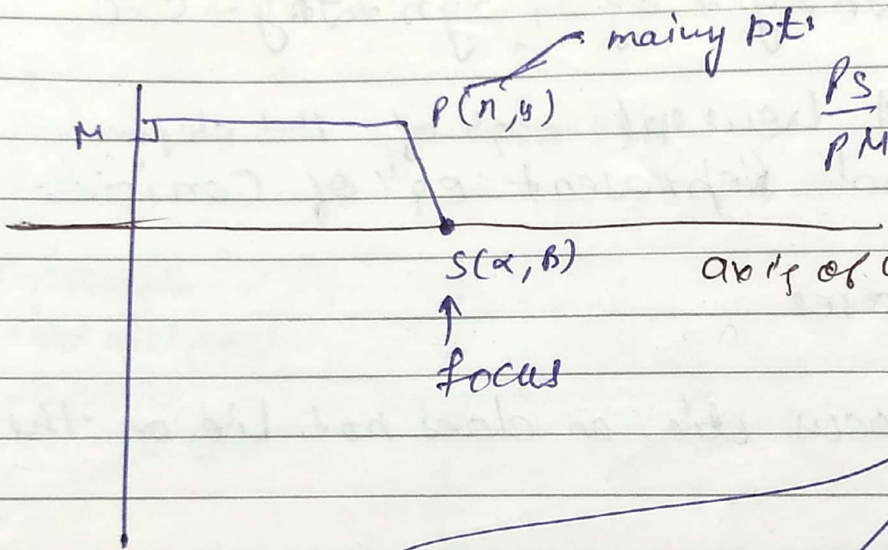
20/09/17

Conic section

Conic section is locus of point which moves us that the ratio of its dist. from fixed Point (focus), to fixed line (Directrix) is always constant.



$\frac{PS}{PM} = e$ (Eccentricity of Conic)

Directix $= lx + my + n = 0$

$e = 1$ ∴ Locus of P is Parabola

$0 < e < 1$ Locus of P is Ellipse

$e > 1$ Hyperbola.

* Line ~~Conic~~ Passes through the focus and ⊥ to diretix. and axis of Conic.

* Point at which Conic meet its axis is called vertex of Conic

$\boxed{PS = ePM}$

$$PS = ePM$$
$$PS^2 = e^2 PM^2$$
$$(x-\alpha)^2 + (y-\beta)^2 = e^2\left(\frac{lx+my+n}{\sqrt{l^2+m^2}}\right)^2$$

$$Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + C = 0$$
OR
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

~~represent~~ General eq^n of 2nd degree ~~it~~ ~~is~~ and Represent eq^n of Conic.

Now two Cases

(1) When focus ~~lie~~ on does not lie on the directrix

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$\Delta \neq 0$	$\Delta \neq 0$	$\Delta \neq 0$	$\Delta \neq 0$
If $a = b$ & $h = 0$ then eq^n represent Circle	$h^2 = ab$ in this case eq^n represent Parabola	$h^2 < ab$ represent ellipse	$h^2 > ab$ Hyperbola
$e = 0$	$e = 1$	$0 < e < 1$	$e > 1$

$$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$$

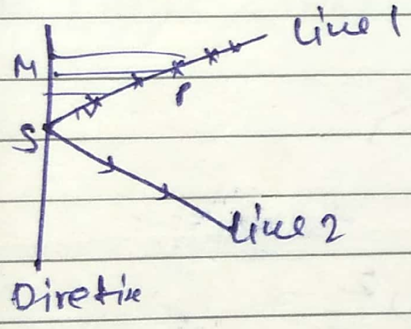
Case II

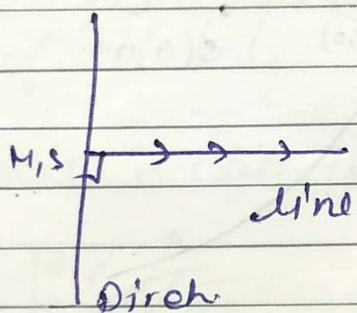
When focus lie on the directrix.

In this Case $\Delta = 0$

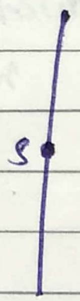
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$e > 1$	$e = 1$	$e < 1$
$\frac{PS}{PM} = e$	$PS = PM$	$\frac{PS}{PM} = e < 1$
In this case 2 distinct and real lines		Imaginary line





Real and Consided lines

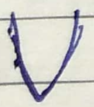


* $xy = 0$ either $x = 0$ on $y = 0$

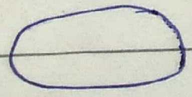
$x = 0 \downarrow$ y axis

$y = 0 \downarrow$ x-axis

Joint eq^n of axis

(i) 

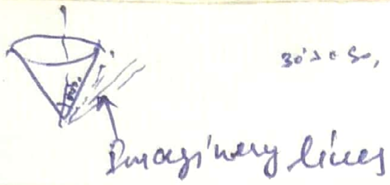
(ii) 

(iii) 

(iv) Parabola

$x^2 + y^2 = 0$

$x = 0$ & $y = 0$



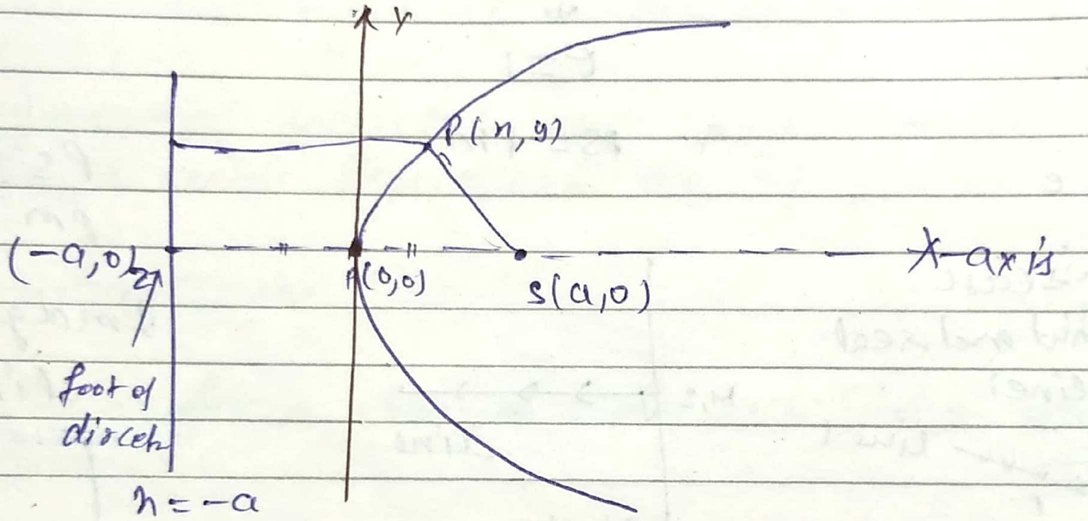
30°≠90, it have Imaginary lines.

Parabola = ∥ to the axis

Hyperbola

Parabola !

$$\frac{PS}{PM} = e = 1$$



$$PS^2 = PM^2$$

$$(x-a)^2 + (y-0)^2 = (x+a)^2$$

$$\cancel{x^2} + \cancel{a^2} - 2xa + y^2 = \cancel{x^2} + \cancel{a^2} + 2xa$$

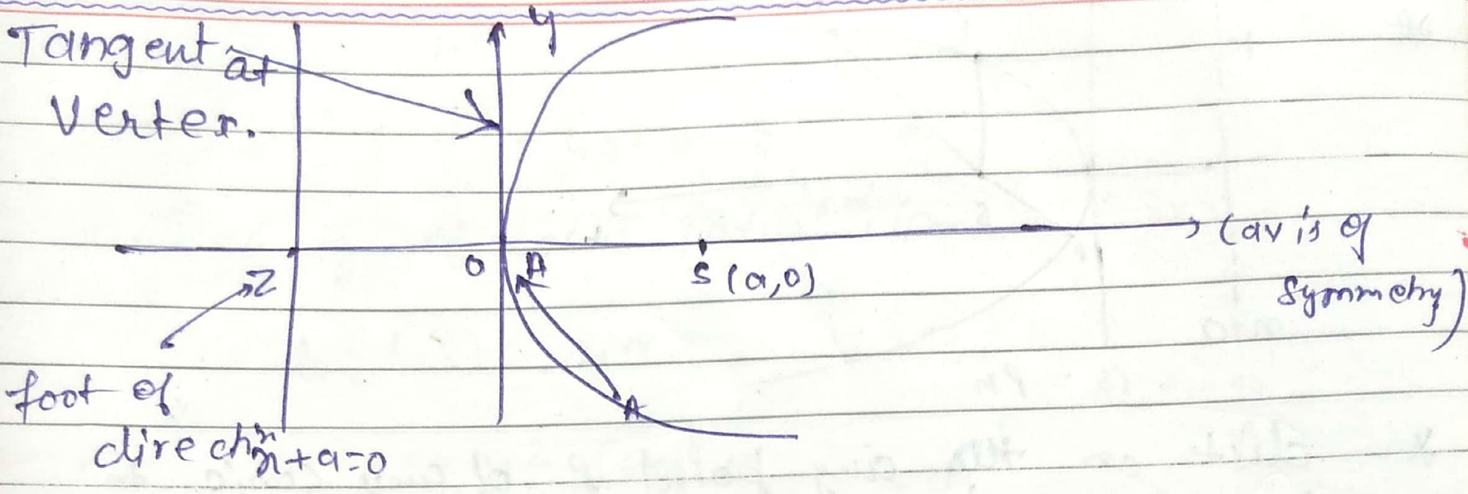
$$y^2 = 4ax.$$

$a > 0$

Because y has even power so

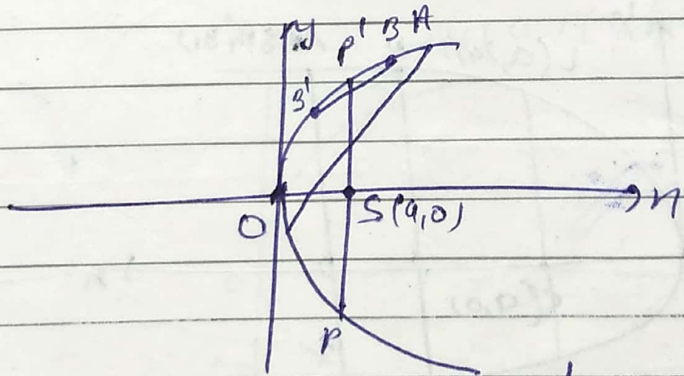
So, curve symmetric about x - axis.

Revise st line
Circle



* Line passes through focus and ⊥ to directrix is called axis (axis of symmetry). of P. and foot of ⊥ from focus upon directrix is called foot of directrix

* Conic meet axis is called vertex

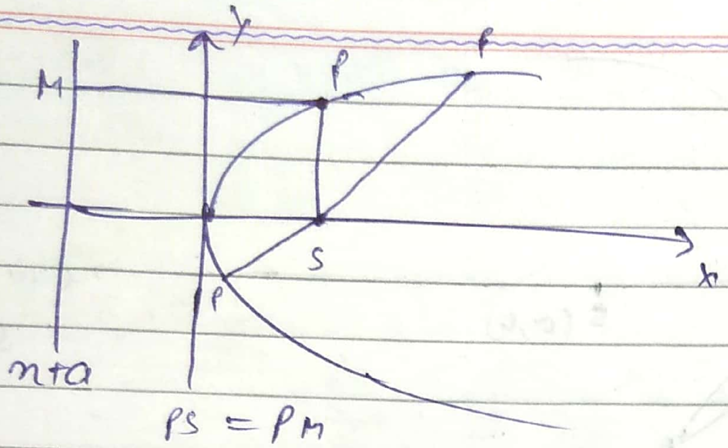
*


focal chord.

* line joining of any two point is the chord of the conic
If chord passes through focus then it is called focal chord of Parabola

BB' = simple chord
PP' is focal chord

*

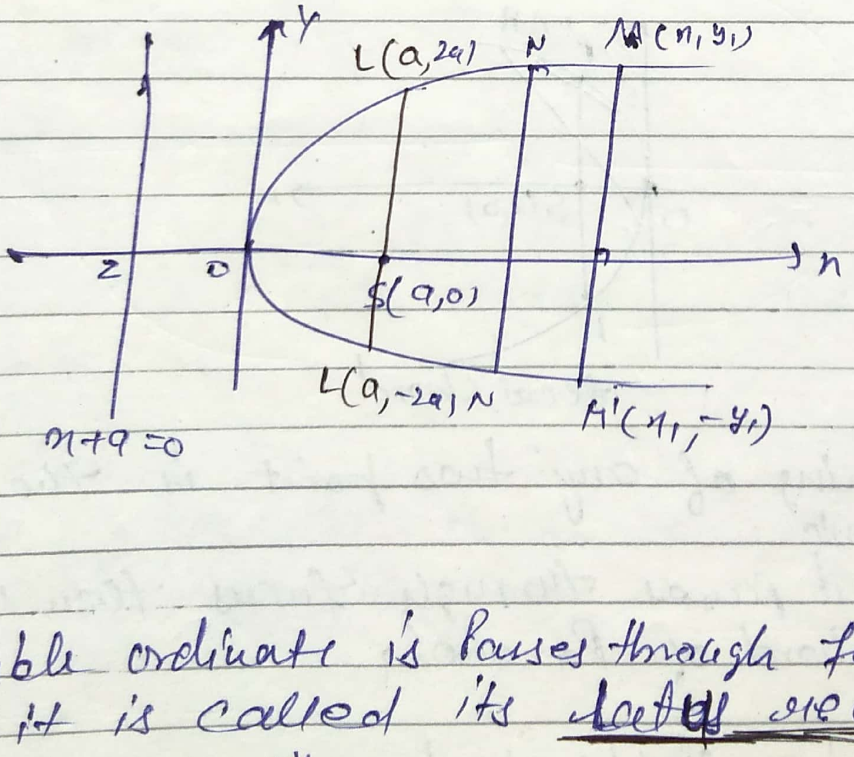


PS = PM

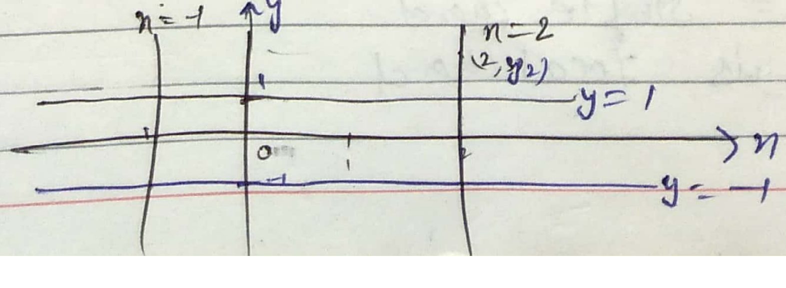
* dist on the any point P of any conic to the focus is called focal dist of the conic and is equal to dist. of Point P from the directrix (Called focal directrix property).

* <u>Double ordinate:</u>

Chord of Parabola which is ⊥ to the axis of Symmetry is called its double ordinate



If double ordinate is passes through focial of P. then it is called its <u>latus rectum</u>



$y^2 = 4ax$

$y^2 = 4a^2 \Rightarrow y = \pm 2a$.

extremity of ~~Locus~~ Latus rectum are LNM.
and

$l(L.R) = \boxed{4a}$

La

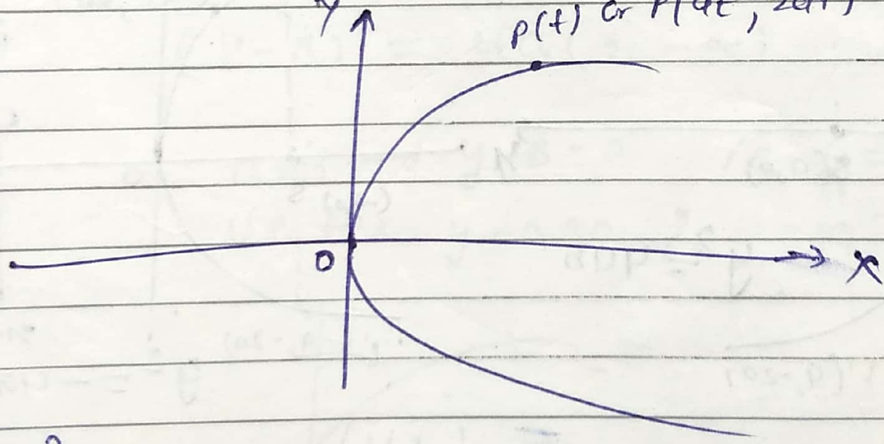
$L(L.R) = 4a$

$y^2 = \underbrace{(4a)}_{L(L.R)} x$

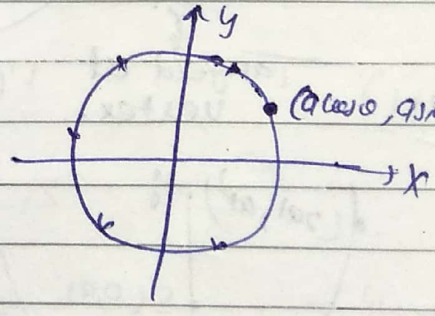
* Two parabola are same to be equal ~~if~~ if their lenth of LR. are same.

* Parametric eqn i.e any point of the Curve

Parametric ~~form~~



$x^2 + y^2 = a^2$

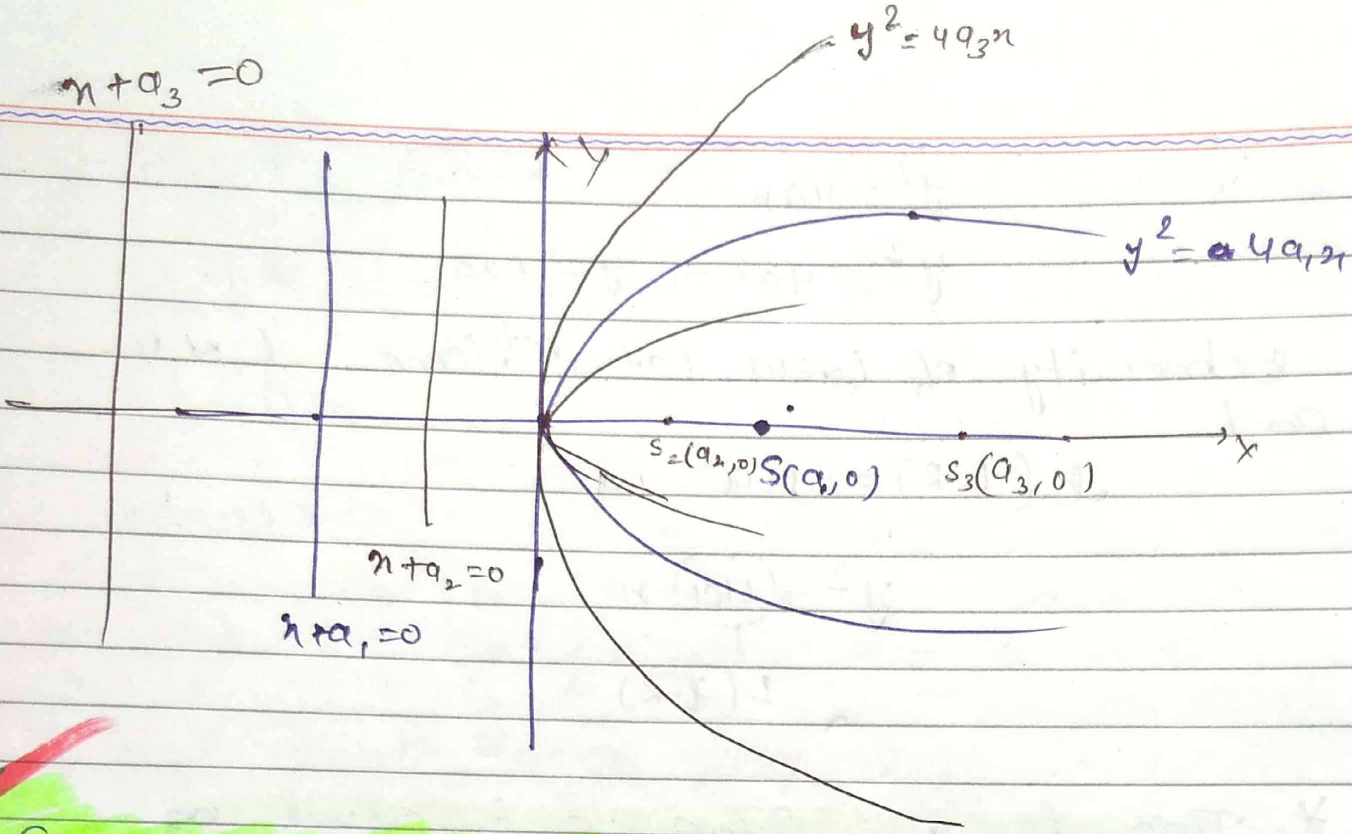


$y^2 = 4ax$.

