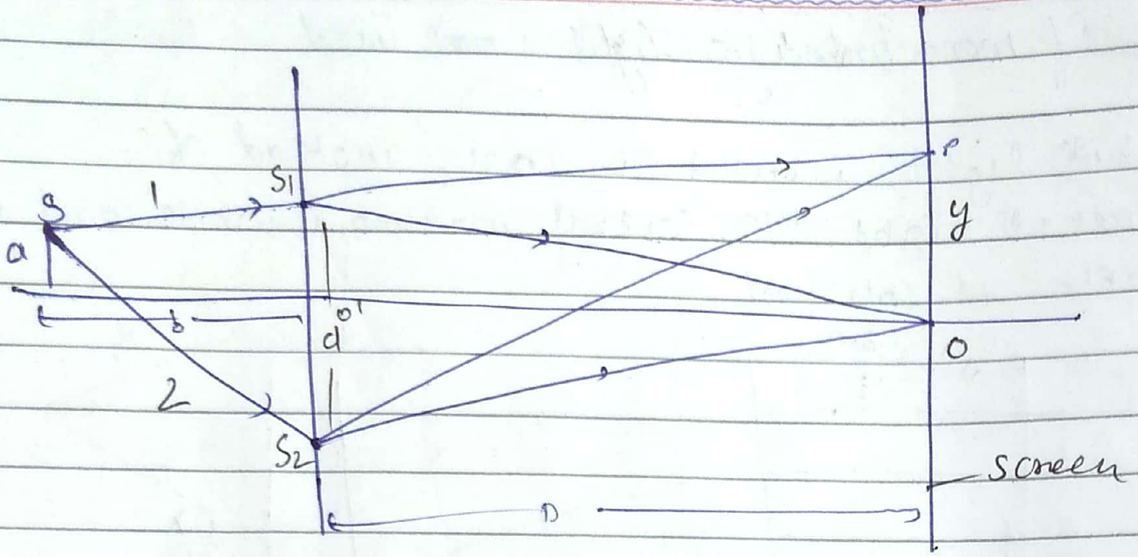
$\lambda_1 = 5000 A$

$\lambda_2 = ?$

max $y_n = \frac{nD\lambda}{d}$ min $y_n \frac{(2n-1) D\lambda}{2d}$

$$\frac{3D\lambda_1}{d} = \frac{5D\lambda}{2d}$$

Case: II If Source is not place symmetrically to the slits i.e the light ray have path diff. before reaching the slits.



$$SS_2 - SS_1 = \frac{da}{b}$$

$$S_2P - S_1P = \frac{dy}{D}$$

net path diff at point P

$$\text{Path diff} = \frac{da}{b} + \frac{dy}{D}$$

$$\text{fringe width} = \frac{D\lambda}{d}$$

for maxima

$$\text{path diff} = n\lambda$$

$$\frac{da}{b} + \frac{dy}{D} = n\lambda \Rightarrow \frac{dy}{D} = n\lambda - \frac{da}{b}$$

$$y = \frac{nD\lambda}{d} - \frac{Da}{b}$$

central maxima $= n = 0$

$$y_0 = -\frac{Da}{b}$$

$$y_1 = \frac{D\lambda}{d} - \frac{Da}{b}$$

$$y_2 = \frac{2D\lambda}{d} - \frac{Da}{b}$$

Lace
Ex S+ 2-6
2,3 6-1 1-14

$$\frac{dy}{D} = n\lambda - \frac{da}{b}$$

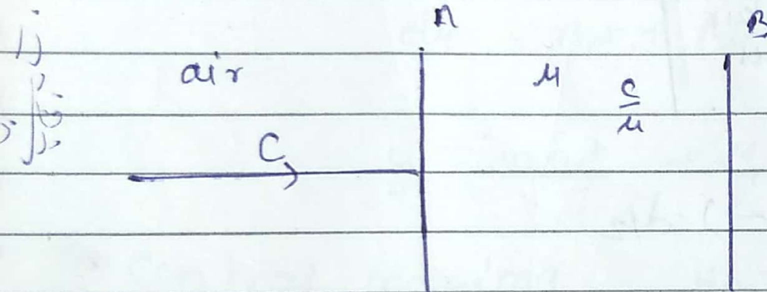
$$y = \frac{nD\lambda}{d} - \frac{Da}{b}$$

