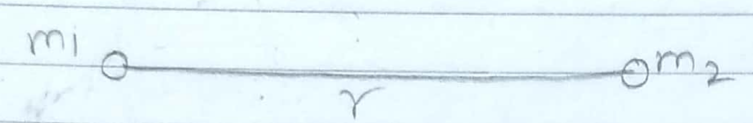
01/06/17

Chapter:

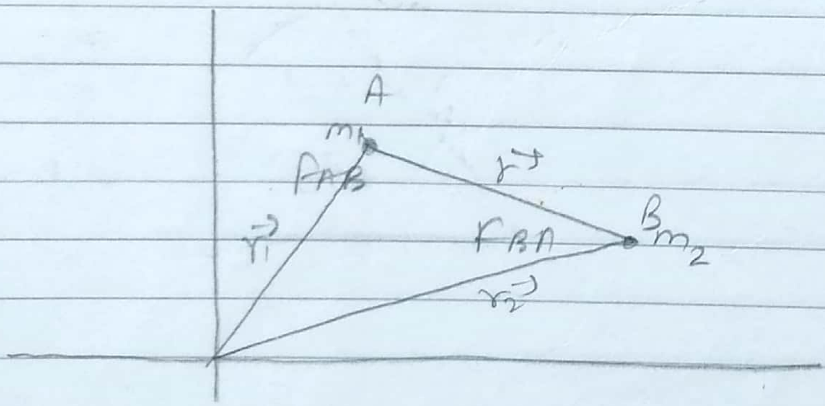
Gravitation

* Newton law of Gravitation:



$$F = \frac{G m_1 m_2}{r^2}$$ (G = universal Grav. Constant)

* Gravitational force act along line of joining
* It is a Conservation force
* It is always attracted
* It is Independent of Median



$$\frac{G m_1 m_2}{r^2}$$

$$\vec{r} = \vec{r_2} - \vec{r_1}$$

$$|\vec{r}| = r$$

$$\vec{F_{AB}} = -\vec{F_{BA}}$$

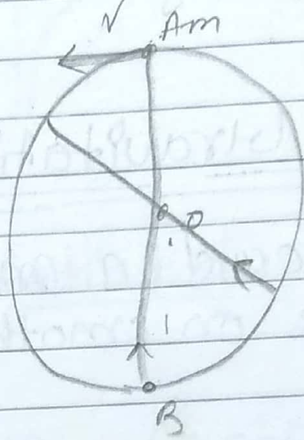
$$\vec{F_{BA}} = \frac{G m_1 m_2}{r^2}(-\hat{r})$$

$$F_{BA} = \frac{-G m_1 m_2}{r^2}\hat{r}$$

$$\vec{F_{AB}} = -\vec{F_{BA}}$$

Que: Two Particles of mass (m) are moving on a Circle of Radius (R) due to mutual gravitation force. find speed of each particle.

Ans:



$$f = \frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$\boxed{\frac{Gm}{4R} = v^2}$$

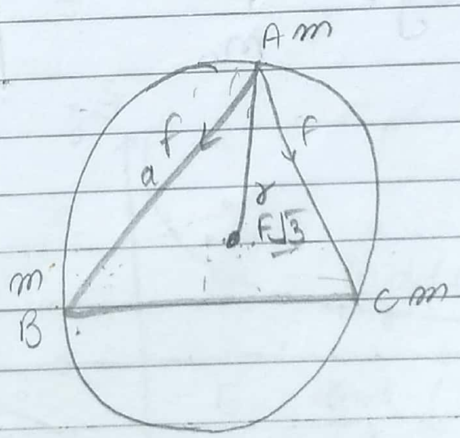
Que: Three Particles are moving on a Circle of radius (R) such that they always formed an equilateral triangle. find speed of each mass.

Ans:

$$\frac{a}{\sqrt{3}} = r$$

$$a = \sqrt{3}r$$

$$f = \frac{Gm^2}{a^2}$$



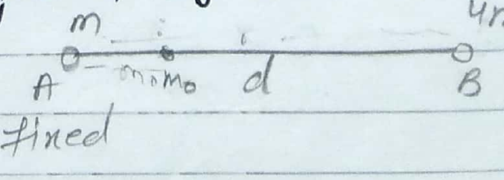
$$\frac{a}{\sqrt{3}} = r$$

$$a = \sqrt{3}r$$

$$f\sqrt{3} = \frac{mv^2}{r}$$

$$\frac{Gm^2}{a^2}\sqrt{3} = \frac{mv^2}{r}.$$

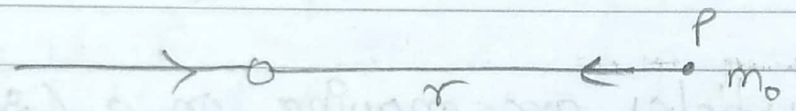
Que: mass m and 4m are seprated by distance (d). where a third mass (m_0) should be placed show that net force on it becomes zero.



Ans: $\frac{Gmm_0}{r^2} = \frac{4Gmm_0}{(d-r)^2}$ $\quad r = \frac{d}{3}$ Ans.

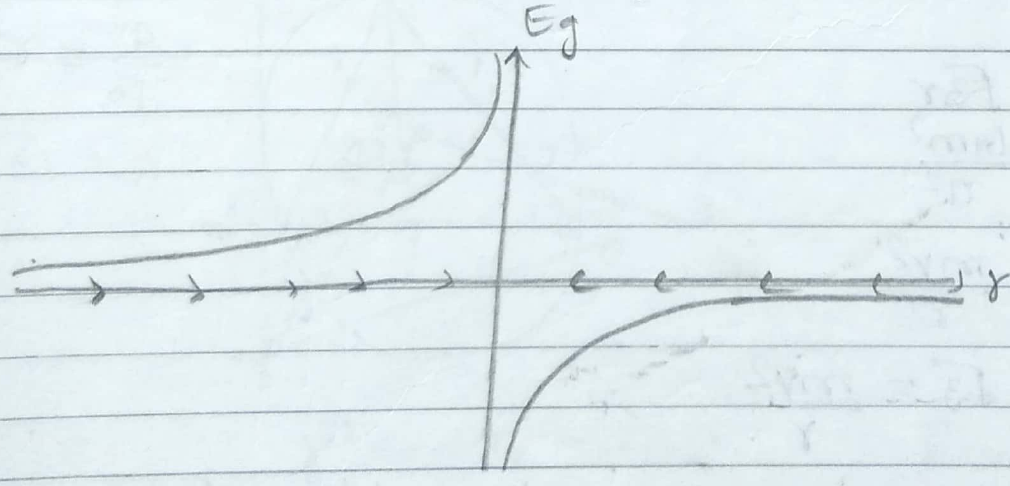
* equib is unstable along line of Joining.
* Stable perpendicular along ~~ling~~ line of Joining.

* ~~Gto~~ **Gravitational field :** (E_g)
It is a region around a mass in which it apply gravitational force on another mass.



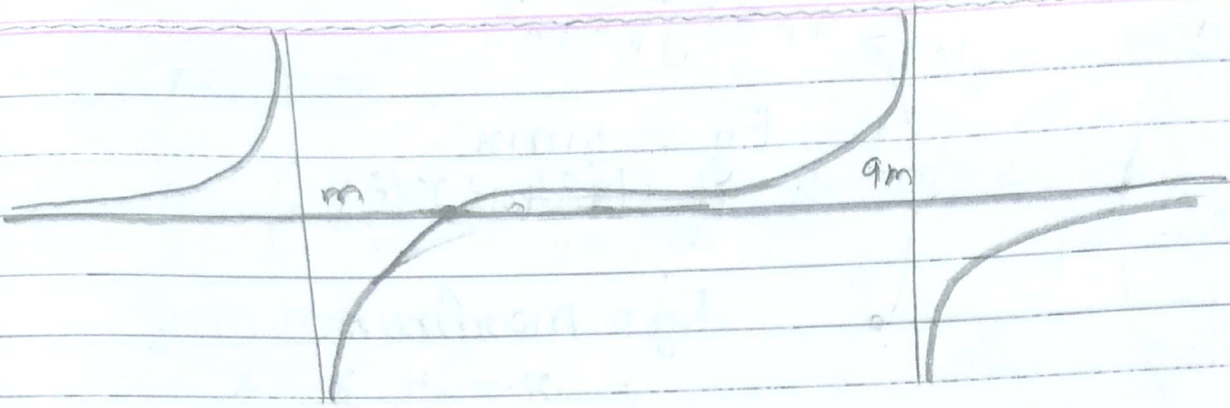
$$f = \frac{GMm_0}{r^2}$$

$$E_g = \frac{f}{m_0} = \boxed{E_g = \frac{GM}{r^2}} \text{ (radialy inward)}$$



Que:

H.W DO-2 complete.
J.M - 24 -



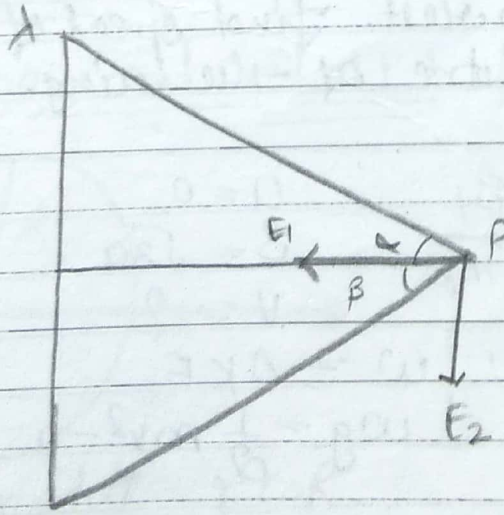
* Gravitational field and potential due to different bodies :

(i) Point charge :

m ———— r $E_g = \frac{GM}{r^2}$, $\Delta V = -E\Delta r$

———— r ———— ∞ $V_g = \frac{-GM}{r}$ $V = 0$

(ii) Uniform Rod :



$$E_2 = \frac{G\lambda}{r}(\cos\beta - \cos\alpha)$$

$$E_1 = \frac{G\lambda}{r}(\sin\alpha + \sin\beta)$$

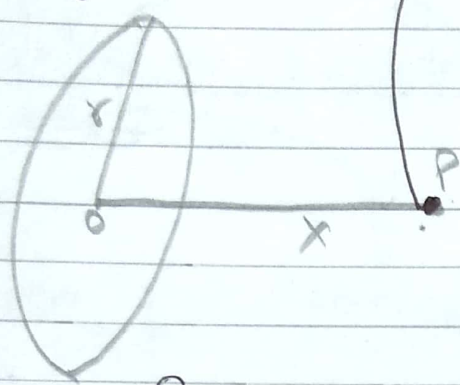
long rod $\Rightarrow$ $E_1 = \frac{2G\lambda}{r}$ $\alpha = \beta = 9$

$$E_2 = 0$$

(iii) **Ring :-**

$$V_P = -\frac{Gm}{\sqrt{r^2+x^2}}$$

$$E_g = \frac{Gmx}{(r^2+x^2)^{3/2}}$$

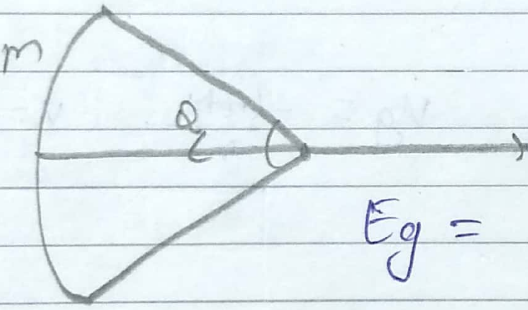


E_g = maximum

$= x = \pm \frac{r}{\sqrt{2}}$

Potential: $V_0 = -\frac{Gm}{r}$

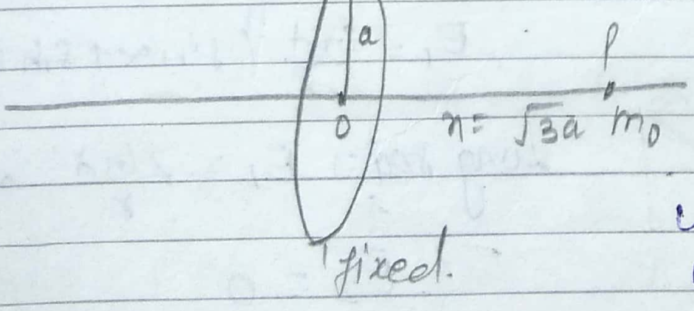
* Gravitational field due to part of ring!



$$\frac{2K\lambda}{r} \sin\frac{\theta}{2}$$

$$E_g = \frac{2G\lambda}{r} \sin\frac{\theta}{2}$$

Que! m_0 is released from rest. Find speed of m_0 when it is at m the centre of the ring.



$u = 0$

$s = \sqrt{3}a$

$v = ?$

$W = \Delta KE$

$W_g = \frac{1}{2}mv^2 - 0$

$= -m_0(V_0 - V_P) = \frac{1}{2}mv^2$

$W_{elect} = -q\Delta V$

$W_{grav} = -m\Delta V$

↓

~~$-m_o(-Gm$~~

$$-m_0\left(-\frac{Gm}{a}+\frac{Gm}{2a}\right)=\frac{1}{2}m_0v^2$$

$$\frac{Gm\,m_0}{2a}=\frac{1}{2}m_0v^2$$

$$\boxed{v=\sqrt{\frac{Gm}{a}}}$$

$$V_0=-\frac{Gm}{a}$$

$$V_P=-\frac{Gm}{\sqrt{a^2+3a^2}}=\frac{-Gm}{2a}$$

$$W_{ext}=q\Delta V$$

$$\boxed{W_{ext}=m\Delta V}$$

* Disc:

$$E=\frac{\sigma}{2\varepsilon_0}\left(1-\frac{x}{\sqrt{r^2+x^2}}\right)$$

$$E_g=\sigma 2\pi G\left(1-\frac{x}{\sqrt{r^2+x^2}}\right)$$

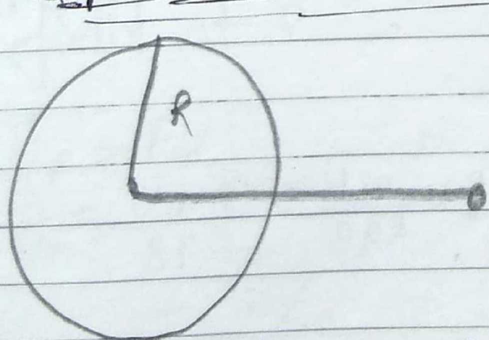
$$K=G$$

$$\frac{1}{4\pi\varepsilon_0}=G$$

$$\boxed{\frac{1}{\varepsilon_0}=4\pi G}$$

* Sphere:

(i) Spherical shell: (Hollow shell).



(i) $r>R$ Outside

$$E_g=\frac{Gm}{r^2}$$

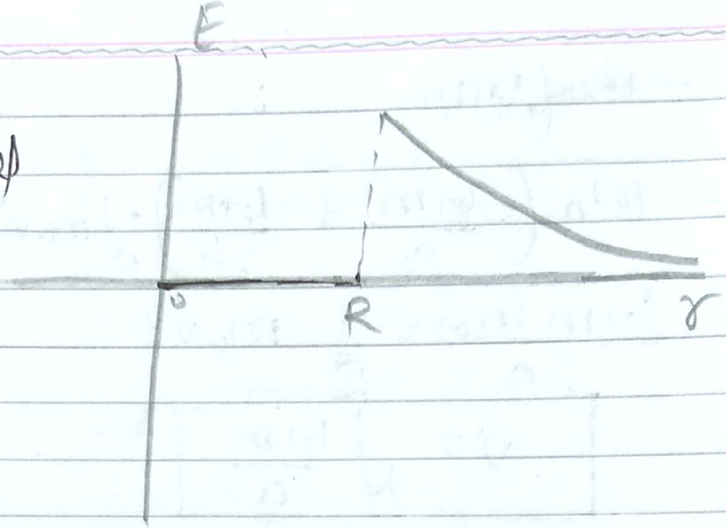
$$\boxed{V=-\frac{Gm}{r}}$$

(ii) Inside $r<R$

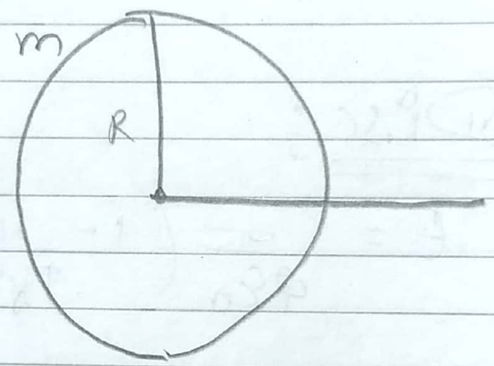
$$E_g=0$$

$$\boxed{V=\frac{-Gm}{R}}$$

Graph of Spherical shell (~~inside~~



* Solid sphere :



(i) Outside point.