$x = at^2$

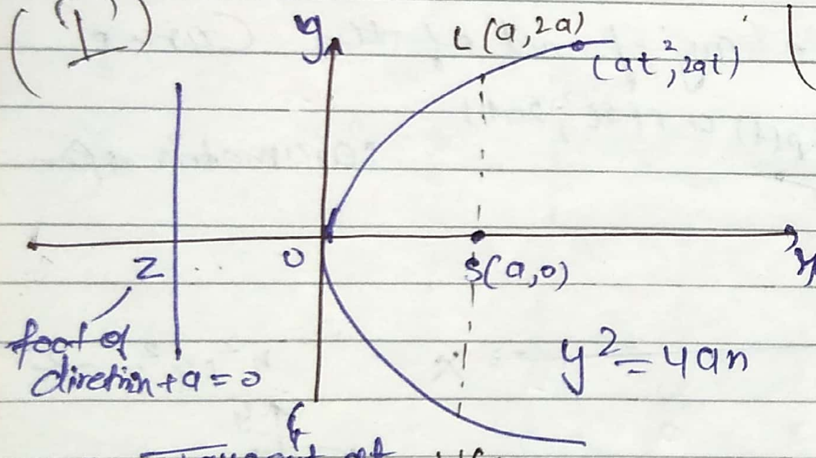
$\therefore y^2 = 4a^2t^2$

$y = 2at$

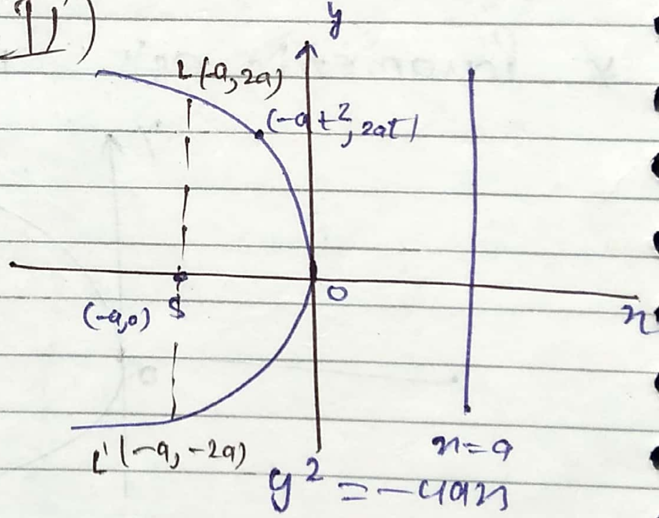


* for standard parabola:

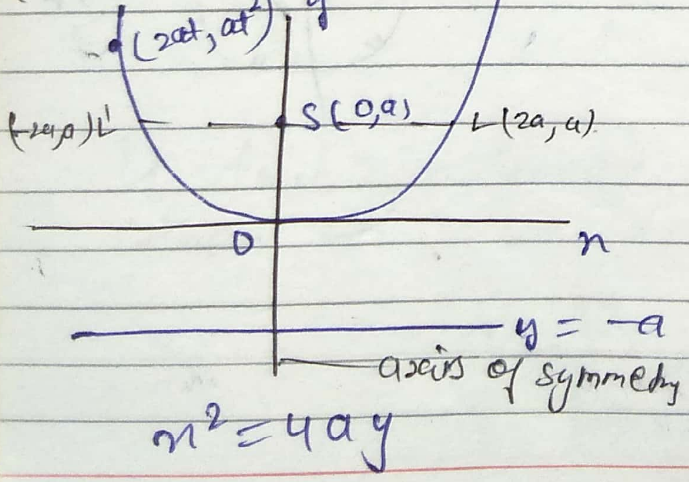
(I)



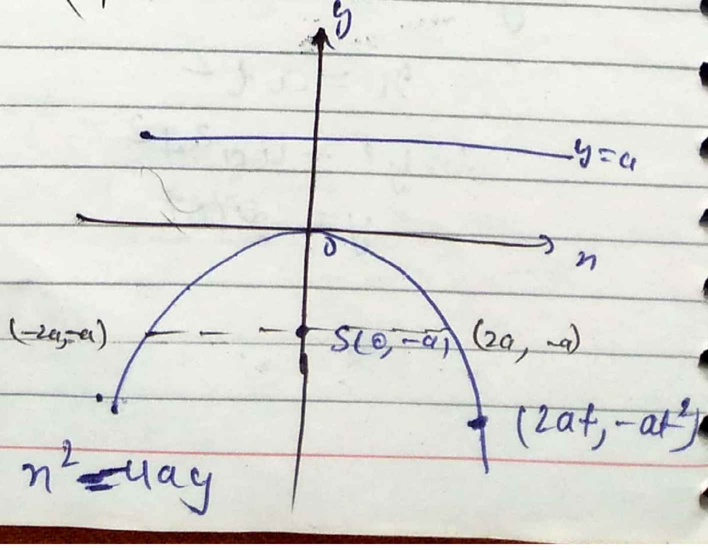
(II)



(III)



(IV)



* $\perp$ dist from focus to directrix $= \frac{1}{2}$ L.Rectum.

* Vertex is middel point of focus ~~&~~ / foot of directrix

* PoI of Axis & Directrix is called foot of directrix

$$y^2 = 4ax$$

$$(y-0)^2 = 4a(x-0)$$

(i) $y - 0 = 0 \Rightarrow$ axis of Parabola.

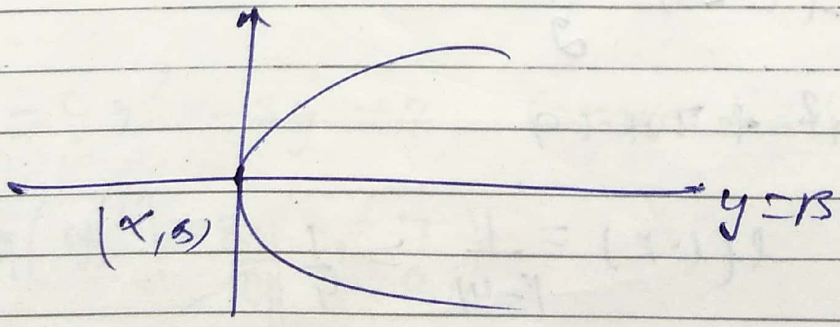
(ii) $y - 0 = 0$ & $x - 0 = 0$ Vertex of Parabola.

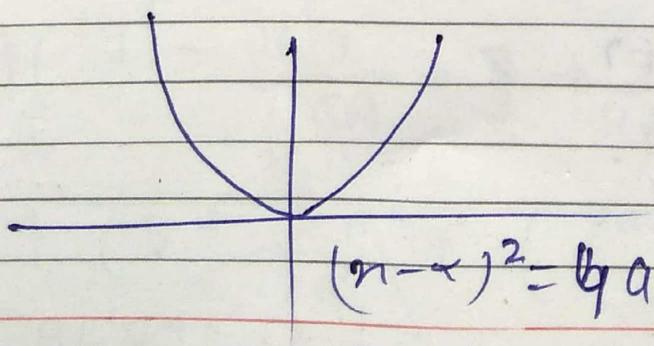
(iii) l (L.R) & Type of St. P.

* $$(y-\beta)^2 = 4a(x-\alpha)$$

axis $\rightarrow y - \beta = 0$ i.e. $y = \beta$

vertex $y - \beta = 0$ & $x - \alpha = 0$ (α, β) vertex





$$(x-\alpha)^2 = 4a(y-\beta)$$

$(y-\beta)^2 = 4a(x-\alpha)$

$y^2+\beta^2-2y\beta = 4ax - 4a\alpha$

$4ax = y^2 - 2y\beta + \beta^2 - 4a\alpha$

$x = \frac{1}{4a}\cdot y^2 - \frac{2y}{4a}\beta + \frac{\beta^2 - 4a\alpha}{4a}$

$l(L.R) = \frac{1}{|A|}$ $\boxed{x = Ay^2 + By + C}$ — for I & II

$\boxed{y = An^2 + Bn + C}$ for III & IV

$l.(L.R) = \frac{1}{|A|}$

III → $A>0$, IV → $A<0$

* $\begin{cases} y = 2x^2 - 3x + 4 \\ \text{III},\ l(L.R) = \frac{1}{2} \end{cases}$

$\begin{cases} y = -4x^2 + 7x + 9 \\ \text{IV} = l(L.R) = \frac{1}{|-4|} = \frac{1}{4} \end{cases}$

$\begin{cases} x = 9y^2 - 8y + 7 \\ \text{I},\ l(LR) = \frac{1}{9} \end{cases}$

$$\left\{\begin{array}{l} x = -7y^2 - 3 \\ \text{(ii)} \quad l(L \cdot R) = \frac{1}{7} \end{array}\right.$$

$*$ $y = ax^2 - bx + c$

$= a \cdot \left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$

$= a\left[x^2 + 2\frac{b}{2a} \cdot b + \frac{c}{a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right]$

$= a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b^2}{4a^2}\right)\right]$

$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = \frac{1}{a} \cdot \left(y - \frac{4ac - b^2}{4a}\right)$

$x = 9y^2 - 8y + 7$ Convert in Perfect sq

$9\left(y^2 - \frac{8y}{9} + \frac{7}{a}\right)$

$= 9\left(y^2 - \frac{16y}{2a} + \frac{7}{a} + \frac{y^2}{4a^2} + \frac{7}{a} - \frac{y^2}{4a^2}\right)$

$= 9\left(y + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b}{4a^2}\right)$

$= 9\left(y + \frac{b}{2a}\right)^2 +$

Ans $\quad x = 9y^2 - 8y + 7$

$$= 9\left[y^2 - 2\cdot\frac{4}{9}y + \frac{7}{9}\right)$$

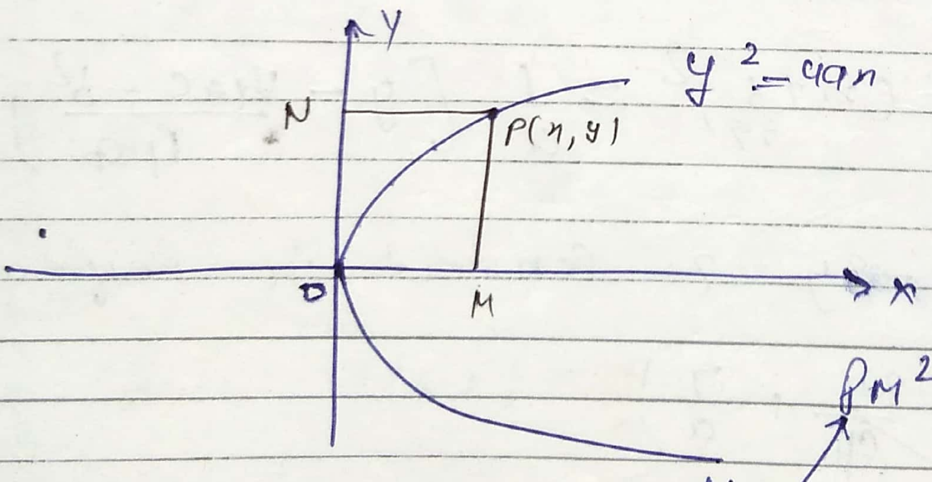
$$= 9\left[y^2 - 2\left(\frac{4}{9}\right)y + \frac{16}{81} + \frac{7}{9} - \frac{16}{81}\right)$$

$$= 9\left(\left(y - \frac{4}{9}\right)^2 + \frac{47}{81}\right)$$

$$= 9\left(y - \frac{4}{9}\right)^2 + \frac{47}{9}$$

$$\left(y - \frac{4}{9}\right)^2 = \frac{1}{9}\left[x - \frac{47}{9}\right)$$

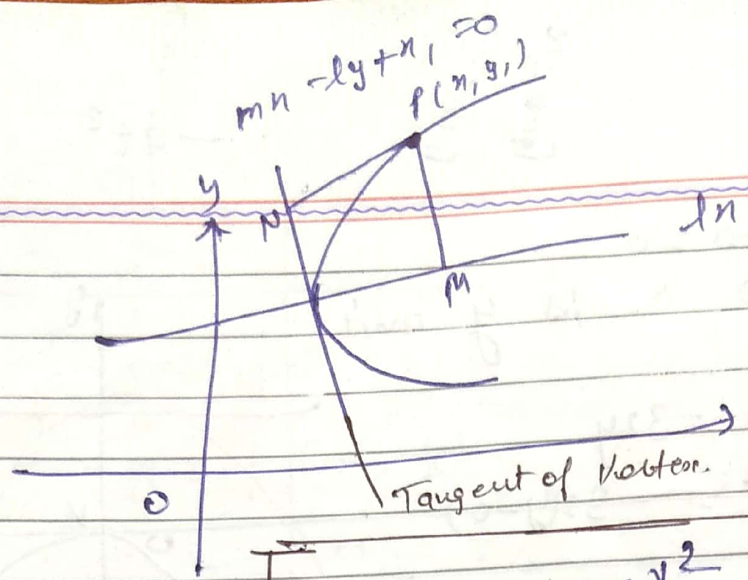
* <u>Re-defination of Parabola</u> :



$PM^2 = l(L \cdot R)PN$

dist. of any point P of the conic from its axis

dist of any point P on the conic from tangent at vertex

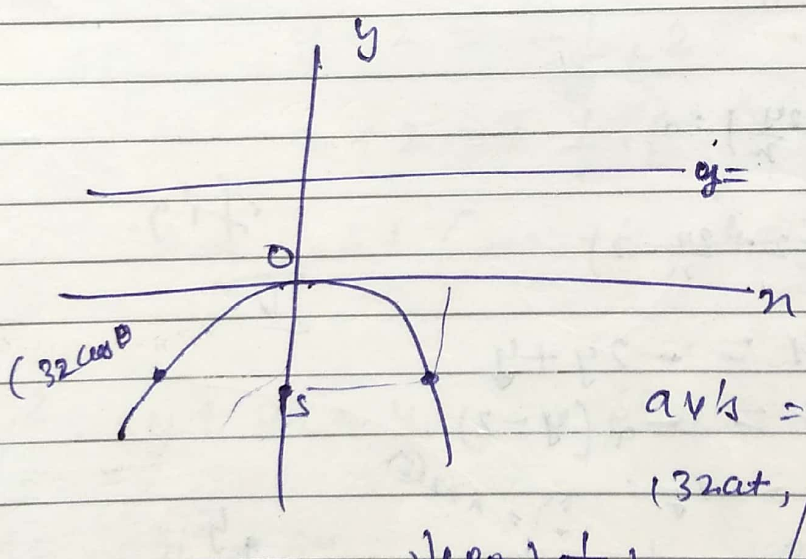


Very Imp → $\boxed{\left(\frac{lx + my + n}{\sqrt{l^2 + m^2}}\right)^2 = l(L \cdot R)\left|\frac{mx - ly + n_1}{\sqrt{m^2 + l^2}}\right|}$

Q. Find Everything

$x^2 + 4x$ $\qquad x^2 = -32y$

x^2



$32y + x^2$.

$axis =$

$(32at, /at^2,$

$y^2 = -32y^2$ $\quad$ $V(0,0)$ $\quad$ $\frac{1}{32})$

Paramatrix eq. form $\quad$ $32\cos\theta$

(L·R) Extremity Points..

$-at^2$

(1) axis $= x - 0 = 0$

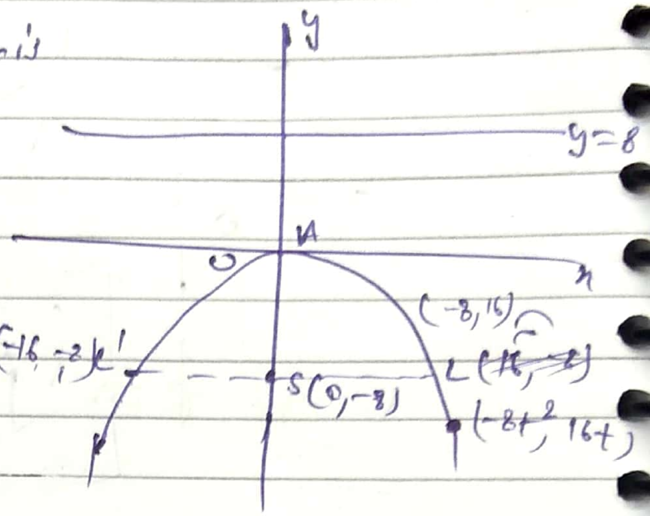
$x = 0$ is y axis

$x^2 = -32y$

$(x-0)^2 = -32(y-0)$

(2) vertex $x - 0 = 0$ & $y - 0 = 0$

i.e $= (0,0)$



(3) L(L.R) $= 32 = 4a$

$= a = 8$

Q. $x^2 + 4x + 2y = 0$

$= \cancel{x\left(x + 4 + \frac{2y}{x}\right) = 0}$

$\cancel{x\left(x + 2\frac{y^2}{x} + 2 + \frac{2y}{x} - 2\right)}$

Fig. IV

$x^2 + 4x + 4 = -2y + 4$

$(x+2)^2 = -2(y-2)$

$x^2 = -2y$. $x = x+2$

axis $x + 2 = 0$

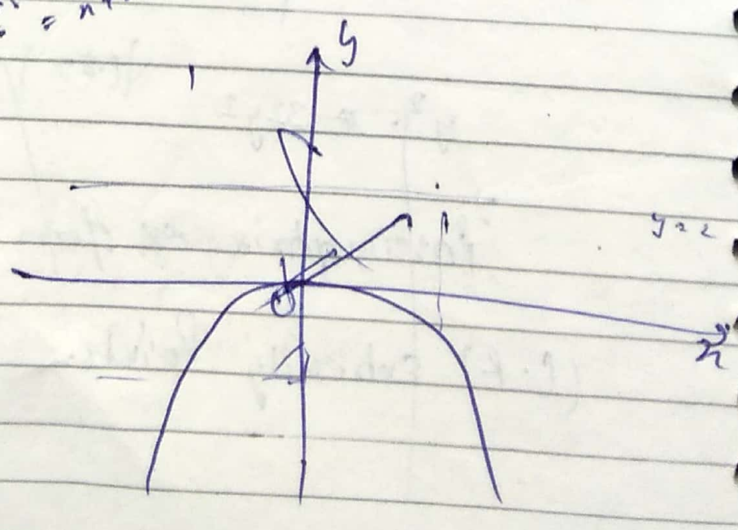
$x = -2$

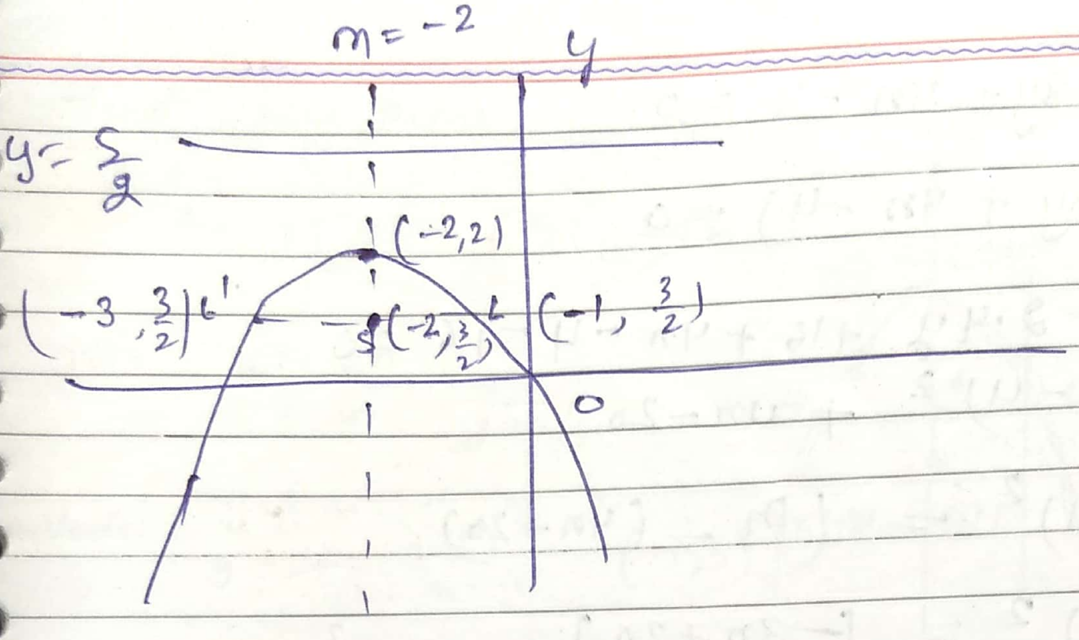
Vertex. $y - 2 = 0$ $x + 2 = 0$

$y = 2$ $x = -2$

$(-2, 2)$

L(L.R) $=$.





