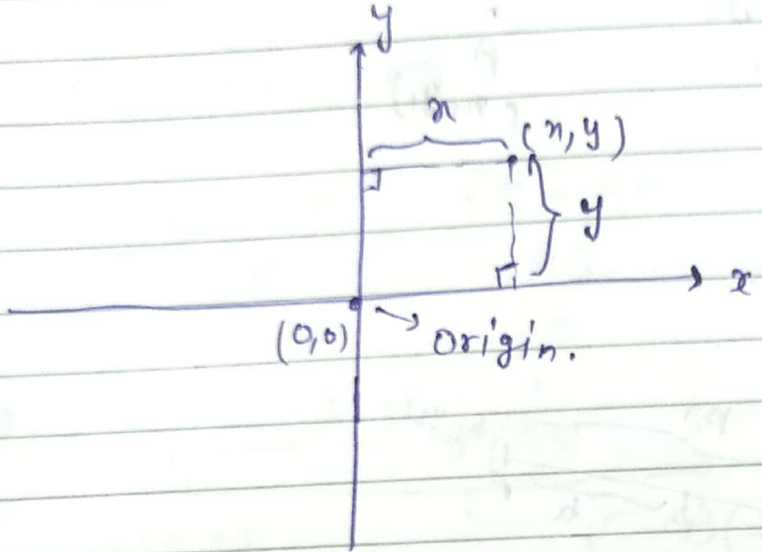
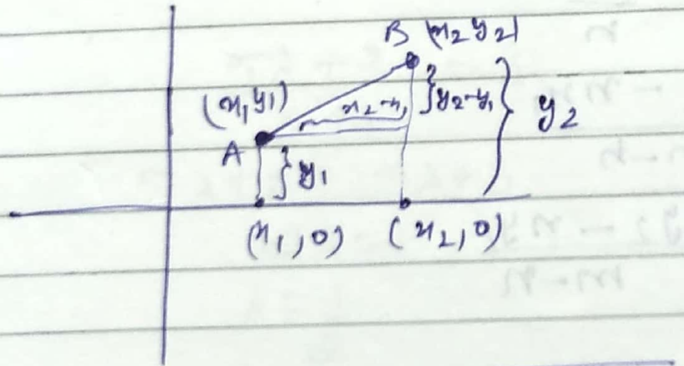
SBG STUDY

28/08/17

Straight line



$x \Rightarrow$ abscissa of the point $\Rightarrow$ distance of y axis
$y =$ ordinate of the point $\Rightarrow$ dist. of x axis

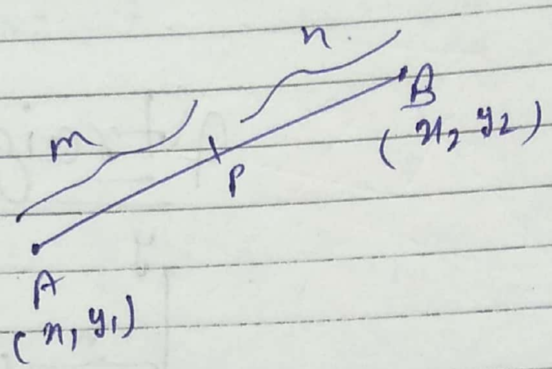


$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

at x - axis, at every point $y = 0$
at y - axis at every point $x = 0$

* Section formula:

(i) Internal division:

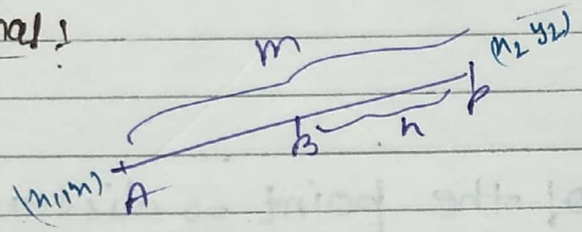


$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$\frac{AP}{PB} = \frac{m}{n}$$

(ii) External:

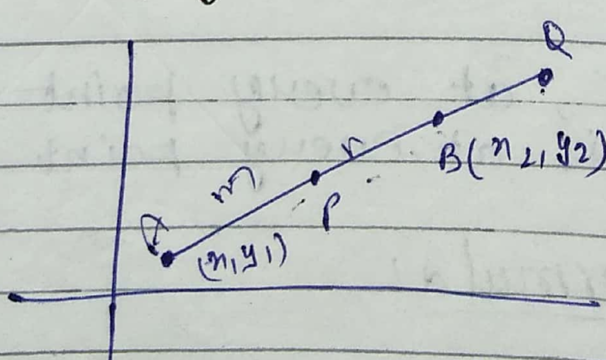


$$\frac{AP}{PB} = \frac{m}{n}$$

$$x = \frac{mx_2 - nx_1}{m-n}$$

$$y = \frac{my_2 - ny_1}{m-n}$$

* Harmonic Conjugate:



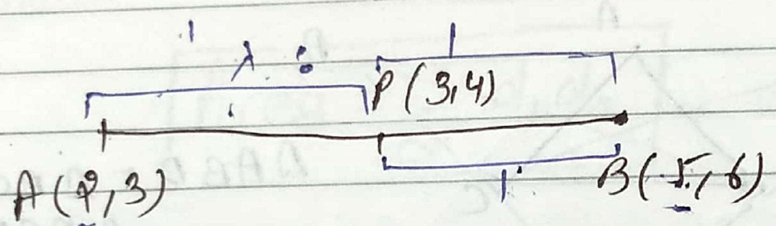
$$\frac{AP}{PB} = \frac{m}{n} \qquad \frac{AQ}{QB} = \frac{m}{n}$$

P divides internally and Q divides externally then P & Q are called Harmonic Conjugate of each other w.r.t A & B.

$\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$ ⇒ AP, AB, and AQ are in HP.

$$\frac{AB - AP}{AP} = \frac{AQ - AB}{AQ}$$

Que.



find Harmonic Conjugate of P.

$\frac{AB - AP}{AP}$ =

$$\frac{5\lambda + 2}{\lambda + 1} = 3$$

$$5\lambda + 2 = 3\lambda + 3$$

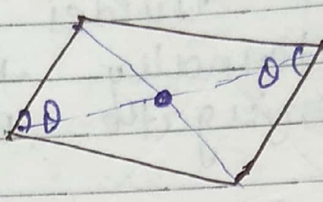
$$2\lambda = 1$$

$$\lambda = \frac{1}{2}$$

$$\left(\frac{1 \times 5 - 2 \times 2}{1 - 2}, \frac{6 - 6}{1 - 2} \right)$$

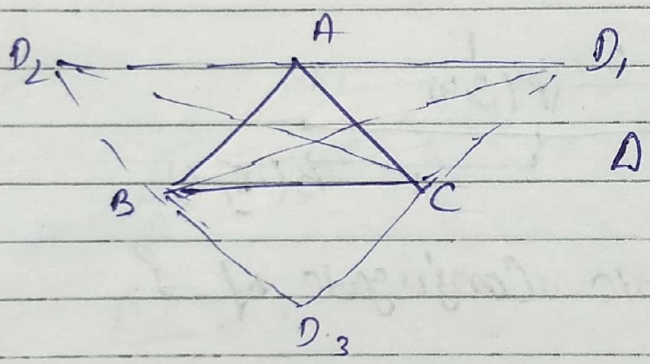
$$(-1, 0)$$

* Quadrilaterals:



→ **Parallelogram:**

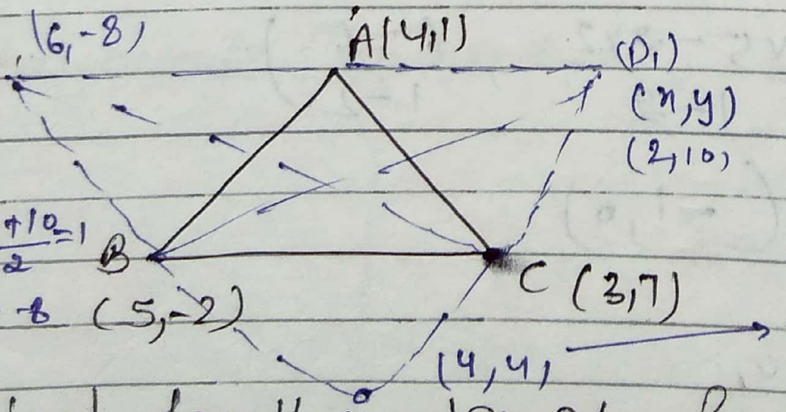
* opposite sides are parallel and equal
* opposite angle are equal
* diagonals are not equal
* diagonals bisect each other.



$\Delta ABD = \Delta BCD$
$= \Delta ACD$

$$\boxed{\begin{array}{l} \text{Area} = ab\sin\theta \\ \text{Area} = \dfrac{d_1 d_2}{\sin\theta} \end{array}}$$

Que.



$\frac{x+2}{2} = 4$ $\quad$ $\frac{y+10}{2} = 1$

$x = 6$ $\quad$ $y = -8$

$\frac{x+2}{2} = 3$

$x = 4$

$\frac{y+10}{2} = 7$

$y = 4$

x, y

2, $\frac{1}{2}$

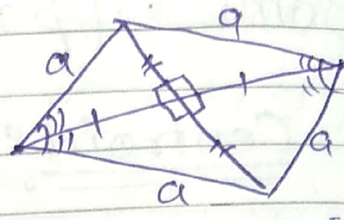
find fourth vertex of Parallogram.

$$\left(\frac{x+5}{2}, \frac{y-2}{2}\right) = \left(\frac{4+3}{2}, \frac{7+1}{2}\right)$$

$$x = 2, \quad y = 10$$

Rectangle = Parallelogram
square = Rhombus.

(2) Rhombus:



$\left[4 \times \frac{1}{2} \times \frac{d_1}{2} \times \frac{d_2}{2}\right]$

* All sides are equal
* opposite sides are parallel.
* opposite angles are equal
* diagonal are not equal
* ~~diagonal bisect each~~ other.
* diagonals are Perpendicular - Bisector of each other.
* Diagonals are angle Bisector of sides of rhombus.

$$\boxed{\text{Area} = \frac{1}{2} d_1 d_2}$$

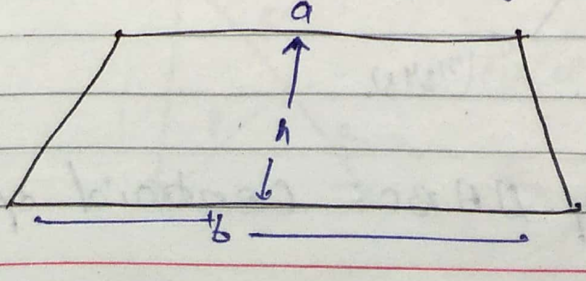
$\theta = 90$ diagonals ~~are~~

* Rectangle:
* $\theta = 90°$
* Diagonals are equal

* Square:
* $\theta = 90°$
* Diagonals are equal

* Trapezium:

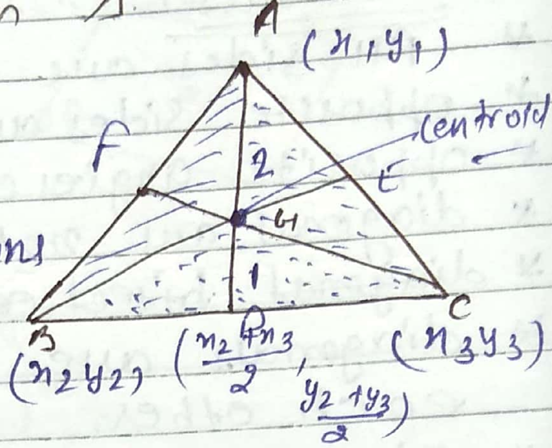
* Exactly one Parallel of sides are Parallel.



$A = \frac{1}{2}(a+b) \times h.$

* Some Spacial point in Δ

(1) Centroid:



* Point of intersection of medians from vertex.
* centroid devides median into 2:1

$$\frac{AG}{GD} = \frac{2}{1}$$

(2) ~~Median~~: * median divides Δ into two
* ~~Centroid~~ equal parts

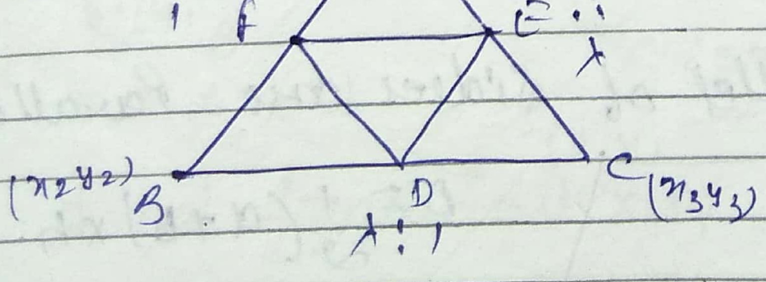
$\Delta ABD = \Delta ABC$

* centroid divide Δ into three equal parts

$\Delta AGB = \Delta AGB = \Delta BGC$

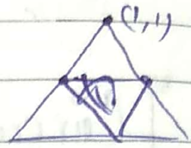
$$\boxed{G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)}$$

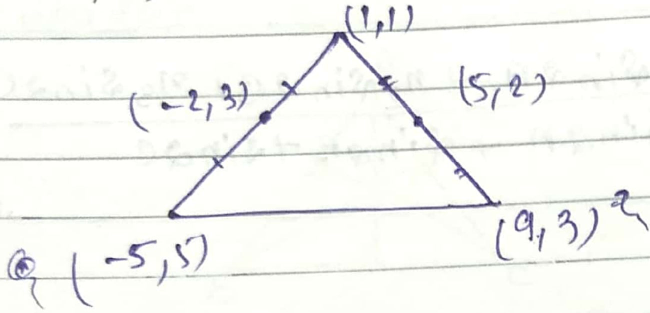
$$\boxed{\Delta ABC = \frac{4}{3}(\text{area formed by medians})}$$



centroid of ΔABC = centroid of ΔDEF

Q. If a vertex of Δ be $(1,1)$ and middle point of two sides throw it $(-2,3)$ and $(5,2)$ the find Centroid.





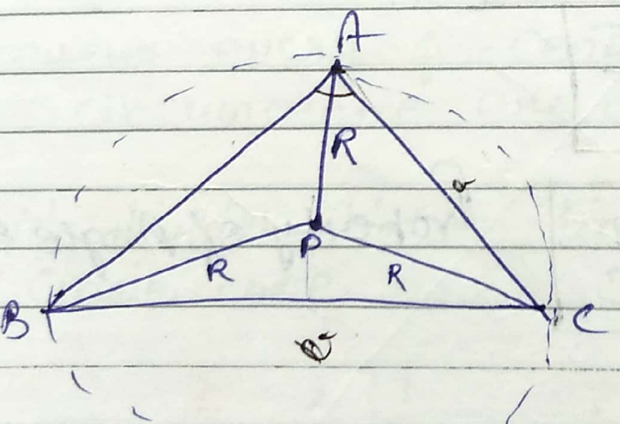
$\frac{x+1}{2} = -2$, $x+1 = -4$
$x = -4-1 = -5$

$\frac{y+1}{2} = 3 = 5$

$\Rightarrow \left(\frac{5}{3}, 3\right)$

Circumcentre :

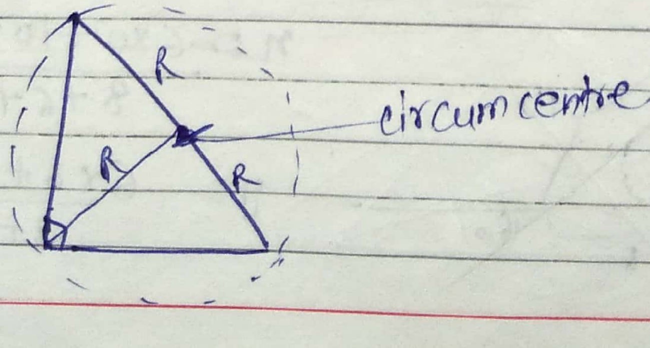
Point of Intersection of Perpendicular Bisector of sides of Δ.