$r > R$

$E_g = \frac{Gm}{r^2}$

$$\boxed{V_g = \frac{-Gm}{r}}$$

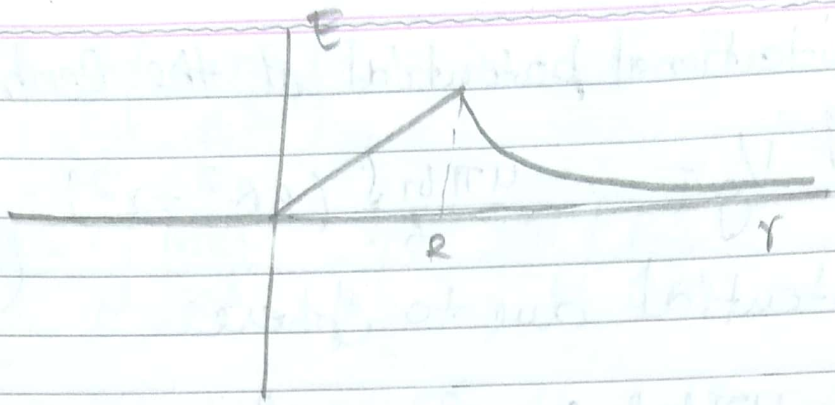
(ii) Inside point : $(r < R)$

$$\boxed{E_g = \frac{4\pi G}{3} \rho r.}$$

$\frac{\rho r}{3\epsilon_0}$

$$= \frac{4\pi G}{\cancel{3}} \frac{m}{\frac{4}{\cancel{3}}\pi R^3} . r$$

$$\boxed{E_g = -\frac{Gmr}{R^3}}$$ in term of density.



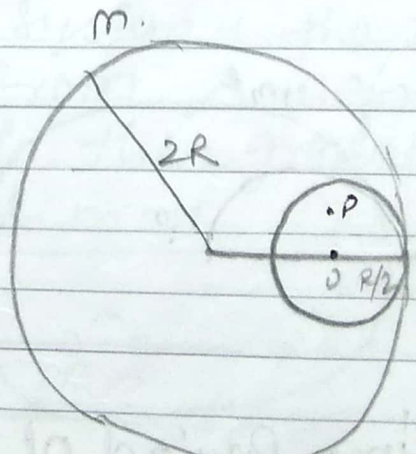
$$\frac{\rho}{6\varepsilon_0}(3R^2-r^2)$$

$$\boxed{V_g = \frac{4\pi G}{6}(3R^2-r^2)}$$ in term of density

$$V_g = \frac{4\pi G}{6} \times \frac{m}{\frac{4\pi}{3}R^3}(3R^2-r^2)$$

$$\boxed{V_g = \frac{Gm}{2R^3}(3R^2-r^2)}$$ in term of mass.

Que: find Gravitational field at P



Ans: $E_p = E_0,$

$$E_1 = \frac{Gmr}{8R^3} = \frac{Gm}{8R^3} \times \frac{3R}{2}$$

$$E_1 = \frac{3Gm}{16R^2}$$

(iii) Find Gravitational potential at the Centre of Cavity

Ans: $V_g = -\frac{4\pi G \rho}{6}(3R^2 - r^2)$ $\left(\rho = \frac{m}{\frac{4\pi}{3}R^3}\right)$

Potential due to sphere

$$V_1 = -\frac{4\pi G\rho}{6}\left(12R^2 - \frac{9R^2}{4}\right)$$

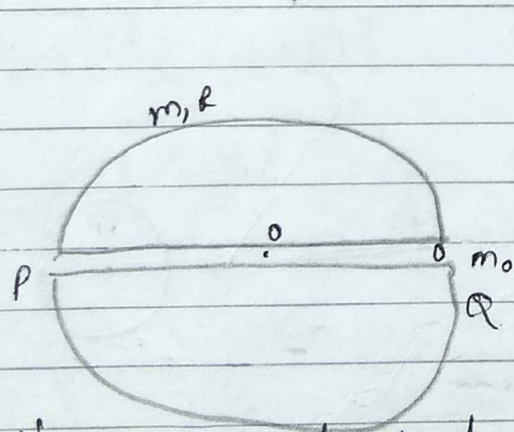
$$V_2 = -\frac{4\pi G\rho}{6}\left(\frac{3R^2}{4} - 0\right)$$

$$V_{O_1} = V_1 - V_2$$

Ques

O - centre
& surface

Que:



Find speed of mass at the centre of sphere

Ans: $-m_0(V_0 - V_Q) = \frac{1}{2}m_0 v^2$

$$E_r = \frac{Gm}{R^3} \cdot r$$

$$f = m_0 E_r$$

$$f = \left(\frac{Gm m_0}{R^3}\right) r$$

Find time Period of SHM:

Ans $V_{max} = A\omega$

$V_{max} = R\omega$

$T = \frac{2\pi}{\omega}$

Conduct H.W

$R = 6\times10$

Earth Time period $= 84.6$ m.

$$a = \left(\frac{Gm}{R^3}\right)r$$

$$\omega = \sqrt{\frac{Gm}{R^3}} \qquad T = 2\pi\sqrt{\frac{R^3}{Gm}}$$

Time Period of Earth $= 84.6$ m.

(J.M) Que

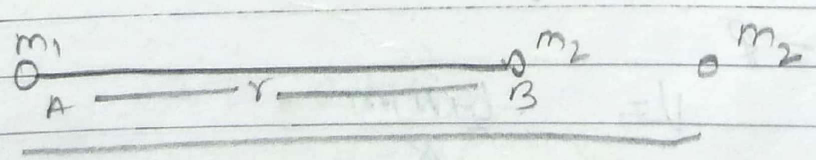
(15) $m = \rho\frac{4\pi}{3}r^3 \qquad qE = \rho\frac{4\pi}{3}r^3 g$

$$6\pi\eta r v = \rho\frac{4\pi}{3}r^3 g.$$

39 (2)

* Gravitational potential Energy:

Gravitational P.E of two point mass system is the work done to take mass from a reference point to infinity in the gravitational field of another point mass.



$$f = \frac{Gm_1m_2}{r^2}$$

$$W = -\int_{r_0}^{\infty}\frac{Gm_1m_2\,dr}{r^2} = -\left[Gm_1m_2\left(\frac{1}{\infty} - \frac{1}{r_0}\right)\right]$$

(a) $W = \frac{Gm_1m_2}{r_0}$

$W_{elect} = -\Delta U = -q\Delta V$

(b) $\Delta U = \frac{Gm_1m_2}{r_0}$

$U_f - U_i = \frac{Gm_1m_2}{r_0}$

$0 - U_0 = \frac{Gm_1m_2}{r_0}$

$PE = mV$

$V = -\frac{Gm_1}{r}$

$$\boxed{U_0 = \frac{Gm_1m_2}{r_0}}$$

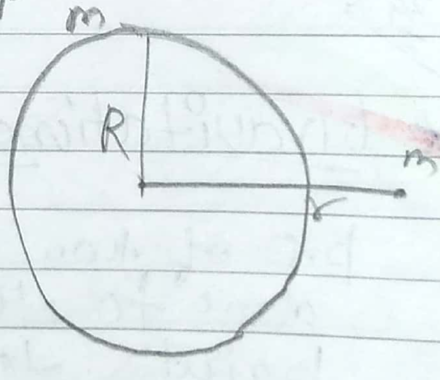
$PE = m_2V$

$= -\frac{Gm_1m_2}{r}$

$W_{grav.} = -\Delta U = -m\Delta V$

$W_{ext} = m\Delta V = \Delta U$

Que: Find potential energy of system if solid sphere



(a) $r > R$

$V = \frac{GM}{r}$

$U = mV$

$= -\frac{GMm}{r}$

b) $r = R$

$V = -\frac{GMm}{R}$

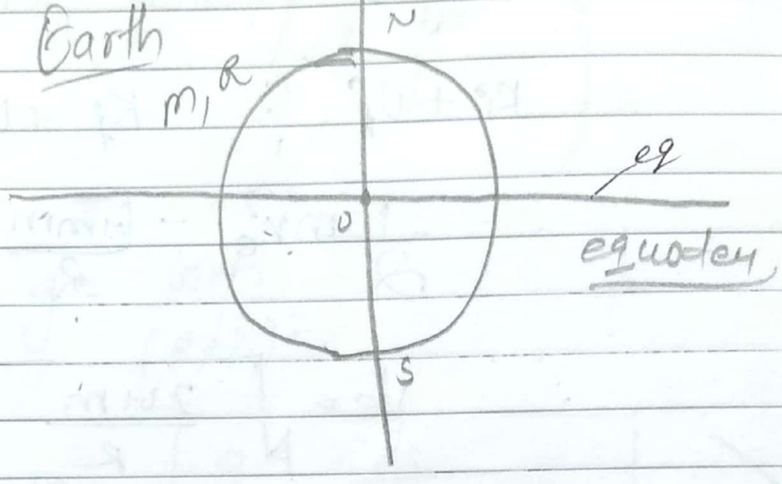
c) $r < R$

$V = \frac{GM}{2R^3}(3R^2 - r^2)$

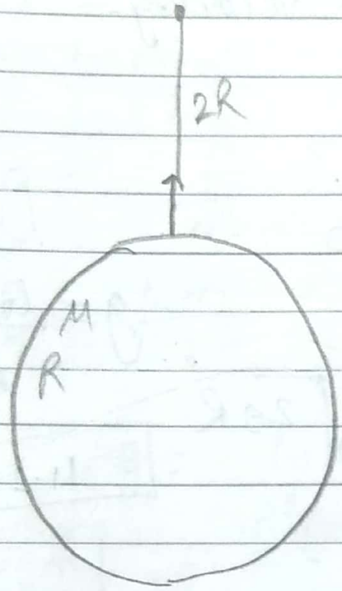
$U = \frac{Gm_1m}{2R^3}(3R^2 - r^2)$

* Potential at centre :-

$$V_{centre} = -\frac{3GMm}{2R}$$



Que :



An object of mass (m) projected vertically upward from earth surface so, that it reaches to height. 2R find initial speed of of the particles.