$2 - \frac{1}{2} = \frac{3}{2}$

$2 + \frac{1}{2} = \frac{5}{2}$

$4a = 2$

$a = \frac{1}{2}$

Parametric - non-std form.

$y - 2 = -\frac{1}{2}t^2$ $\qquad a = \frac{1}{2}$

$x + 2 = 2 \cdot \frac{1}{2} \cdot t$

Any point $\left(-2 + t,\ 2 - \frac{t^2}{2}\right)$

(1) $y^2 - 8y + 4x = 4$ Find every thing.

(2) $x = -y^2 + 6y$ "

① $y^2 - 8y + 4x - 4 = 0$

$(y^2 - 8y + 4x - 4) = 0$

$y^2 - 2\cdot 4y + 16 + 4x - 4 - 16 = 0$

$(y-4)^2 + 4x - 20$

$(y-4)^2 = (x - (4x - 20))$

$(y-4)^2 = (-3x + 20)$

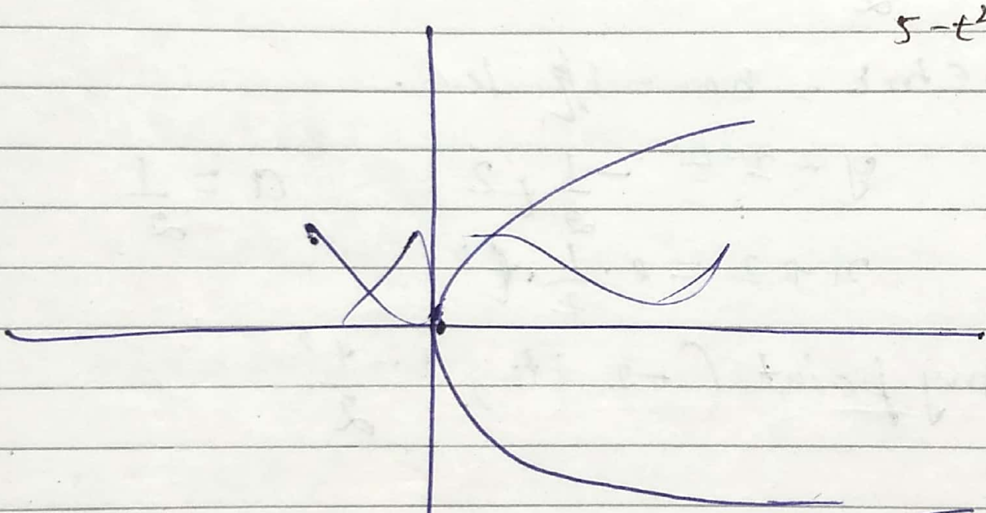
$(y-4)^2 = (-3x + 20)$

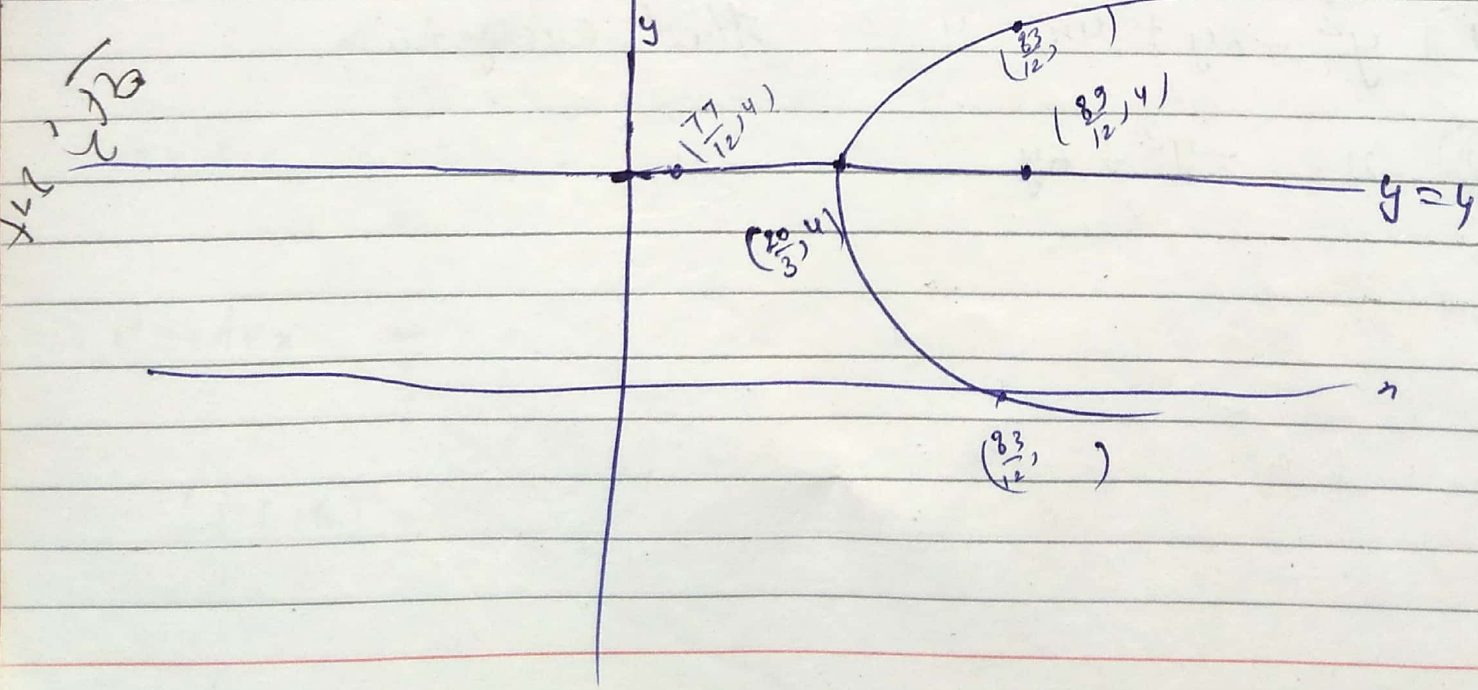
$(y-4)^2 = -4(x-5)$

$x - 5 = -1 \cdot t^2$

$x - 4 = 2 \cdot 1\, t$

$5 - t^2, \; 4 + 2t$



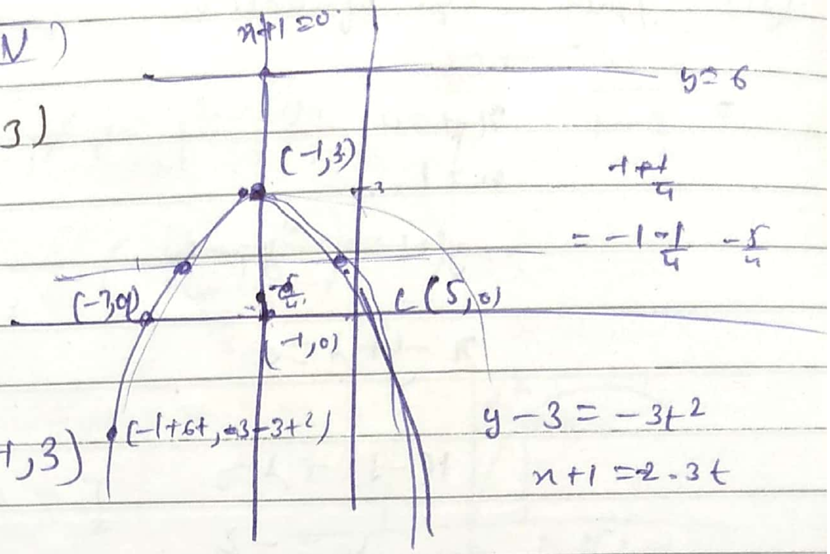


HW Revise - Tangent

$y = mx + c$ Circle Tangent

Q. find Every thing (IV)

$$(x+1)^2 = -(12(y-3))$$



axis $= x+1=0$

$x = -1$ y axis

Vertex $y-3=0$

$y = 3$ $x+1=0$

$x=-1$

$(-1,3)$

$y-3 = -3t^2$

$x+1 = 2\cdot 3t$

$(at^2, 2at)$

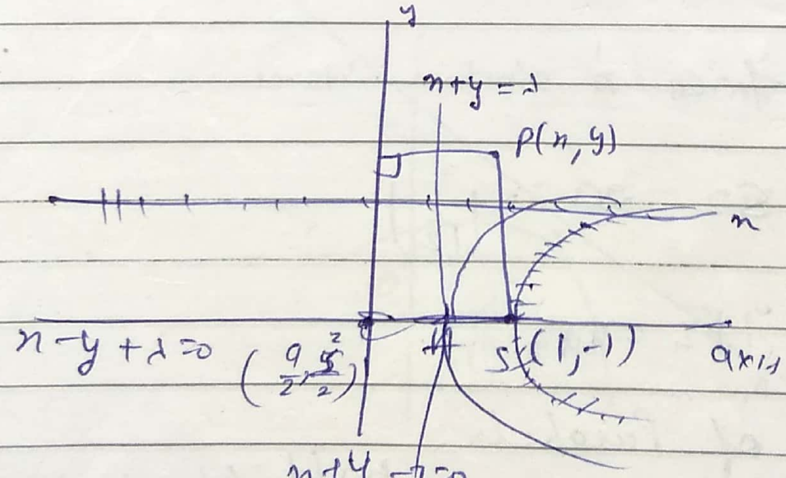
$4a = -12$

$a = \frac{-12}{4} = 3$

L.R $= \frac{1}{4a}$ $a = -3$

Q. find eqⁿ of Parabola where focus is $S(1,-1)$

Directrix $x+y-7=0$



$x = -y+7$

$x = -(y-7)$

$x+y-7=0$

$S(1,-1)$

$\left(\frac{x+y-7}{\sqrt{x^2+y^2}}\right)^2 = L.R \left|\frac{\cancel{x}+y+x}{\sqrt{x^2+y^2}}\right|$

$y =$

$$PS^2 = PM^2$$

$$(x-1)^2 + (y+1)^2 = \left(\frac{x+y-7}{\sqrt{2}}\right)^2$$

(ii) find eq^n of axis

$=$ $x \not{-} 1 = 0$

$x \neq 1$

$\cancel{y+1=0}$ $\cancel{y = -1}$

$x - y + \lambda = 0$

$1(-1) + \lambda = 0$

$\lambda = -2$

i.e $x - y - 2 = 0$

(iii) find foot of directrix

$x + y - 7 = 0$ solve

$x - y - 2 = 0$

(iv) find L.(L.R)

$\cancel{x+y-7=0}$ $=$ focus से directrix distance $= 2a$

$\cancel{x-y-2=0}$

$\cancel{8}2 = 2a = \left|\frac{7}{\sqrt{2}}\right|$

$L.(L.R) = 7\sqrt{2} = 49$

(v) find vertex of Parabola

mid pt of S & Z.

$\cancel{x-1=0}$ $= x = 1$ $(1, -1)$

$\cancel{y+1=0}$ $y = -1$

$= \left(\frac{\frac{9}{2}+1}{2}, \frac{\frac{5}{2}+(-1)}{2} \right)$

Write eqⁿ of ~~Vertex~~ tangent at vertex

$$\left(x - \frac{\left(\frac{9}{2}, 1\right)}{2}\right) = \left(\frac{-1-\frac{5}{2}}{1-\frac{9}{2}}\right)\left(y - \left(\frac{\frac{5}{2}+(-1)}{2}\right)\right)$$

$$m = \frac{-1-\frac{5}{2}}{1-\frac{9}{2}}$$

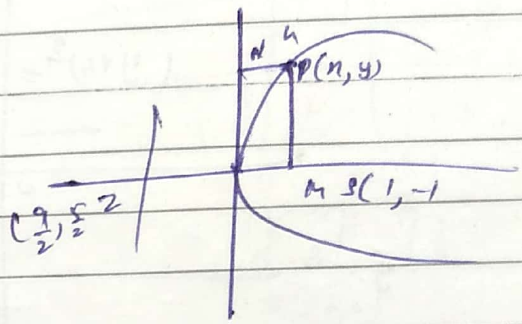
$$=$$

$$\left(\frac{11}{4}, \frac{3}{4}\right)$$

$x + y = \lambda$

i.e. $\lambda = \frac{7}{2}$

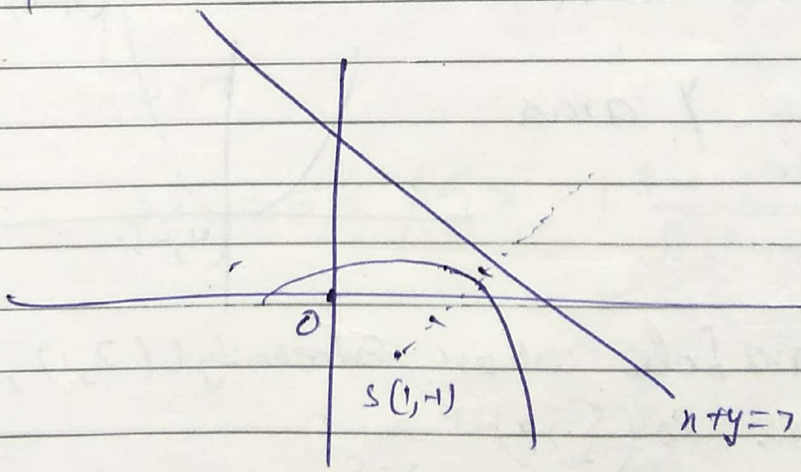
$\frac{11}{4} + \frac{3}{4} = \lambda \therefore \lambda = \frac{7}{2}$



(i) eqⁿ of P in another way

$PM^2 = (4a)\ PN$

$$\left|\frac{y-2-2}{\sqrt{2}}\right|^2 = 7\sqrt{2}\left|\frac{x+y-\frac{7}{2}}{\sqrt{2}}\right|$$

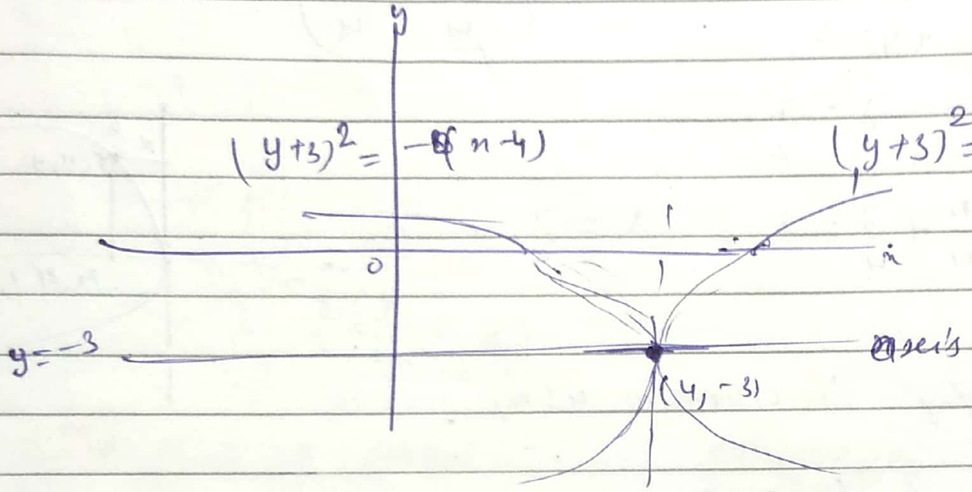


Que: find eq^n of P whose vertex is (4, −3)

A(4, −3)

whose L(L'R) = 8 & axis is

(i) axis is ∥ to x-axis



= A(4, −3)

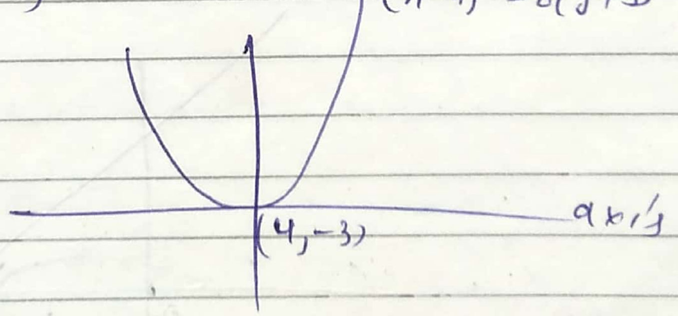
$4x + 8y = 0$

$(x-4)^2 + (y+3)^2 = -8$ / $4a^2$

$(y+3)^2 = -8(x-4)$

$(x-4)^2 = 8(y+3)$

⇒ Parallel to y axis

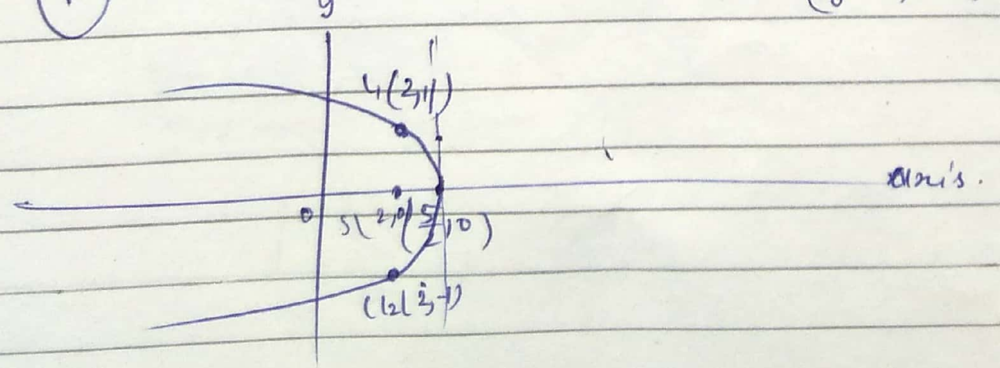


Que: find Parabola whose Extremity $L_1(2,1)$, $L_2(2,-1)$ and vertex is $\left(\frac{5}{2}, 0\right)$

$x = \frac{5}{2}$

(II)