$\omega = \frac{D\lambda}{d}$

No. of fringes shift $n = \frac{y_0}{\omega}$

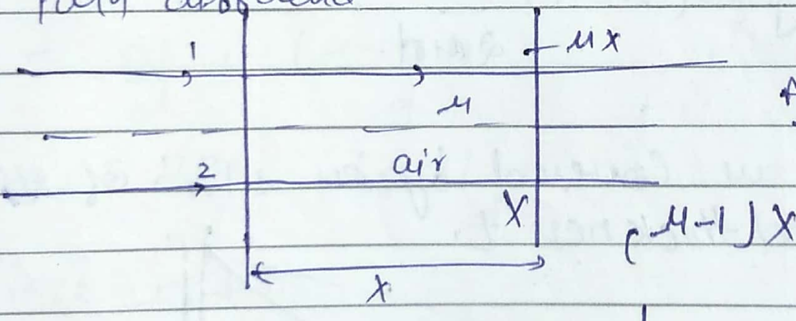
* **Case-III ⇒ YDSE in a medium of Refractive Index (μ):**

Optical Path: It is the distance travelled by the light wave in vaccum or air in the same time in a medium of refractive index μ.



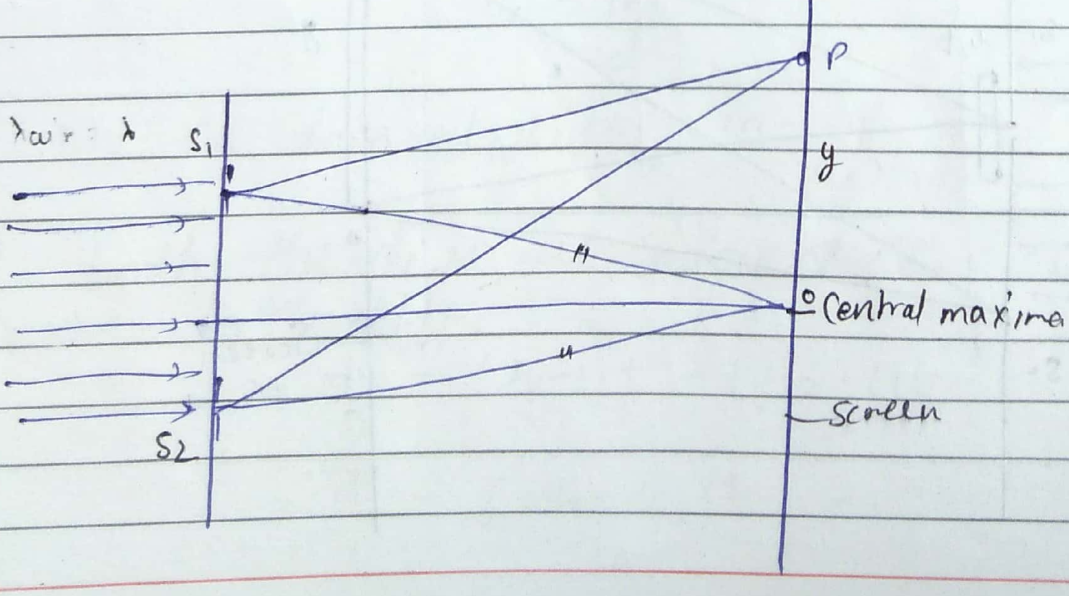
$$t = \frac{x}{c/\mu} = \frac{\mu x}{c}$$