Ans:

$$PE_i + KE_i = -\frac{GMm}{3R} = \frac{1}{2}mv^2$$

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 - \frac{GMm}{3R}$$

$$v = 2\sqrt{\frac{Gm}{3R}}$$

* Escape Velocity : When an object is projected from earth surface so that it does not return to the earth surface.

OR

It leave gravitational field of earth that velocity is called scape velocity.

$$K_i + U_i = K_f + U_f$$

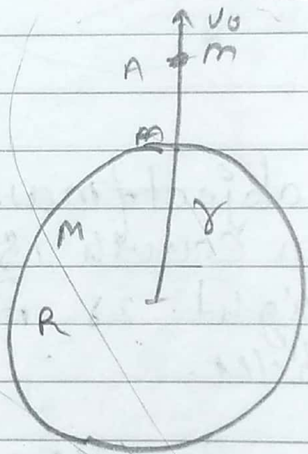
$$\frac{1}{2} m v_e^2 - \frac{GMm}{R} = 0$$

$$g = \frac{GM}{R^2}$$

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

$$\boxed{= 11.2\ km/sec.}$$

Que: Find V_o so that its scape from gravitational field

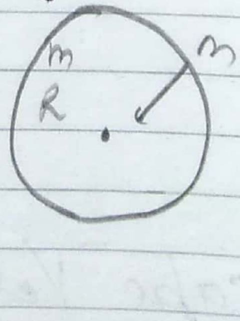


Ans: $K_i + U_i = K_f + U_f$

$$\frac{1}{2} m v_o^2 - \frac{GMm}{r} = 0 + 0$$

$$V = \sqrt{\frac{2GM}{r}} = \sqrt{2gr}$$

* Acceleration due to gravity: (g).



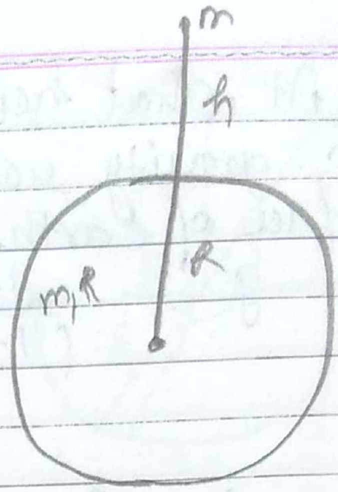
$$F = \frac{GMm}{R^2}$$

$$a = \frac{f}{m}$$

$$g = \frac{f}{m}$$

$$\boxed{g_s = \frac{GM}{R^2}}$$

(ii) Above Earth Surface:



$$r = R + h$$

$$F = \frac{GMm}{r^2}$$

$$g = \frac{F}{m}$$

$$g = \frac{GM}{r^2} \Rightarrow g = \frac{GM}{(R+h)^2}$$

$$g = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} \qquad \boxed{g = \frac{g_s}{\left(1+\frac{h}{R}\right)^2}}$$

If $h <<< R$

$$g = g_s\left(1+\frac{h}{g}\right)^{-2}$$

$$\boxed{g = g_s\left(1-\frac{2h}{R}\right)}$$

$$\frac{g}{g_s} = 1 - \frac{2h}{R} \Rightarrow \frac{2h}{R} = 1 - \frac{g}{g_s}$$

$$\frac{2h}{R} = \frac{g_s - g}{g_s} \Rightarrow \frac{\Delta g}{g_s} = \frac{2h}{g}$$

fraction Charge.

Race :- 12,

S = III 1, 2, 6, 7, 9, 13, 15,

Q: At what height above earth surface acc. due to gravity remains 25%. of its value at the surface of earth.

Ans: $g = \dfrac{g_s}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \dfrac{g_s}{4} = \dfrac{g_s}{\left(1+\frac{h}{R}\right)^2}$

$\left(1+\frac{h}{R}\right)^2 = 4 \Rightarrow 1+\frac{h}{R} = 2$

$h = R.$

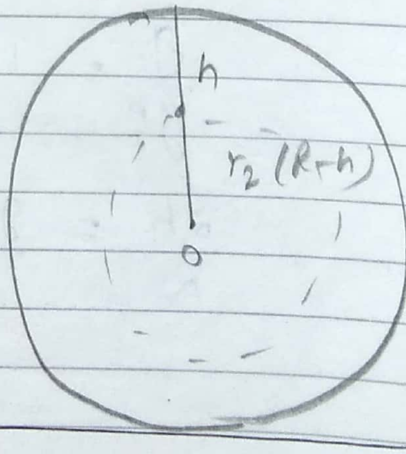
Que: At what height above surface acc. due to gravity changes by one Percent.

Ans: $\dfrac{\Delta g}{g_s} \times 100 = \dfrac{2h}{R} \times 100$

$1 = \dfrac{2h \times 100}{R}$

$h = \dfrac{R}{200}$ $\quad h = 32$ km.

(iii) below earth surface:



$E_g = \dfrac{GM}{R^3} \cdot r$

$F = mE_g = \dfrac{GMmr}{R^3}$

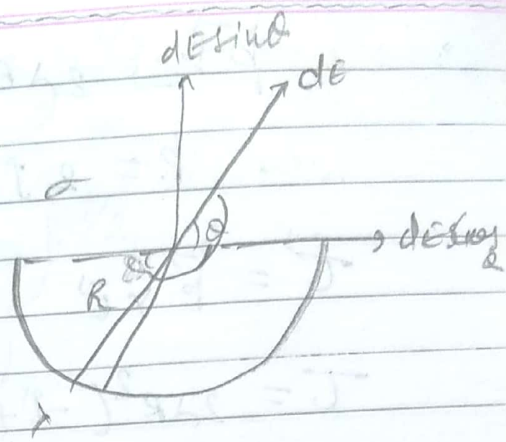
$g = \dfrac{F}{m} = \dfrac{GM}{R^3} \cdot r$