$AP = PB = PC$.

$$\boxed{R = \frac{abc}{4\Delta}}$$

$$= \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

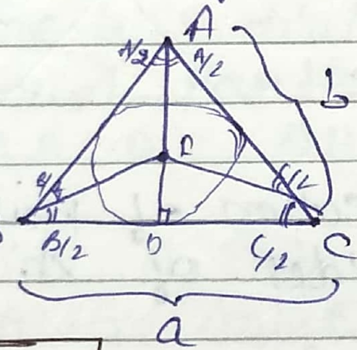


In Right angle Δ, circumcentre is the middle point of hypotenous.

$$\boxed{\text{Circum Centre} :- \frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}}$$

* In centre :

Point of intersection of angle Bisectors of triangle



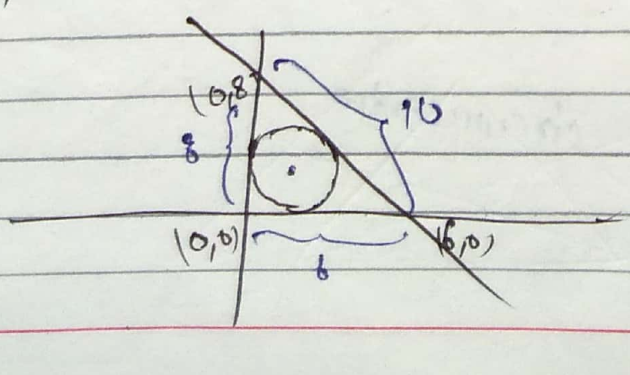
$$\boxed{r = \frac{\Delta}{s}}$$

$$\boxed{\frac{AI}{ID} = \frac{b+c}{a}}$$

Imp $\boxed{\frac{BD}{DC} = \frac{AB}{AC}}$ Property of Angle Bisector

$$\boxed{\text{In centre} = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)}$$

Q:

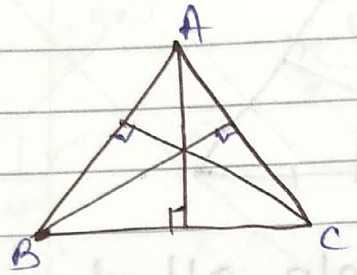


$$x = \frac{6\times0 + 10\times0 + 8\times6}{8+6+10} = 2$$

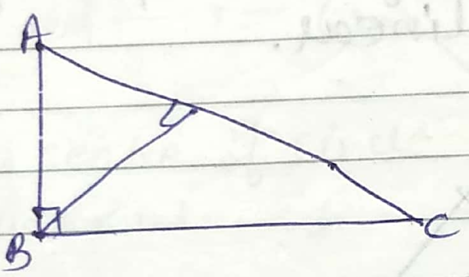
$$y = \frac{6\times8 + 10\times0 + 0\times8}{8+6+10} = 2$$

* Orthocentre :

Point of Intersection of Altitudes from vertex.



$$\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}$$



In right angle Δ, Orthocentre is the vertex where right angle formed

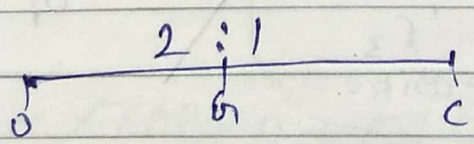
orthocentre of $\Delta BOC \rightarrow A$

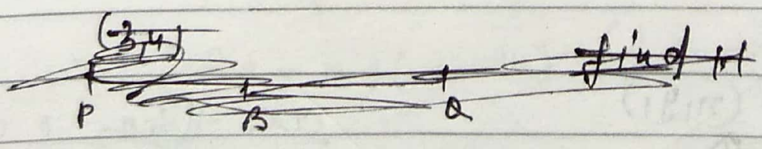
$\Delta COA \rightarrow B$

$\Delta AOB \rightarrow C$.

* for ~~every~~ every Δ centroid, orthocentre and circumcentre are always collinear.

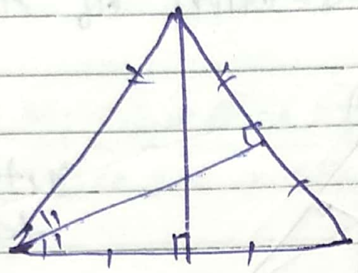
* Centroid divides divides line joining Orthocentre and Circumcentre



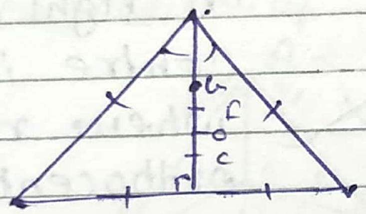
Q. 

30/08/17

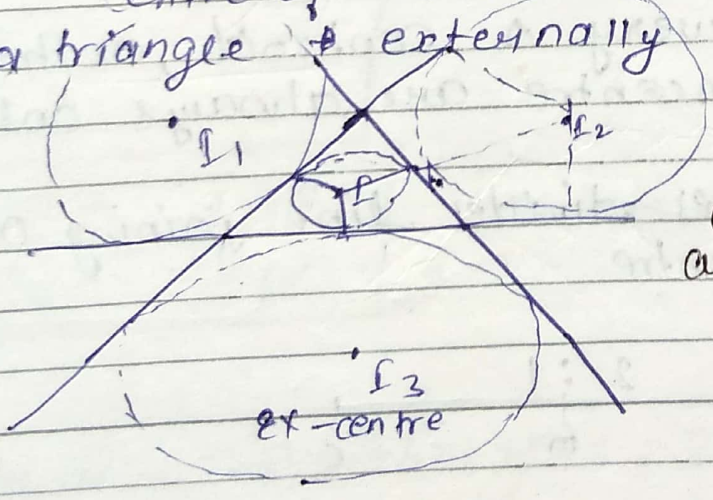
* In equilateral Δ
all four points (G, C, O, I) concide with each other



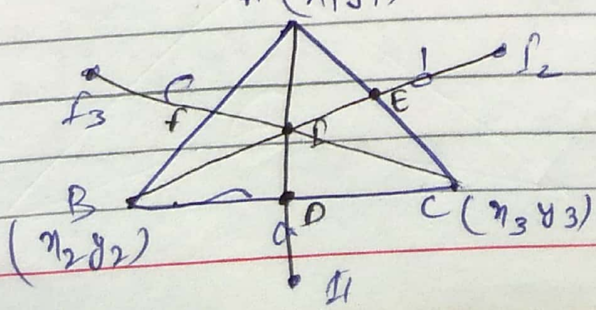
* In Isoceles triangle all four point
G, C, O, I are collinear.



* **Ex-centre:-**
centre of circle touching all the sides of a triangle externally



(I, I_1) (I, I_2) (I, I_3)
are Harmonic conjugate of each other

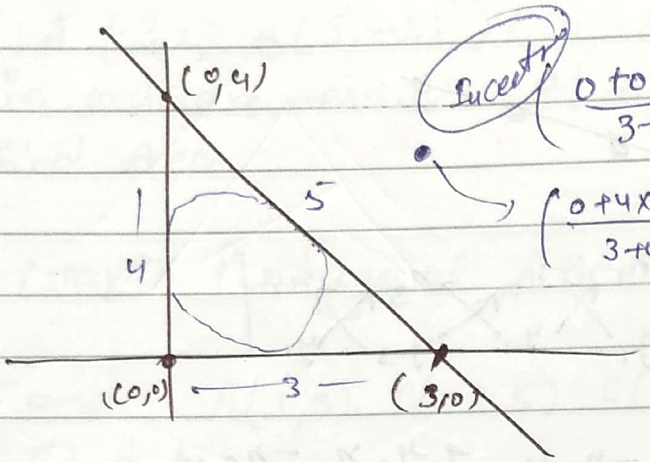


$= \frac{ax_1 + bx_2 + cx_3}{a+b}$

* — $\frac{1}{2}$(Sum of 11 side) + C

$$\boxed{= \left(\frac{a x_1 + b x_2 + c x_3}{a+b+c}, \frac{a y_1 + b y_2 + c y_3}{a+b+c} \right)}$$ Learn

Q.

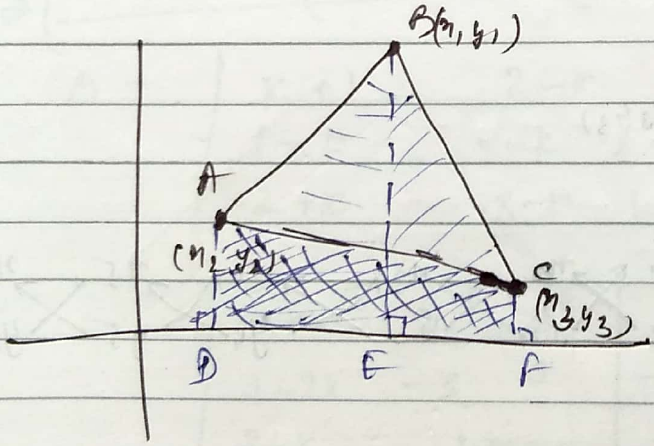


Incentre $\left(\frac{0+0+12}{3+4+5}, \frac{12+0+0}{3+4+5} \right) = (1,1)$

$\left(\frac{0+4\times3-5\times0}{3+4-5}, \frac{3\times4+0\times4-5\times0}{3+4-5} \right) = (6,6)$

Find the centre of circle touching the sides of Δ in 1 Quadrant

* Area of Triangle :



$\Delta ABC = \square ABED + \square BCFE - \square ACFD$

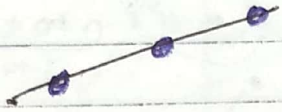
$= \frac{1}{2}(x_1 - x_2)(y_2 - y_1) + \left(\frac{1}{2}(x_3 - x_1)(y_1 + y_3) - \frac{1}{2}(x_3 - x_2)(y_2 - y_3)\right)$

$\Delta ABC = \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

* Area is a non negative Quantity

* if three point are collinear

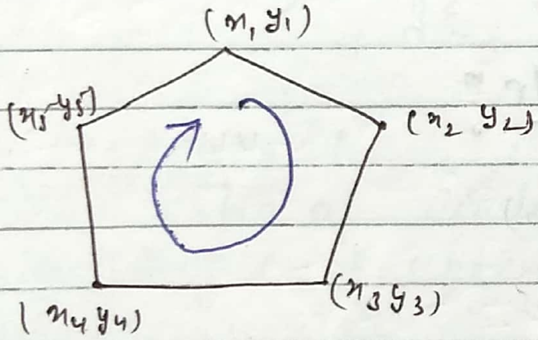
$\boxed{\Delta = 0}$



$$= \Delta ABC = \frac{1}{2}\left| \begin{matrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{matrix} \right|$$

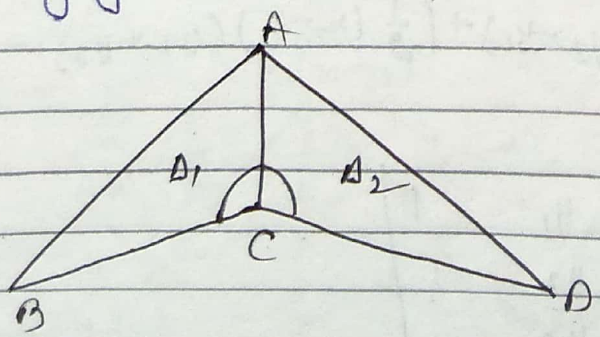
$$= \frac{1}{2}\left| x_1 y_2 - x_2 y_1 + y_3 x_2 - x_3 y_2 + y_1 x_3 - x_1 y_3 \right|$$

*



$$= \frac{1}{2}\left| \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_5 & y_1 \end{matrix} \right|$$

= formula is valid only for convex Polygon



$\Delta = \Delta_1 + \Delta_2$

Q. if points $(k+1, 2-k)$, $(1-k, -k)$ and $(2+k, 3-k)$ are collinear then find. k.

Q. if $A(3,2)$ $B(5,-2)$, $C(1,-3)$ and $D(-4,-1)$ are in in order are vertices of a Quadrilateral ABCD find Area.

Q. A convex Pentagonal ABCDE Area equal to $\frac{45}{2}$ and $A(1,3)$ $B(-2,5)$ $C(-3,-1)$, $D(0,-2)$ $E(2,t)$ Find t

Ans: $= \frac{1}{2}\begin{vmatrix} k+1 & 1-k & 2+k \\ 2-k & -k & 3-k \end{vmatrix} = 0$

$\frac{1}{2}\left(\cancel{-k(k+1) - ((1-k)(2-k))} + \right.$

$$\Delta = \begin{vmatrix} k+1 & 2-k & 1 \\ 1-k & -k & 1 \\ 2+k & 3-k & 1 \end{vmatrix} = 0 \qquad \begin{matrix} C_1 \to C_1 - C_2 \\ C_2 \to C_2 - C_3 \end{matrix}$$

$$\begin{vmatrix} 2k & 2 & 0 \\ -1-2k & -3 & 0 \\ 2+k & 3-k & 1 \end{vmatrix} = 0$$

$-3(2k) - 2(-1-2k) = 0$

$-6k + 2 + 4k = 0$

$2k = 2 \qquad k = 1$

Q2
Sol:

$$\Delta = \frac{1}{2}\begin{vmatrix} 3.5 & 1 & -4 & 3 \\ 2-2 & -3 & -1 & 2 \end{vmatrix}$$

$$\Delta = \frac{1}{2}\left|-6-10-15+2-1-12-8+3\right|$$

$$\Delta = \frac{47}{2}$$

(3)

$$= \frac{1}{2}\begin{vmatrix} 1 & -2 & -3 & 0 & 2 & 1 \\ 3 & 5 & -1 & -2 & t & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left|5+6+2+15+6+0+0+4+6-t\right| = \frac{45}{2}$$

$$= \left|\frac{44-t}{\cancel{2}}\right| = \frac{45}{\cancel{2}}$$

$$|44-t| = 45$$

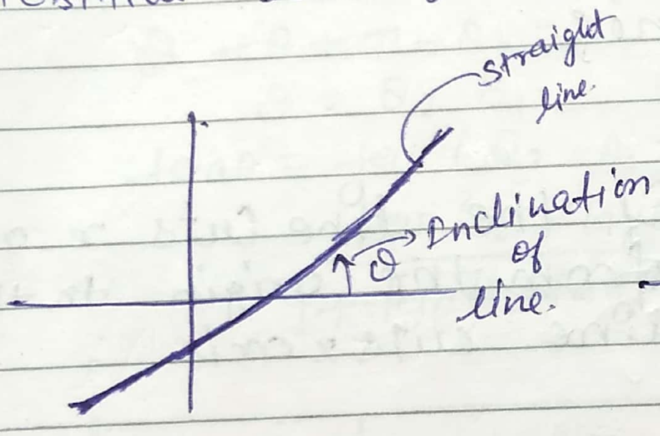
$$44-t = \pm 45$$

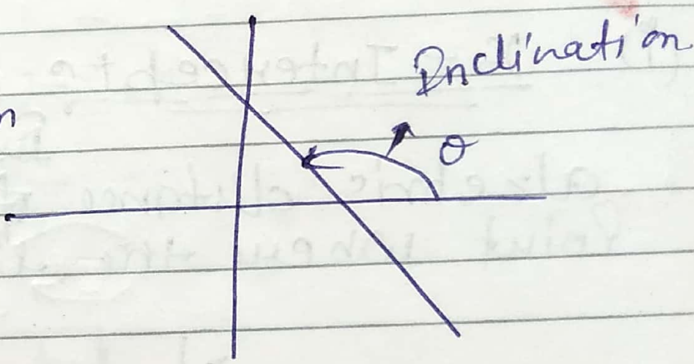
$$t = -1$$

$$t = 89$$

Straight line

* **Inclination of a line :** if a straight line intersect x-axis then the Inclination of the line is defined as the ~~major of the~~ measure of the smallest non negative angle which the line max with the positive dir^n of x-axis

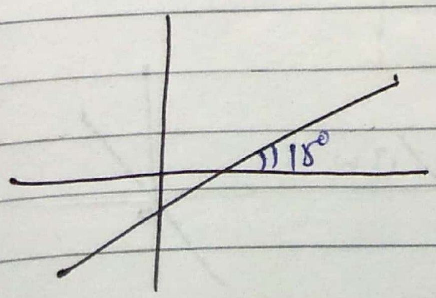




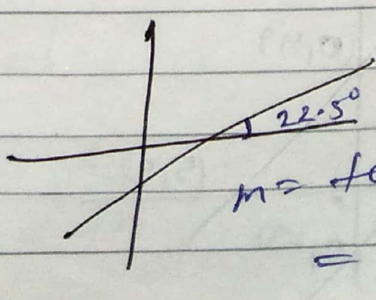
* **Slope or gradient of a line :**
If the inclination of the line is θ and $\theta \neq \frac{\pi}{2}$ then the slope ~~of~~ or greadent of the line is defined as $\boxed{m = \tan\theta}$ $\theta \neq \frac{\pi}{2}$

slope/gradient.

