S.H.M

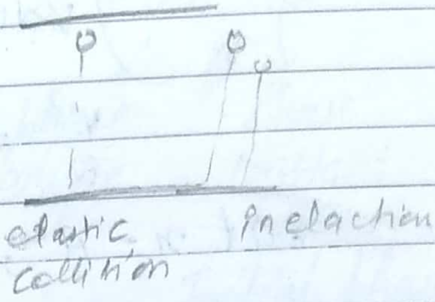
S.H.M :-

Periodic — repeat path after fixed time

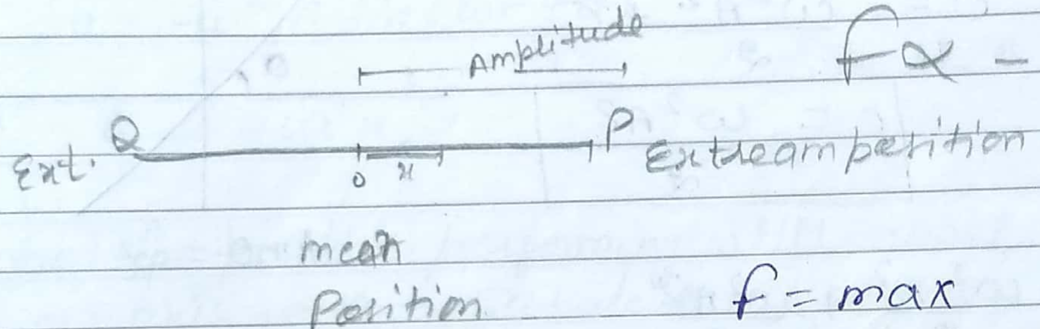
Time period - T

frequency $= \frac{1}{T}$

oscillatory. To & Fro motion.



S.H.M



$F \propto -x$

$F = -kx$

Mean position:

$F = 0$

$a = 0$

$v = max$

$KE = max$

$PE = min$ (0)

Extreme position:

$f = max$

$a = max$

$v = 0$

$KE = 0$

$PE = max$

$a = -\left(\frac{K}{m}\right)x \qquad \omega^2 = \frac{K}{m}$

$a = -\omega^2 x \qquad \omega = \sqrt{\frac{K}{m}} = 2\pi f = \frac{2\pi}{T}$

ω → angular frequency.

$|a_{max}| = \omega^2 A$

* Speed in S.H.M:

$$v\frac{dv}{dx} = -\omega^2 x$$

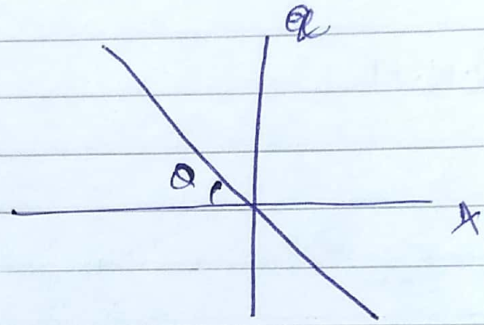
$$\int v\,dv = -\omega^2 \int x\,dx$$

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C$$

at $x = A$, $v = 0$

$$0 = \frac{\omega^2 A^2}{2} + C$$

$$\boxed{C = \frac{\omega^2 A^2}{2}}$$



$\tan\theta = \omega^2$

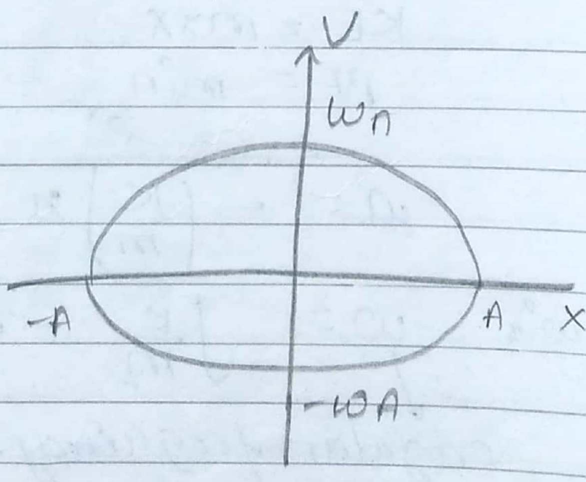
Speed

$$\frac{v^2}{\cancel{2}} = -\frac{\omega^2 x^2}{\cancel{2}} + \frac{\omega^2 A^2}{\cancel{2}}$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$\boxed{v = \omega\sqrt{A^2 - x^2}}$$