$(y-0)^2 = -2\left(x - \frac{5}{2}\right)$



www.iiae

Q. find length of L.(L.R) of P.

$4a = \frac{25}{}$

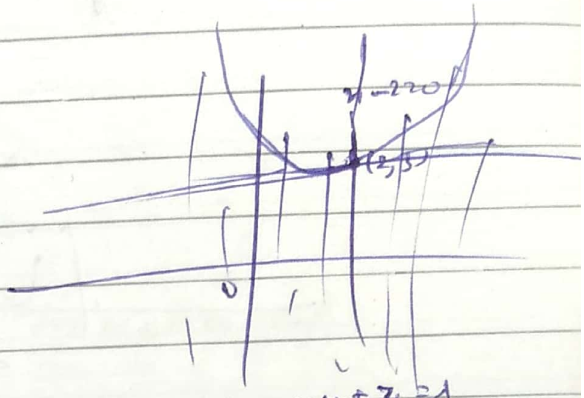
$a = \frac{25}{4}$

$$25[(x-2)^2+(y-3)^2] = [3x-4y+7]^2$$

$x-2=0 \quad x=2$

$(y-3)=0$

$y=3$



$3x-4y+7=\lambda$

$6-12+7=\lambda$

$\lambda=1$

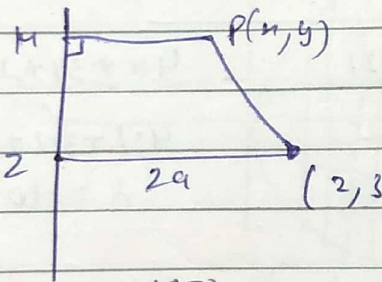
$$(x-2)^2+(y-3)^2 = \frac{1}{25}(3x-4y+7)^2$$

$$(x-2)^2+(y-3)^2 = \left(\frac{3x-4y+7}{5}\right)^2$$

$$PS^2 = PM^2$$

$$=$$

$$= \left(\frac{3x-4y+7}{\sqrt{5}}\right)^2$$

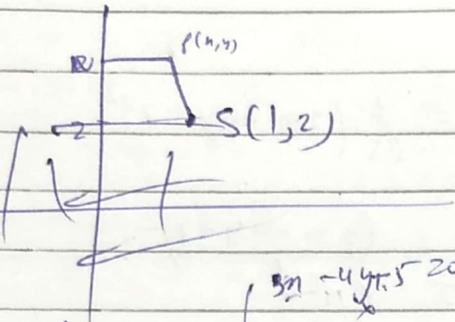


$$SZ = 2a = \left|(3,2)\right| = \left|\frac{3\cdot2-4\cdot3+7}{\sqrt{3^2+4^2}}\right|$$

$$= \frac{1}{5}$$

$$4a = \frac{2}{5}$$

Q. for some Parabola tangent at Vertex $3x - 4y = 5$ & $S(1,2)$ find eq^n of parabola



$3x - 4y = 5$

$-4y = 8 - 3x + 5$

$-4y = 3x - 5$

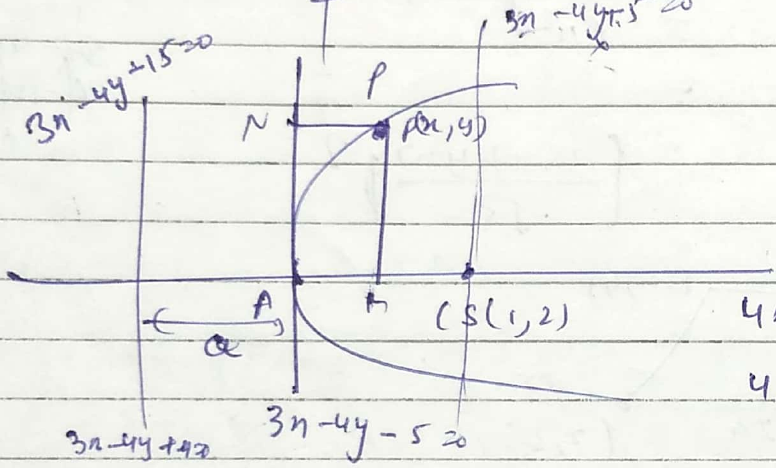
$$\boxed{(4y-1)^2 = 5(3x-2)^2}$$

$4y - 1 = 0$

$y = \frac{1}{4}$

$3x = 5$

$x = \frac{5}{3}$



$4x + 3y + \lambda = 0$ i.e $4x + 3y - 10 = 0$

$4\cdot 1 + 3\cdot 2 + \lambda = 0$

$\lambda = -10$

$$SA = a = \left| \frac{3\cdot 1 - 4\cdot 2 - 5}{5} \right|$$

$$= 2$$

$$PM^2 = l(L\cdot R)\cdot PN$$

$$\left(\frac{4x + 3y - 10}{5}\right)^2 = 8\left(\frac{3x - 4y - 5}{5}\right)$$

Hyperbola
New Rehabilitation — Point of trisection 2 : 1

A —— 1 : 2 —— B

A —— 2 : 1 —— B

(0;1) ⇒ 6, 7, 8, 9, 10, 11
H.W

(M-2)

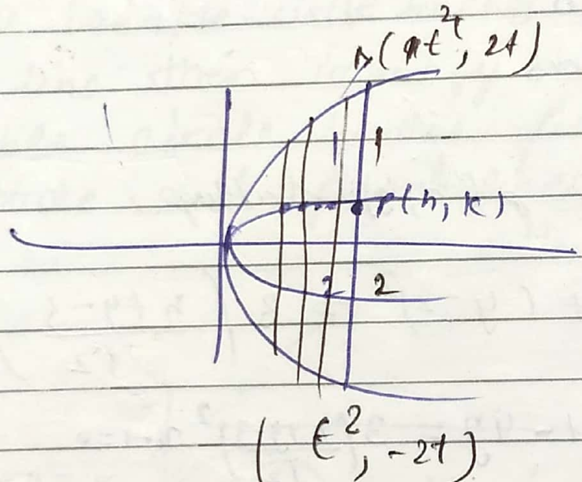
$$\left|\frac{M-(-5)}{5}\right| = 2$$

$$M = \pm 10 - 5$$

$$M = \begin{cases} 5 \\ -15 \end{cases}$$

$3x - 4y - 15 = 0$

Que: Find locus of Point of trisection of double ordinate of P
$y^2 = 4x.$ $a = 1$



Be Moving Point

$h = t^2$

$$k = \frac{1(-2t) + 2 \cdot 2t}{1+2} = \frac{2t}{3}$$

Eliminate t $\qquad t = \frac{3k}{2}$

$(h, k) \to (x, y)$ $\qquad h = \left(\frac{3k}{2}\right)^2 \Rightarrow 9k^2 = 4h$

$$9y^2 = 4x$$

Q. Indentify locus of Conic.

$$\sqrt{\frac{x}{3}} + \sqrt{\frac{y}{2}} = 1$$

$\frac{x}{3} + \frac{y}{2} = 1$

$$\frac{x}{3} + \frac{y}{2} + 2\sqrt{\frac{xy}{6}}$$

$= \frac{x}{3} + \frac{y}{2} + 2\sqrt{\frac{xy}{6}}$

$$2\sqrt{\frac{xy}{6}} = \left(1 - \frac{x}{3} - \frac{y}{2}\right)$$

$$4 \cdot \frac{xy}{6} = 1 + \frac{x^2}{9} + \frac{y^2}{4} - \frac{2x}{3} - y + \frac{xy}{3}$$

$a = \quad b = \quad h = \quad f = \quad g = \quad 4 =$

Parabola $\quad \Delta \neq 0$

$h^2 = ab$

Q. Identify locus of $P(x, y)$ satisfying

$$(x-1)^2 + (y-2)^2 = 3\left(\frac{x+y-3}{\sqrt{2}}\right)^2$$

$\cancel{x^2 + 1 - 2x + y^2 + 4 - 4y = 3\left(\frac{x+y-3}{\sqrt{2}}\right)^2}$ $\quad x-1=0 \quad y-2=0$

$x = 1, y = 2 \quad (1, 2)$

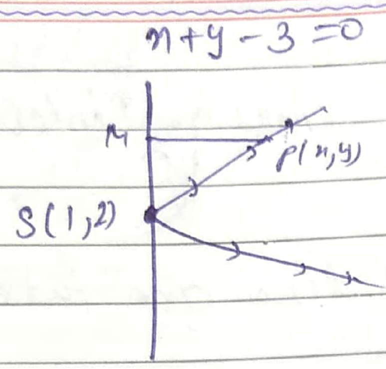
$4a = 3 \quad a = 3/2$

$\cancel{x^2 + 1 - 2x + y^2 + 4 - 4y = 3\left(\frac{x^2 + y^2 + 2xy - (3)^2}{2}\right)}$

$\cancel{x^2 + 1 - 2x + y^2 + 4 - 4y = \frac{3x^2 + 3y^2 + 6xy - 27}{2}}$

$\cancel{2x^2 + 2 - 4x + 2y^2 + 8 - 8y = 3x^2 + 3y^2 + 6xy - 27}$

Ans:



$PS = \sqrt{3}PM$

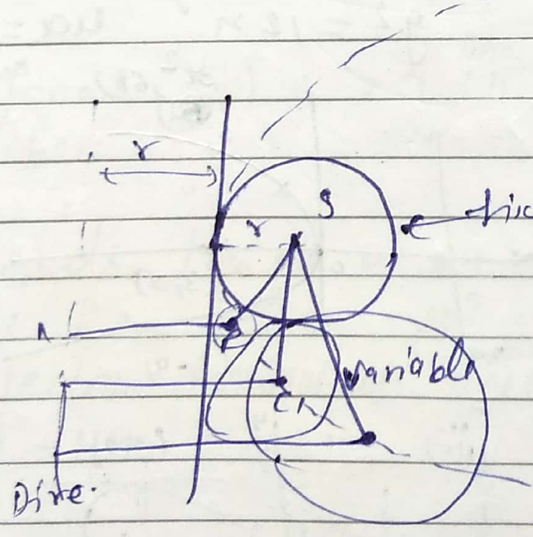
$\frac{PS}{PM} = \sqrt{3}$

Very Imp.

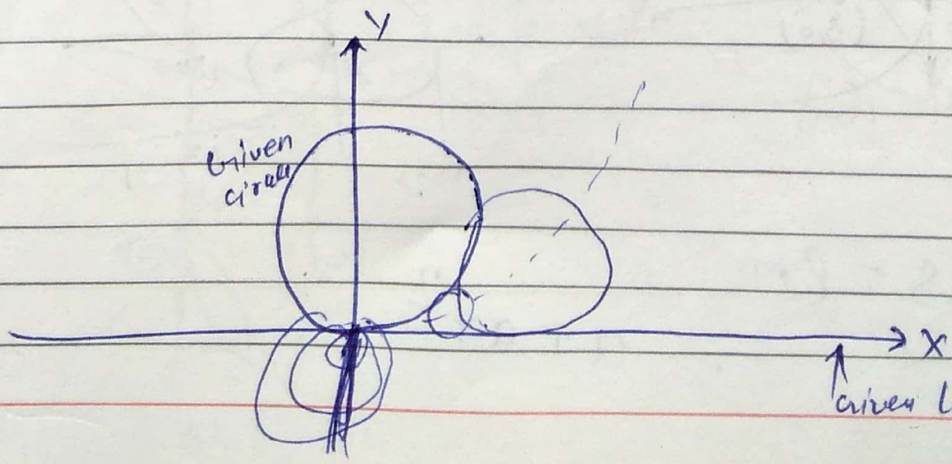
Important

Que. If a variable circle touches a fixed circle and a fixed line then identify the locus of centre of variable circle in the following condition

(1) ~~fixed circle~~ and fixed lines are ~~no~~ intersecting and touches fixed circle



$CC_1 = r_1 + r$.

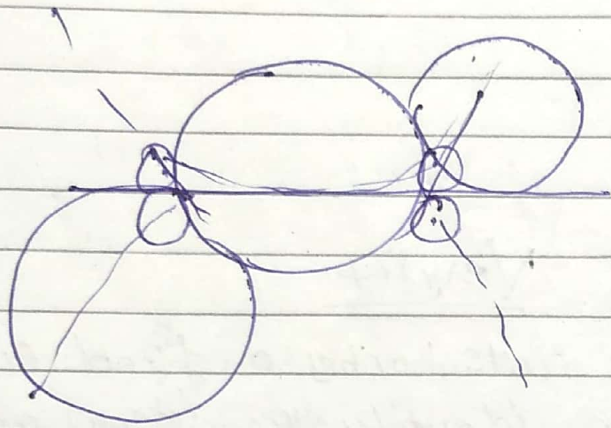


Ans Parabola and a ray

Q: fixed circle and fixed lines are non intersecting.

Ans Locus – Parabola

③ fixed circle and fixed line are intersecting



Ans Pair of Parabola

Que find Points on Parabola having focal Radius

(i) 4 — ②

(ii) 3 — $P(0,0)$

(iii) 2 — ✗

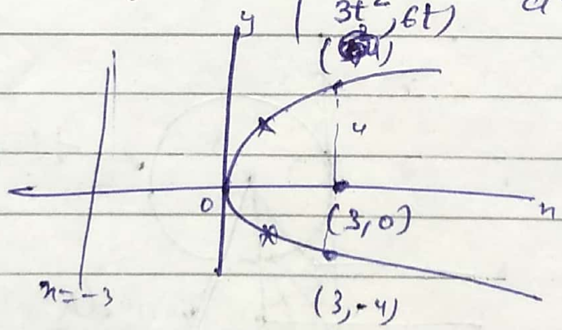
use Parametric

$y^2 = 12x$ $\quad 4a = 12$

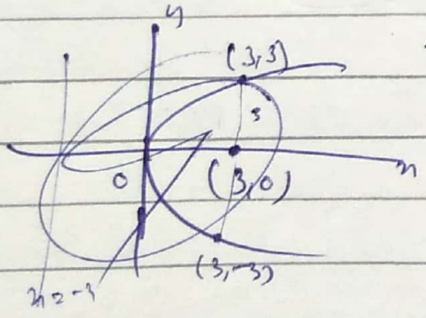
$a = 3$

$(3t^2, 6t)$

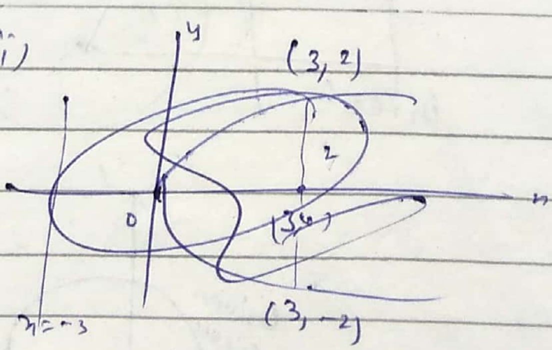
(i)



(ii)



(iii)



$PS = PM$

$= a + at^2$

(i) $a + at^2 = 4$

$3t^2 + 3 = 4$

$3t^2 = 1$

$t = \pm \frac{1}{\sqrt{3}}$

$P(3t^2, 6t)$

$\boxed{P\left(1, \pm \frac{6}{\sqrt{3}}\right)}$

* minimum focal dist. for the ~~Parabola~~ of any pt. on the P is a i.e vertex.

** if focal dist < a (case iii) then no such pt is Possible.

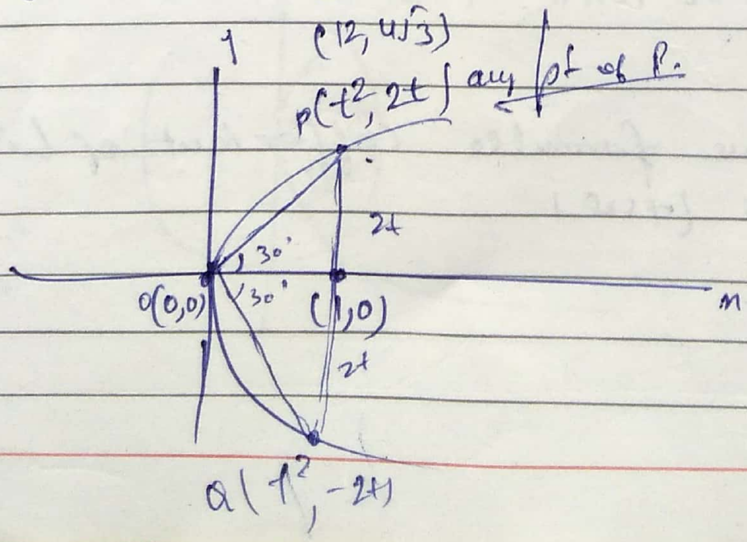
** if focal dist. > a (case i) then 2 such Pt is Possible

Q. find side Length of an equllateral △ which inside P.: ~~$y^2 = 4x$~~ $y^2 = 4x$. and whose One vertex consided vertex consider vertex of P.

$y^2 = 4x$. $a = 1$

$4a = 4$
$a = 1$

$(8\sqrt{3})$



$PQ = 4t = OP$.

$$4t = \sqrt{(t^2-0)^2 + (2t-0)^2}$$

$$16t^2 = t^4 + 4t^2$$

$$t^4 - 12t^2$$

$$t^2 = 0,\ t^2 = 12$$

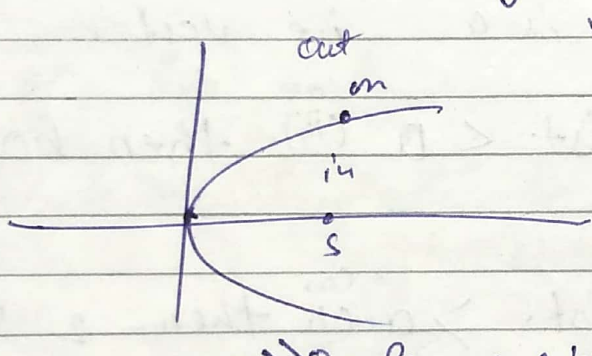
$$t = 2\sqrt{3}$$

side $= 4t = 8\sqrt{3}$

w.r.t

* **Position of point relative to Parabola !**

$P(x_1, y_1)$ $\qquad$ $S(x_1, y_1) = y^2 - 4ax$.

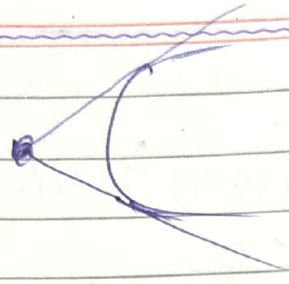


$S(x_1, y_1)$
- $> 0 \Rightarrow P(x_1, y_1)$ lie outside Parabola
- $< 0 \Rightarrow$ " " inside "
- $= 0 \Rightarrow$ " " on "

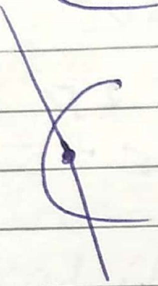
सबसे पहले Big Power write. then Power subtra—

* By appling above formulla cofficient of highest power will be (+ve)

*

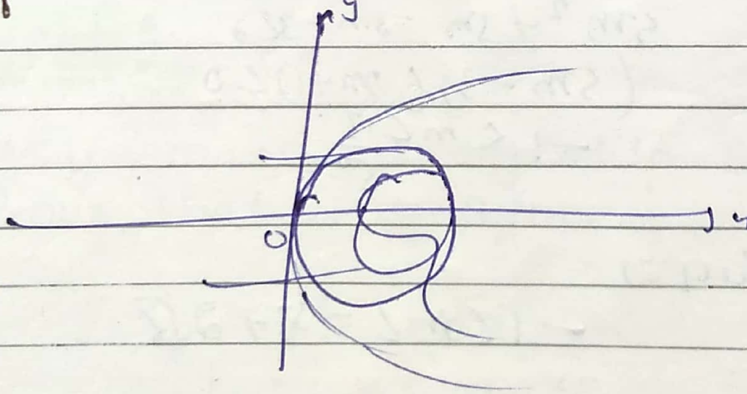
(1)  when pt lie outside the curve the two Real & dist. tangent will be drawn

(2)  if Point on the curve Real and Coincident tangents

(3)  if pt. inside the curve No. tangent will be draw

*

Que if $(-2m, m+1)$ is an interior pt of smaller region Bounded by circle and Parabola then find Possible Values of m.



$4a^2$

$x^2+y^2=4$ &

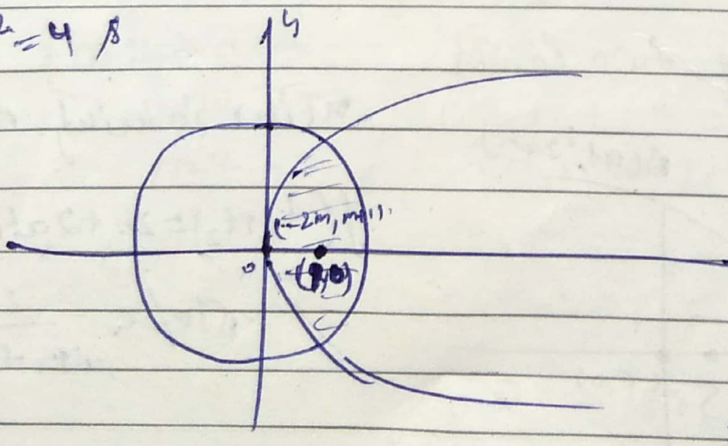
$y^2=4x$

$4a=4$

$a=1$

$-2m$

$(-2m)$



$-2m$

$(-2m)^2+(m+1)^2=4$

$4m^2+m^2+1+2m=$

$4m^2+m^2+2m=3$

$S(x,y) = y^2 - 4x$

$S(-2m, m+1) = (m+1)^2 - 4(-2m)$

$m^2 + 1 + 2m + 8m < 0$

$m^2 + 10m + 1 < 0 \quad \text{—(i)}$

$m^2 + 10m + 25 < 24$

$(m+5)^2 < (2\sqrt{6})^2$

$-2\sqrt{6} < m+5 < 2\sqrt{6}$

$-2\sqrt{6} - 5 < m < -5 + 2\sqrt{6}$

$S(x,y) \quad x^2 + y^2 - 4$

$S(-2m, m+1) = (m+1)^2 + 4m^2 - 4 < 0$

$= m^2 + 1 + 2m + 4m^2 - 4 < 0$

$5m^2 + 2m - 3 < 0$

$5m^2 + 5m - 3m - 3 < 0$

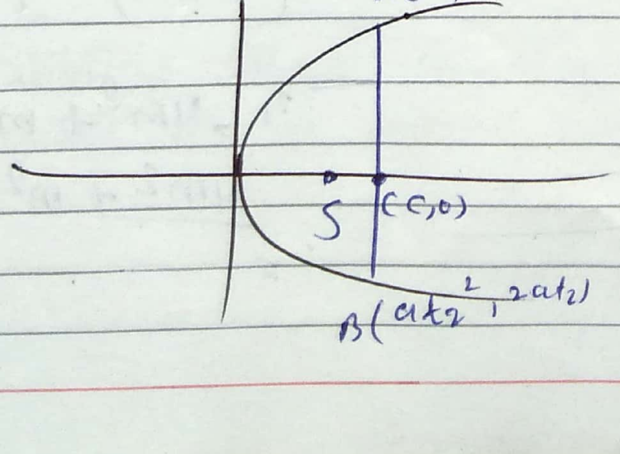
$(5m-3)(m+1) < 0$

$-1 < m < \frac{3}{5}$

Common Answer ⇒

$-1 < m < -5 + 2\sqrt{6}$

Que: Chord joining two Points.