* At $\theta = \frac{\pi}{2}$ m is not defined



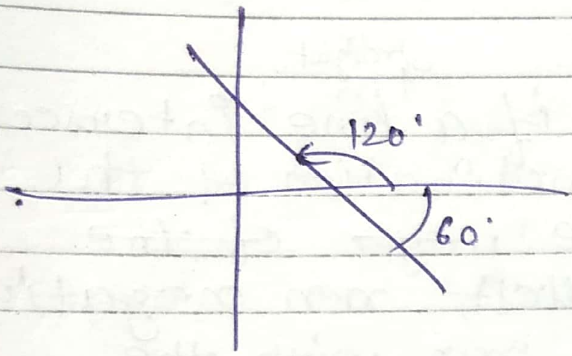
$m = \tan 15°$
$= 2 - \sqrt{3}$



$m = \tan 22.5°$
$= \sqrt{2} - 1$

$\tan 75 = 2+\sqrt{3}$

$\tan 67.5^\circ = \sqrt{2}+1$



$\theta = 120^\circ$

$m = \tan 120^\circ = -\sqrt{3}$

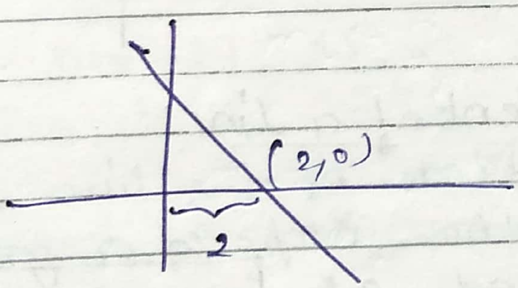
$\theta = -60^\circ$

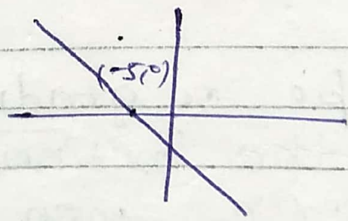
$m = \tan(-60)$

$m = -\sqrt{3}$.

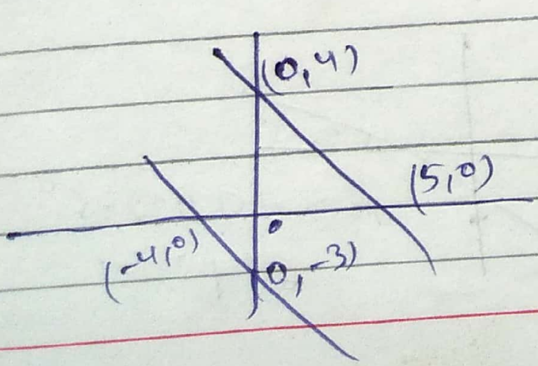
* Intercept of a line :-

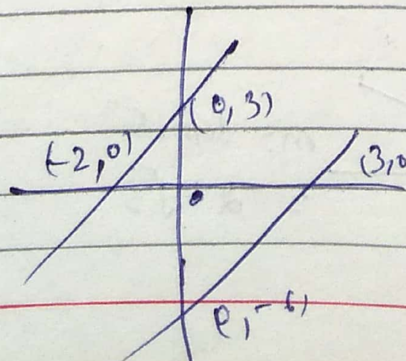
(i) **x - Intercept :-** If the line cuts x axis alzebric distance from the origin to the Point where the line cuts x axis

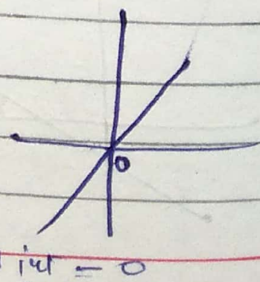




(ii) **Y - Intercept :-** If the line cuts y axis alzebric dist from the origin to the Point where the line Cut y axis



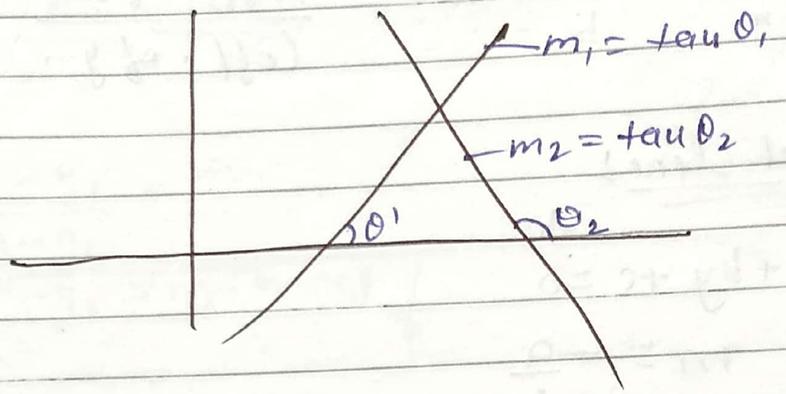




x int = 0

y int = 0.

* Angle b/w two lines :



$$\theta + \theta + \pi - \theta_2 = \pi$$

$$\theta = \theta_2 \; \theta_1$$

$$\tan\theta = \tan(\theta_2 - \theta_1)$$

$$\frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_1 \tan\theta_2}$$

$$\boxed{\tan\theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|}$$

* General eq^n of line :-

Every single Degree equation in x and y. Represent a straight line. i.e it is called a linear eq^n.

$$\boxed{ax + by + c = 0}$$

$$m = \tan\theta = \frac{dy}{dx}$$

$$a + b\frac{dy}{dx} + 0 = 0$$

$$m = \frac{dy}{dx} = \frac{-a}{b} = -\frac{\text{Coff. of } x}{\text{Coff. of } y}$$

* eqⁿ of Parallel line:

$$ax + by + c = 0$$

$$m = \frac{-a}{b}$$

$$\Rightarrow \boxed{ax + by + d = 0}$$

* eqⁿ of Perpendicular line:

$$ax + by + c = 0$$

$$m = -\frac{a}{b}$$

$$m_1 m_2 = -1$$

$$-\frac{a}{b} \times m_2 = -1$$

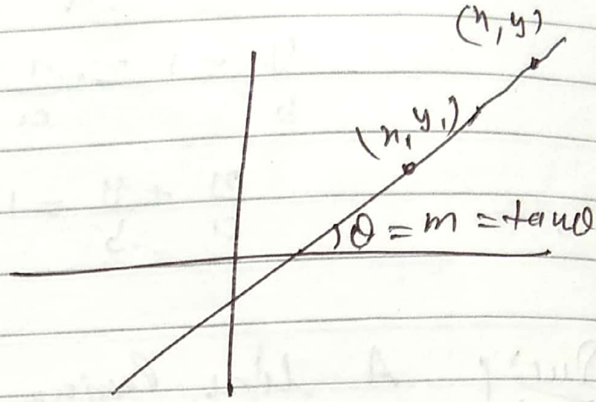
$$m_2 = \frac{b}{a}$$

$$\boxed{bx - ay + \lambda = 0}$$

* Various form of a Straight:

* Various form of a straight line :

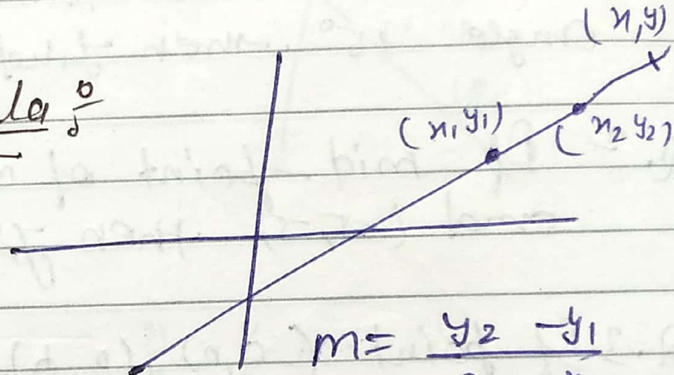
(1) Point - slope form :



$$\frac{y-y_1}{x-x_1} = m$$

$$\boxed{y - y_1 = m(x - x_1)}$$

* (2) two point formula :

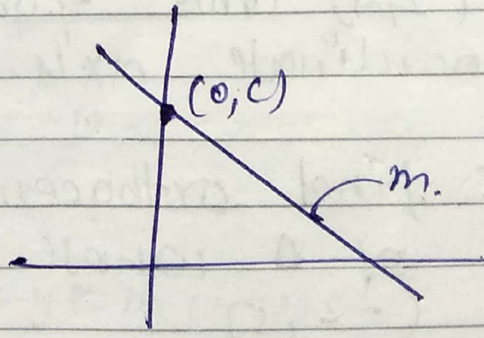


$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

* Slope - Intercept form :

~~Soth~~ slope and y - Intercept are given :-

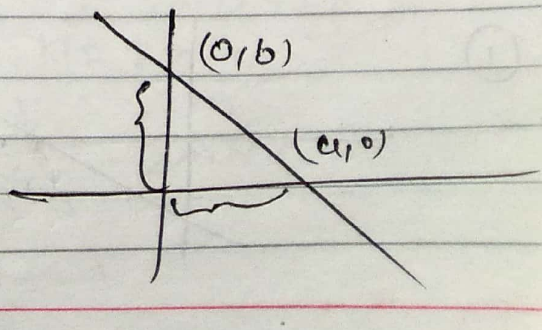


$$y - c = m(x - 0)$$

$$y - c = mx$$

$$* \boxed{y = mx + c}$$

* Double Intercept form :



$$y - b = \frac{b - 0}{0 - a}(x - 0)$$

$$y - b = \frac{-b}{a}(x)$$

$$\frac{y}{b} - 1 = \frac{-x}{a}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

Que:1 A line Passing through (1,0) and (2,1) is rotated about Point (1,0) anticlock wise by angle 15°. then find its new eqⁿ.

Q.2 If mid-point of sides of Δ are (2,1) (-5,7) and (-5,-5) then find eqⁿ of sides

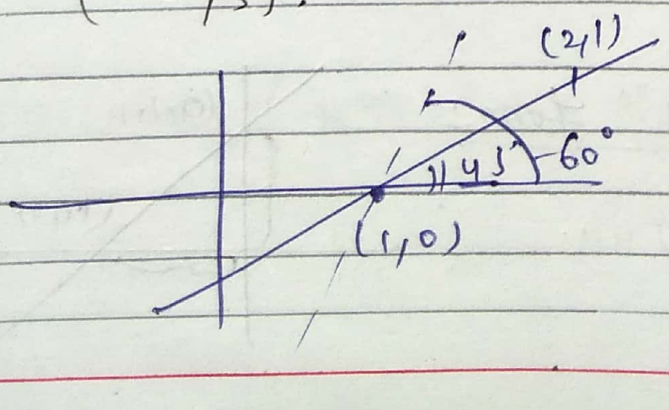
Q.3 If point (a,0), (0,b), (1,1) are collinear. find value of $\frac{1}{a}+\frac{1}{b}$

Q.4 find eqⁿ of line Passing through (4,-3) and sum of Intercept is 5.

Q.5 find no. of st. lines Passing through (2,4) and forming a Δ of Area 16 with coordinate axis

Q.6 find orthocentre and Circum centre of Δ whose vertices are (0,1) (2,3) and (-2,5).

(1)

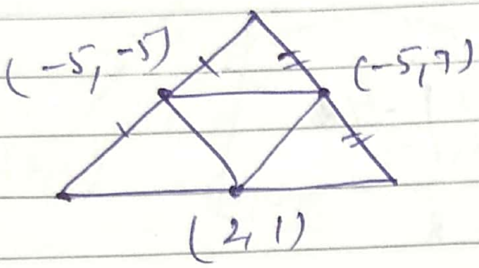


$$m = \frac{1-0}{2-1} = 1$$

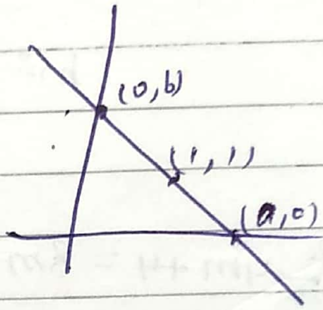
$$\tan\theta = 1$$

$$\theta = 45°$$

$y - 0 = \sqrt{3}\,(x-1)$ Ans

② 

③ $(a,0)\ (0,b)\ (1,1)$



$\frac{1}{a} + \frac{1}{b}$

$\frac{x}{a} + \frac{y}{b} = 1$

$(4, -3)$

$a + b = 5$

$b = 5 - a$

$\frac{4}{a} - \frac{3}{b} = 1$

$4b - 3a = ab$

$4(5-a) - 3a = a(5-a)$

$20 - 4a - 3a = 5a - a^2$

$a^2 - 12a + 20 = 0$

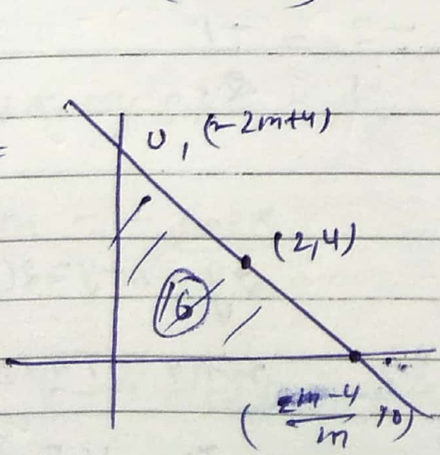
$(a-10)(a-2)$ $a = 2, 10$

$b = 3, -5$

$\frac{x}{2} + \frac{y}{3} = 1$

$\frac{x}{10} + \frac{y}{-5} = 1$

④ =



$y - 4 = m(x-2)$

$mx - y - 2m + 4 = 0$

$x = \frac{2m-4}{m}$

$y = -2m + 4$

$$\Delta = \left| \frac{1}{2}\left(\frac{2m-4}{m}\right)(-2m+4) \right| = 16$$

$$\left| \frac{2(m-2)(m-2)}{m} \right| = 16$$

$$\frac{(m-2)^2}{|m|} = 8$$

$$m^2+4-4m = 8|m|$$

$m^2-4m+4 = 8m$

$m^2-12m+4=0$ $\begin{cases} m_1 \\ m_2 \end{cases}$

$D>0$

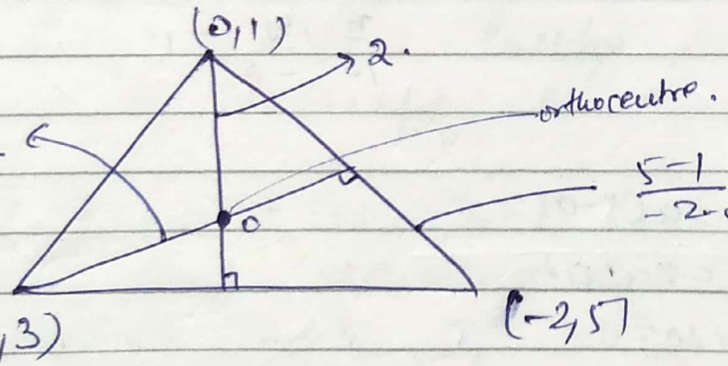
$m^2+4-4m = -8m$

$m^2+4m+4=0$

$(m+2)^2=0$

$m=-2$

(6)



$$\frac{5-1}{-2-0} = -2$$

orthocentre $\frac{5-3}{-2-2} = \frac{-2}{4} = \frac{-1}{2}$

$m_1 m_2 = -1$

$m_2 = \frac{-1}{m}$

$y-1 = 2(x-0)$

$y = 2x+1$

$y-3 = \frac{1}{2}(x-2)$

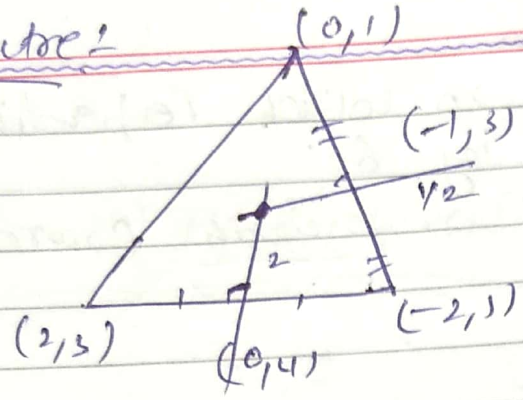
$2y-6 = x-2$

$2y = x+4 = 2(2x+1)$

$x+4 = 4x+2$

$x = 2/3 \quad y = 7/3$

Circumcentre:



Right ∠ = hypo mid pt

Scalar ⇒ scalen

$y-3 = \frac{1}{2}(x+1)$

$y-4 = 2(x-0)$

$2y-6 = x+1$

$2y = x+7$

$y = 2x+4$

$2(2x+4) = x+7$

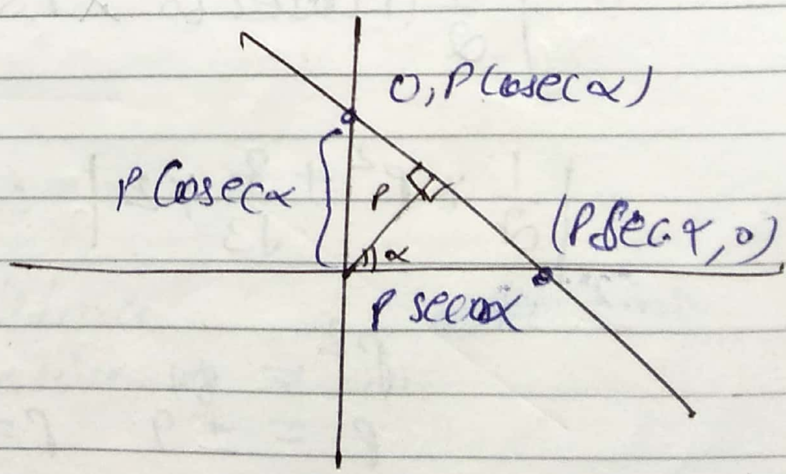
$4x+8 = x+7$

$x = -\frac{1}{3}$

$y = \frac{10}{3}$

Normal form:

$\frac{x}{p\sec\alpha} + \frac{y}{p\,\mathrm{cosec}\,\alpha} = 1$



$x\cos\alpha + y\sin\alpha = p$

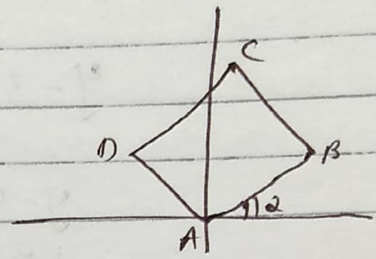
$m = -\frac{\cos\alpha}{\sin\alpha} = -\cot\alpha = \tan\left(\frac{\pi}{2}+\alpha\right)$

$p \rightarrow$ distance ≥ 0

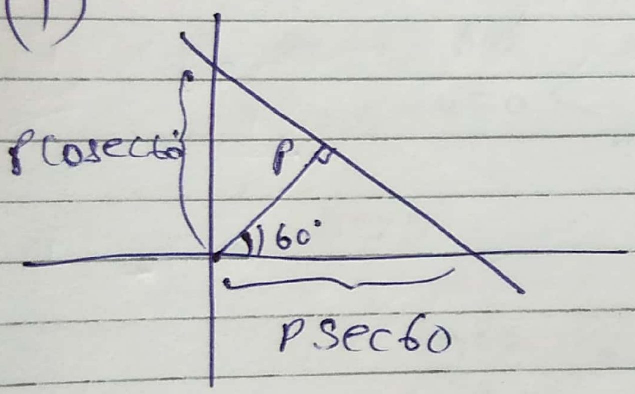
$\alpha \in [0, 2\pi)$

Q.1 find eq^n of a line on which Perpendicular from Origin is inclined by $60°$ and line form a Δ with coordinate axis, of Area $54\sqrt{3}$

Q.2 ~~find~~
ABCD is a sq. of side A
find eq^n of BD.



(1)



$$= \left| \frac{1}{2}(P\,cosec 60° \times P \sec 60°) \right| = 54\sqrt{3}$$

$$\left| \frac{1}{2} \times P^2 + \frac{2}{\sqrt{3}} \times 2 \right| = 54\sqrt{3}$$

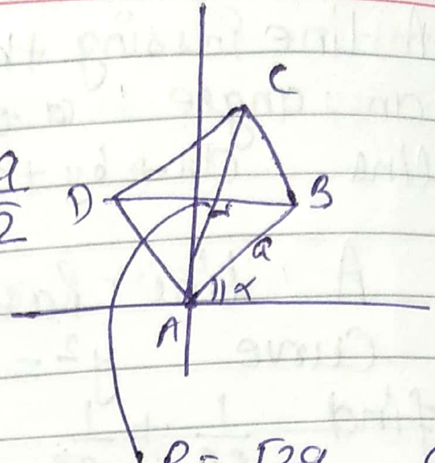
$$P^2 = 81$$
$$P = \pm 9 \quad P = 9.$$

$$x \cos 60° + y \sin 60° = 9$$

$$x + \sqrt{3}y = 18.$$

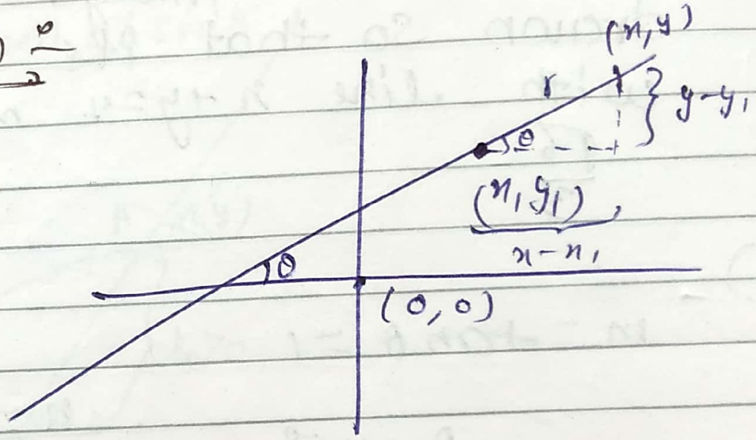
(2)

$x\cos(\alpha+45)+y(\sin(\alpha+45))=\frac{a}{\sqrt{2}}$



$P=\frac{\sqrt{2}a}{2}=\frac{a}{\sqrt{2}}$

* Parametric form :-



$\sin\theta=\frac{y-y_1}{r}$

$\cos\theta=\frac{x-x_1}{r}$

$$\boxed{r=\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}}$$

Parameter / variable.

r ⟶ Algebric distance

r can be Positive, negative or zero

Que find the point on the line $x-y+4=0$
which is at a dist. of $4\sqrt{2}$ from (5, 9).

Ans

(2) A line passing through $A(x, y_1)$ and making an angle θ with x axis intersect a line $ax + by + c = 0$ at P find AP.

(3) A line passing through $P(1,0)$ intersect curve $y^2 = 4x$ at A & B the find $\frac{1}{PA} + \frac{1}{PB}$

(4) In what dir^n through the point $(1,2)$ must be drawn so that its intersection point P with line $x + y = 4$ may be at a dist of $\frac{\sqrt{6}}{3}$

(1)

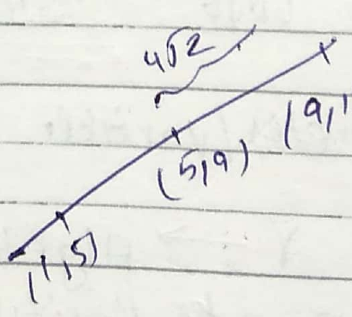
$m = \tan\theta = 1$

$\theta = 45^\circ$ $\qquad \frac{x-5}{\cos 45^\circ} = \frac{y-9}{\sin 45} = \pm 4\sqrt{2}$

$\cancel{xy} - 5 = y - 9 = \pm 4$

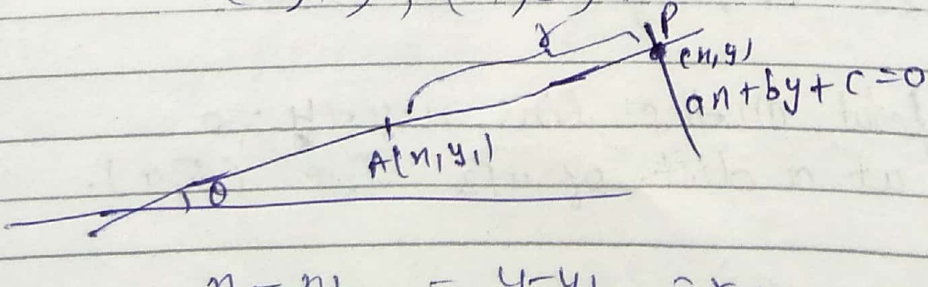
$x = 9 \qquad y = 13$

$x = 1 \qquad y = 5$



$(9, 13), (1, 5)$.

(2)



$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r.$

$x = r\cos\theta + x_1$

$y = r\sin\theta + y_1$

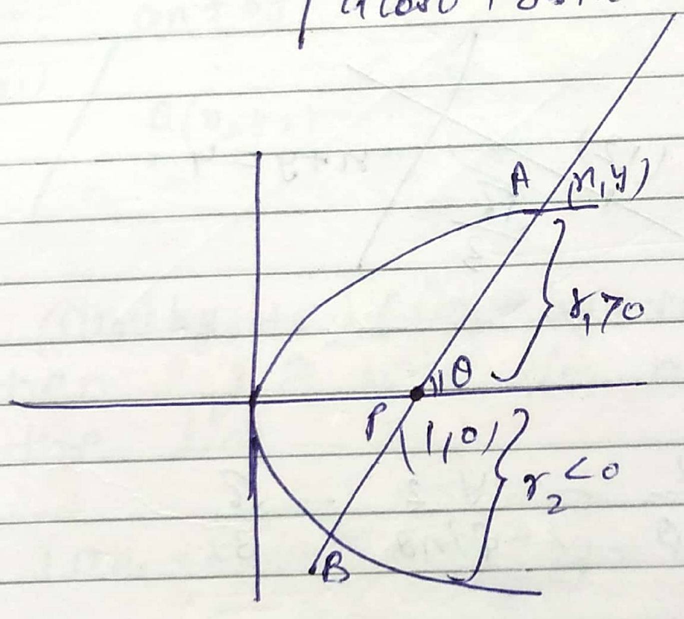
$ax + by + c = 0$

$= a(r\cos\theta + x_1) + b(r\sin\theta + y_1) + c = 0$

$$r = \frac{-ax_1 - by_1 - c}{a\cos\theta + b\sin\theta}$$

$$AP = |r| = \left| \frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta} \right|$$

(3)



$$\frac{x-1}{\cos\theta} = \frac{y-0}{\sin\theta} = r$$

$x = r\cos\theta + 1$

$y = r\sin\theta$

$r^2 \sin^2 = 4r\cos\theta + 4$

$r^2\sin^2\theta - 4r\cos\theta - 4 = 0$ r_1, r_2

$\boxed{\begin{matrix}\text{Diff. of roots}\end{matrix} \; |\alpha - \beta| = \frac{\sqrt{D}}{|a|}}$

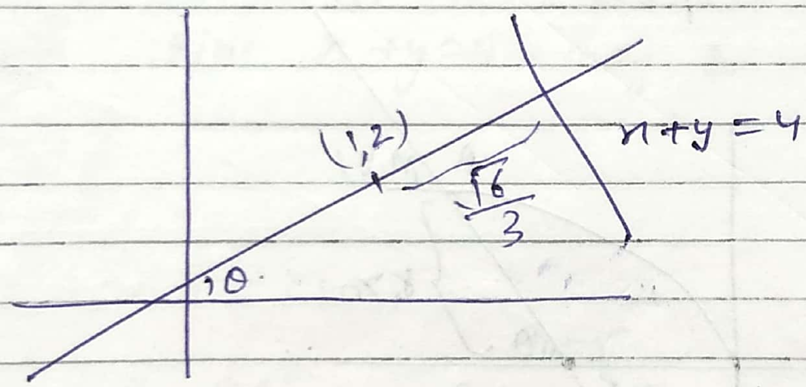
$$\frac{1}{PA} + \frac{1}{PB}$$

$$\frac{1}{|r_1|} + \frac{1}{|r_2|} = \left|\frac{1}{r_1} - \frac{1}{r_2}\right| = \left|\frac{r_2 - r_1}{r_1 r_2}\right| = 1$$

$$r_1 r_2 = \frac{-4}{S^2}$$

$$|r_1 - r_2| = \frac{\sqrt{D}}{|a|} = \frac{\sqrt{16C^2 + 16S^2}}{S^2} = \frac{4}{S^2}$$

(4)



$$\frac{x-1}{\cos\theta} = \frac{y-2}{\sin\theta} = \frac{\sqrt{6}}{3}$$

$$x = \frac{\sqrt{6}}{3}\cos\theta + 1$$

$$y = \frac{\sqrt{6}}{3}\sin\theta + 2$$

$$\frac{\sqrt{6}}{3}\cos\theta + 1 + \frac{\sqrt{6}}{3}\sin\theta + 2 = 4$$

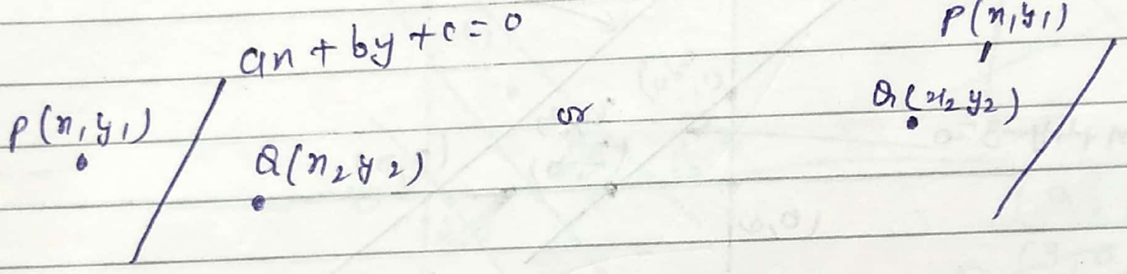
$$\cos\theta + \sin\theta = \frac{3}{\sqrt{6}}$$

$$\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{3}{\sqrt{2}\times\sqrt{6}} = \frac{\sqrt{3}}{2}$$

$$\sin(\theta+45) = \sin 60 \text{ or } \sin 120^\circ$$

$$\theta + 45 = 60 \text{ or } 120$$
$$\theta = 15 \text{ or } 75$$

* Position of a point w.r.t a line:



* If $(ax_1+by_1+c)(ax_2+by_2+c) = (+)ve$ then P & Q will lie at same side of the line.

* If $(ax_1+by_1+c)(ax_2+by_2+c) = (-)ve$ the P and Q will lie at opposite side of line.

$$\lambda = -\frac{(ax_1+by_1+c)}{ax_2+by_2+c} > 0$$

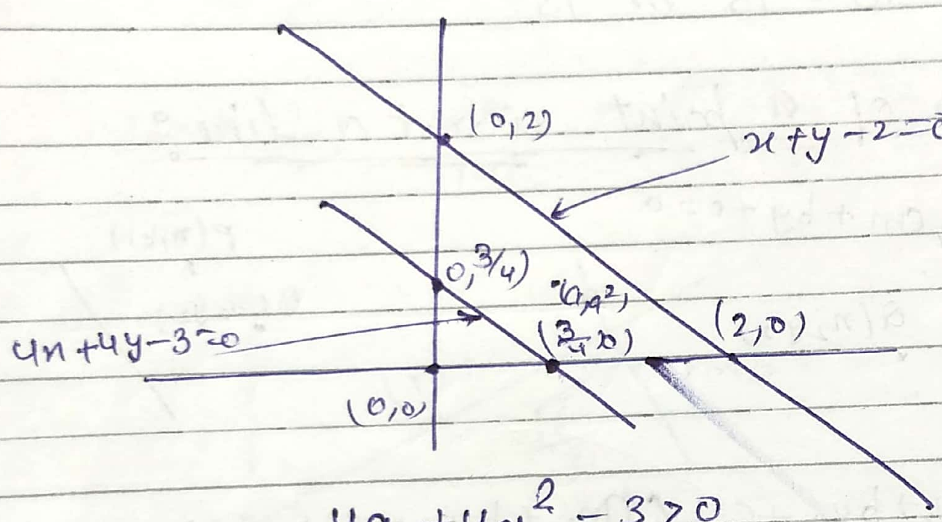
Ex: (15,1) $3x-4y-5=0$ $P = (1,2), (-1,2), (3,4), (5,6), (15,1)$

$\ominus$ $\ominus$ $\ominus$ $\ominus$ $\oplus$

Put value P(1,2) in eqⁿ of line

Que: If (a, a^2) lie inside the region b/w the line $x+y-2=0$ and $4x+4y-3=0$ then find range of a.