$V_{max} = A\omega$



$$v^2 = \omega^2 A^2 - \omega^2 x^2$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$\frac{v^2}{(\omega A)^2} + \frac{x^2}{(A)^2} = 1$$

* Equation of S.H.M:

$$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

$$\int \frac{dx}{\sqrt{A^2 - x^2}} = \omega \int dt$$

$$\sin^{-1}\left(\frac{x}{A}\right) = \omega t + C$$

$$\frac{x}{A} = \sin(\omega t + C)$$

$$\boxed{x = A\sin(\omega t + \theta)}$$

(ϕ)
$\omega t + \theta$ = phase of particle

$\phi = \omega t + \theta$

phase angle (ωt) — Phase constant or Initial phase (θ)

$$v = A\omega\cos(\omega t + \theta)$$

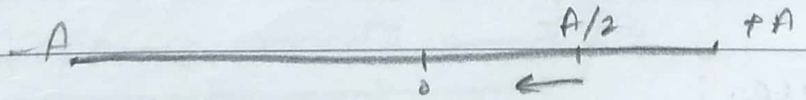
$$a = -\omega^2 A\sin(\omega t + \theta)$$

$$a = -\omega^2 x$$

Que! A particle performing SHM about origin on x-axis with amplitude (A) angular frequency (ω) find its equation of motion if

i) at $t = 0$ it is at $x = \frac{A}{2}$ and going towards mean position.

Ans:



$t = 0$. $x = \frac{A}{2}$, $v = -ve$.

$$x = A\sin(\omega t + \theta)$$

$$\frac{A}{2} = A\sin\theta$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$v = A\omega\cos(\omega t + \theta)$

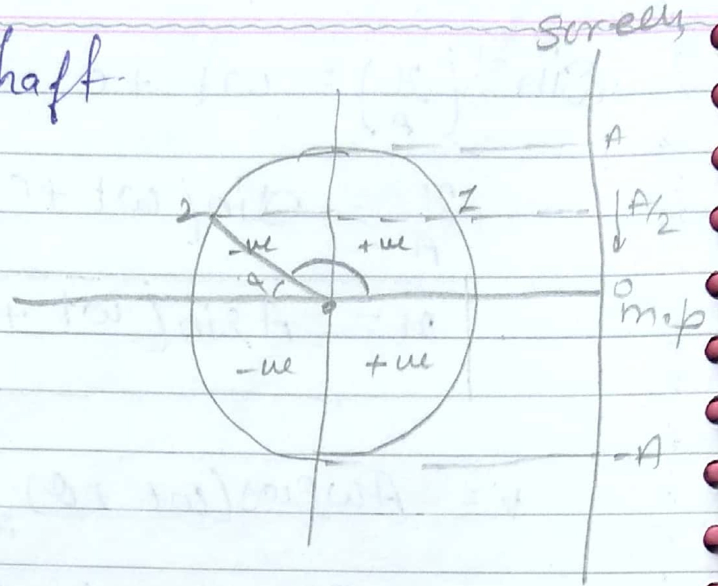
$t = 0$

$v = A\omega\cos\theta$

$$x = A\sin\left(\omega t + \frac{5\pi}{6}\right).$$

M-2 **phases:** Crank shaft

ϕ = phase of particle



$\sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$

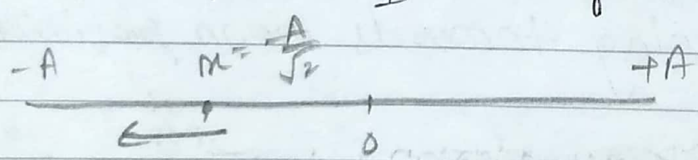
$\phi = \frac{5\pi}{6} = \omega t + \theta$