Line joining AB

$y(t_1 + t_2) = 2x + 2at_1t_2$

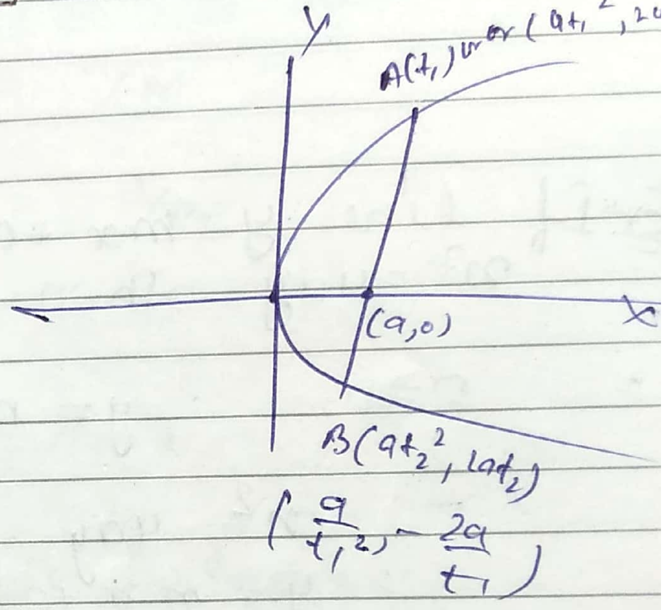
$\text{slope} = \frac{2}{t_1 + t_2}$

Straight line / Circle | Pair of circle / Homogenisation

Suppose line joining AB meet axis (c, 0) then

$y=0, \; x=c$

$$\boxed{t_1 t_2 = -\frac{c}{a}}$$ Important



(1) if $t_1 t_2 = -1$

$c = a$.

i.e AB become focal chord

$t_2 = -\frac{1}{t_1}$

* **Tangent :**

Ex: if line $y = mx + c$ is tangent to curve $y^2 = 4ax$ then prove that $c = a/m$

$C : - y^2 = 4ax$

$L = y = mx + c$

$(mx + c)^2 = 4ax$

$m^2x^2 + 2cmx + c^2 = 4ax$

$m^2x^2 + (2cm - 4a)x + c^2 = 0 \; \begin{matrix} x_1 \\ x_2 \end{matrix}$

$D = (2cm - 4a)^2 - 4m^2c^2$

$16a^2 - 16acm$

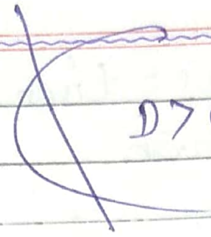
$= 16a(a - cm.$

$D = b^2 - 4ac$

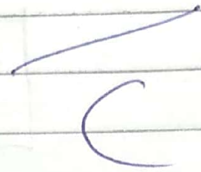
tangent $D = 0$

$a - cm = 0$

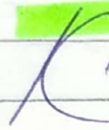
$c = \frac{a}{m}$



$D > 0$



$D < 0$



$D = 0$

Ex: If line $y = mx + c$ is tangent to curve $x^2 = 4ay$ then prove that $c = -am^2$

= $D = 0$ $\quad y = mx + c$

$$\left.\begin{aligned} x^2 &= 4ay \\ y &= mx + c \end{aligned}\right\} \text{①-②}$$

$x^2 = 4a(mx + c)$

$x^2 - 4amx - 4ac = 0$

tangent $D = 0$

$16a^2m^2 + 16ac = 0$

$16a[am^2 + c] = 0$

$c = -am^2$

* If line $y = mx + c$ is tangent to the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then prove that $c^2 = a^2m^2 + b^2$

$y = mx + c$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\qquad \frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$

$b^2x^2 + a^2(mx + c)^2 = a^2b^2$

$$(b^2+a^2m^2)x^2 + (2a^2cm)x - a^2b^2 = 0$$

$$D = 0$$

$$c^2 = a^2m^2+b^2$$

Curve	$y = mx+c$ is tangent then
* $y^2 = 4ax$	$c = a/m$
* $y^2 = -4ax$	$c = -a/m$
* $x^2 = 4ay$	$c = -am^2$
* $x^2 = -4ay$	$c = am^2$
* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$c = \pm\sqrt{a^2m^2+b^2}$
* $x^2+y^2 = a^2$	$c = \pm a\sqrt{m^2+1}$
* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$c = \sqrt{a^2m^2-b^2}$
* $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$	$c = \pm\sqrt{-a^2m^2+b^2}$

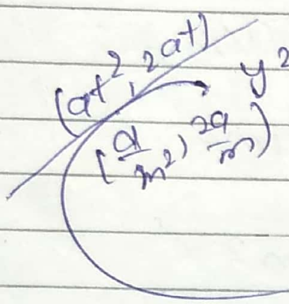
Different forms of tangent in case of Parabola :

(i) **Slope form :**

Parabola $y^2 = 4ax$

slope of tangent m

eq^n of tangent $y = mx + \frac{a}{m}$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$



$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2at} = \frac{1}{t}$

$m = \frac{1}{t}$

eg: Find eq^n of tangent of Parabola $y^2 = 16x$ of slope 2.

$y = mx + \frac{a}{m}$ $\qquad 4a = 16$, $a = 4$

$y = 2x + \frac{a}{2}$ $a=4$ $\qquad c = \frac{a}{m}$

$y = 2x + \frac{4}{2}$ $\qquad y = 2x + 2.$ $\qquad c = 2.$

Ex: if line $y = mx + 3$ tangent to Curve $y^2 = 8x$

$c = \frac{a}{m} = \cancel{\frac{2}{3}}$

$c = \frac{a}{m}$ $\qquad 3 = \frac{2}{m}$ $\qquad m = \frac{2}{3}.$

Que: find eq^n of tangent to Parabola
$x^2 = 20y$
which is parallel to line $y = -2x + 100$
or
$\perp$ular to line $x - 2y + \pi = 0$
or
inclined at arc $\tan(-2)$ with x axis

$x^2 = 20y$ $\qquad\qquad$ $\tan^{-1}(-2) = \theta$

$\tan\theta = -2$

$m = -2$

eq^n of tangent , $a = 5$

$y = mx - am^2$

$y = -2x - 5(-2)^2$

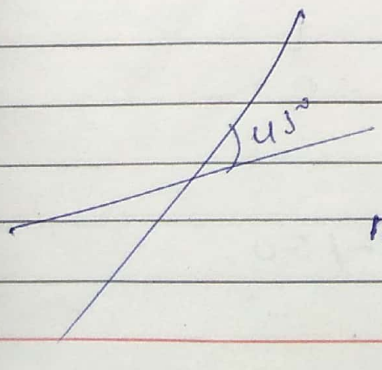
$y = -2x - 5(4)$

$y = -2x - 20$

Que: find eq^n of tangent to Parabola
$y^2 = 4x$ which makes $45°$ angle with line $2x - y + 1000 = 0$

$y^2 = 4x$ $\qquad\qquad$ $a = 1$,

$\tan 45° = 1$ $\qquad\qquad$ $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$



$1 = \left|\frac{m - 2}{1 + 2m}\right|$ $\qquad\qquad$ $m = -3$ or $\frac{1}{3}$

$m = -3$ $\qquad\qquad$ $y = -3x + \frac{1}{-3}$

$m = \frac{1}{3}$ $\qquad\qquad$ $y = \frac{1}{3}x + 3$

Que! find eqⁿ of Common tangent to the Parabolas $y^2 = 4x$ and $x^2 = -32y$

$y^2 = 4x$ $\qquad\qquad$ $x^2 = -32y$

$y = mx + \boxed{c}$

$y^2 = 4x$ $\qquad\qquad$ $x^2 = -32y$

$c = \frac{1}{m}$ $\qquad\qquad$ $c = 8m^2$

$\frac{1}{m} = 8m^2 \Rightarrow m^3 = \frac{1}{8}$

$m = \frac{1}{2}$ $\qquad$ $c = 2$

$y = \frac{1}{2}x + 2$

Que! find eqⁿ of tangent to the Parabola $y^2 = 4ax$ which passes through Point $(-1, 2)$

$y - 2 = m(x + 1)$

$y - 2 = m(x + 1)$

$y = mx + \boxed{2 + m}$

$c \Rightarrow$

$c = \frac{a}{m}$

$2 + m = \frac{1}{m} \Rightarrow m^2 + 2m - 1 = 0$

$$m = \frac{-2 \pm \sqrt{8}}{2} = -1 + \sqrt{2} \text{ or } -1 - \sqrt{2}.$$

tangent

$$y = (-1 + \sqrt{2})x + 2 + (-1 + \sqrt{2})$$

or

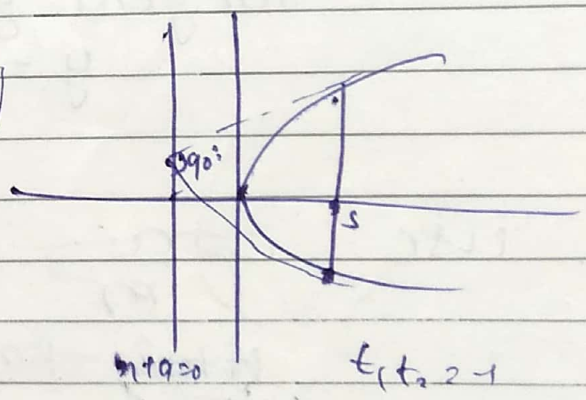
$$y = (-1 - \sqrt{2})x + 2 + (-1 - \sqrt{2})$$

Que: find the eq^n of tangent of the Parabola $y^2 = 12x$. which passes point $(2, 5)$

$y^2 = 12x \qquad 4a = 12 \qquad a = 3$

$$y - 5 = m(x - 2)$$

$$y = mx \underbrace{\boxed{-2m + 5}}_{c}$$



$$C = \frac{a}{m}$$

$$-2m + 5 = \frac{a}{m}$$

$$= -2m^2 + 5m - a = 0$$

$$= -2m^2 + 5m - 3 = 0$$

$$2m^2 - 5m + 3 = 0$$

$$2m^2 - 2m - 3m + 3 = 0$$

$$2m(m - 1) - 3(m - 1) = 0$$

$m - 1 = 0 \qquad 2m - 3 = 0$

$m = 1 \quad , \quad m = \frac{3}{2}$

$\frac{a}{n^2}, \frac{2a}{n}$

$y = n + 5 - 2$ or

$y = \frac{3}{2}n + 5 - 3$

Qu: find the locus of point from which two tangent drawn on the parabola $y^2 = 4an$ having slope m_1 & m_2 satisfying the condition

(1) $m_1 m_2 = -1$

(2) $m_1 m_2 = 1$

(3) $m_1 + m_2 = 0$

(4) angle b/w tangent α.

Let Point (h, k)

tangent $y - k = m(n - h)$

$y = mn + \underbrace{k - mh}_{c}$

use $c = \frac{a}{m}$

$hm^2 - km + a = 0 \; \begin{matrix} m_1 \\ m_2 \end{matrix}$

$m_1 + m_2 = \frac{k}{h}$

$m_1 m_2 = \frac{a}{h}$ $\quad |m_1 - m_2| = \left|\frac{\sqrt{k^2 - 4ah}}{h}\right|$

(1) $m_1 m_2 = -1$

$\frac{a}{h} = -1$

$h = -a$

Locus $\boxed{x = -a}$

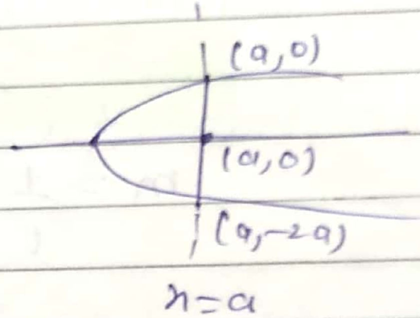
Directer circle of $y^2 = 4ax$

(2) $m_1 m_2 = 1$

$\frac{a}{h} = 1$

$a = h$

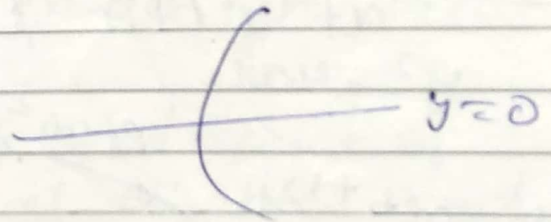
Locus $x = a$, $|y| > 2a$



(3) $m_1 + m_2 = 0$

$\frac{k}{h} = 0$ $\quad k = 0$

Locus $y = 0$, $x < 0$



(4) $\tan\alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$(1 + m_1 m_2)^2 \tan^2\alpha = (m_1 - m_2)^2$

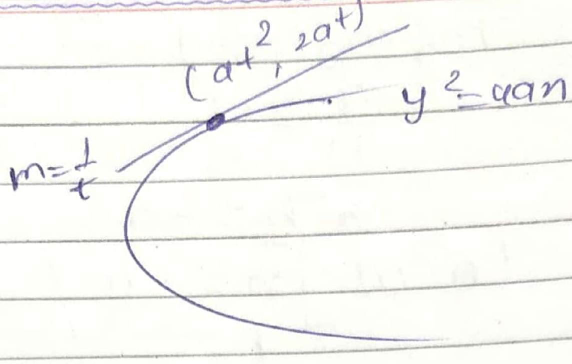
$\left(1 + \frac{a}{h}\right)^2 \tan^2\alpha = \frac{k^2 - 4ah}{h^2}$

= Locus $(ax + a)^2 \tan^2\alpha = y^2 - 4ax$

Parametric form :

Parabola $y^2 = 4ax$

Point $P(at^2, 2at)$



Eqⁿ of tangent

$$\boxed{\begin{gathered} T = 0 \\ ty = x + at^2 \end{gathered}}$$

$m = \frac{1}{t}$

$x^2 \to xx_1$

$y^2 \to yy_1$

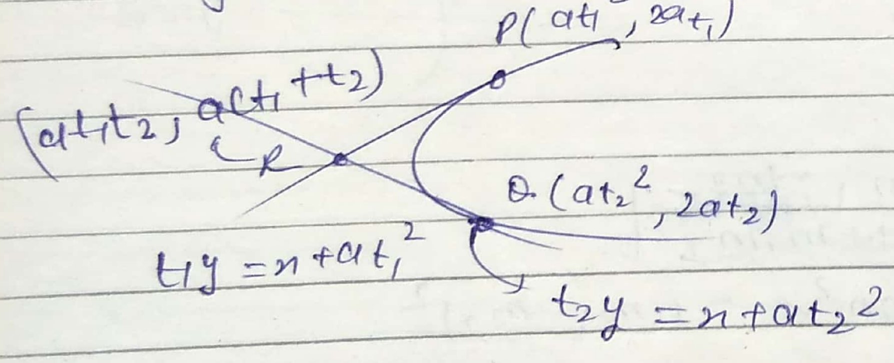
$x \to \frac{x + x_1}{2}$

$y \to \frac{y + y_1}{2}$

$xy \to \frac{xy_1 + yx_1}{2}$

$c \to c$

Note : ① Point of intersection of tangents at $P(t_1)$ & $Q(t_2)$ to parabola $y^2 = 4ax$

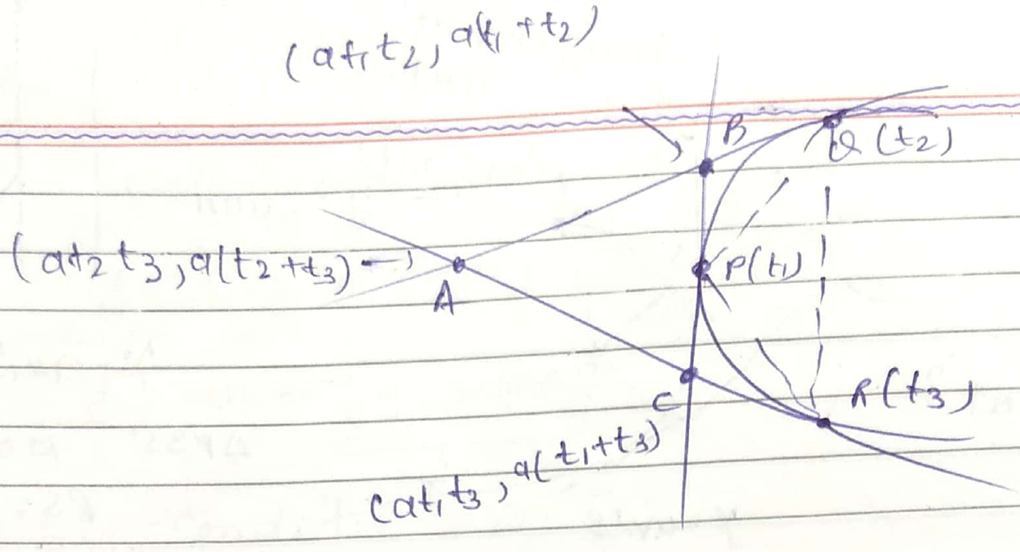


$\Rightarrow R(at_1t_2, a(t_1 + t_2))$

→ x coordinate of P, R, Q are in G.P

→ Y coordinate of P, R, Q are in A.P

(2)



triangle formed by tangent ΔABC

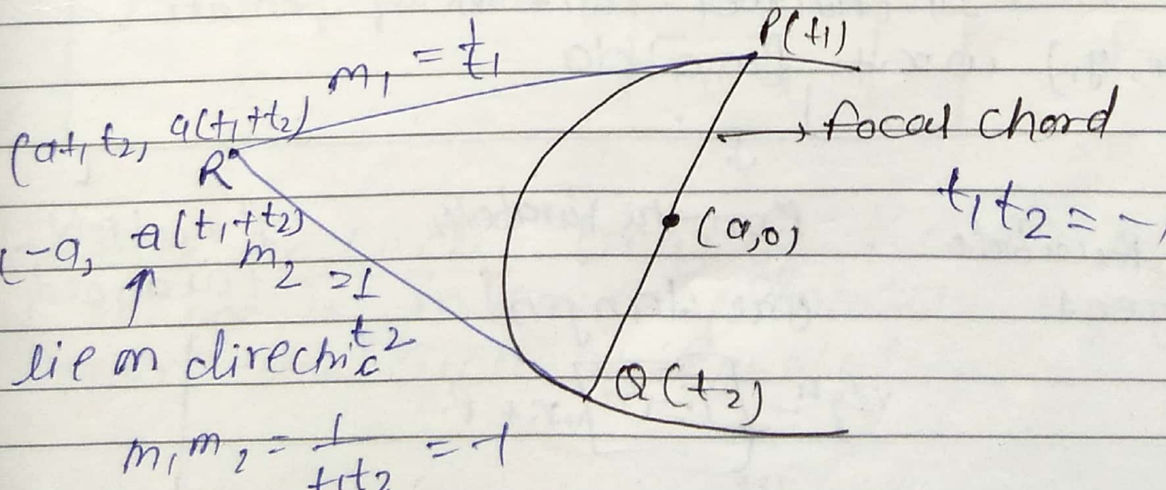
point of contacts ΔPQR

Area of $\Delta PQR = 2\,\Delta ABC$

ortho centre of ΔABC lies on directrix

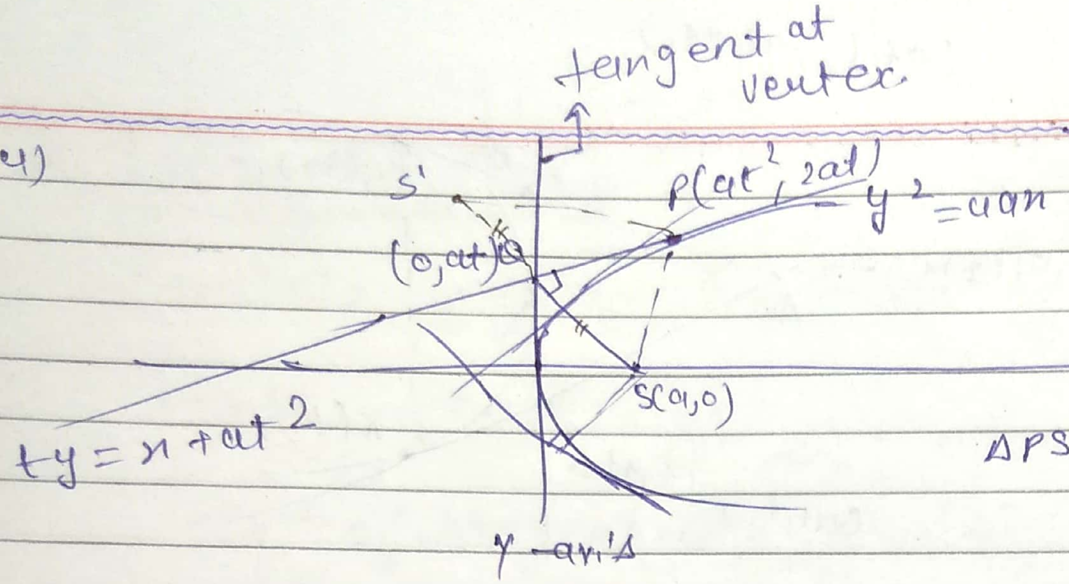
in all parabolas.