$x+y-2=0 \quad = \quad a+a^2=2 \Rightarrow a^2+a-2=0$



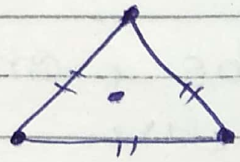
$4a+4a^2-3>0$

$a+a^2-2<0$

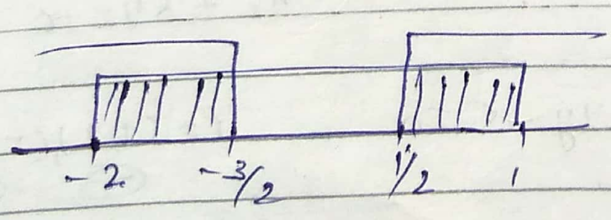
$(a+2)(a-1) < 0$

$a \in (-2, 1)$

$(2\alpha+3)(2\alpha-1)>0$



$\alpha \in (-\infty, \frac{3}{2}) \cup (\frac{1}{2}, \infty)$



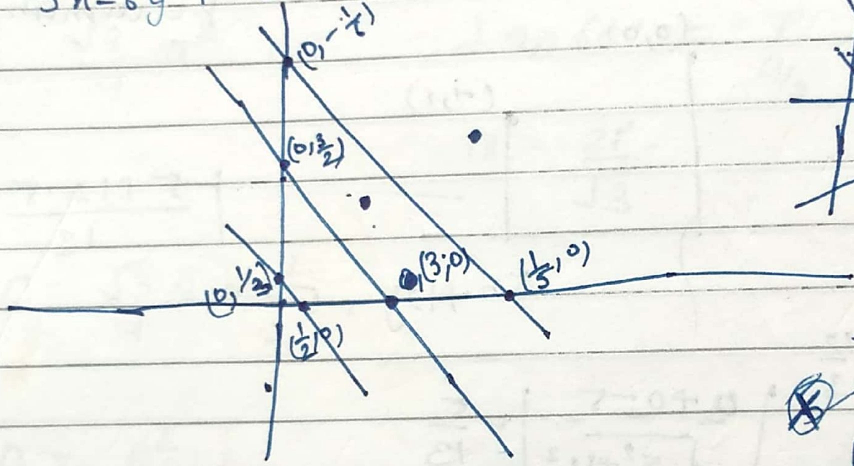
$a \in (-2, \frac{-3}{2}) \cup (\frac{1}{2}, 1)$.

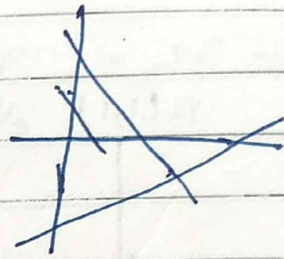
Que: If the point (α, α^2) lies inside the Δ formed by lines $2x+3y-1=0$, $x+2y-3=0$ and $5x-6y-1=0$, then find range of α.

Ans: $\alpha \in \left(-\frac{3}{2}, -1\right) \cup \left(-\frac{1}{2}, 1\right)$.

1. $2x+3y-1=0$, $x+2y-3=0$

$5x-6y-1=0$





$\left(0, \frac{1}{3}\right)$ $\left(\frac{1}{2}, 0\right)$

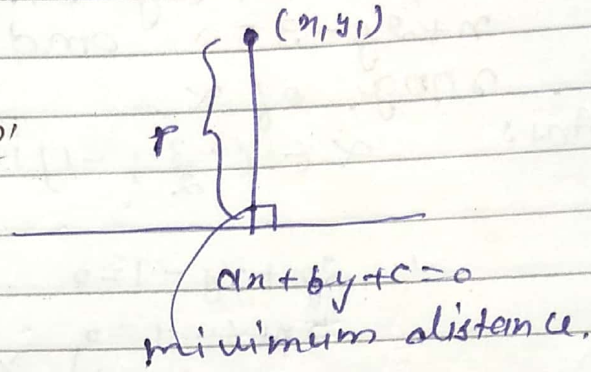
$(3, 0)$ $\left(0, \frac{3}{2}\right)$

$\left(\frac{1}{5}, 0\right)$ $\left(0, -\frac{1}{6}\right)$

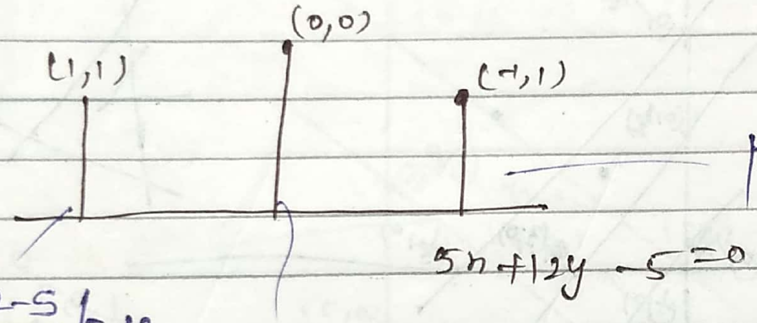
from

* Perpendicular dist. of a point from a line.

$$\boxed{P = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|}$$ — Imp.



Ex:

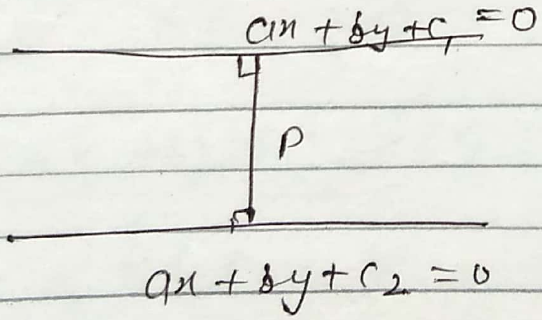


$\left| \frac{-5 + 12 - 5}{13} \right| = \frac{21}{13}$

$\left| \frac{5 + 12 - 5}{13} \right| = \frac{12}{13}$

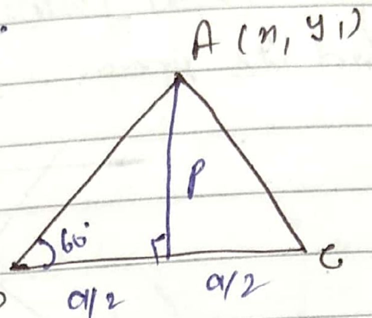
$\left| \frac{0 + 0 - 5}{\sqrt{5^2 + 12^2}} \right| = \frac{5}{13}$

* Distance b/w two Parallel lines:



$$\boxed{P = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|}$$ — Imp.

* If line is fixed :
* Then $\perp$ distance is minimum.



* For equilateral Δ

~~Poi~~ Point A and eq^n of BC is given

$\Delta = \frac{\sqrt{3}}{4} a^2$

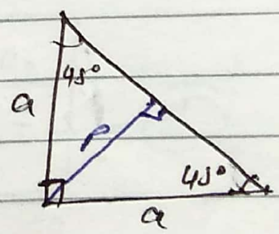
$\tan 60° = \frac{P}{a/2} = \frac{2P}{a} = \sqrt{3}$

$a = \frac{2P}{\sqrt{3}}$

$\Delta = \frac{\sqrt{3}}{4} \times \frac{4P^2}{3}$

$$\boxed{\Delta = \frac{P^2}{\sqrt{3}}}$$

* for Right Isoceles triangle :



$\Delta = \frac{1}{2} \times a \times a$

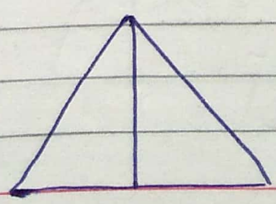
$\Delta = \frac{1}{2} \times 2P^2$

$\Delta = P^2$

$\sin 45° = \frac{P}{h} = \frac{1}{\sqrt{2}}$

$a = \sqrt{2}P$

Q:⇒ If one vertex of equilateral Δ is origin and other two vertices lies on $3x - 4y + 7 = 0$ then find its Area.

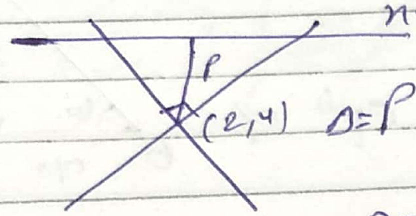


~~$\tan 6°$~~ $= \frac{P}{}$

$\Delta = \frac{P^2}{\sqrt{3}} = \frac{49}{25\sqrt{3}}$

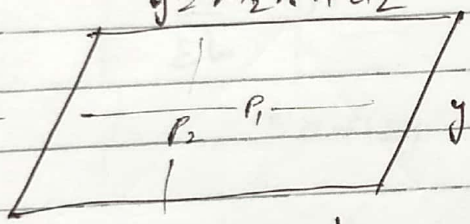
$P = \left| \frac{0 + 0 + 7}{5} \right| = \frac{7}{5}$

(2) If two mutually perpendicular lines passing through (2,4) forming a Isocles Δ with $x-y+1=0$ find its area.



$\Delta = p^2 = \frac{1}{2}$

$p = \left| \frac{2-4+1}{\sqrt{2}} \right| = \frac{1}{\sqrt{2}}$

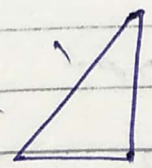


$\Delta = \frac{P_1 P_2}{\sin\theta}$

$P_1 = \left| \frac{c_1 - c_2}{\sqrt{1+m_1^2}} \right|$

$P_2 = \left| \frac{d_1 - d_2}{\sqrt{1+m_2^2}} \right|$

$\tan\theta = \left| \frac{m_1 - m_2}{1+m_1 m_2} \right|$



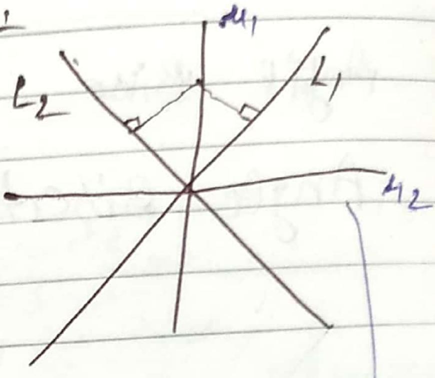
$\sin\theta = \frac{|m_1 - m_2|}{\sqrt{m_1^2 + m_2^2 - \cancel{2m_1m_2} + 1 + m_1^2 m_2^2 + \cancel{2m_1m_2}}}$

$\sin\theta = \frac{|m_1 - m_2|}{\sqrt{1+m_1^2}\sqrt{1+m_2^2}}$

learn

$$\boxed{A = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|}$$

Angle Bisector of two lines:



It is the locus of the point which is equidistance from two given lines.

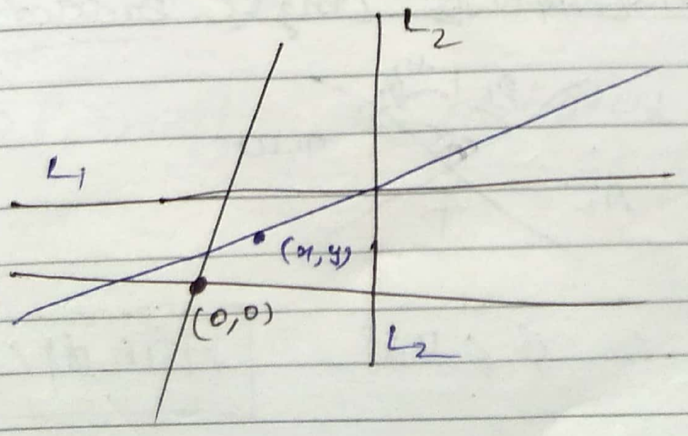
$L_1 = a_1x + b_1y + c_1$

$L_2 = a_2x + b_2y + c_2$

$$\left| \frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} \right| = \left| \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \right|$$

$$\boxed{\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \left(\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \right)} \quad (1)$$

Bisector Containing Origin:



$L_1 = 3x + 4y - 5 = 0$

$L_2 = 5x - 12y + 7 = 0$

$L_1 = -3x - 4y + 5 = 0$

* make the sign of Constant term in L_1 & L_2 same or (+)ve.

then Positive sign in eq (1) will give the angle Bisecter which contain origen.

* ~~Angle Bise~~

Angle Bisector which Contain the Point (α, β)

$(-1, 3)$ (Suppose)

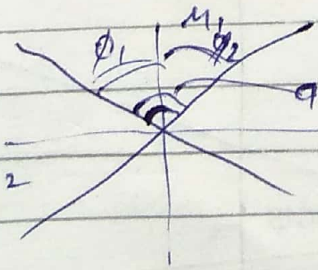
$L_1 = -3x - 4y + 5 = 0$
$L_2 = -5x + 12y - 7 = 0$

$L_1 = -3 + 12 - 5 > 0$
$-5 - 36 + 7 < 0$

make sign of $a_1\alpha + b_1\beta + c$ and $a_2\alpha + b_2\beta + c_2$ of same sign than (+ve) sign in eq (1) will give angle Bisecter which Contain (α, β)

* Acute and obtuse angle Bisector.

Method 1:



for actute ~~Bis~~ Bisector

~~2~~$\phi < 90°$ $\phi < 45°$ $\boxed{|\tan\phi| < 1}$

$M-3$ $\left[\begin{array}{llll} & & \text{obtuse} & \text{acute} \\ a_1 a_2 + b_1 b_2 = +ve & & + ve & - ve. \\ a_1 a_2 + b_1 b_2 = -ve & & - ve & + ve \end{array}\right.$

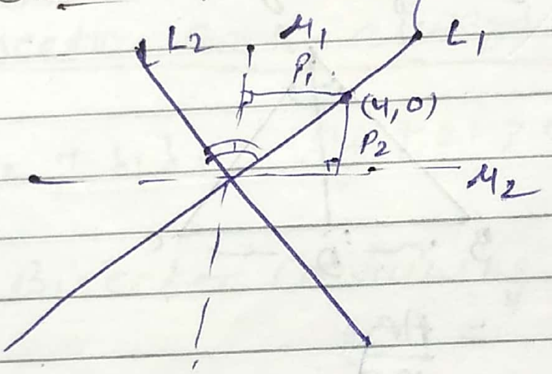
* find M_1 and M_2 by eq ①
then select any of L_1 and L_2 and M_1, M_2

then find $\tan\theta$ b/w them

$\tan\theta > 1 \Rightarrow$ ~~acu~~ obtus.
$\tan\theta < 1 \Rightarrow$ acute.

* ~~for~~ ~~acute Bisector~~ :-

Method - 2



छोटी Dis acute
बड़ी obtus.

dist ⇒ acute

find eqⁿ of bisector from eqⁿ ①

then choose a line from L_1 or L_2 and
take a random Point on that line
then find dist. from M_1 & M_2

$P_1 > P_2 \Rightarrow$ then M_1 is obtute angle Bisector

$P_2 > P_1 \Rightarrow$ then M_2 is obtus angle Bisector

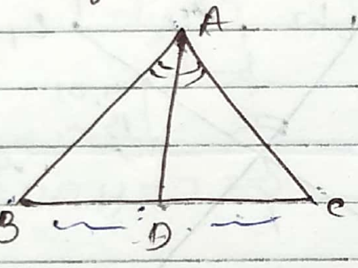
Method - 3

Make c_1 & c_2 (+ve) then find $a_1a_2 + b_1b_2$

Sign of $a_1a_2 + b_1b_2$ in eq^n ① will give obtuse angle Bisector.