$\theta = \frac{5\pi}{6}$

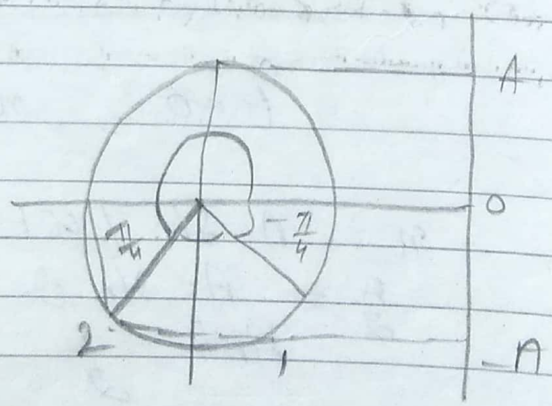
$x = A\sin\left(\omega t + \frac{5\pi}{6}\right)$ Ans.

(ii) at $t = 0$

$x = \frac{-A}{\sqrt{2}}$ and its speed is decreasing. Find equation of S.H.M.



$v = -ve$



$\omega t + \theta = \frac{5\pi}{4}$

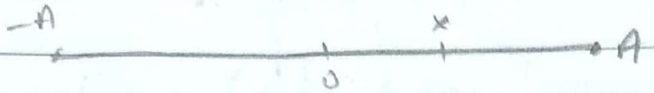
$\theta = \frac{5\pi}{4}$

$x = A\sin\left(\omega t + \frac{5\pi}{4}\right)$

$V_{max} = A\omega$ At mean position

Date: 27/05/17

* Energy S.H.M : $TE = KE + PE$



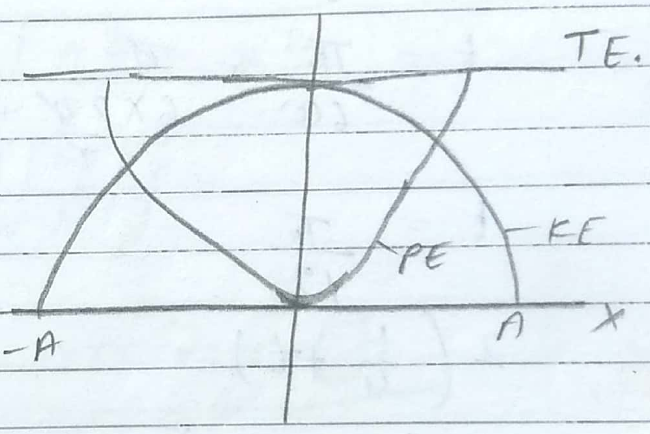
$$V = \omega\sqrt{(A^2 - x^2)}$$

$$KE_x = \frac{1}{2} m\omega^2 (A^2 - x^2)$$

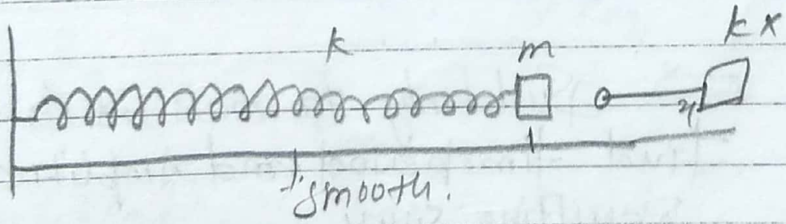
$$KE\ max = \frac{1}{2} m\omega^2 A^2$$ – mean position

$$PE_x = \frac{1}{2} m\omega^2 x^2$$

$$TE = \frac{1}{2} m\omega^2 A^2$$



* Spring - Block System :