Optical path difference



$ct = \mu x$

optical path



Geometrical Path diff.

$$S_2P - S_1P = \frac{dy}{D}$$

Optical Path diff.

$$\mu(S_2P - S_1P) = \mu\frac{dy}{D}$$

For maxima $\mu\frac{dy}{D} = n\lambda \Rightarrow y_n = \frac{nD\lambda}{\mu d}$

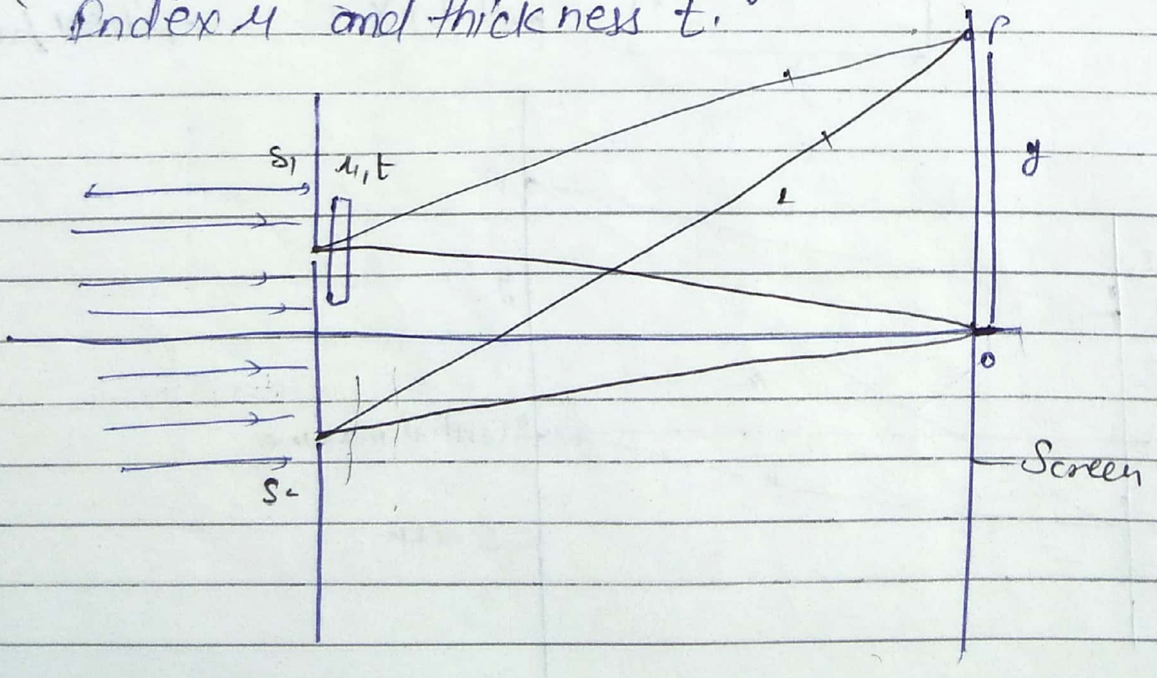
$$y_1 = \frac{D\lambda}{\mu d} \qquad y_2 = \frac{2D\lambda}{\mu d}$$

$$\boxed{\omega_{medium} = \frac{D\lambda}{\mu d} = \frac{\omega_{air}}{\mu}}$$

minima $\mu\frac{dy}{D} = (2n-1)\lambda/2$

$$y = (2n-1)\frac{D\lambda}{2\mu d}.$$

(ii) If slits are covered by a slab of refractive index μ and thickness t.