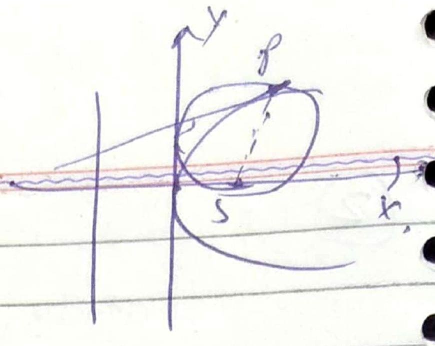
(3) Point of Intersection at end point of focal chord intersect at Directrix and these tangents are $\perp$cular to each other.



$m_1m_2 = \frac{1}{t_1t_2} = -1$

(4)





$\Delta PSS'$ is oscller
$PS = PS'$

* Any tangent to a Parabola and Lr on it from the focus meet on the tangent at the Vertex Q

* Image of focus w.r.t any variable tangent of Parabola lies on directrix of Parabola

eg: Let the tangent to parabola $y^2 = 4ax$. meet the axis T and tangent at vertex A in Y. if rectangle TAYG is completed (H.W) then find locus of G.

Point form:
Ist checked Position of Point $P(x_1, y_1)$ w.r.t Parabola

P inside Parabola
No tangent

P on the Parabola
one tangent
$eq^n = \boxed{T=0}$ w.r.t P.

T=0
P

P outside the Parabola

→ Let eqⁿ of tangent

$$y - y_1 = m(x - x_1)$$

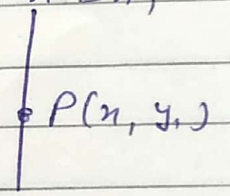
$$y = mx + \underbrace{\boxed{y_1 - mx_1}}_{c}$$

apply condition on c.

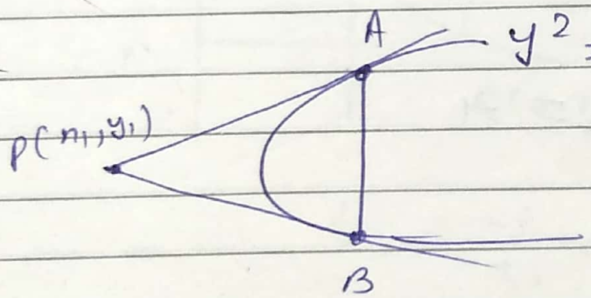
Note: In this case we get two values of m. If we get one value of m then other tangent is vertical tangent its eqⁿ

$$x = x_1$$

$x = x_1$



Note:



$y^2 = 4ax$

COC: $\boxed{T = 0}$ w.r.t P

Length AB =

$$= \left| \frac{\sqrt{y_1^2 + 4a^2}\ \sqrt{y_1^2 - 4ax_1}}{a} \right|$$

$$\boxed{\text{Area of } \Delta ABP = \left| \frac{(S_1)^{3/2}}{2a} \right|}$$ — valid for all Parabola

* Director Circle: Locus of P.O.I of ⊥ular Tangents in case of Parabola Directrix is Director circle of Parabola.

Q-1 L-5
12, 13, 14, 15, 16,
18, 19, 22, 23, 24, 25

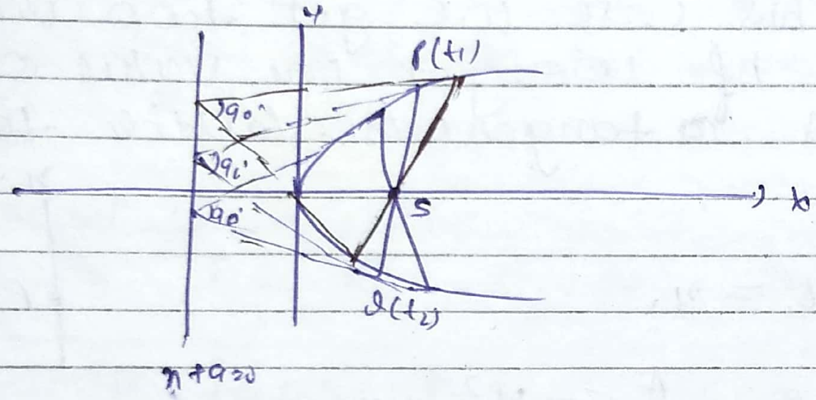
Let Parabola $y^2 = 4ax$

Director circle $x = -a$

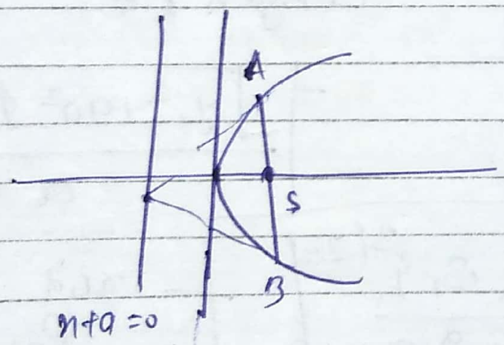
Parabola $x^2 = 4ay$

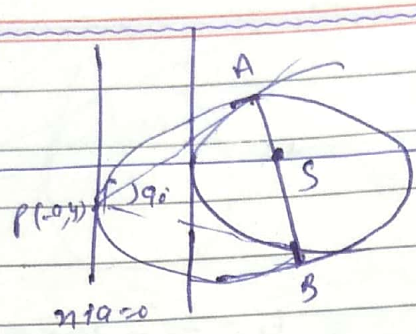
Director circle $y = -a$

*



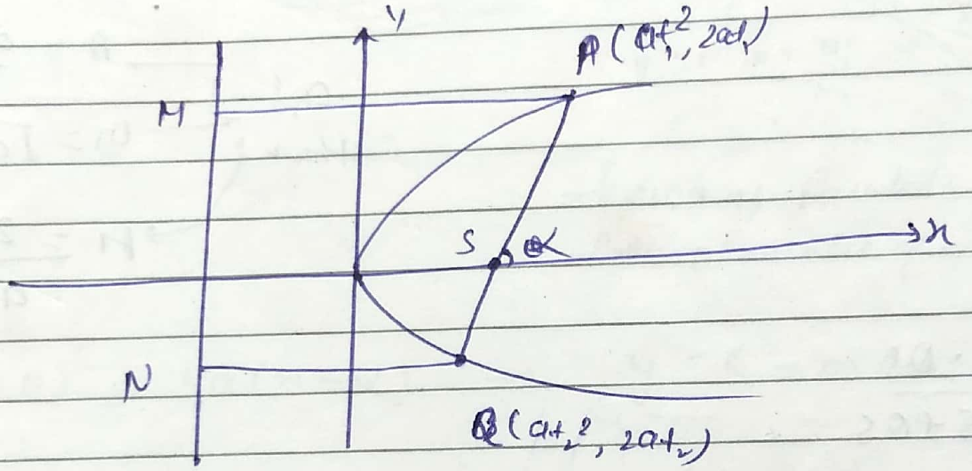
$t_1 t_2 = -1$





$\Rightarrow$ a circle on any focal chord on diameter touches the directrix

A circle on any focal radius as a diameter touches the tangent at vertex



$$t_2 = \frac{-1}{t_1}$$

$PS = PM = at_1^2 + a$
$QS = QN = at_2^2 + a$

PS and QS are length segment of focal chord.

$$PS + QS = 2a + at_1^2 + \frac{a}{t_1^2}$$

$$= a\left(t_1^2 + \frac{1}{t_1^2} + 2\right)$$

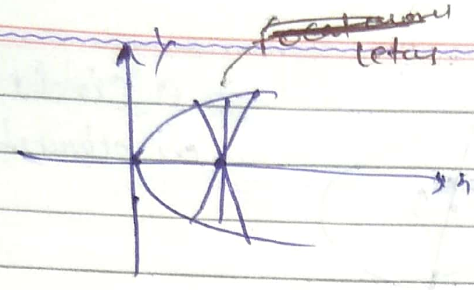
$$PQ = a\left(t_1 + \frac{1}{t_1}\right)^2 = 4a\,\text{cosec}^2\alpha$$

$$PQ \geq 4a$$

$$\frac{t_1^2 + \frac{1}{t_1^2}}{2} \geq \sqrt{t_1^2 \cdot \frac{1}{t_1^2}}$$

$$t_1^2 + \frac{1}{t_1^2} \geq 2$$

सबसे छोटी length ~~focal chor~~ Latus Rectum की होगी



$$\tan\alpha = \frac{2}{t_1+t_2} = \frac{2}{t_1-\frac{1}{t_1}}$$

$$t_1 - \frac{1}{t_1} = 2\cot\alpha.$$

$$\left(t_1 - \frac{1}{t_1}\right)^2 = 4\cot^2\alpha$$

$$\left(t_1 + \frac{1}{t_1}\right)^2 = \operatorname{cosec}^2\alpha$$

a, b then $A = \frac{a+b}{2}$, $G = \sqrt{ab}$, $H = \frac{2ab}{a+b}$

$$= \frac{2\,PS\cdot QS}{PS+QS}$$

$$= \frac{2\,(at_1^2+a)(at_2^2+a)}{a(t_1^2+t_2^2+2)}$$

$$= \frac{2a^2\,[\,t_1^2t_2^2 + t_1^2 + t_2^2 + 1)}{a\,(\cancel{t_1^2+t_2^2+2})}$$

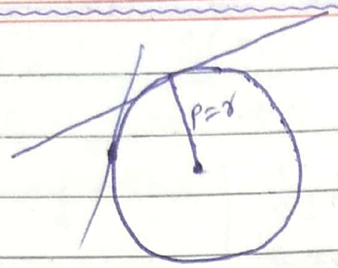
$$= 2a$$

$$= l \text{ (semi latus rectum)}$$

☆☆ Hence H.M of length segment of focal chord is the semi L.R. of Parabola li $\qquad\qquad\qquad\qquad b^2-4ac$

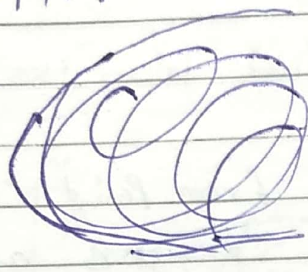
$y = mx + a\sqrt{1+m^2}$

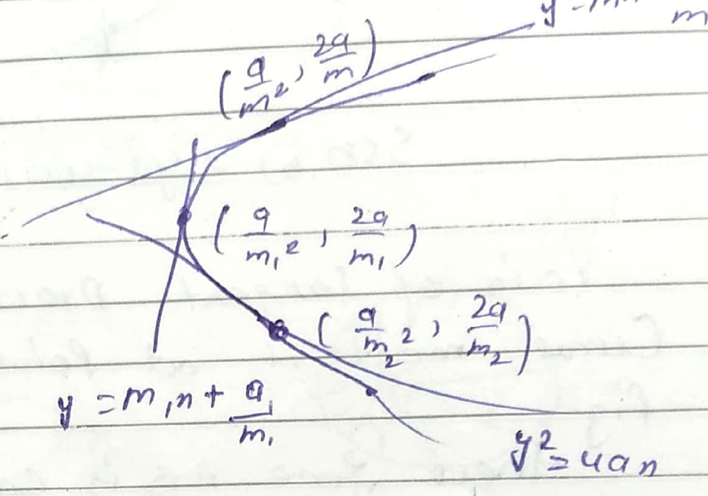
* $\qquad x^2+y^2=a^2$



$y = m_1 x + a\sqrt{1+m^2}$

touches P





touches Parabola $y = m_2 x + \frac{a}{m_2}$

for all value of m

$(y-\beta)^2 = 4a(x-\alpha) \rightarrow y-\beta = m(x-\alpha) + \frac{a}{m}$

Curve $\qquad\qquad$ tangent

$(x-\alpha)^2 + (y-\beta)^2 = a^2 \qquad (y-\beta) = m(x-\alpha) \pm a\sqrt{1+m^2}$

$y^2 = -4ax \qquad y = mx - \frac{a}{m}$

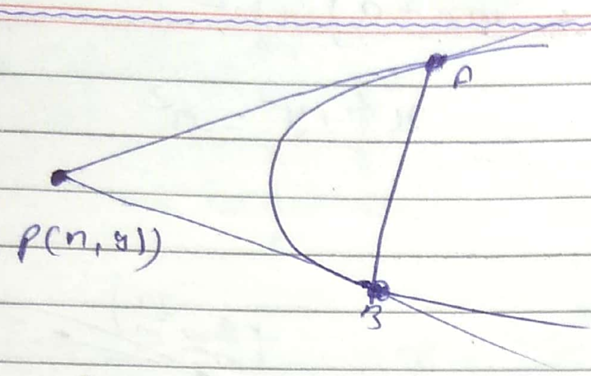
$(y-\beta)^2 = 4a(x-\alpha) \rightarrow y-\beta = m(x-\alpha) + \frac{a}{m}$

$x^2 = 4ay \rightarrow y = mx + am^2$

*

T=0 tangent

*



$S(x,y) = y^2 - 4ax$

Pair of tangents drawn from point $P(x_1, y_1)$ to the curve meet it at points A and B as shown in the fig.

therefore AB is called chord of contact to the parabola w.r.t to point P.

* eq^n of pair of tangent (i.e joint eq^n of PA and PB)

$= SS_1 = T^2$

$S = S(x, y) = y^2 - 4ax$

$S_1 = S(x_1, y_1) = y_1^2 - 4ax_1$

$T = T(x_1, y_1) =$

$yy_1 = 4a\frac{(x + x_1)}{2}$

standard substitution

$x^2 \Rightarrow xx_1$

$y^2 \Rightarrow yy_1$

$x \to \frac{x + x_1}{2}$

$y = \frac{y + y_1}{2}$

$c \to c$

$xy \to \frac{xy_1 + yx_1}{2}$

Valid for only conic

$T = (x_1, y_1) = yy_1 - 2a(x + x_1).$

* eq^n of chord of contact AB

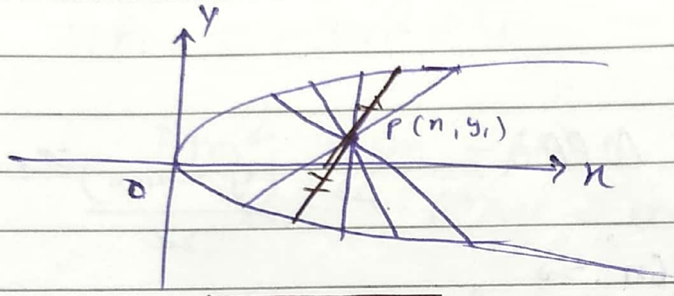
$\boxed{T = 0}$

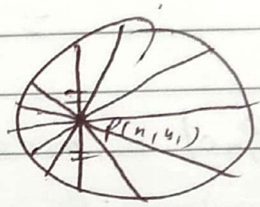
$$\Delta_{PAB} = \frac{S(x_1, y_1)^{3/2}}{2a}$$

$$\text{Length of } AB = \left| \frac{\sqrt{y_1^2+4a^2}\ \sqrt{y_1^2-4ax_1}}{a} \right|$$

* Chord whose mid. Point is given

$x^2+y^2=a^2$





eqn ⇒ $\boxed{T = S_1}$

Que: $y^2 = 16x$

(1, 2) find eqn of chord whose eq is

$T = S_1$ $\quad 4a = 16 \quad a = 4$

$= y^2 = 16x$

$y^2 - 16x = 0 \quad yy_1 = 16(x + x_1)$

$2y - 16\left(\frac{x+1}{2}\right) = (2^2 - 16\cdot 1)$

Que: From P Pair of Tangent drawn to the curve find it's eqn of C.O.C

4y-

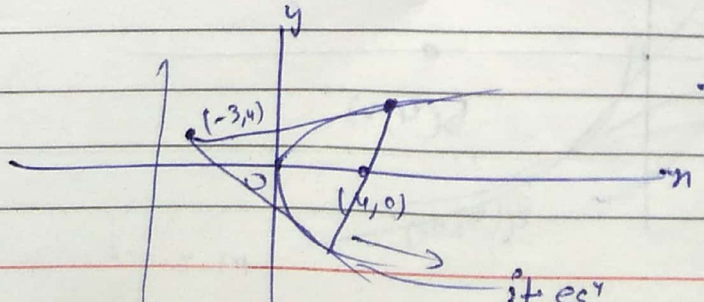
(1) $y^2 - 16x = 0 \qquad P(-3, 4)$

$T = 0$

$= 4y - 16\left(\frac{x+3}{2}\right) = 0$

$4y - 8(x - 3) = 0$

$y = 2(x - 3)$



(2) find length of CC. $-3,4)$

$$\frac{\sqrt{y_1^2+4a^2}\sqrt{y_1^2-4ax_1}}{a}$$

$$=\frac{\sqrt{16+\cancel{1}4\cdot 16}\sqrt{4^2-16(-3)}}{4}$$

$=$

(3) find Area of ΔPAQ. $AB=\frac{(y_1^2-4ax)^{3/2}}{2a}$

$= y^2-16x = 0$

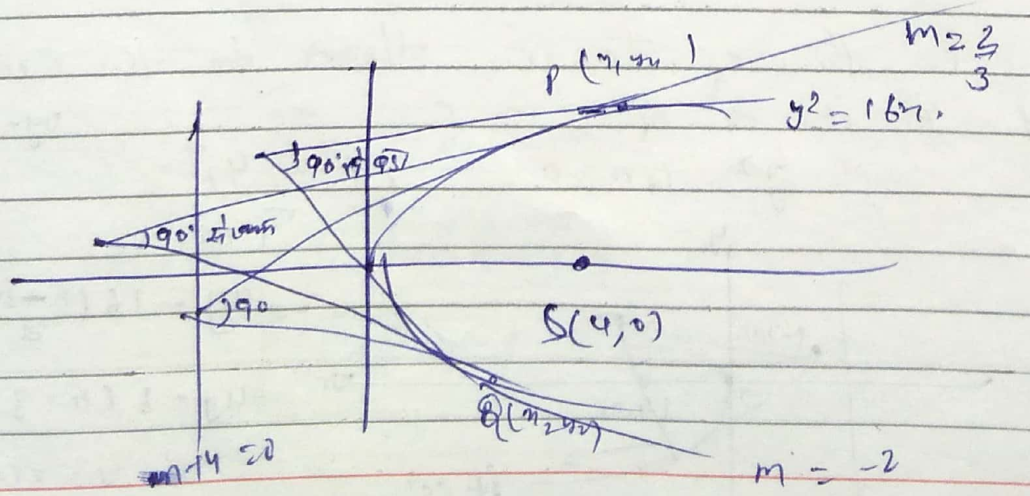
$= 16-(16(-3)$

$$= \frac{(16+48)^{3/2}}{8} =$$

(4) find eqn of Pair of tangents

$SS_1 = T^2$ $y = mx + c$

$$=(y^2-16x)(16-(16(-3))) = y\cdot 4-16\frac{(x-3)}{2}$$



Q. write eqⁿ of tangent drawn from Pt (P) to
(~~..~~) Parabola shw

$$y = mx + \frac{4}{m}$$

$$4 = m(-3) + \frac{4}{m}$$

$$4m = -3m^2 + 4$$

$$3m^2 + 4m - 4 = 0$$

$$3m^2 + 6m - 2m - 4 = 0$$

$$(3m-2)(m+2) = 0$$

Parabola

Q. Prove that passing through $(4a, 0)$ to the Parabola $y^2 = 4ax$ subtand right angle at its vertex.

$$y - 0 = m(x - 4a)$$

$$y = mx - 4ma$$

$$mx - y = 4am$$

$$\frac{mx - y}{4am} = 1$$

$$y^2 - 4ax\left(\frac{mx - y}{4am}\right) = 0$$

$$\boxed{y^2 - x^2 + \frac{xy}{m} = 0}$$

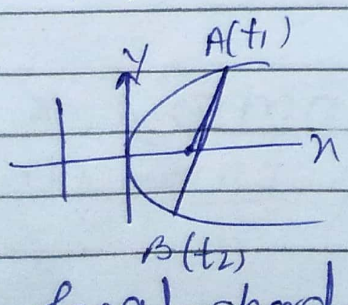
Cofficient of x^2 + Cofficient of $y^2 = 0$

$\therefore$ OA & OB are $\perp^r$

Hence Prove.