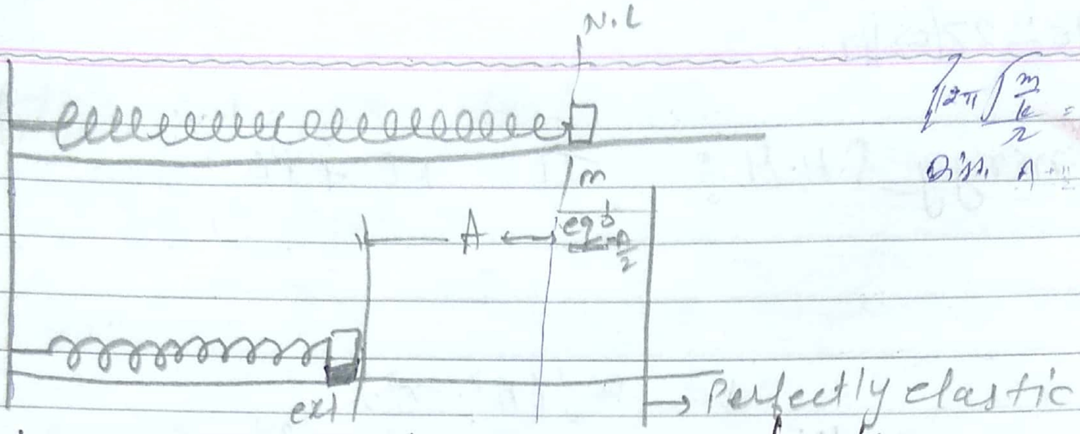
$$f = -kx$$

$$a = -\left(\frac{k}{m}\right)x \qquad \omega = \sqrt{\frac{k}{m}}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

velocity = 0 Extreme Position.

Que:



$\left[\frac{2\pi\sqrt{\frac{m}{k}}}{2} = \pi\sqrt{\frac{m}{k}}\right]$

D.M. A = $\boxed{\frac{3A}{2}}$

find time taken to return to initial position.

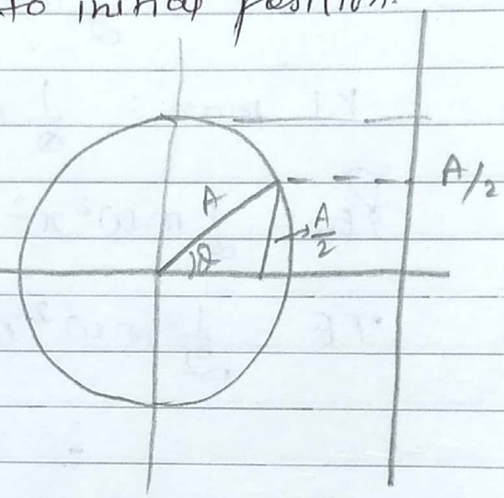
Ans: $\theta = \frac{\pi}{6} = \omega t$

$t = \frac{\pi}{6\omega} = \frac{\pi}{6 \times \frac{2\pi}{T}}$

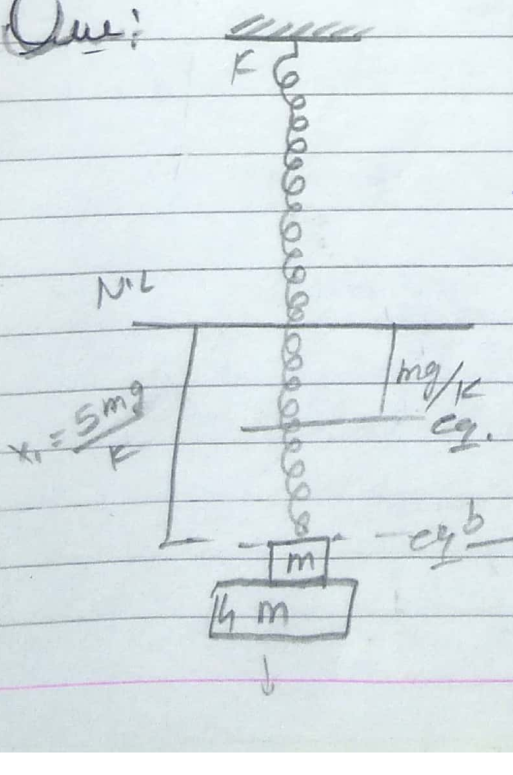
$t = \frac{T}{12}$

$2\left(\frac{t}{4} + t\right)$

$2\left(\frac{T}{4} + \frac{T}{12}\right) = \frac{2}{3}T = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$