Geometrical Path diff.

$$S_2P - S_1P = \frac{dy}{D}$$

optical path diff.

$$= (\mu - 1)d$$

Net path diff

$$= \frac{dy}{D} - (\mu - 1)t$$

for maxima

$$\frac{dy}{D} - (\mu - 1)t = n\lambda$$

$$\frac{dy}{D} = n\lambda + (\mu - 1)t$$

$$y = \frac{nD\lambda}{d} + \frac{(\mu - 1)tD}{d}$$

Central maxima $y_0 = \frac{(\mu - 1)tD}{d}$

$$y_1 = \frac{D\lambda}{d} + \frac{(\mu - 1)tD}{d}$$

$$y_2 = \frac{2D\lambda}{d} + \frac{(\mu - 1)tD}{d}$$

frinze width

$$\omega = \frac{D\lambda}{d}$$

$\mu = 10$ $= 5$
$\omega = 2cm$

$y = 11$ 5.5
$\omega = 2$

No. of frinzed shift $n = \frac{y}{\omega}$

if both the slits are covered

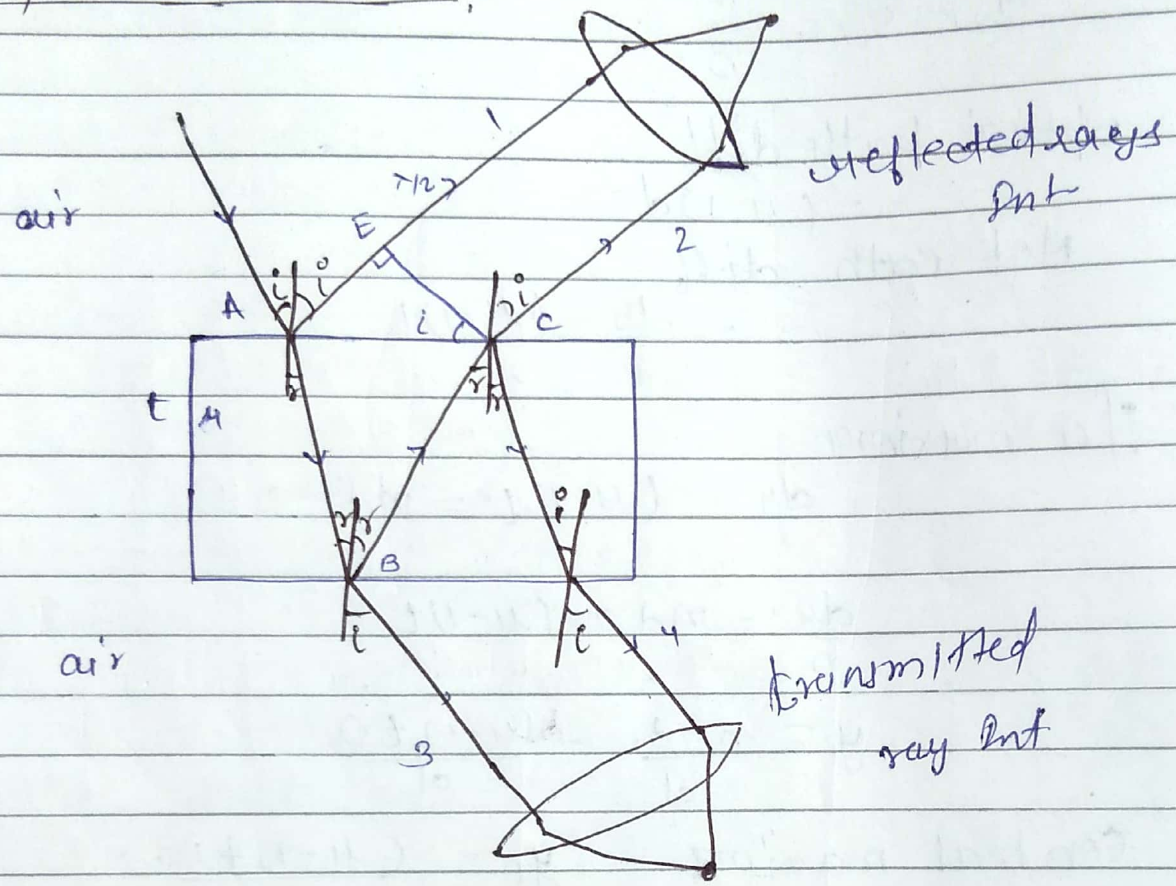
Net path diff.