$g = \dfrac{GM}{R^3}(R-h) = \boxed{g = g_s\left(1-\frac{h}{R}\right)}$

05/06/17 Ex: S-2

(1) $dE = \frac{2K\lambda}{R}$

$= \frac{2K\sigma R d\theta}{R} = 2K\sigma d\theta$



$E_y = \int dE \sin\theta$

$= 2K\sigma \int_0^{\pi} \sin\theta d\theta$

$= 2K\sigma (\cos\theta]_0^{\pi}$

$= 4K\sigma$ Ans

2) $dq = \lambda R d\theta$

$dp = dq\, 2R$

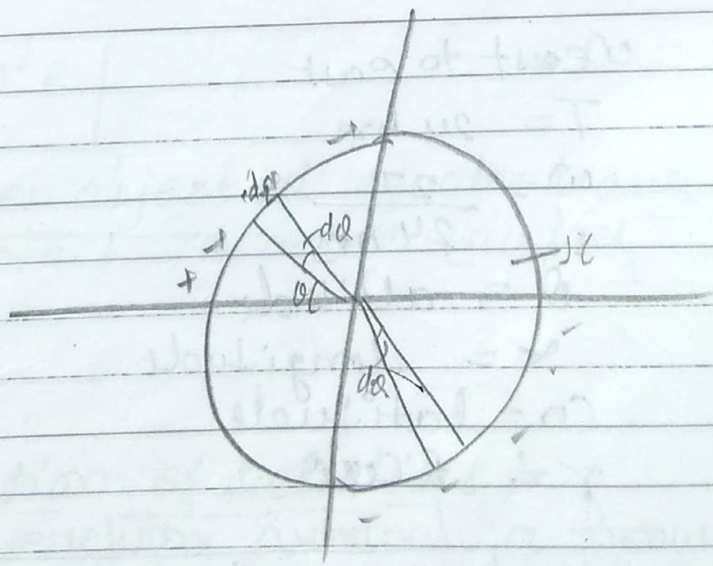
$= \lambda R\, 2R d\theta$

$= 2\lambda R^2 d\theta$

$p_x = \int dp \cos\theta$

$p_x \int dp \cos\theta$

$= 2\lambda R^2 \int_0^{\pi/2} \cos d\theta$



$P_x = 2\lambda R^2$

$P_y = \int dp \sin\theta$

$= 2\lambda R^2 \int_0^{\pi/2} \sin\theta d\theta$

$= 2\lambda R^2$

acc. = 0 inertial

earth — Non-inertial frame

$\vec{p} \Rightarrow 2\lambda R^2(-\hat{i}+\hat{j})$

$p = 2\sqrt{2}\lambda R^2$

$a = r\alpha$

$f = ma$

$\tau = I\alpha$

$\frac{f}{\tau} = \frac{MR}{I}$

$\vec{\tau} = \vec{p} \times \vec{E}$

$\tau = 2\lambda R^2(-\hat{i}+\hat{j}) \times E_0\hat{i}$

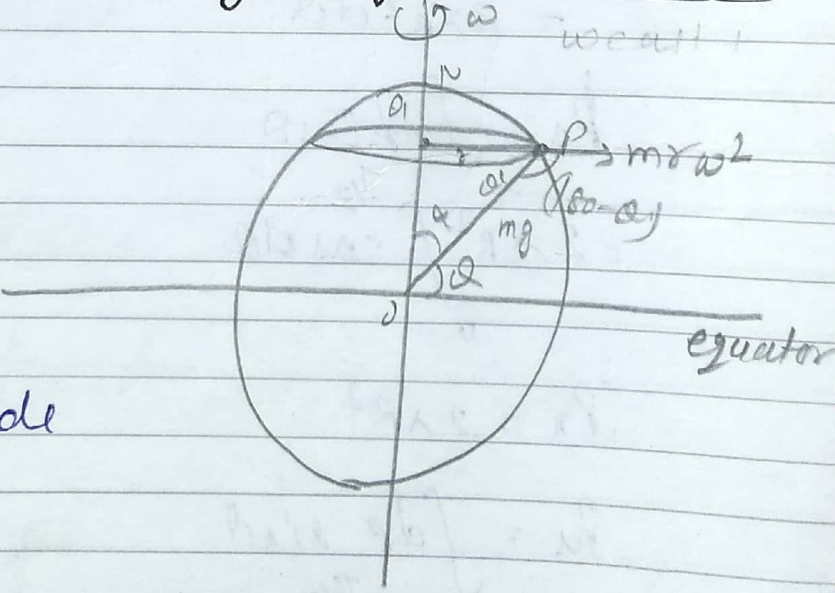
$\tau = -2\lambda R^2 E_0 \ \hat{k}$

15)

$r = \sqrt{R^2+h^2}$ $\qquad f = \frac{\sigma^2}{2\varepsilon_0} \times \pi(R^2-h^2)$.

05/06/17

(iv) Variation in acc. due to gravity: due to rotation of earth:



west to east

$T = 24$ hrs

$\omega = \frac{2\pi}{24 \text{ hrs}}$

θ = altitude

α = longitude

co-latitude

$r = R\cos\theta$

$F_{Net} = \sqrt{(mg)^2 + \underbrace{(mr\omega^2)}_{\text{neglected}} + 2mg\ mr\omega^2 \cos(180-\theta)}$

$F_{net} = (m^2g^2 - 2m^2 g r\omega^2 \cos\theta)^{1/2}$

$$f_{net} = mg\left(1 - \frac{2r\omega^2 \cos\theta}{g}\right)^{1/2}$$

$$f_{net} = mg\left(1 - \frac{2R\omega^2}{g} \cos^2\theta\right)^{1/2}$$

$$f_{net} = mg\left(1 - \frac{\omega^2 R}{g} \cos^2\theta\right)$$

$$mg' = mg\left(1 - \frac{\omega^2 R}{g} \cos^2\theta\right)$$

$$\boxed{g' = g - \omega^2 R \cos^2\theta}$$

Poles - North Pole

$\theta = 90°$

$g' = g = 9.8\ m/s^2$

equator $\theta = 0$

$$\boxed{g' = g - \omega^2 R}$$

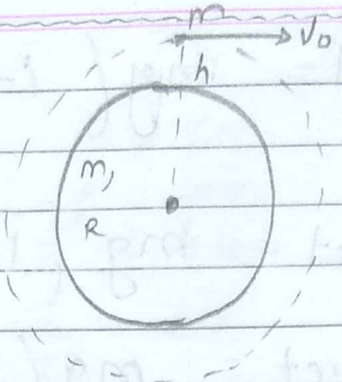
If $g' = 0$, then object at equator leave contact to earth surface

$$\boxed{\omega = \sqrt{\frac{g}{R}}}$$

* Settelites / motion of settelites :

A light object revolving around a heavy object due to gravitational force only called settelite.

* Artificial Settelites:



radius of orbit (r)

$r = R + h$

Or

1) Orbital velocity:

$$\left[g = \frac{GM}{r^2} \right] \qquad \frac{GMm}{r^2} = \frac{mv_0^2}{r}$$

$$v_0 = \sqrt{\frac{GM}{r}}, \quad v_0 = \sqrt{gr}$$

* Time Period of Settelites:

$$T = 2\pi r$$
$$V = \sqrt{gr}$$

$$T_1 = \frac{2\pi r}{v_0} =$$

$$= \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

$$= T \propto r^{3/2}$$

$$T^2 \propto r^3$$

Time Period of 2 settelites

$$\boxed{\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}}$$

* Energy of Settelites:

$TE = KE + PE$

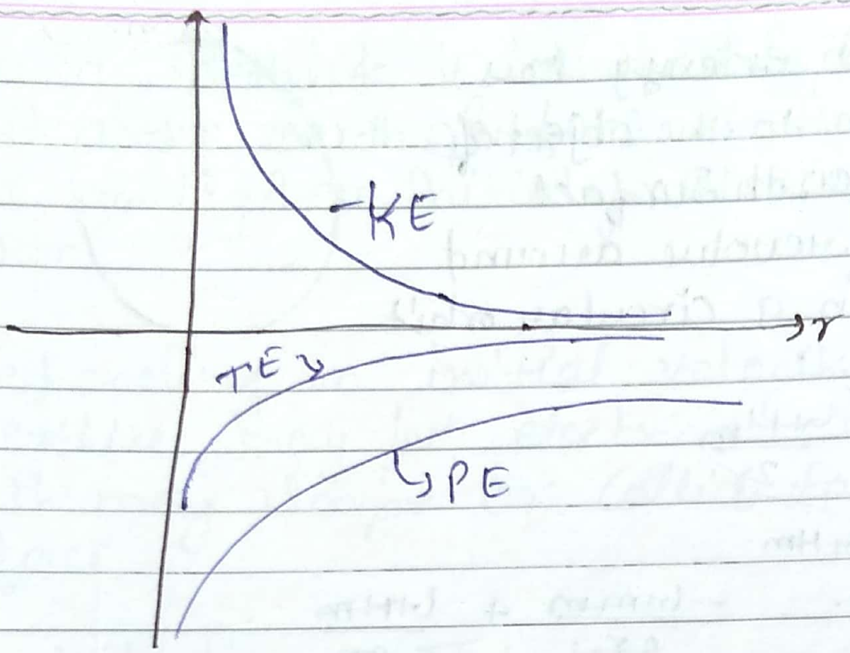
$$KE = \frac{1}{2} m v_0^2$$

$$KE = \frac{GMm}{2r}, \quad PE = -\frac{GMm}{r}$$

$$T.E = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$\boxed{T.E = -\frac{GMm}{2r} = \frac{P.E}{2}}$$

HCV: P. No- 225
Q. 2, 3, 5, 8, 9, 10, to 20, 21, 22
27



radius ↑ = PE ↓
or all ↑

$g_{app} = g - \omega^2 R = 0$

* Angular momentum of settelite (L) =

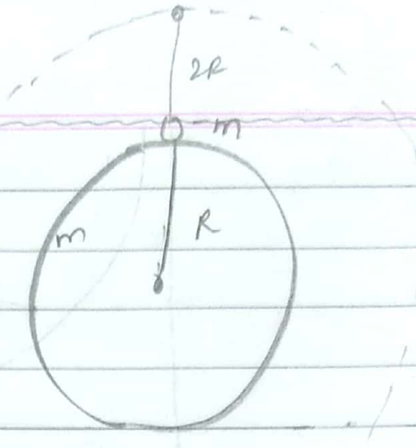
$$L = mvr$$
$$L = mv \cdot r$$

* Work done to change orbit of settelite:
from r_1 to r_2:

$$\text{Work done} = TE_f - TE_i$$

$$W = -\frac{GMm}{2r_2} + \frac{GMm}{2r_1}$$

Que] How much energy has to be given to the object of mass (m) on earth surface show that it revolve around the earth in a circular orbit of radius 3R.