Que:



$\Rightarrow$ Suddenly 4m mass is Removed find timeperiod and Amplitude of Resulting S.H.M.

Ans: $v = 0$ $T = 2\pi\sqrt{\frac{m}{K}}$

$A = \frac{4mg}{K}$

Inertial frame = Velocity = 0

ii) Find Maximum Speed when 4m is Removed.

Ans:

$$\omega = \sqrt{\frac{K}{M}} \qquad V_{max} = 4g\sqrt{\frac{m}{K}}$$

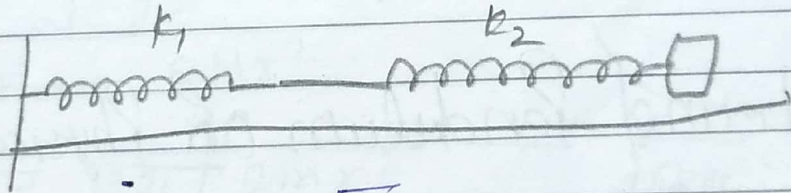
iii) Find maximum Compression impraction:

$$\frac{3mg}{K}$$

(IV) Find Speed of the Block at natural length!

$$V = \omega\sqrt{A^2 - x^2}$$

$$= \left(4g\sqrt{\frac{m}{K}}\right)\sqrt{}$$



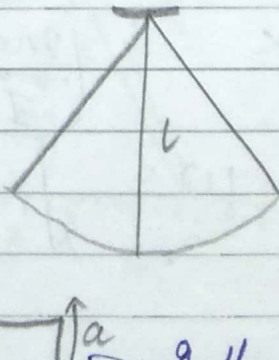
$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

Series: $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$

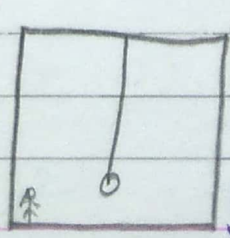
Parallel: $k_{eq} = k_1 + k_2$.

Simple Pendulam:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

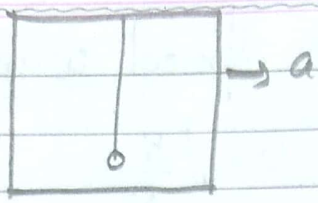


$$T = 2\pi\sqrt{\frac{l}{g_{effective}}}$$



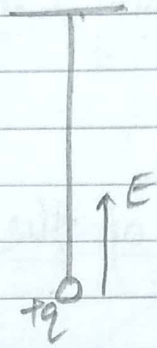
$\uparrow a \rightarrow g_{eff} = g + a$

$\downarrow \rightarrow g_{eff} = g - a$.



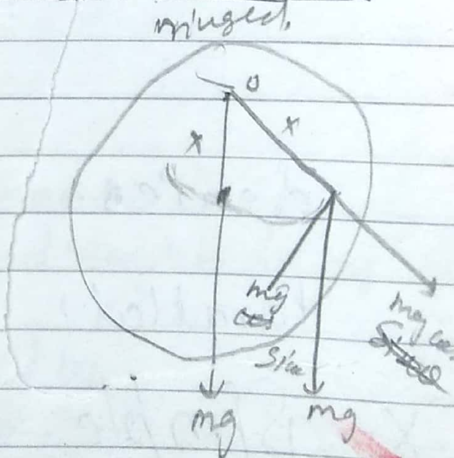
$$g_{eff.} = \sqrt{g^2 + a^2}$$

$$T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$$



$$F_{net} = mg + qE$$
$$mg_{eff.} = mg - qE$$
$$g_{eff.} = g - \frac{qE}{M}$$
$$T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$$