both thickness same

$$= \frac{dy}{D} - (\mu_1 - 1)t + (\mu_2 - 1)t$$

$$= \frac{dy}{D} - (\mu_2 - \mu_1)t$$

* Thin film Interference:



*> when light wave reflect from Denser medium the phase of the wave changes by 180°

$$\Delta\phi = \frac{2\pi}{\lambda}\ \text{Path diff.}$$

$$\pi = \frac{2\pi}{\lambda}\ \text{Path diff}$$

$$\text{Path diff.} = \lambda/2$$

* when light ray reflect from rarer medium it phase does not change

* During refraction phase of the wave does not change

Optical path travelled by ray 2 in the slab

$= \mu (AB + BC)$

$= 2\mu AB$

$= \frac{2\mu t}{\cos r}$

ΔABD

$\cos r = \frac{t}{AB}$ $\quad AB = \frac{t}{\cos r}$

ΔACE

$\sin i = \frac{AE}{AC}$

$AE = 2AD \sin i$

$AE = 2AD \mu \sin r$

$= 2\mu t \tan r \sin r$

$= \frac{2\mu t \sin^2 r}{\cos r}$

ΔABD $\quad \tan r = \frac{AD}{t}$ $\quad AD = t \tan r.$

$\cancel{\Delta\phi = \frac{2\pi}{\lambda} \text{Path diff.}}$

Path diff. $= \frac{2\mu t}{\cos r}(1 - \sin^2 r)$

$= 2\mu t \cos r$

Net Path diff $= 2\mu t \cos r - \frac{\lambda}{2}$

* for Constructive inter. or maxima.

$2\mu t \cos r - \frac{\lambda}{2}$

~~for Constructive Int~~ or maxima

$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

$2\mu t \cos r = (n + \frac{1}{2})\lambda$ $\quad$ —(1)

$t = -$

Destructive Int -

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \quad (11)$$

Transmitted ray

path diff. $2\mu t \cos r$

for maxima

$$2\mu t \cos r = n\lambda$$

for minima

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

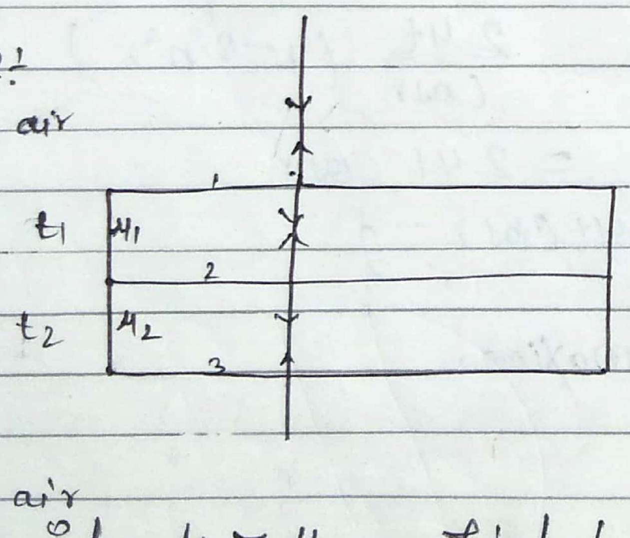
In thin film Int. the wavelengths producing maxima in Reflected Rays produce minima in transmitted rays.

for normal incidence

$r = 0$

$\cos r = 1$

Que:



(Q) if $\mu_1 > \mu_2$ find t_1 show that light rays reflected from 1 and 2 makes constructive inter.