$-ve \Rightarrow$ $-$ obtus.

* finding eq^n of angle Bisector of triangle.



$$\frac{BD}{DC} = \frac{AB}{AC}$$

Q. $4x + 3y - 7 = 0$ $\quad 24x + 7y - 31 = 0$

find angle Bisector which contain/doesn't contain origin. Identify acute and obtuse ~~alt~~ angle Bisector.

$O(0,0)$. $\Rightarrow L_1 = 4x + 3y - 7 = 0$

$L_2 = 24x + 7y - 31 = 0$

$$\frac{4x + 3y - 7}{\cancel{5}} = \pm\left(\frac{24x + 7y - 31}{\cancel{25}\,5}\right)$$

$20x + 15y - 35 = 24x + 7y - 31$

$4x - 8y + 4 = 0$

$x - 2y + 1 = 0$ Containing origin. (Obtuse) Containing Point $(-1,2)$

$20x + 15y - 35 = -24x - 7y + 31$

$44x + 22y - 66 = 0$

$2x + y - 3 = 0$ Does not contain origin. (acute).

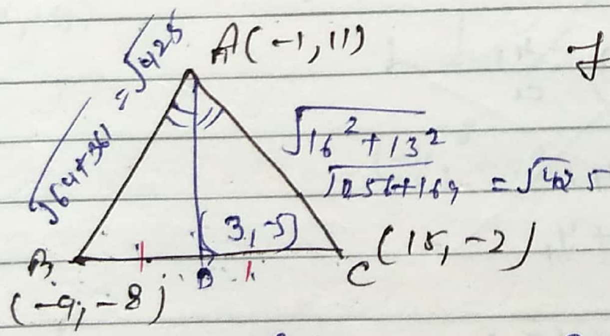
Angle Bisector are always Perpendicular

M-3 $\quad a_1a_2 + b_1b_2 = 96 + 21 > 0$

Q: Identify Bisector containing Point $(-1,2)$

$= -4 + 6 - 7 < 0$

$-24 + 14 - 31 < 0$

Q. 

Find eqn of angle Bisector of $\angle A$

$\frac{BD}{CD} = \frac{AB}{AC} \qquad \frac{BD}{CD} = \frac{1}{1}$

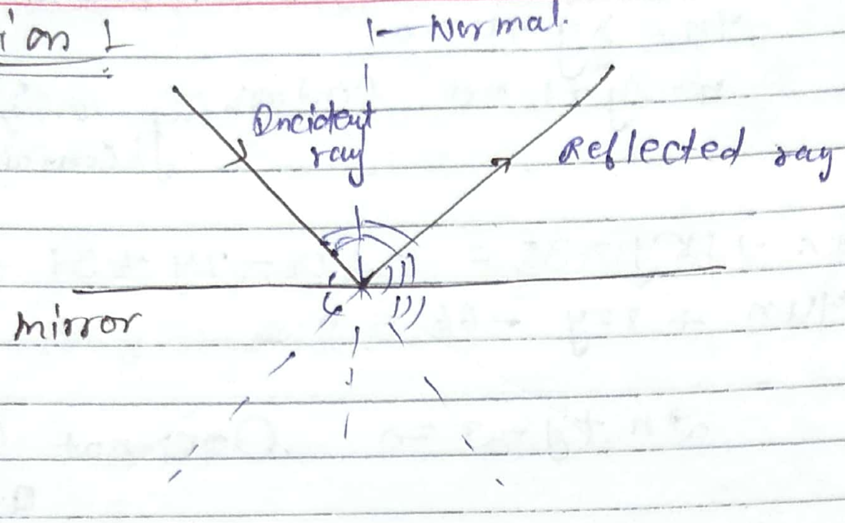
$\Rightarrow$

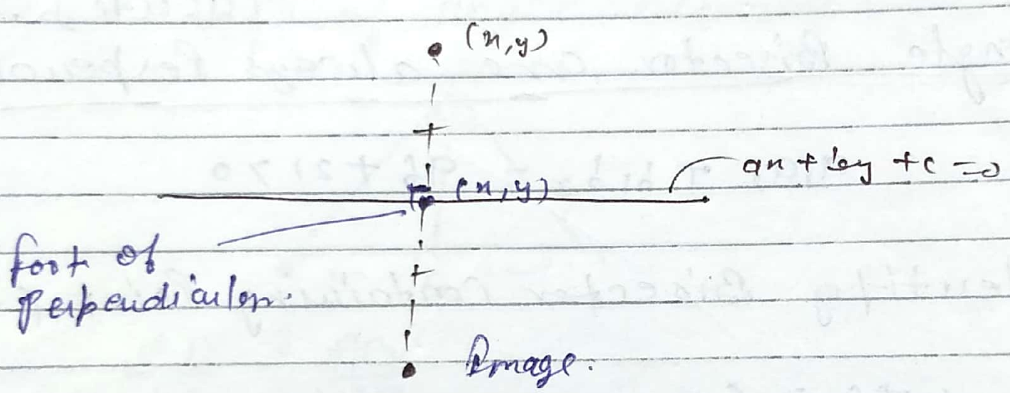
$y - 11 = \frac{11+5}{-1-3}(x+1)$

$y - 11 = -4x - 4$

$4x + y = 7$

* Reflection :





$$\frac{y - y_1}{x - x_1} = \frac{b}{a}$$

$$\frac{y - y_1}{b} = \frac{x - x_1}{a} = \lambda$$

$$x = a\lambda + x_1$$

$$y = b\lambda + y_1$$

$$a(a\lambda + x_1) + b(b\lambda + y_1) + c = 0$$

$$\lambda(a^2 + b^2) + ax_1 + by_1 + c = 0$$

$$\lambda = \frac{-ax_1 - by_1 - c}{a^2 + b^2}$$

Imp

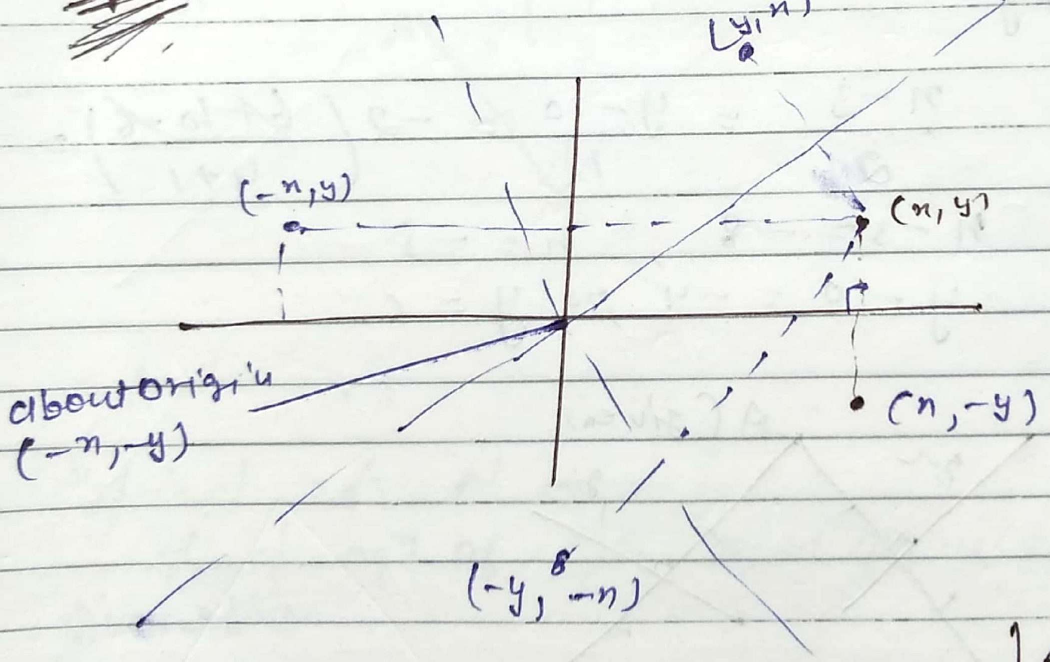
$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{(ax_1+by_1+c)}{a^2+b^2}}$$

For foot of perpendicular.

* For Image

Imp

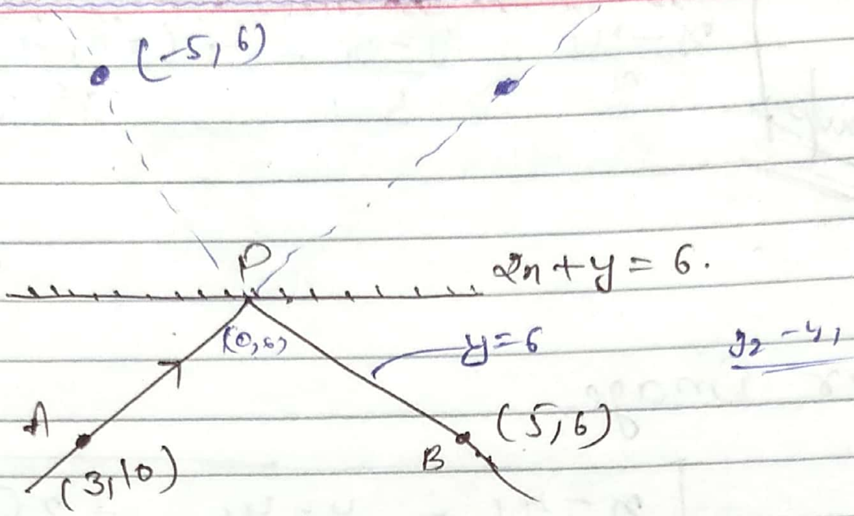
$$\boxed{\frac{x-x_1}{a} = \frac{y-y_1}{b} = -\frac{2(ax_1+by_1+c)}{a^2+b^2}}$$



Learn

x-axis	=	(x, -y)
y-axis	=	(-x, y)
y = x	=	(y, x)
y = -x	=	(-y, -x)
Origin	=>	(-x, -y).

Ques



find eq^n of AP and PB.

6 + 10 = 6

16.

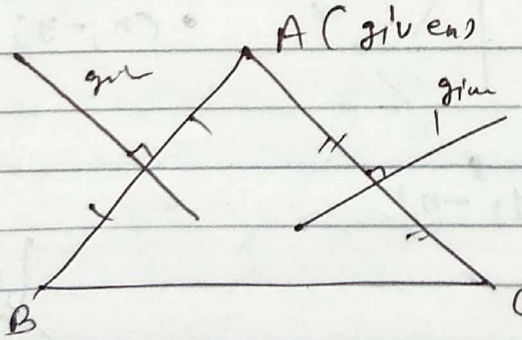
formula Apply

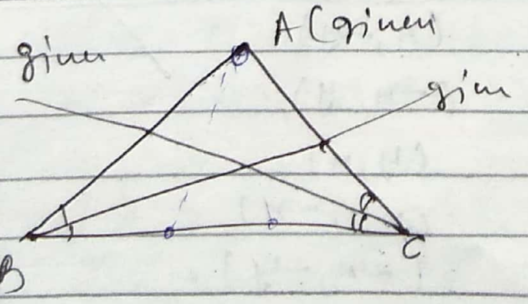
$$\frac{x-3}{2} = \frac{y-10}{1} = -2\left(\frac{\cancel{6}+10-\cancel{6}}{4+1}\right) = -4$$

$x - 3 = -8$, $x = -5$

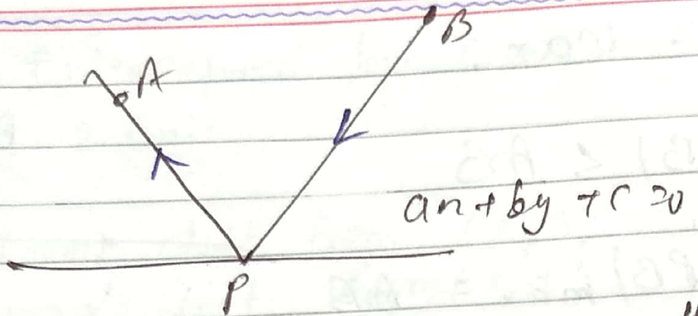
$y - 10 = -4$, so $y = 6$.

Q.





Q.



Que

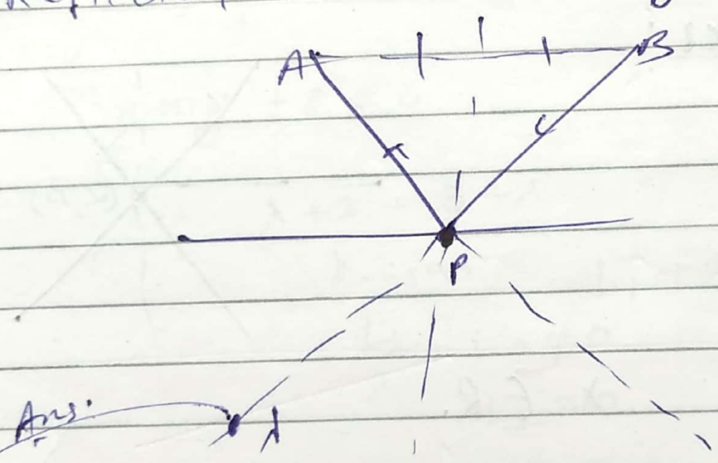
find Point P on line such that

(1) $PA + PB \Rightarrow \min$

(2) $|PA - PB| \rightarrow \min.$

(3) $|PA - PB| \rightarrow \max.$

(1) Reflected path is always least path.



find eqⁿ A' and
find eqⁿ of A'B and solve with the given line.

(2) $|PA - PB| = \min \Rightarrow 0$

$PA = PB$

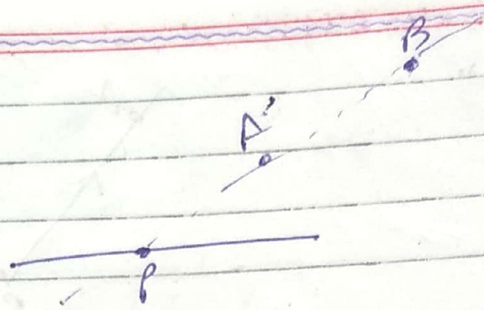
then Point P lies on Perpendicular Bisector of AB

then solve with given line.

H.W. : 1, 2, 3, 4, 5, 6, 7, 10, 11, (20, 21, 22) →.

Drpp.