* Compound Pendulum or Physical Pendulum:



Rigid body S.H.M.

$$\tau_0 = mgx\sin\theta \qquad T = 2\pi\sqrt{\frac{I_0}{mgx}}$$

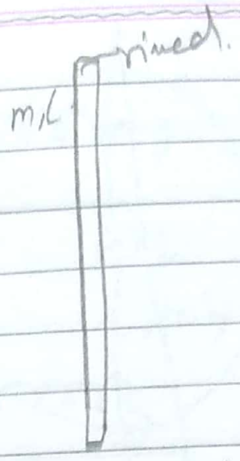
$$\tau = (mgx)\theta$$
$$I_0\alpha = (mgx)\theta$$
$$\alpha = \left(\frac{mgx}{I_0}\right)\theta$$
$$\omega = \sqrt{\frac{mgx}{I_0}} \qquad T = 2\pi\sqrt{\frac{I_0}{mgx}}.$$

Que!
Ans!

$$I_0 = \frac{ml^2}{3}$$

$$x = \frac{l}{2}$$

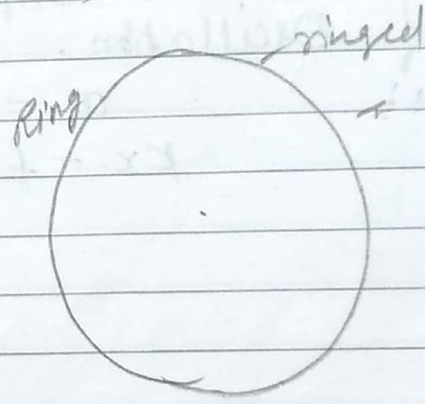
$$2\pi\sqrt{\frac{2l}{3g}}$$



Length of equivalent Simple pendulum

only for MG force

$$2\pi\sqrt{\frac{l_0}{g}}$$ Ans.



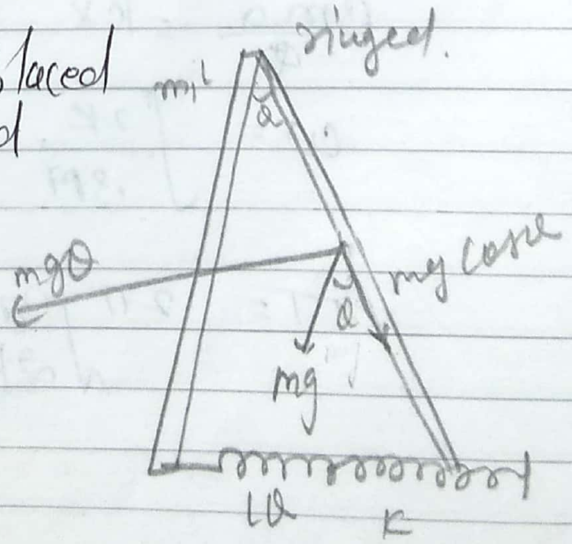
Que!
Ans! $T = 2\pi\sqrt{\frac{r}{g}}$

Ring Moment inertia => $2Mr^2$

$$T = 2\pi\sqrt{\frac{2mr^2}{mgr}}$$

$\left\{ \text{Disc} : \frac{mr^2}{2} + \frac{mr^2}{2} \right\}$

Que: A Rod is slightly displaced & Released find time Period.