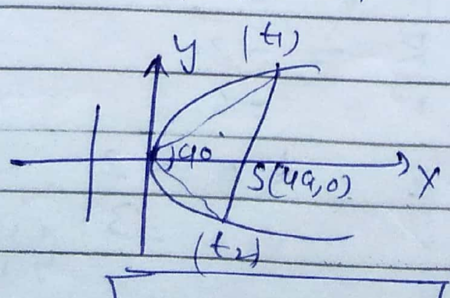
$$t_1 t_2 = -\frac{c}{a}$$

I. $t_1 t_2 = -1$



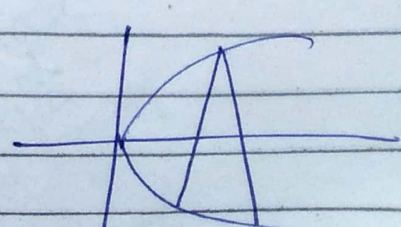
focal chord

$t_1 t_2 = -4$

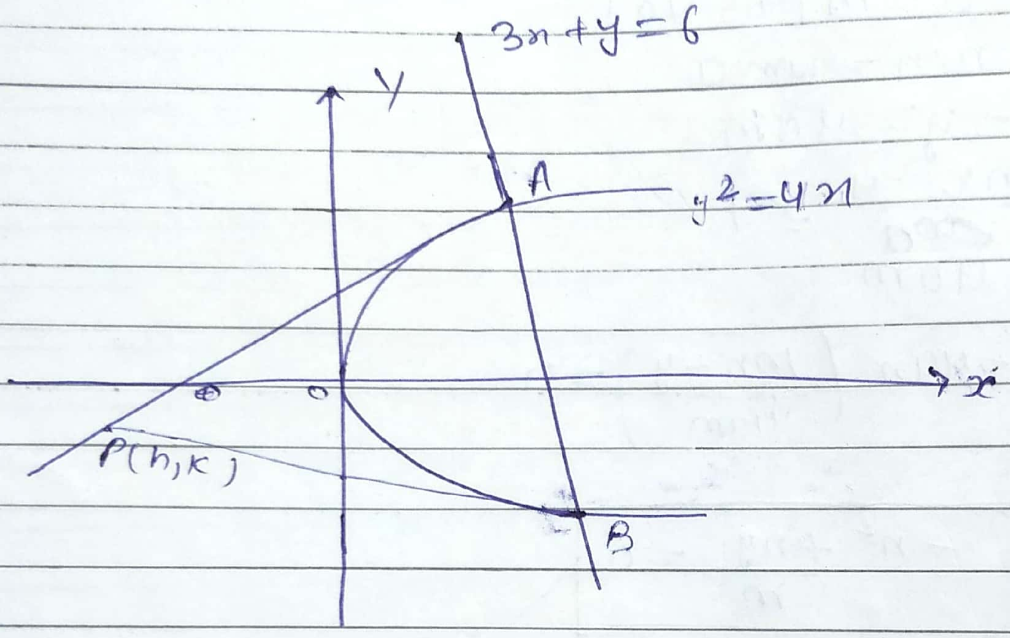


$$\boxed{t_2 = -\frac{4}{t_1}}$$

$t_1 t_2 = 2$



Que: Line $3x+y=6$ intersect Parabola $y^2=4ax$ at a & b find coordinate of P.O.I of tangent drawn at A & B



AB is chord of Contact to the Parabola w.r.t Point (P)

$\therefore$ it eqⁿ is

$$T=0$$

$$yk = 4^2\left(\frac{x+h}{2}\right)$$

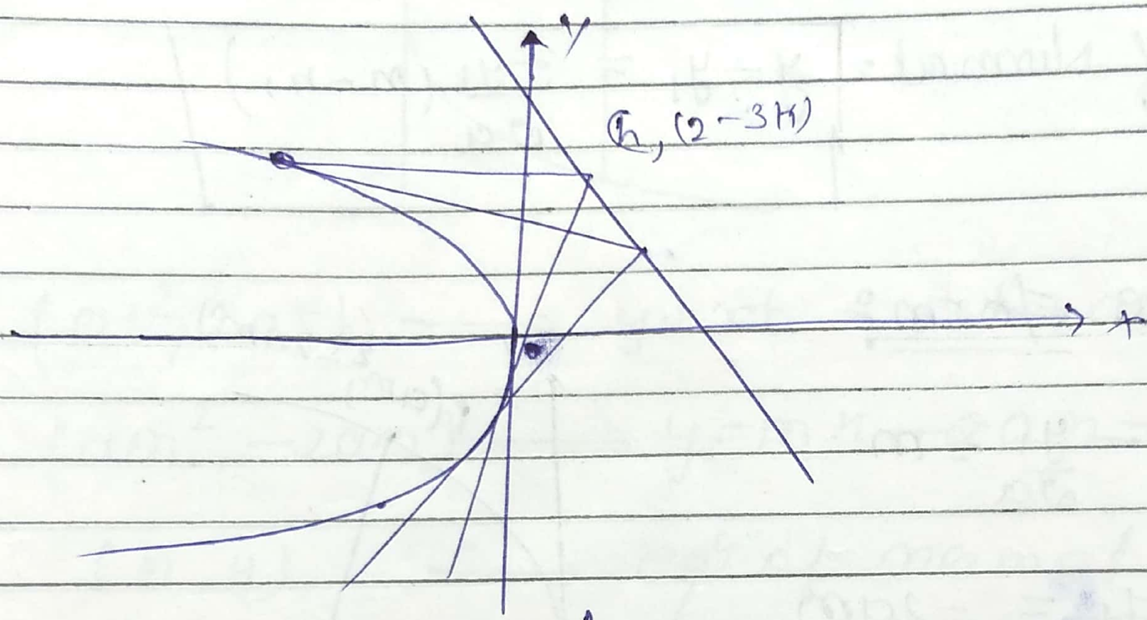
$$yk-2x-2h=0 \quad \text{are identical}$$

Now, corner it with $3x+y-6=0$

$$\frac{-2}{3}=\frac{k}{1}=\frac{-2h}{-6}$$

$$h=-2 \qquad k=\frac{-2}{3}$$

Q Pair of tangent are drawn to the parabola $y^2 = -4x$ from every point on the line $3x+y=2$ shows that their chord of contact passes through fixed point.



$$y(2-3h) = \cancel{-4}^{2}\frac{(x+h)}{\cancel{2}}$$

$$y(2-3h)+2(x+h)=0$$

$$2y-3hy+2x+2h=0$$

$$\underbrace{2(y+x)}_{L_1}+h\underbrace{(-3y+2)}_{L_2}=0$$

$$L_1+\lambda L_2=0$$

$$2x+2y=0 \quad -(1)$$

$$2-3y=0 \quad -(2) \text{ solve both eq}^n$$

& get Co-ordinate.

* Normal:

(1) Point form:

$$2yy' = 4a$$

$$y' = 2a/y$$

$$y^2 = 4ax$$

$$2yy' = 4a$$

$$\boxed{y' = \frac{2a}{y}}$$

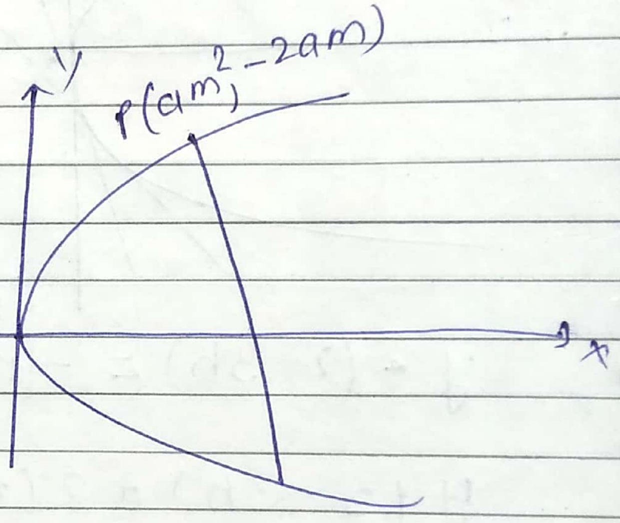
$(y')_P = \frac{2a}{y'}$ = Slope of T at Point P

$\therefore$ Slope at normal is $= -y_1/2a$

eq^n of Normal →

eq^n of Normal = $\boxed{y - y_1 = \frac{-y_1}{2a}(x - x_1)}$

(2) Slope form:



$$-\frac{y_1}{2a} = m$$

$$y_1 = -2am$$

$$y^2 = 4ax$$

$$(-2am)^2 = 4ax$$

$$x = am^2$$

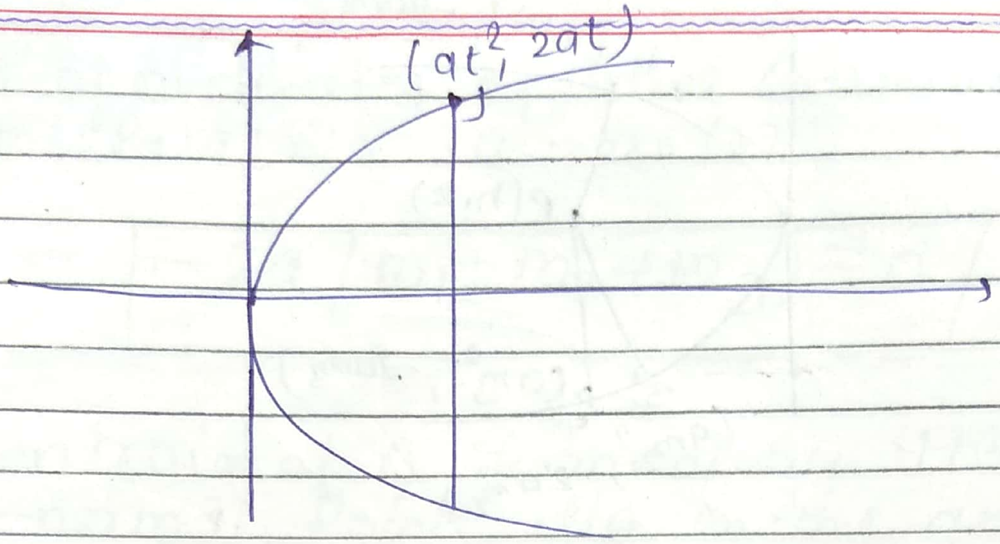
$$y + 2am = m(x - am^2)$$

$$\boxed{y = mx - 2am - am^3}$$

(3) Parametric form:

$$y - 2at = -\frac{2at}{t}(x - at^2)$$

$$\boxed{y + xt = 2at + at^3}$$



$(at^2, 2at) \longrightarrow y + xt = 2at + at^3$

$(am^2, -2am) \longrightarrow y = mx - 2am - am^3$

$(x, y) \longrightarrow$ eqⁿ of normal is

$$\boxed{\left(y - y_1 = \frac{-y_1}{2a}(x - x_1) \right)}$$

* <u>More about Normal in slope form:</u>

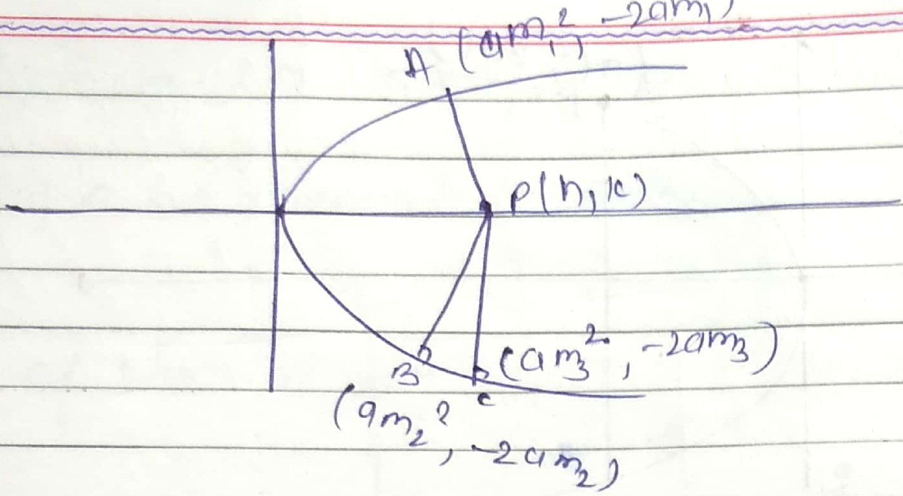
Suppose Normal to the Parabola passes through $P(h, k)$

$y = mx - 2am - am^3$

$k = mh - 2am - ax^3$

$am^3 + (2a - h)m + k = 0$ ①

$m_1 \quad m_2 \quad m_3$



i.e at most three real normal can be drawn from Point $P(h, k)$ to the Parabola

$$m_1 + m_2 + m_3 = 0 \quad \text{—} \ (2)$$

$$m_1 m_2 + m_2 m_3 + m_1 m_3 = \frac{2a - h}{c} \quad \text{—} \ (3)$$

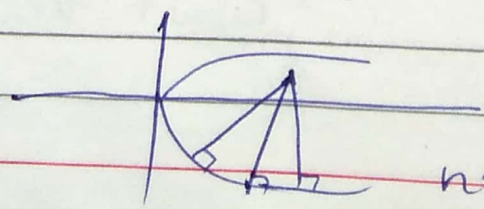
$$m_1 m_2 m_3 = -\frac{k}{a} \quad \text{—} \ (4)$$

Here m_1, m_2, m_3 are the slopes of three Congurrent normals.

* At most Three normals Can be drawn
* At least one normal Can be drawn.
* foot of Normal of three Concurrent normals are called co-normal Points (Point A, B, & C)

* Algebric sum of slope of three Concurrent nomals is zero (0).

$$m_1 + m_2 + m_3 = 0$$

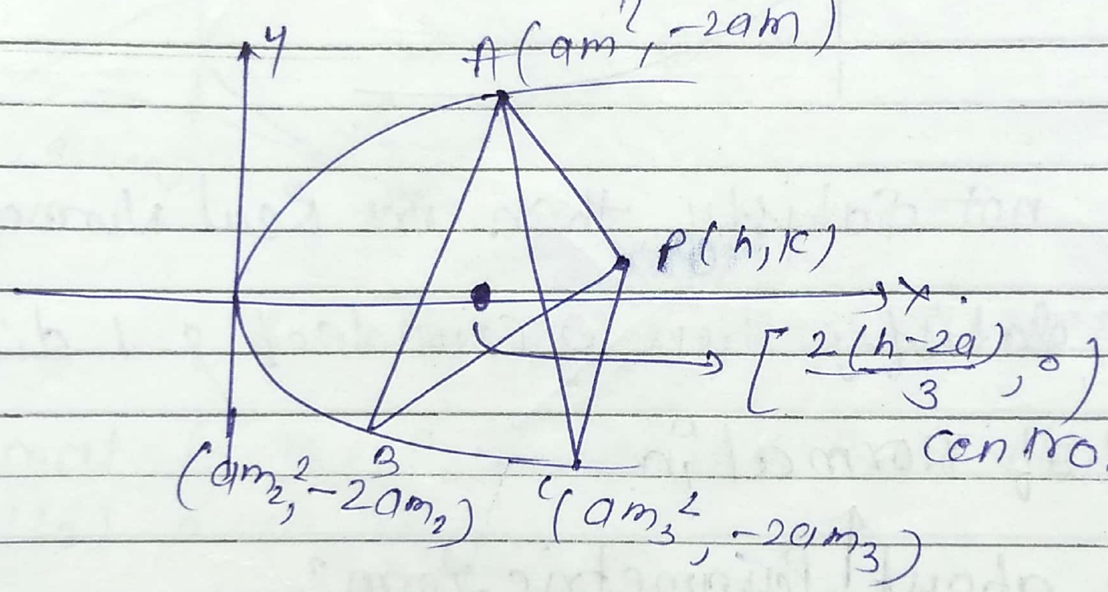


not Possible

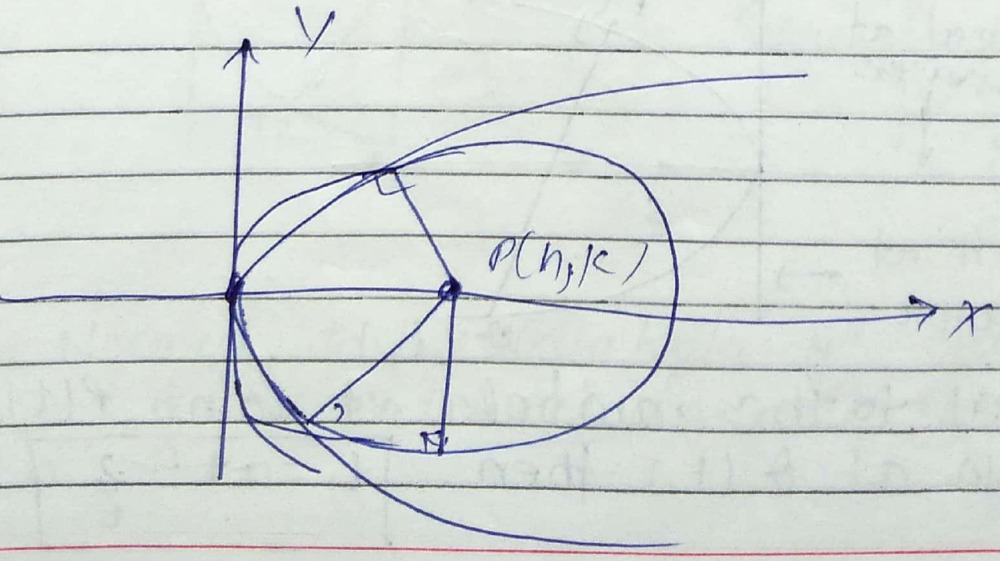
* Sum of ordinates (→ y-coordinate) of three co-normal Points on the Parabola is zero (0).

$$\boxed{-2a(m_1+m_2+m_3)=0}$$

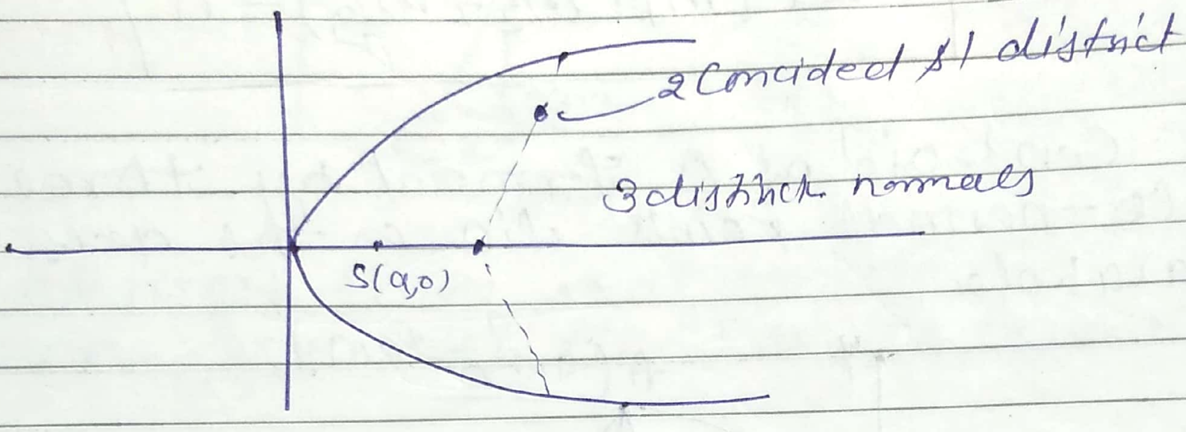
* Centroid of Δ formed by three Co-normal Points lie on the axis of Parabola.