Ans! $E = \frac{-GMm}{2r}$

$= \cancel{-GMm}$

$W.D = \frac{-GMm}{2r_2} + \frac{GMm}{2r_1}$

$=$

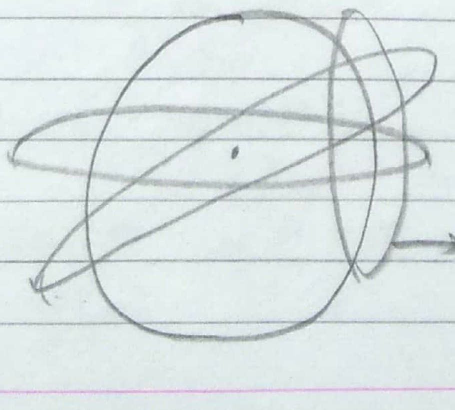
$K_i + V_i = K_f + V_f$

$K_i - \frac{GMm}{R} = \frac{-GMm}{6R}$

$K_i = \frac{GMm}{R} - \frac{GMm}{6R}$

$K_i = \frac{5GMm}{6R} = \frac{5}{6} m g_s R$

* Launching of Settelite: When a Settelite Revolve around a earth orbit of Settelite is called stable orbit. if Centre of Orbit coincide to the Centre of earth.

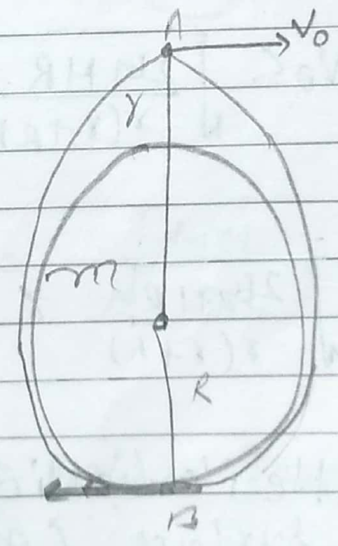


unstable.

When a Settilide is launched in a particular orbit above earth surface it is taken to that point and given initially velocity in horizontal direction.

* Depending on initial velocity orbit of Setthise may be electrical circular or it may scape or collide to the earth surface

* In this Process Total Energy and angular momentum remains conserved.



Que: Find minimum value of V_0 so that it does not colide to the earth surface

$$TE_A = TE_B$$

$$\frac{1}{2}mV_0^2 - \frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$L_A = L_B$$

$$mV_0 r = mvR$$

$$v = \frac{V_0 r}{R}$$

$$\frac{V_0^2}{2} - \frac{GM}{r} = \frac{v^2}{2} - \frac{GM}{R}$$

$$\frac{GM}{R} - \frac{GM}{r} = \frac{v^2 - V_0^2}{2}$$

max → appari

$$2GM\left(\frac{1}{R}-\frac{1}{r}\right) = v_0^2\left(\frac{r^2}{R^2}-1\right)$$

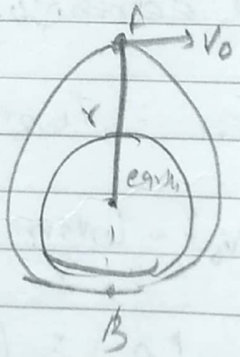
$$2GM\left(\frac{r-R}{Rr}\right) = v_0^2\left(\frac{r^2-R^2}{R^2}\right)$$

$$\frac{2GM}{r} = v_0^2\left(\frac{r+R}{R}\right)$$

$$\boxed{V_0 = \sqrt{\frac{2GMR}{r(r+R)}}}$$

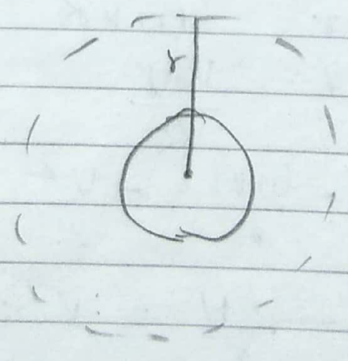
i) if $V_0 < \sqrt{\frac{2GMR}{r(r+R)}}$ Settelite collide to earth surface.

ii) If $\sqrt{\frac{2GMR}{r(r+h)}} < V_0 < \sqrt{\frac{Gm}{r}}$

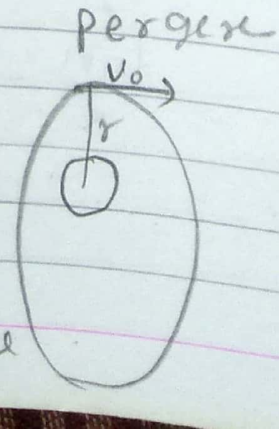


Sattelite collide to earth surface (Appcar.

iii) If $V_0 = \sqrt{\frac{Gm}{r}}$

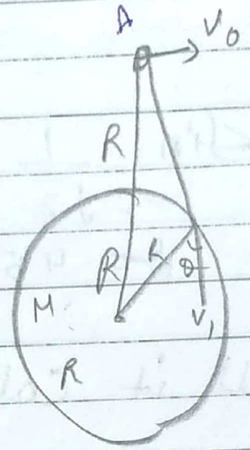


iv) $\sqrt{\frac{GM}{r}} < V_0 < \sqrt{\frac{2Gm}{r}}$



(v) $v_0 = \sqrt{\frac{2Gm}{r}}$. it scape along parabolic path.

(vi) $v_0 > \sqrt{\frac{2Gm}{r}}$ It scape along hyperbolic.



Que:

$v_0 = \sqrt{\frac{Gm}{7R}}$ will it.

Ans: ~~Valid~~ collide to earth Surface

$$V_0 < \sqrt{\frac{2GMR}{r(r+R)}}$$

$r = 2R$

$$v = \sqrt{\frac{2GMR}{2R \times 3R}} = \sqrt{\frac{GM}{3R}}$$

(ii) Find Speed of Settelite when it crush into earth.

Ans) $K_i + U_i = K_f + U_f$

$$\frac{1}{2} m v_0^2 - \frac{GMm}{2R} = \frac{1}{2} m v^2 - \frac{GMm}{R}$$

$$v = \sqrt{\frac{8GM}{7R}}$$

(iii) In which direction / or At what angle from verticle it collide to earth Surface

07/06/17 H.C.V : obt 1, 2

Ans: $L_A = L_B$

$m V_0 2R = m V \sin\theta R$

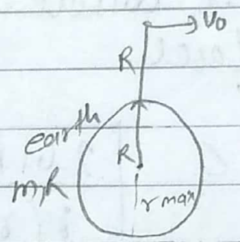
$$\sin\theta = \frac{2V_0}{V} = \frac{2\sqrt{\frac{GM}{7R}}}{\sqrt{\frac{8GM}{7R}}} = \frac{2}{\sqrt{8}}$$

$$\sin = \frac{1}{\sqrt{2}}$$

$$\theta = 45°$$

Que: will it collide to earth

i)



$$r = \sqrt{\frac{3GM}{5R}}$$

$$V_0 = \sqrt{\frac{3GM}{5R}}$$

$$= \sqrt{\frac{2GMR}{r(r+R)}} = \sqrt{\frac{2GMR}{2R \times 3R}} = \sqrt{\frac{GM}{3R}}$$

$$V_0 > \sqrt{\frac{GM}{3R}}$$

ii) what will be the Path of satellite

$$\sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{2R}}$$

$$V_0 > \sqrt{\frac{GM}{2R}} \qquad V_e = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{4M}{R}}$$

find max. distance of sattilite from earth centre.

$L = mv \cdot r$ $\qquad \cancel{KE} + \cancel{PE} = \cancel{KE} + \cancel{PE}$

$= \cancel{mx}$

$\cancel{KE_i + U_i} = \cancel{KE_f + U_f}$.