Ans!

$$\tau_0 = mg\frac{l}{2}\theta + kl^2\theta$$

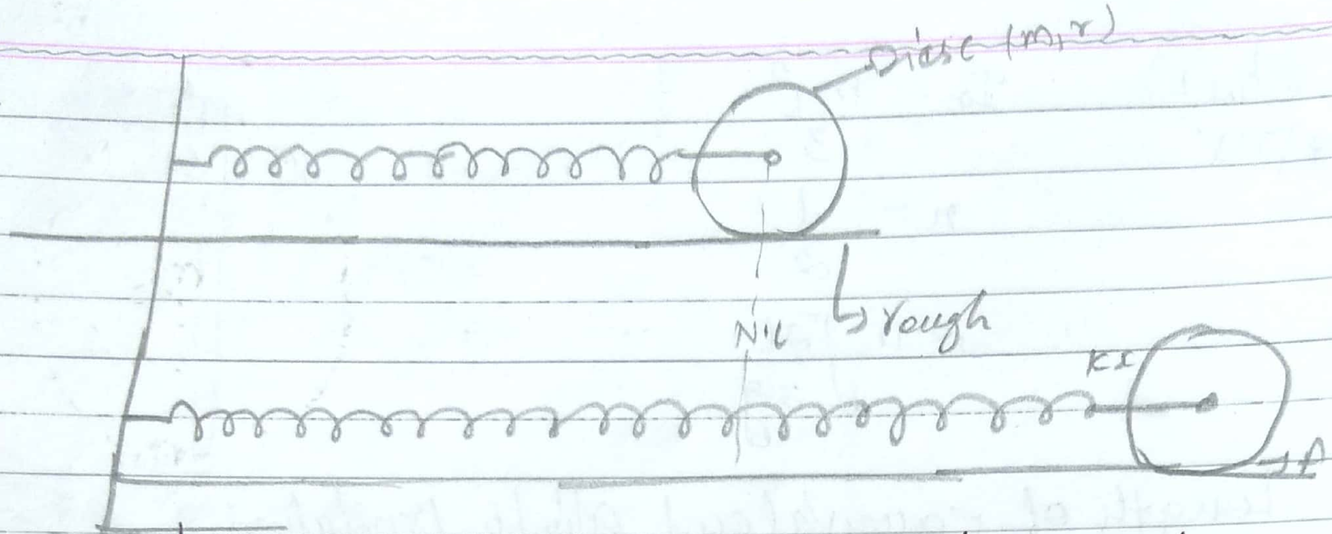
$$\tau = \left(mg\frac{l}{2} + kl^2\right)\alpha$$

$$\alpha = \left(\frac{mgl/2 + kl^2}{I_0}\right)\theta$$

$$T = 2\pi\sqrt{\frac{I_0}{mg\frac{l}{2} + kl^2}}$$

$[a = r\alpha]$

Que:



» Disc is slightly pulled away and Released So, that it does not slip on the Surface. find time period of Oscillation.

Ans:

$$a = r\alpha \qquad fr = I\alpha$$

$$Kx - f = ma$$

$$fr = \frac{mr^2}{2}\alpha$$

$$f = \frac{m}{2} r\alpha$$

$$Kx - \frac{ma}{2} = ma \qquad f = \frac{ma}{2}$$

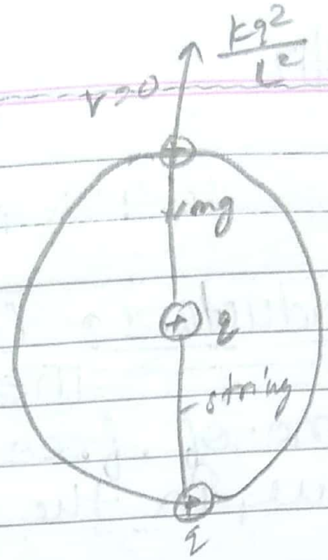
$$\frac{3ma}{2} = kx \qquad a = \left(\frac{2k}{3m}\right)x$$

$$\omega = \sqrt{\frac{2K}{3M}}$$

$$T = 2\pi\sqrt{\frac{3m}{2K}} \quad \text{Ans}$$

Exercise
(O-1) Electrostatic

(9)



Ans: $K_i + U_i = K_f + U_f$

$$\frac{1}{2}mv^2 + \frac{kq^2}{l} = 0 + \frac{kq^2}{l} + mg2l.$$

$$u = \sqrt{4gl}.$$

Date: 29/05/17

SBG STUDY