for constructive int

$$2\mu t \cos r = \frac{\quad}{2}$$

$$n\lambda = 4\mu t \cos r.$$

Ans: $2\mu_1 t_1 - \frac{\lambda}{2} = n\lambda$

$$2\mu_1 t_1 = (n + \tfrac{1}{2})\lambda$$

$$t_1 = \frac{(n+\frac{1}{2})\lambda}{2\mu_1}$$

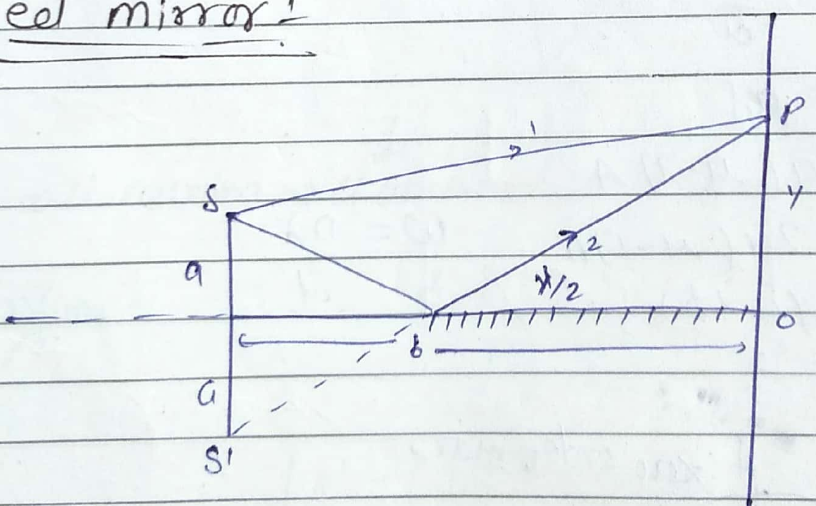
(II) if $\mu_1 < \mu_2$ find t_1 show that light waves reflected at ~~one~~ 1 and 2 interfere constractively. $[2\mu_1 t_1 = (n-\frac{1}{2})\lambda]$

Path diff $= 2\mu_1 t_1$

$$2\mu_1 t_1 = n\lambda$$

$$t_1 = \frac{n\lambda}{2\mu_1}$$

* Lloyed mirror:



$d = 2a$

$D = b$

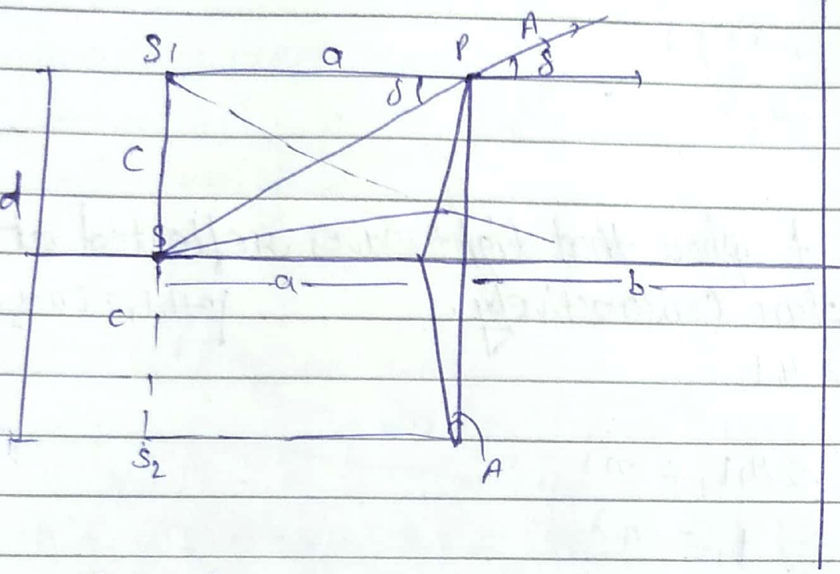
$$\frac{dy}{D} + \frac{\lambda}{2} = n\lambda \text{ — maxima}$$