(3) $|PA - PB| = max.$



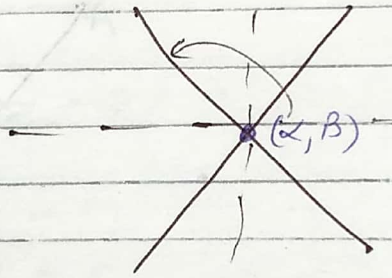
$|PA - PB| < AB$

$|PA - PB|_{max} = AB$

P, A, B are Collinear.

find eqⁿ of AB and solve in given line.

family of lines :



$\boxed{L_1 + \lambda L_2 = 0}$

$0 + \lambda \cdot 0 = 0 \qquad \lambda \in R.$

Let P be the point of Intersection L_1 & L_2 then the eqⁿ of the line passing through then PoI (1) given by

Q. $L_1 = 3x + 4y + 6 = 0$
$L_2 = x + y + 2 = 0$
find the eqⁿ of line passing through PoI of L_1 & L_2 and
(1) Passes Through (2, −3)
(2) Parallel to $x + 2y + 3 = 0 \qquad m = \frac{-1}{2}$
(3) Perpendicular to $2x - 3y + 1 = 0 \qquad m = \frac{2}{3}$

(4) Area intercepted by line and coordinate axis is 2 unit

(5) at minimum dist from (2, 3)
(6) is at max dist. from (2, 3)

Ans: eqⁿ of line
$$3x + 4y + 6 + \lambda(x + y + 2) = 0$$

(1) $6 - 12 + 6 + \lambda(2 - 3 + 2) = 0$
$$\lambda = 0$$

Put $\lambda = 0$
$$3x + 4y + 6 = 0$$

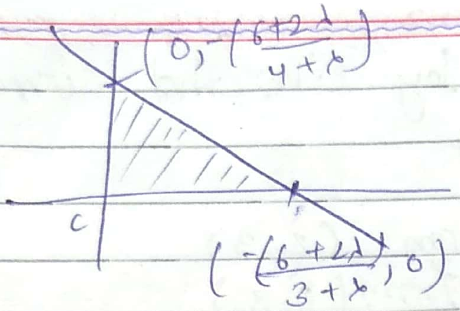
(2)
$$\lambda + 2 = 3 + \lambda$$
$$\lambda \to \infty \qquad L_1 + \lambda L_2 = 0$$
$$\frac{L_1}{\lambda} + L_2 = 0$$
$$L_2 = 0$$

(2) $x(3+\lambda) + y(4+\lambda) + 6 + 2\lambda$

$$m = -\left(\frac{3+\lambda}{4+\lambda}\right) = -\frac{1}{2}$$
$$6 + 2\lambda = 4 + \lambda$$
$$\lambda = -2.$$

(3)
$$-\left(\frac{3+\lambda}{4+\lambda}\right) = -\frac{3}{2}$$
$$6 + 2\lambda = 3\lambda + 12$$
$$\lambda = -6.$$

(4)



$\Delta = \left| \frac{1}{2} \left(\frac{(6+2\lambda)^2}{(3+\lambda)(4+\lambda)} \right| = 2.$

$4\lambda^2 + 36 + 24\lambda = \left| 4(\lambda^2 + 7\lambda + 12) \right| =$

$\Delta = 2$

$|4\lambda^2 + 28\lambda + 48| \quad 4\lambda^2 + 36 + 24\lambda$

$|x| = a \qquad 4\lambda = -12$

$\lambda = -3$

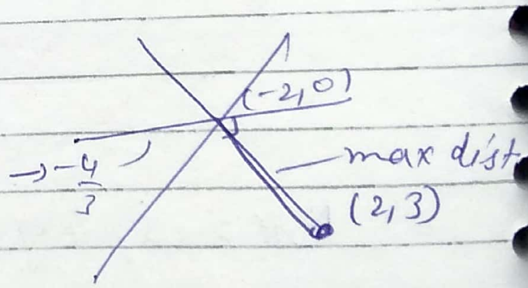
$x = \pm a$

$8\lambda^2 + 52\lambda + 84 = 0$

$4\lambda^2 + 26\lambda + 42 = 0$

$2\lambda^2 + 13\lambda + 21 = 0$

$2\lambda^2 + 7\lambda + 6\lambda + 21 = 0$



$L_1 = 3x + 4y + 6 = 0$

$L_2 = 3x + 3y + 6 = 0 \qquad y = 0, \quad x = -2.$

$y - 0 = \frac{-4}{3}(x+2)$

$y = \frac{-4}{3}x - \frac{8}{3}$ Ans

$L_1 + \lambda L_2 = 0$ [L_1 and L_2 का intersection है $\lambda =$]

Q: $x(a+2b) + y(a+3b) = a+b$
Passes through a fix point find that Point.

$a(x+y-1) + b(2x+3y-1) = 0$

$(x+y-1) + \left(\frac{b}{a}\right)\cdot(2x+3y-1) = 0$

$x+y-1=0$
$2x+3y-1=0$
$2x+2y-2=0$

$y+1=0$ $\quad y=-1$, $x=2$

Point $\Rightarrow (2,-1)$

Q. if a, b, c are in AP then $ax+by+c=0$ Passes through a fixed find that Point.

$2b = a+c$ (because AP).

$ax + \left(\frac{a+c}{2}\right)y + c$

$2ax + ay + cy + 2c = 0$
$a(2x+y) + c(y+2) = 0$
$2x+y + \left(\frac{c}{a}\right)(y+2) = 0$

$2xy = 0$ $\quad x = 1$
$y = 2$

$a - 2b + c = 0$

$ax + by + c = 0$

firstly const. same.

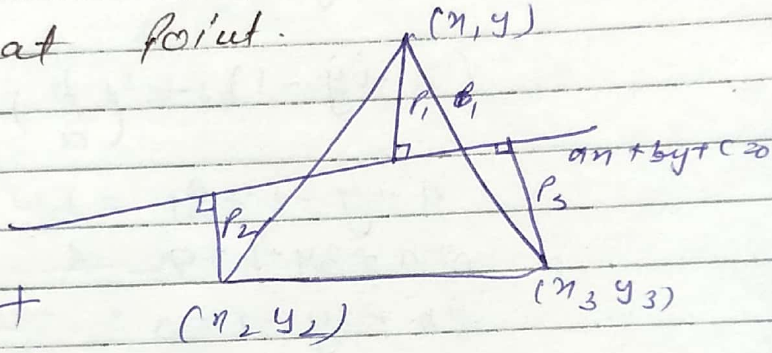
then compair eq.

find a x, y.

$x = 1$

$y = -2.$

Q: If sum of Algebric dist from vertices of a triangle is zero then line passes through a fix point find that point.



$p_1 + p_2 + p_3 = 0$

$$= \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} + \frac{ax_2 + by_2 + c}{\sqrt{a^2+b^2}} + \frac{ax_3 + by_3 + c}{\sqrt{a^2+b^2}} = 0$$

$$= a(x_1 + x_2 + x_3) + (y_1 + y_2 + y_3) + 3c = 0$$

$$a\left(\underbrace{\frac{x_1 + x_2 + x_3}{3}}_{x}\right) + b\left(\underbrace{\frac{y_1 + y_2 + y_3}{3}}_{y}\right) + c = 0$$

fixed point is centroid.

Q. If $a^2 + 4b^2 + 4ab - 9c^2 = 0$
then line $ax + by + c = 0$ passes through fixed point
find that point.

$$(a+2b)^2 - (3c)^2 = 0$$
$$(a+2b+3c)(a+2b-3c) = 0$$

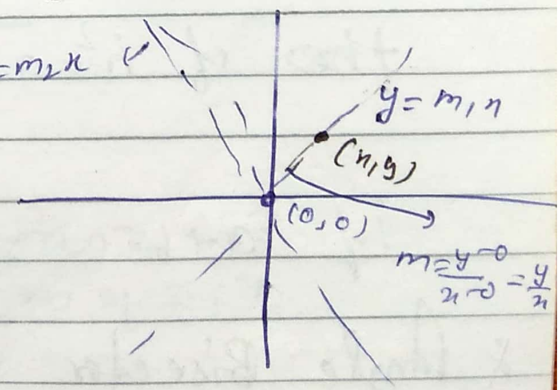
$a + 2b + 3c = 0$ / $a + 2b - 3c = 0$

$\frac{a}{3} + \frac{2}{3}b + c = 0$ $\quad -\frac{a}{3} - \frac{2}{3}b + c = 0$

$\left(\frac{1}{3}, \frac{2}{3}\right)$ $\quad \left(\frac{-1}{3}, \frac{-2}{3}\right)$.

* Pair of Lines:

(i) Pair of lines passing through origin:



$$(m_2 x - y)(m_1 x - y) = 0$$

$$m_2 m_1 x^2 + y^2 - (m_1 + m_2)xy = 0$$

It is a second degree Homogenious eq^n in x & y.

$ax^3 + bx^2y + cxy^2 + dy^3 = 0 \Rightarrow$ Represent three lines passing through origin.

general pair of line: $ax^2 + 2hxy + by^2 = 0$

$$a + 2h\left(\frac{y}{x}\right) + b\left(\frac{y}{x}\right)^2 = 0$$

$$bm^2 + 2hm + a = 0 <^{m_1}_{m_2}$$

$$m_1 + m_2 = \frac{-2h}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{\sqrt{D}}{b}}{1 + a/b}\right| = \left|\frac{\sqrt{D}}{a+b}\right| =$$

$$= \frac{\sqrt{4h^2 - 4ab}}{|a+b|} = 0$$

$$\boxed{\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}}$$ lean

$= 0$

~~the~~ if $h^2 = ab$, then lines are Coincidental lines.

If $a+b=0$ then line are Perpendicular.

* Angle Bisector of these lines are given by

$$\boxed{\frac{x^2 - y^2}{a-b} = \frac{xy}{h}}$$ lean

$y = m_1 x$

$y = m_2 x.$

$$\left|\frac{m_1 x - y}{\sqrt{1+m_1^2}}\right| = \left|\frac{m_2 x - y}{\sqrt{1+m_2^2}}\right|$$

S.B.S. and find value of m_1 and m_2.

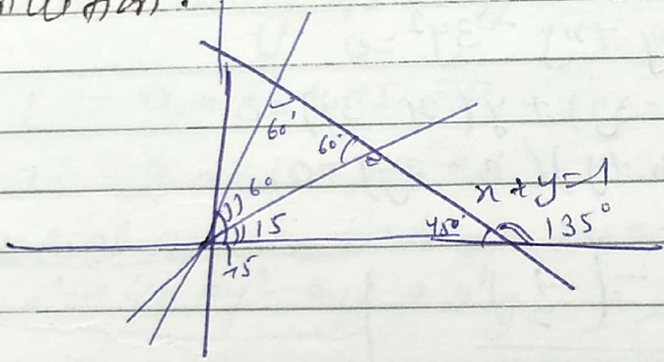
Que. Prove that $x^2 - 4xy + y^2 = 0$ and $x + y = 1$ encloses a equilateral Δ. also find its Area.

Q. Prove that $3x^2 - 8xy - 3y^2 = 0$ and $x + 2y = 3$ exclose a right isosceles Δ. Find its Area

Q. $ay^3 + by^2x + cyx^2 + dx^3 = 0$ representation three line two of which are perpendicular. then find condition.

$x^2 - 4y + y$ ... $x + y$ ... $-4x = x$

(1)



$m = 4$

$$x^2 - 4xy + y^2 = 0$$

$m = 2 - \sqrt{3}$

$m = 2 + \sqrt{3}$

$$\left(\frac{y}{x}\right)^2 - 4\left(\frac{y}{x}\right) + 1 = 0$$

$$m^2 - 4m + 1 = 0$$

$$m = 2 \pm \sqrt{3}.$$

$$P = \left|\frac{0 + 0 - 1}{\sqrt{1^2 + 1^2}}\right|$$

$$P = \frac{1}{\sqrt{2}}$$

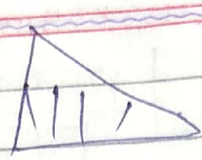
$$\Delta = \frac{p^2}{\sqrt{3}}$$

$$\Delta = \frac{1}{2\sqrt{3}}$$ Ans

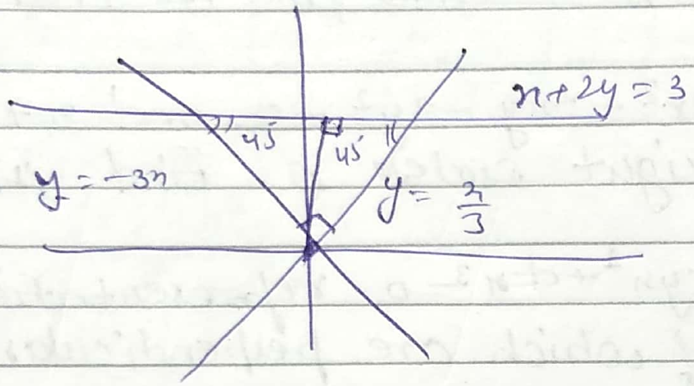
$a + b = 0$

(2) $\overset{a}{3x^2} - 8xy - \overset{b}{3y^2} = 0$

$x - 2y = 3$



$a + b = 0$ So, triangle is Right angle tringle.



$$3x^2 - 9xy + xy - 3y^2 = 0$$

$$3x(x - 3y) + y(x - 3y) = 0$$

$$(3x + y)(x - 3y) = 0$$

$$\tan\theta = \left| \frac{\frac{1}{3} - \left(\frac{-1}{2}\right)}{1 + \left(\frac{1}{3}\right)\left(\frac{-1}{2}\right)} \right|$$

$$\tan\theta = \frac{5/6}{1 - 1/6} = 1 \Rightarrow \theta = 45$$

$$P = \left| \frac{0 + 0 - 3}{\sqrt{1^2 + 2^2}} \right| = P = \frac{3}{\sqrt{5}}$$

$$D = \frac{9}{5}$$

$\frac{d}{a} \Rightarrow$ slope

(3) $a m^3 + b m^2 + c m + d = 0$ — m_1, m_2, m_3

$$m_1 m_2 = -1$$

$$m_1 m_2 m_3 = -\frac{d}{a} \qquad m_3 = \frac{d}{a}$$

$$a\left(\frac{d}{a}\right)^3 + b\left(\frac{b}{a}\right)^2 + c\left(\frac{d}{a}\right) + d = 0$$

$$d^3 + bd^2 + cda + da^2 = 0$$ — Condition for two lines are Ʇlar.

* Pair of line Passing through Different Point.

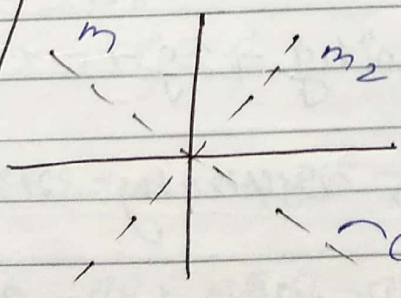
$$L_1 = a_1 x + b_1 y + c_1 = 0$$

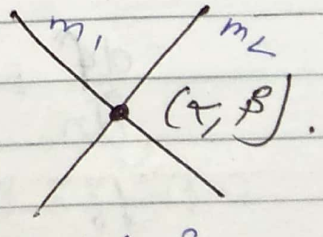
$$L_2 = a_2 x + b_2 y + c_2 = 0$$

general eqⁿ of pair of lines is

$$\boxed{a x^2 + 2h xy + b y^2 + 2gx + 2fy + c = 0}$$

slope is depend only on this Homogenius





$$a x^2 + 2hxy + by^2 = 0$$

$$m_1 + m_2 = \frac{-2h}{b}$$

$$m_1 m_2 = a/b$$

$$\boxed{\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}}$$

Race: Jun

If $h^2 = ab$ then lines are Parallel
If $a + b = 0$, then lines are Perpendicular.

General two degree eq^n will be a pair of line if:

Imp → $$\boxed{\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0}$$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

Partial differenciation method to find PoI

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

Differenciate w.r.t x, assuming y as const.

$$\frac{df}{dx} = 2ax + 2hy + 2g + 0 + 0 = 0 \quad — (1)$$

Diff w.r.t y, assuming x as const.

$$\frac{df}{dx} = 0 + 2hx + 2by + 0 + 2f + 0 = 0 \quad - (2)$$

Solve eq^n (1) and (2) [Point of Intersection]

Ques

(1) $x^2+y^2+2pxy+2qx+ry+8=0$
represent two parallel lines
find p & q

(2) $y^2-xy-6x^2+13x-y-6=0$
find point of intersection and angle b/w lines

(3) $x^2-4xy+4y^2+5x-10y-6=0$
find distance b/w lines.