$\cancel{\frac{1}{2} m v_0^2 +} =$

$m V_0 \, 2R = m V r$

$V = \frac{2 V_0 R}{r}$

$$\frac{1}{2} m v_0^2 - \frac{GMm}{2R} = \frac{1}{2} \frac{m 4 V_0^2 R^2}{r^2} - \frac{GMm}{r}$$

$$\frac{1}{2} m v_0^2 - \frac{2mR^2 V_0^2}{r^2} = \frac{GMm}{2R} - \frac{GMm}{r}$$

$$V_0^2 \left(\frac{1}{2} - \frac{2R^2}{r^2} \right) = \frac{GM}{2R} - \frac{Gm}{r}$$

$$\frac{3GM}{5R} \left(\frac{1}{2} - \frac{2R^2}{r^2} \right) = \frac{Gm}{2R} - \frac{Gm}{r}$$

$$\frac{3}{10R} - \frac{6}{5r^2} = \frac{1}{2R} - \frac{1}{r}$$

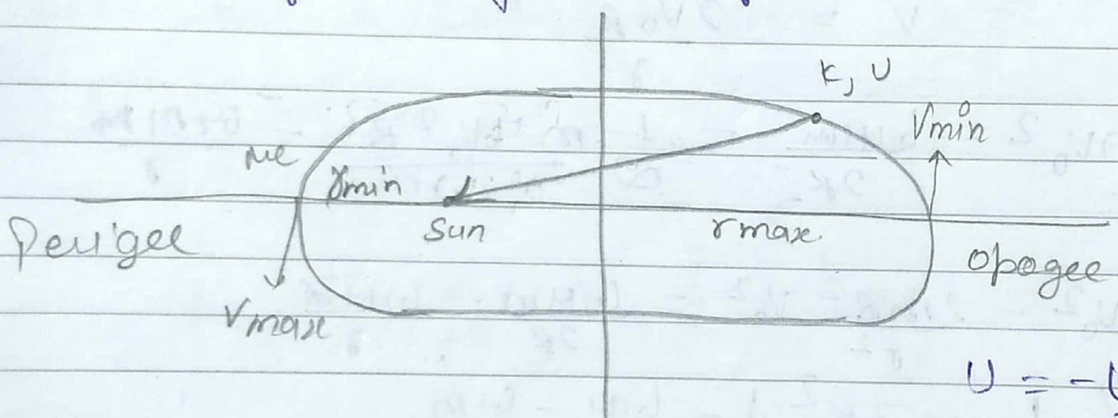
$$\frac{1}{r} - \frac{6R}{5r^2} = \frac{1}{2R} - \frac{3}{10R}$$

$$\frac{5r - 6R}{5r^2} = \frac{2}{10R} \Rightarrow 5Rr - 6R^2 = r^2$$

$$r^2 - 5Rr + 6R^2 = 0$$
$$(r-2R)(r-3R) = 0$$
$$r = 2R$$
$$r = 3R \quad \text{— max. Ar}$$

* Keplar's law of planetory motion:-

Law:
1) Every planet revolve around Sun in eleptical orbit and Sun is always at one of the focus of ellips



$$U = -\frac{GMm_s M_e}{r}$$

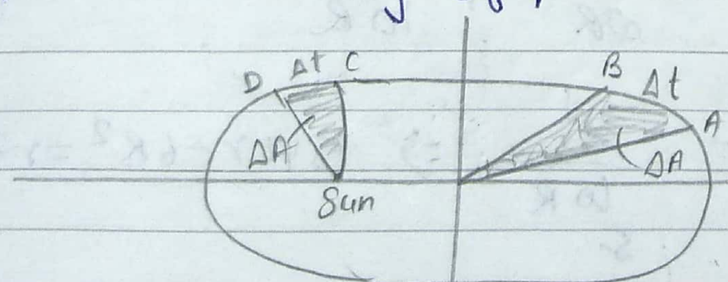
τ earth about sun $= 0$

$L =$ constant.

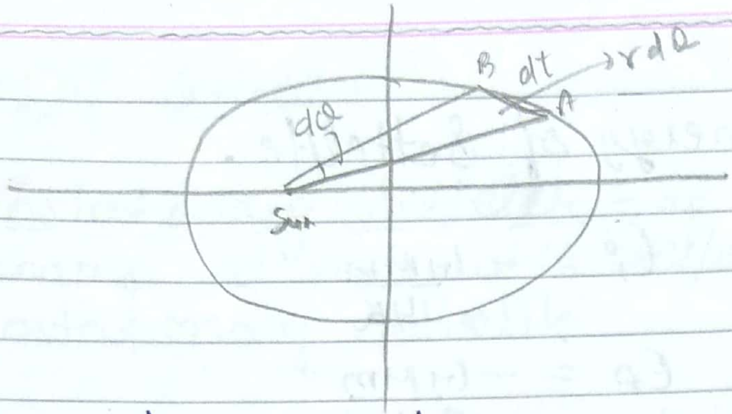
Law 2 line of joining of earth and Sun covers equal area in equal time ~~intv~~ interval

OR

Areal velocity of planet Remains constant.



$$\frac{\Delta A}{\Delta t} = \frac{dA}{dt} \text{ — Areal velocity}$$



$$dA = \frac{1}{2} r r d\theta$$

$$dA = \frac{1}{2} r^2 d\theta \Rightarrow \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

$$= \frac{1}{2} r^2 \omega = \frac{rv}{2}$$

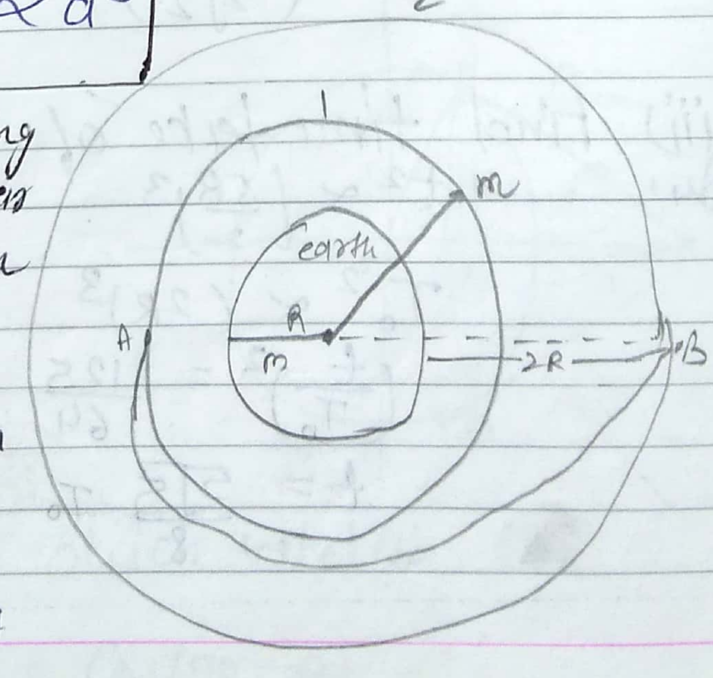
$$\frac{dA}{dt} = \frac{mvr}{2m}$$ → Angular momentum.

Areal velocity $$\boxed{\frac{dA}{dt} = \frac{L}{2m}}$$ mass of earth

Law-3:- Time period of Planet in elliptical orbit depends on the length of semi-major axis.

Time Period → $$\boxed{T^2 \propto a^3}$$

Que: A settilite orbiting around earth in a circular path of Radius 2R when it is at Point A Dt is given and impulse so that it move on an elliptical path from A to B. then again start moving in a circular orbit of radius 3R



PGT
Ex: 3-1 = 1 to 8 + 7
0-1 = 1 to 10

~~Ans~~: find ~~a~~
i) change in Energy of Settelite.
Au:

$$E = -\frac{GMm}{2r} \qquad E_i = -\frac{GMm}{4R}$$

$$E_f = -\frac{GMm}{6R}$$

$$E_f - E_i =$$

(ii) If time Period of Rotation in initial orbit was T_0 find time Period in finial orbit.

$$T = \frac{2\pi}{\sqrt{Gm}} \cdot r^{3/2}$$

$$T^2 \propto r^3$$

$$T_0^2 \propto (2R)^3$$

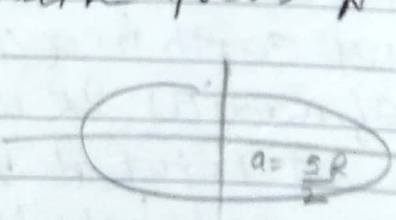
$$T^2 \propto (3R)^3$$

$$\left(\frac{T}{T_0}\right)^2 = \frac{27}{8} = \frac{T}{T_0} = \frac{3\sqrt{3}}{2\sqrt{2}}$$

$$T = \left(\frac{3\sqrt{3}}{2\sqrt{2}}\right) T_0$$

(iii) Find time take of Sattilile from A to B.
Au:

$$t^2 \propto \left(\frac{5R}{2}\right)^3$$



$$T_0^2 \propto (2R)^3$$

$$\left(\frac{t}{T_0}\right)^2 = \frac{125}{64}$$

$$t = \frac{5\sqrt{5}}{8} T_0 \qquad t_{AB} = \frac{5\sqrt{5}}{16} T_0 \text{ Ans}$$

w to E

Co

08/06/17

* Types of Settelites

i) **Geostationary Settelite**:- A settelite which remains at rest with respect to earth called geostationary settelite

* Its angular Velocity and time Period are same as that of earth

* These Settelites revolve around earth in equiterial plane. or above equator.