$$= (2n-1)\frac{\lambda}{2} \text{ minima}$$

$$\omega = \frac{D\lambda}{d}$$

* Fresenl's Biprism :-

A - very small, μ



$\delta = (\mu - 1)A$

$\tan\delta = \frac{c}{a}$

$c = a\delta$

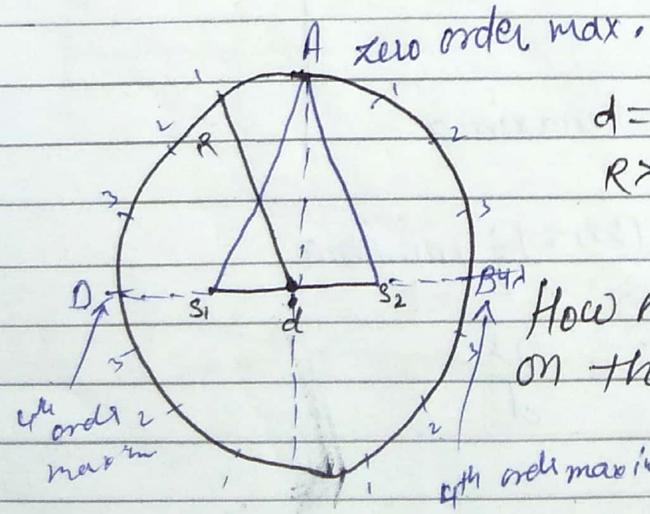
$c = a(\mu - 1)A$

$d = 2a(\mu - 1)A$ $\qquad \omega = \frac{D\lambda}{d}$

$D = (a + b)$

$= \lambda$

Ex :-



$d = 4\lambda$

$R >> d$

16 maxima Ans

How many maximas are observed on the circumfence of

$d\sin\theta = \frac{n\lambda}{d}$

$\theta = 0$

Op-wave:
0-1 : to 51,

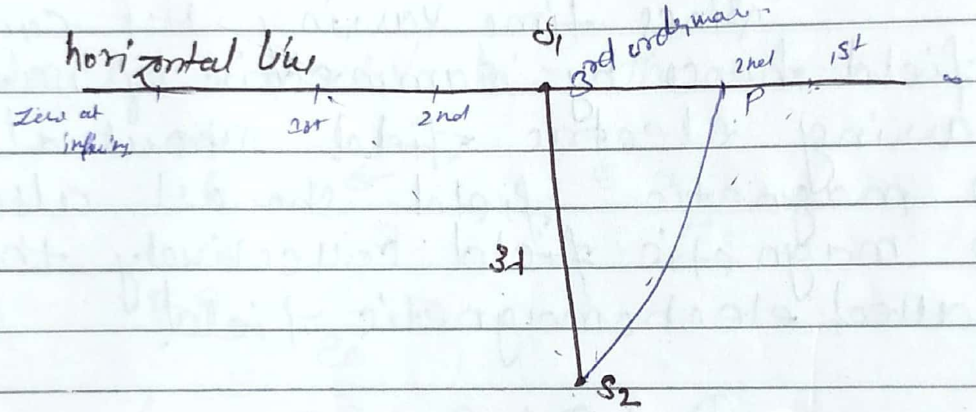
$\phi \sin\theta = \frac{1}{4}$ 1st order

$\phi \sin\theta = \frac{1}{2}$ 2nd order

$\phi \sin\theta = \frac{3}{4}$ 3rd order

$\theta = 90^\circ$ 4th.

Ex:-



How will maxima will be observed on horizontal line.

maxima :- $S_2P - S_1P$

$$\sqrt{x^2 + 9\lambda^2} - x = n\lambda \quad \text{— maxima}$$

$$\sqrt{x^2 + (9\lambda)^2} = (n\lambda + x)$$

$$x^2 + 9\lambda^2 = (n^2\lambda^2) + 2n\lambda x + x^2$$

for minima $n\lambda \rightarrow (n-1)\frac{\lambda}{2}$

$$x = \frac{9\lambda^2 - n^2\lambda^2}{2n\lambda}$$