(1) $p = \cancel{abc+2fg}$

$ax^2+2hxy+by^2+2gx+2fy+c=0$

$\Delta = abc+2fgh-af^2-bg^2-ch^2=0$

$\tan\theta = \dfrac{2\sqrt{h^2-ab}}{|a+b|}$

$h^2=ab$ $\quad p^2=1$

$p=\pm 1$

$1\times1\times8+2\times3\times q+p-1\times3^2-1\times q^2-8\times p^2=0$

$8+6pq-\cancel{p}q-q^2-8p^2=0$

$p=1$ $\quad 8+6q-q-q^2-8=0$

$q^2-6q+9=0$

$(q-3)^2=0$

$q=3$

n

$p = -1 \quad 8 - 6q - 9 - q^2 - 8 = 0$

$q^2 + 6q + 9 = 0$

$q = -3$

(2) p

$y^2 - ny - 6n^2 + 13n - y - 6 = 0$

Diff. w.r.t x

$0 - y - 12n + 13 + 0 + 0 = 0$

Diff w.r.t y

$2y - n + 0 + 0 - 1 + 0 = 0$

$12n + y = 13$

$2y - n - 1 = 0$

$n = 1$

$y = 1$

$y^2 - y(n+1) - 6n^2 + 13n - 6 = 0$

$$y = \frac{n+1 \pm \sqrt{n^2+1+2n+24n^2-25n+24}}{2}$$

$2y = n+1 \pm 5(n-1)$

$2y = 6n \qquad y = 3n$

$2y = -4n+2$

$y = -2n+1$

$2y = -4n+6$

$y = -2$

$$\tan\theta = \frac{2\sqrt{h^2+3}}{|a+b|}$$

$h = \frac{-1}{2}$

$a = -6$

$b = 1$

$\tan\theta = 1$

$\theta = 45^\circ$

3

$t^2 + 5t + 6 = 0$

$(x-2y)^2 + 5(x-2y) - 6 = 0$

$(x-2y+6)(x-2y-1) = 0$

$x - 2y + 6 = 0$

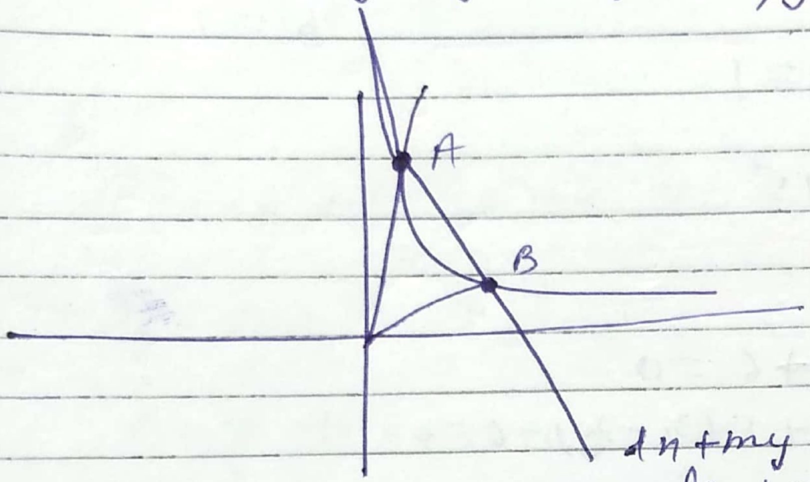
$x - 2y - 1 = 0$

$$D = \left| \frac{6+1}{\sqrt{1^2+2^2}} \right| = \frac{7}{\sqrt{5}}$$

~~Powers Plane~~

Homogenisation :

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$



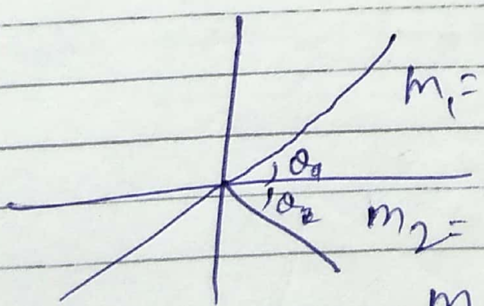
$$lx+my=n$$

$$\frac{lx+my}{n}=1$$

$$ax^2+2hxy+by^2+2gx(1)+2fy(1)+c(1)^2=0$$

$$ax^2+2hxy+by^2+2gx\left(\frac{lx+my}{n}\right)+2fy\left(\frac{lx+my}{n}\right)+c\left(\frac{lx+my}{n}\right)^2=0.$$

This is combined eq^n of OA and OB



$m_1 = \tan\theta$

$m_2 = \tan(-\theta)$

$m_2 = -m_1$

$m_1+m_2=0$

$\frac{-2h}{b}=0 \qquad h=0$

Q. find the eq^n of pair of Line joining origin and POI of line $2x-y=3$ and the curve

$$x^2-y^2+3x-6y+18=0$$

also find angle b/w them

(2) find the value of m if the lines joining the origin to the POI of ~~x^2+y^2+x-2y~~

$$x^2+y^2+x-2y-m=0$$

and $x+y=1$ are at right angle

(3) If angle b/w line joining the origin and the P.O.I of line $x-y=2$ and
$x^2-4xy+2y^2-2x+y+k=0$ is 45°
find k.

(1) $2x-y=3$

$$\frac{2x}{3}-\frac{y}{3}=1$$

$$x^2-y^2+3x(1)-6y(1)+18(1)^2=0$$

$$x^2-y^2+3x\left(\frac{2x-y}{3}\right)-6y\left(\frac{2x-y}{3}\right)+18\left(\frac{2x-y}{3}\right)^2=0$$

$$x^2-y^2+2x^2-xy-4xy+2y^2+2(4x^2+y^2-4xy)=0$$

$$11x^2 + 3y^2 + 18xy = 0$$

OA & OB combined eqⁿ.

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}$$

$h = \frac{18}{2}$

$a = 11$

$b = 3$

$$= \frac{2\sqrt{\frac{18^2}{4} - 33}}{14}$$

(2) $x^2 + y^2 + x - 2y - m = 0$

$x + y = 1$

$x^2 + y^2 + x(x+y) - 2y(x+y) - m(x+y)^2 = 0$

$x^2 + y^2 + x^2 + xy - 2xy - 2y^2 - mx^2 - my^2 - 2mxy = 0$

$x^2(2-m) + y^2(-1-m) + xy = 0$

$a + b = 0$

$2 - m + (-1 - m) = 0$

$2m = 1$

$m = \frac{1}{2}.$

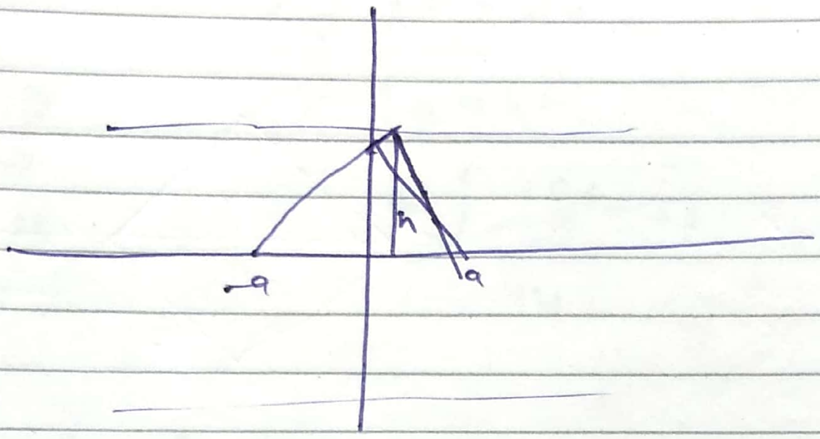
try at Home

(3)

H.W

V. Das.

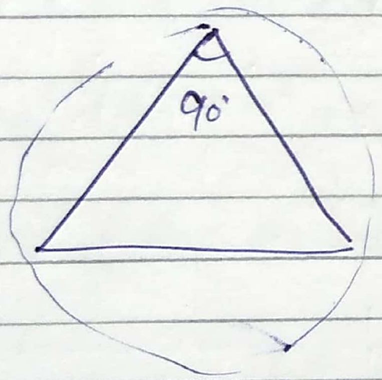
LOCUS.

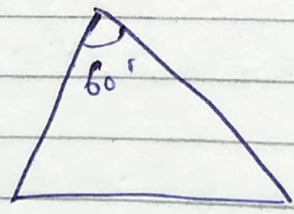


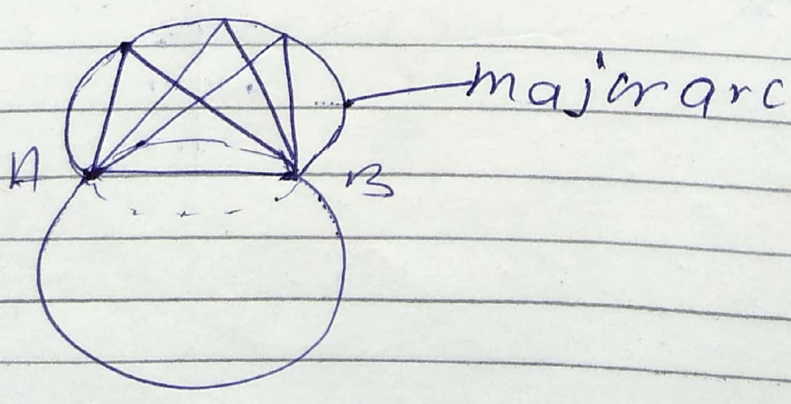
$\Delta PAB = \text{const}$

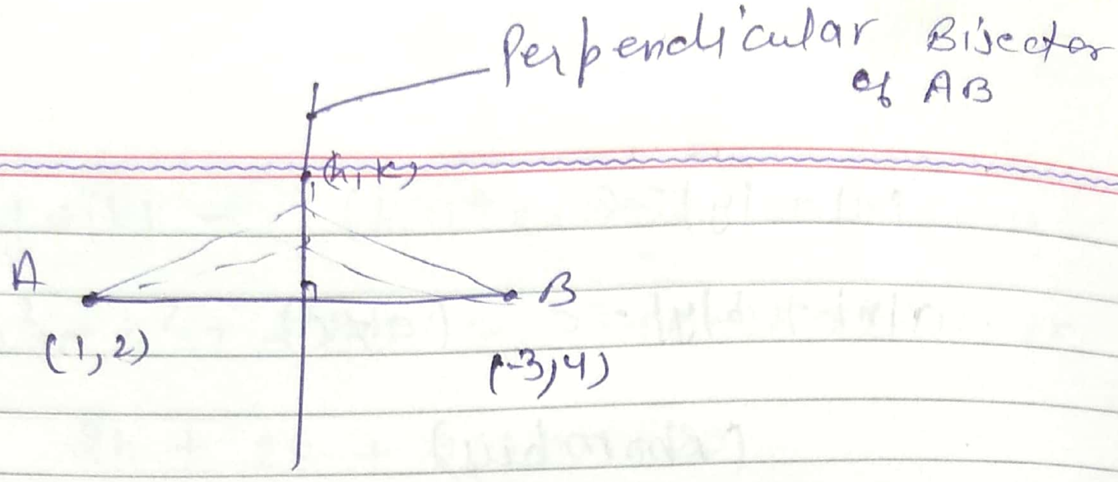
$\frac{1}{2} \times 2a(h) = \text{const}$

$h = \text{const}$





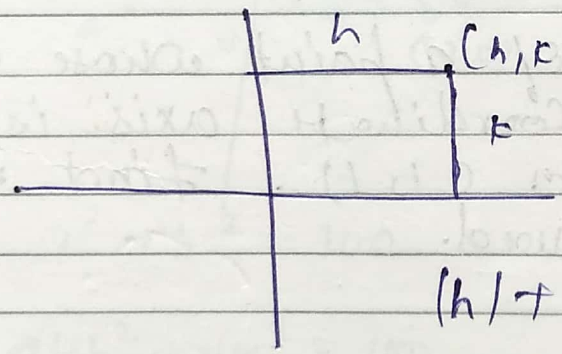




$$(h-1)^2+(k-2)^2=(h+3)^2+(k-4)^2$$

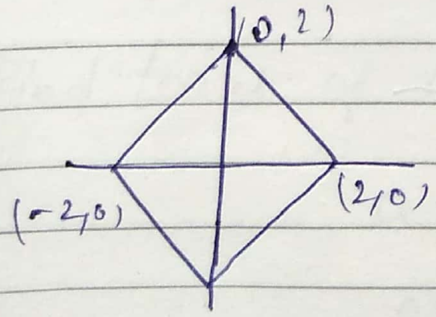
$$\cancel{h^2}+1-2h+\cancel{k^2}+4-4k=\cancel{h^2}+9+6k$$
$$+\cancel{k^2}+16-8k$$

Que: ~~find~~ find locus of **a** point ~~where~~ whose sum of distance from coordinate axis is 2, also find the enclosed area



$$|h|+|k|=2$$

$$|x|+|y|=2$$



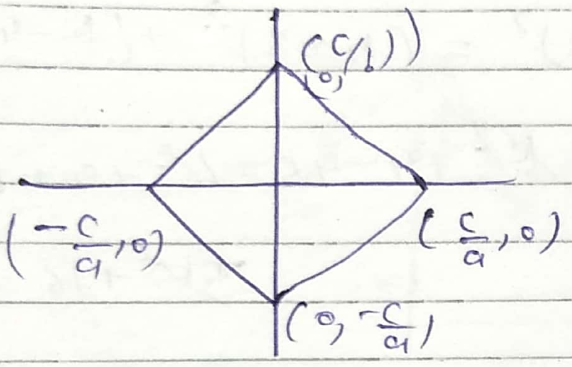
$$x+y=2$$
$$-x+y=2$$
$$-x-y=2$$

$$\Delta=8$$

$|x| + |y| = c$

$a|x| + b|y| = c \quad (a \neq c)$

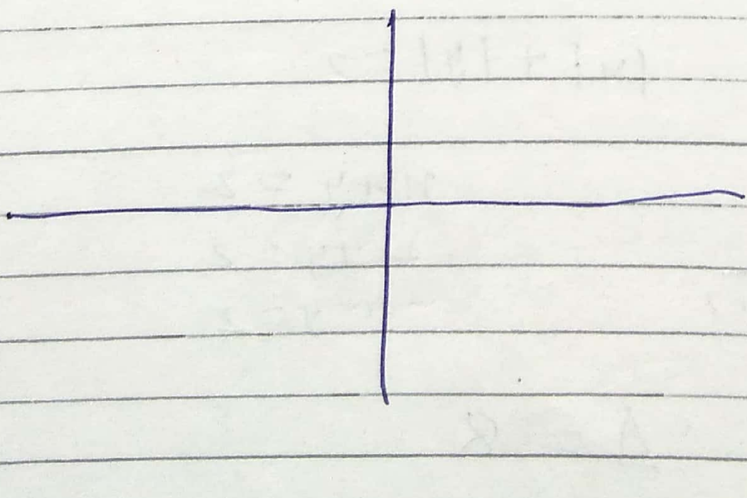
(Rhombus)



$\Delta = \frac{1}{2} \times d_1 \times d_2$

$\Delta = \frac{1}{2} \times \frac{2c}{a} \times \frac{2c}{b} \qquad \Delta = \frac{2c^2}{ab}$

Q. Find locus of a point whose sum of dist. from Coordinate axis is equal to dist from (1, 1). Find the locus in second Quad.



Ans: $|h|+|k| = \sqrt{(h-1)^2+(k-1)^2}$

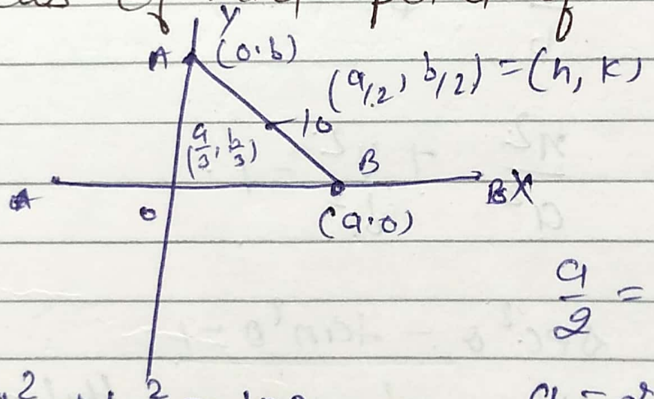
$$h^2+k^2+2|hk| = h^2+1-2h+k^2+1-2k.$$

$$2h+2k+2(hk)=2$$

$$h+k+|hk|=1$$

$$h+k-hk=1$$

Q. A stick of length 10 is sliding on two Perpendicular lines
- find locus of mid point of stick.



$$\frac{a}{2}=h \qquad \frac{b}{2}=k$$

$$a^2+b^2=100 \qquad a=2h \qquad b=2k$$

$$4h^2+4k^2=100$$

$$h^2+k^2=25$$

find locus of Centriod of Δ OAB Locus: (0,0)

$$a=3h \qquad b=3k.$$

$$9h^2+9b^2=100$$

SBG STUDY

* Parametric Locus !

(1) $x = r\cos\theta$ θ = Parameter
$y = r\sin\theta$ r = constant

$x^2 + y^2 = r^2$

(2) $x = a\cos\theta$
$y = b\sin\theta$

$\frac{x}{a} = \cos\theta$

$\frac{y}{b} = \sin\theta$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(3) $x = a\tan\theta$ $\sec^2\theta - \tan^2\theta = 1$
$y = b\sec\theta$ $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ Hyperbola

(4) $x = 2at$ $t = \frac{x}{2a}$
$y = at^2$

$y = a\left(\frac{x^2}{4a^2}\right) \Rightarrow x^2 = 4ay.$