* Their ~~Right~~ height from earth surface aproximately $h \simeq 6R_e \simeq 36000$ km.

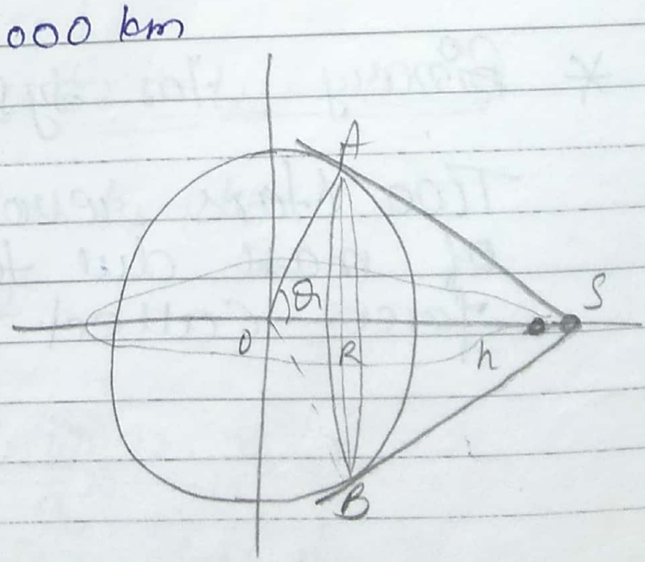
* These settelites are used for Communication on particular area of the earth

* These rotates from west to east

$$h \simeq 6R_e \simeq 36000 \text{ km}$$

$$r = R + h$$

$$T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$



$$\cos\theta = \frac{R}{R+h}$$

$$\theta = \cos^{-1}\left(\frac{R}{R+h}\right)$$

max lattitude to which settelite can Communicate

Co-lattitude = $\cos(90 - \theta)$

Solid angle of Disc. $= 2\pi(1-\cos\theta) = \Omega$

4π Solid angle have area $= 4\pi R^2$

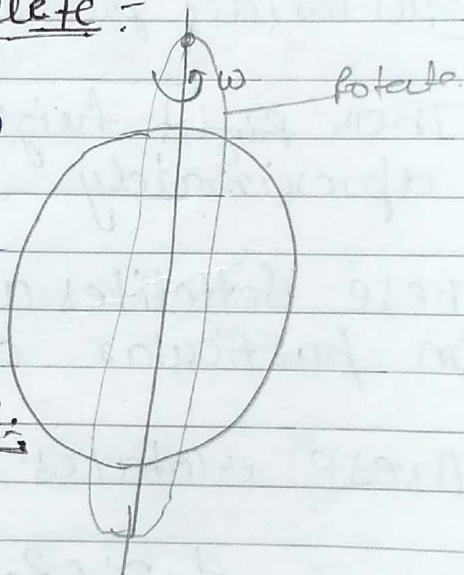
Covered 1 ——— $= R^2$

Area Ω ——— $= 2\pi R^2(1-\cos\theta)$

Area covered $= 2\pi R^2(1-\cos\theta)$

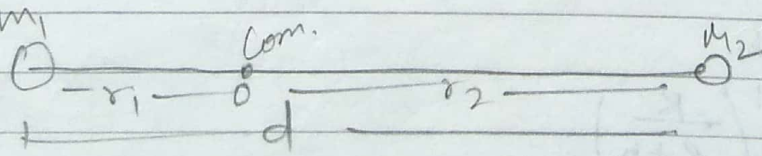
2.) Sun Synchronised Settelete :-

These Settilites revolve in Polar orbit avery part of earth passes in front of the Settelite twice in one rotation. These Settelites are also called Spy Settelite.



* Binary Star System :

Two Stars revolving around their centre of mass due to their mutual gravitational force called Binary Star system.



$$\frac{r_1}{r_2} = \frac{m_2}{m_1} \qquad r_1 + r_2 = d$$

$$r_1 = \frac{m_2 d}{m_1 + m_2}$$

$$r_2 = \frac{m_1 d}{m_1 + m_2}$$

$$\Rightarrow \quad f = \frac{G M_1 M_2}{d^2} \Rightarrow \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$$\boxed{V_1 = \sqrt{\frac{G M_2 r_1}{d^2}}} \qquad \boxed{T_1 = \frac{2\pi r_1}{V_1}}$$

$$\boxed{V_2 = \sqrt{\frac{G M_1 r_2}{d^2}}} \qquad \boxed{T_2 = \frac{2\pi r_2}{V_2}}$$

$$T_1 = T_2$$

$$\frac{r_1}{V_1} = \frac{r_2}{V_2} \qquad \omega_1 = \omega_2$$

$$\frac{1}{\omega_1} = \frac{1}{\omega_2}$$

Kinetic Energy of System: $= K_1 + K_2$.

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= \frac{1}{2} \frac{G M_1 M_2}{d^2} r_1 + \frac{1}{2} \frac{G m_1 m_2}{d^2} \cdot r_2$$

$$= \frac{1}{2} \frac{G M_1 M_2}{d^2} (r_1 + r_2) = \frac{G M_1 M_2}{2d}$$

$m_1 v_1 r_1 = m_2 v_2 r_2$

$$P.E = -\frac{G M_1 M_2}{d}$$

$$T.E = -\frac{G m_1 m_2}{2d}$$

\+ Angular momentum:

$$L_{system} = L_1 + L_2 = I\omega$$

$$= M_1 V_1 r_1 = M_2 V_2 r_2$$

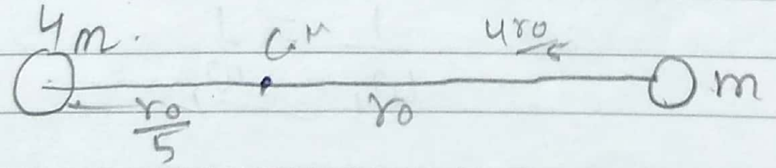
$$= I\omega = I_1\omega + I_2\omega$$

$$\boxed{= (m_1 r_1^2 + m_2 r_2^2)\omega}$$

$$\frac{L_{system}}{L_1} = \frac{L_1 + L_2}{L} = 1 + \frac{L_2}{L_1}$$

$$= 1 + \frac{M_2}{M_1}\left(\frac{r_2}{r_1}\right)^2 = 1 + \frac{M_1}{M_2}$$

Que:



Both are Revolving around centre of mass in circular orbit. Find time Period of Rotation.

$$\frac{G 4m\cancel{m}}{r_0^{\cancel{2}}} = \frac{\cancel{4}\cancel{m} v^2}{\frac{\cancel{r_0}}{5}}$$

$$\frac{G 4M}{5 r_0} = v^2$$

It is independent of position and orientation of tunnel

H.W: Q-1 Complete and Q-2

2π

$$T = \frac{2\pi r_0/5}{v}$$

$$T = \frac{2\pi r_0}{\sqrt{\frac{4GM}{5r_0}}} = T = \frac{2\pi}{\sqrt{5GM}} r_0^{3/2}$$

* Simple Pendulum :

1) $$T = 2\pi\sqrt{\frac{l}{g}} \qquad l <<<<< R_e$$

$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{l} + \frac{1}{R_e}\right)}}$$

2) $$\frac{1}{l} >>>> \frac{1}{R_e} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

3) $l >>>> R_e$

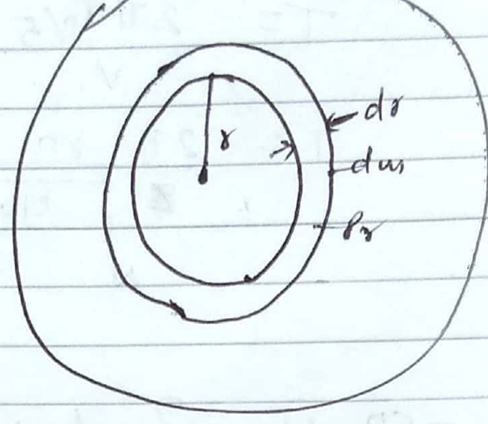
Simple pendulum of infinite length

$$\frac{1}{l} <<<< \frac{1}{R_e}$$

$$T = 2\pi\sqrt{\frac{R_e}{g}} = 2\pi\sqrt{\frac{R_e^3}{GM}} \qquad g = \frac{GM}{R_e^2}$$

core
metal
crust

* **Gravitational Pressure:**



dmg

So

$$\int 4\pi r^2 dr \frac{GM}{R^3} r = P_r 4\pi r^2$$

$$P_r = \frac{\rho G M}{R^3} \int_r^R r\,dr$$

$$\boxed{P_r = \frac{GM\rho}{2R^3}(R^2 - r^2)}$$

SBG STUDY