* Circle Passes through Co-normal point always Passes through the vertex of Parabola



* Point $P(h,k)$ satisfy $27ak^2 < 4(h-2a)^3$ then three normals can be drawn from such point to parabola



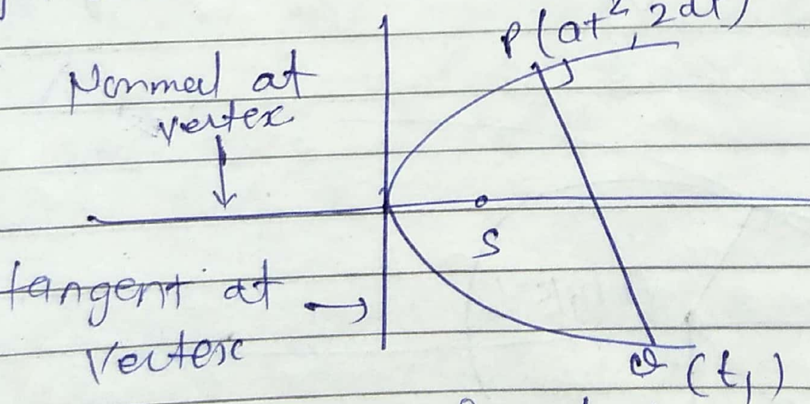
If not satisfy then one Real Normal

if satisfy then 2 consided & 1 district.

* eqn of Normal in

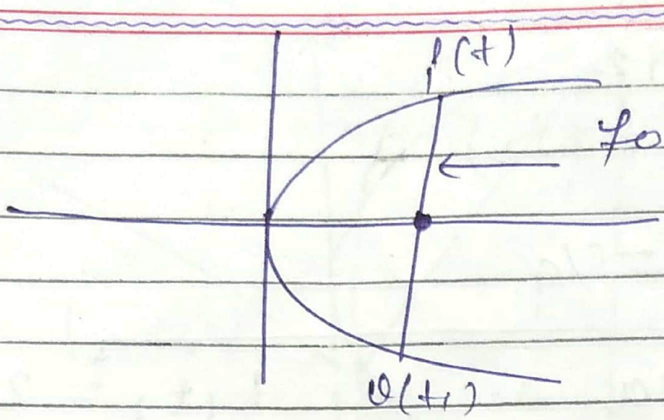
More about ↑ Parametric form:

$y + xt = 2at + at^3$

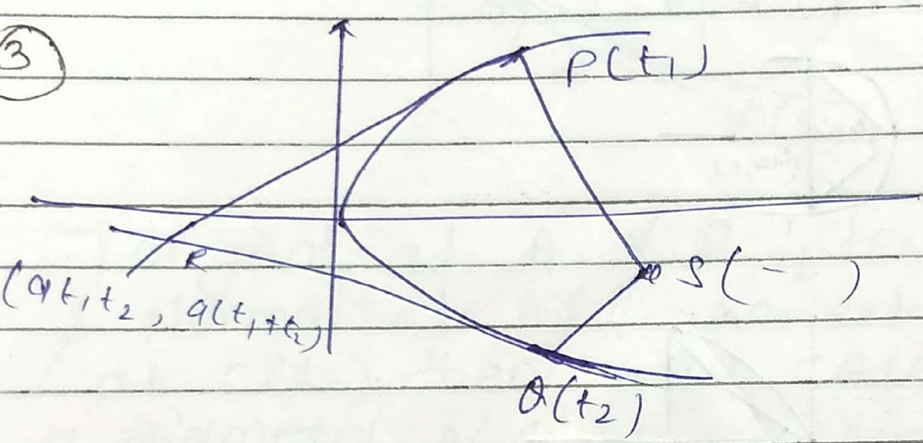


(1) Normal to the Parabola at Point $P(t)$ meet Parabola at $Q(t_1)$ then $\boxed{t_1 = -t - \frac{2}{t}}$

(2)



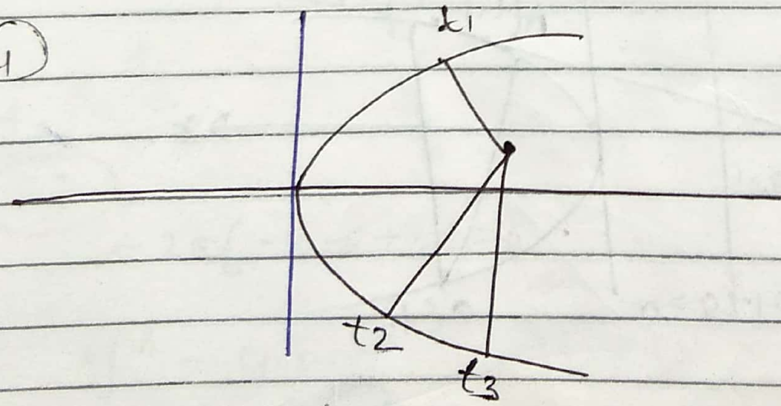
(3)

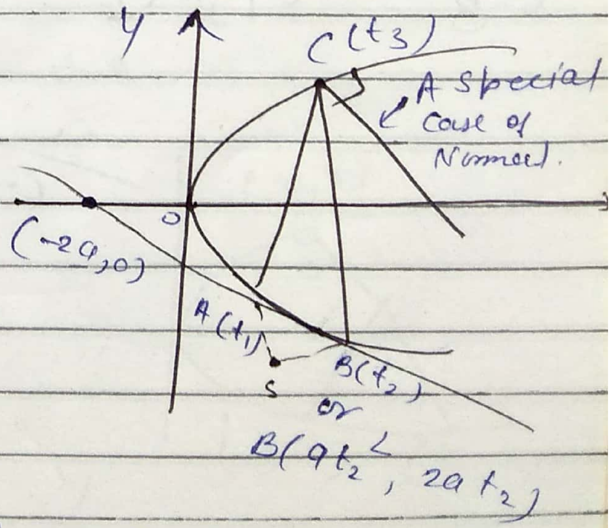


$S\left(a(t_1^2+t_2^2+t_1t_2+2),\ -at_1t_2(t_1+t_2)\right)$

Normal to the Parabola at Point $P(t_1)$ & $Q(t_2)$ meet at Point R.

(4)





If Normal to the parabola $y^2 = 4ax$ at Point $A(t_1)$ & $B(t_2)$ intersect again on the Parabola $C(t_3)$ then.

(i) $t_1 t_2 = +2$

(ii) $t_3 = -(t_1 + t_2)$

$t_1 t_2 = -c/a.$

$c = a$

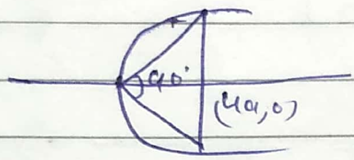
focal chord

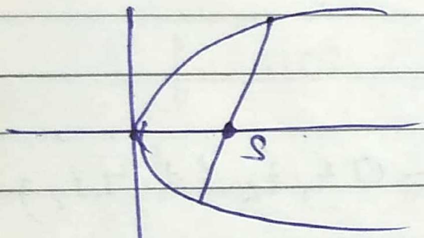
$t_1 t_2 = -1$

$c = 4a$

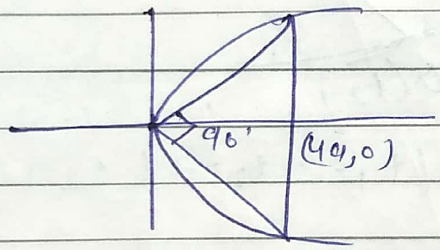
$t_1 t_2 = -4$

$t_1 t_2 = 2.$



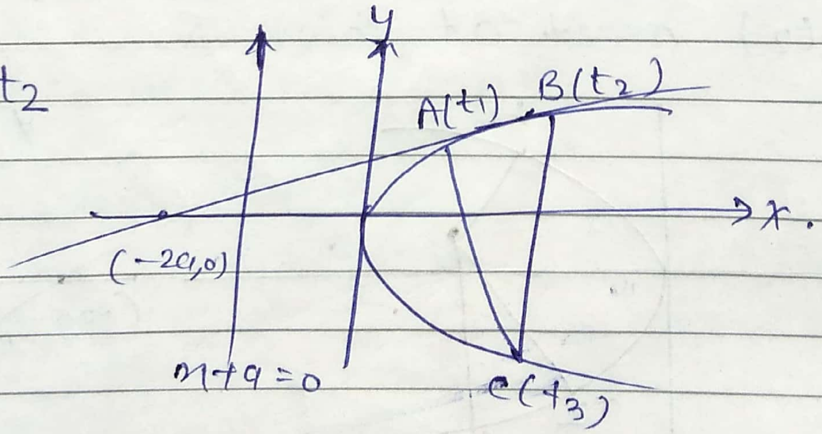


$t_1 t_2 = -1$

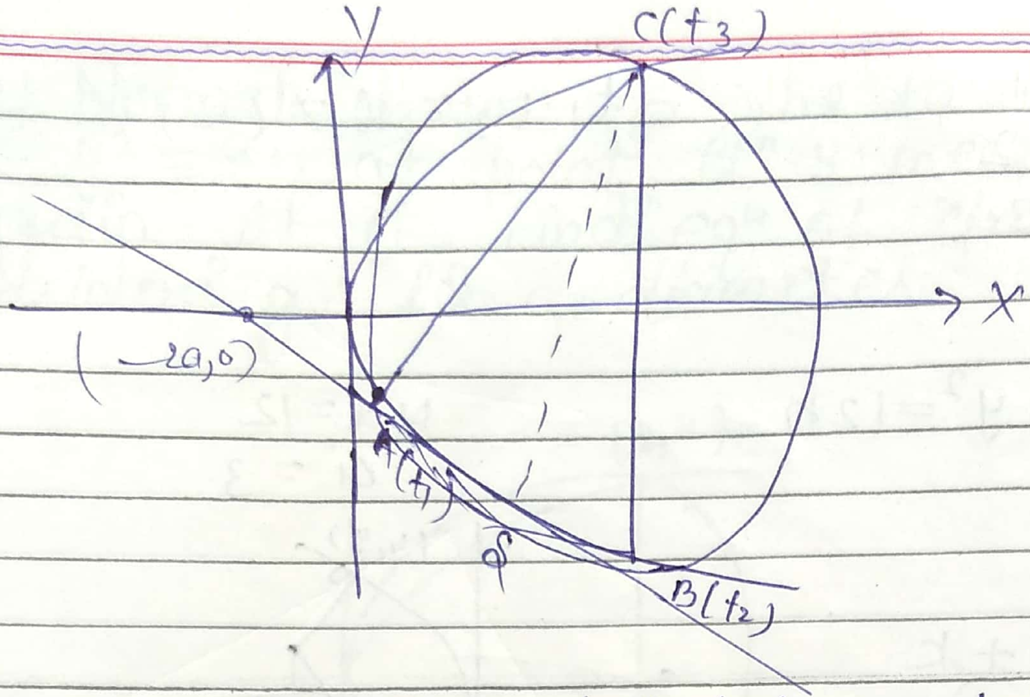


$t_1 t_2 = -4.$

$t_3 = -t_1, -t_2$

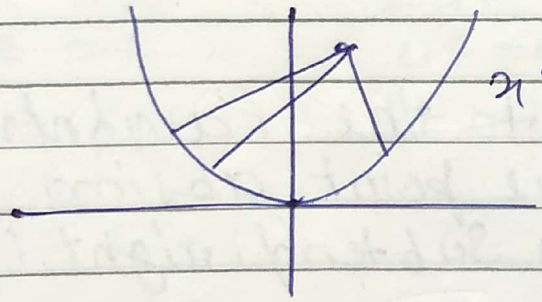


*



Tangent at A & B intersect at Point S & Normal at A & B remeet at Parabola at $C(t_3)$ then quad CASB is cyclic quad & diameter of circle is line CS.

(Q-1 Q-30h)



Q-1 (32)

$-2a(-4-6+x)=0$

$y^2 = 4x$

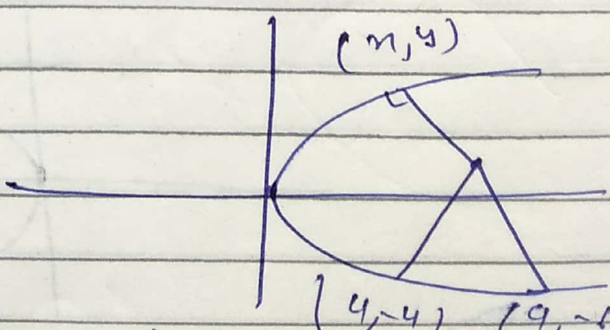
$(-6a)^2 = 4 \cdot 9a$

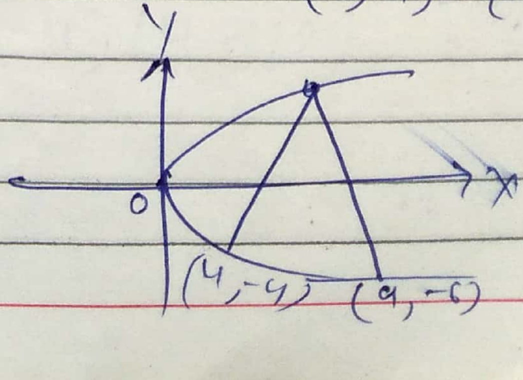
$36a^2 = 36a$

$a = 1$

$y_1 + (-4) + (-6) = 0$

$y_1 = 10$





$t_1^2 = 4$

$t_1 = -2$

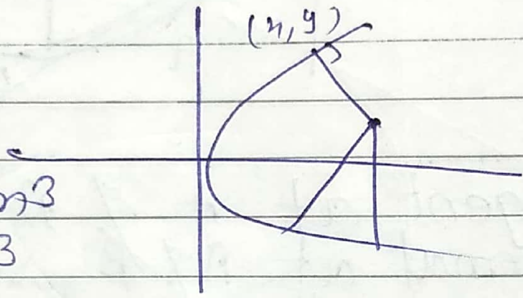
$t_2 = -3$

$t_1 t_2 = 6 \neq (2)$

Q-1
34 $y^2 = 12x$

$4a = 12$

$a = 3$



$m = -1$

$y = -x + k$

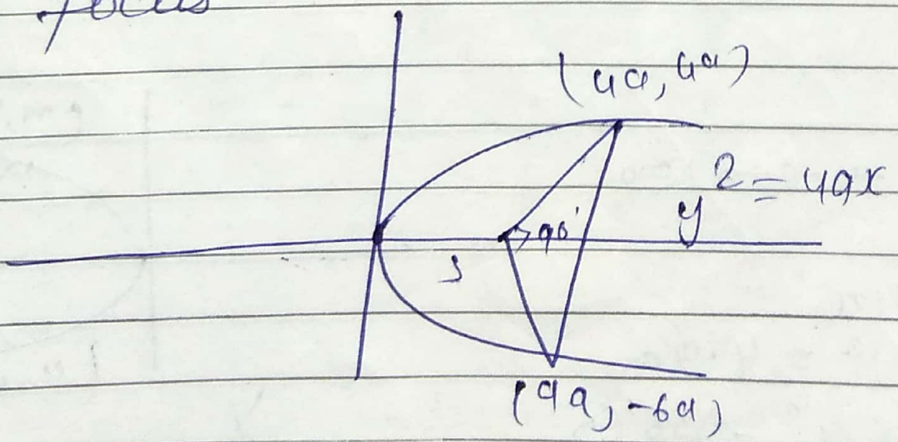
$y = -mx - 2am - am^3$

$k = -2am - am^3$

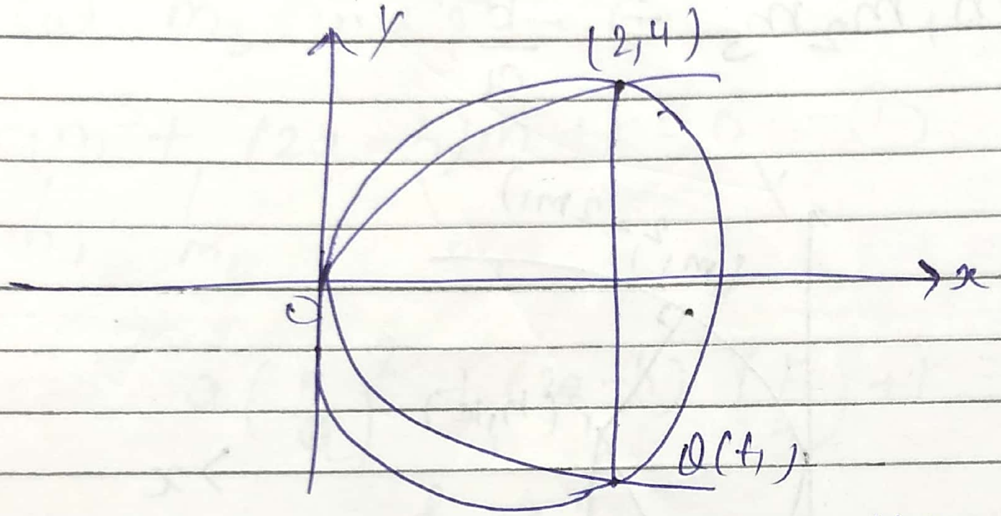
$= -2\cdot 3(-1) - 3(-1)^3$

$= 6 + 3 = 9.$

★★
★**Note:** Normal chord to the Parabola
★★ $y^2 = 4ax$ at the point absicca is equal to ordinate, Subtend right is at its focus.



Que! Normal drawn to the Parabola $y^2 = 8x$ at point $P(2,4)$ meet Parabola again at Q. find eq^n of circle considering PQ as diameter



$4a = 8$

$a = 2$

$at_1^2 = 2$

$t_1^2 = 1$

$t_1 = 1$

$2at = 4$

$4t = 4$

$t = 1$

$t_1 = -t - \frac{2}{t}$

$t_1 = 1$

$-1 - \frac{2}{1} = -3$

$Q \; (at_1^2, 2at_1)$

$(18, -12)$

$(x-2)(x+8) + (y-4)(y+12)$

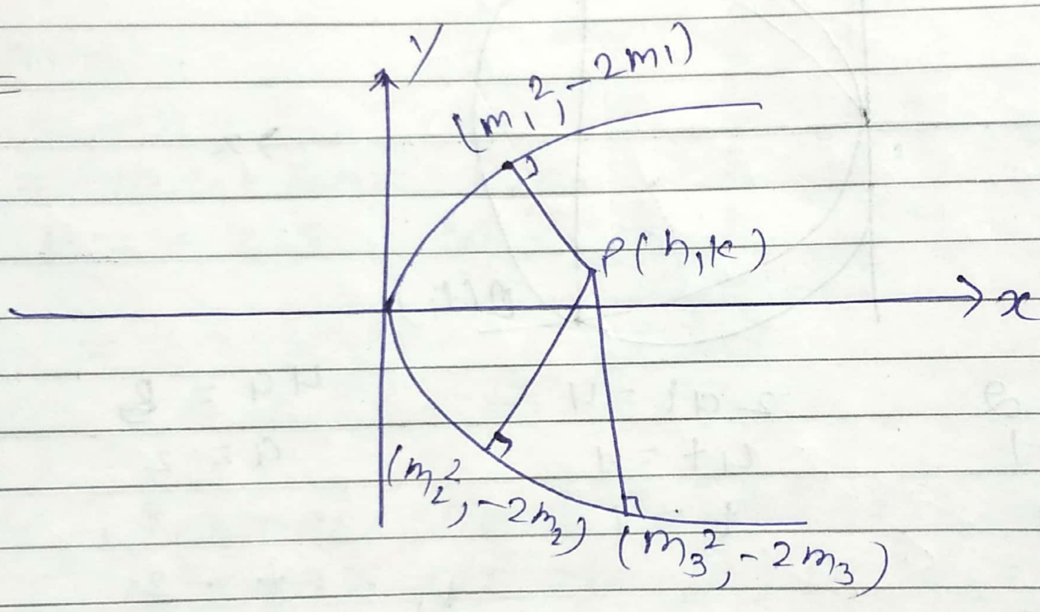
$am_1^3 + (2a-h)m + k = 0$ (i)

$m_1 + m_2 + m_3 = 0$

$$m_1 m_2 + m_2 m_3 + m_1 m_3 = \frac{2a-h}{a}$$

$$m_1 m_2 m_3 = \frac{-k}{a}$$

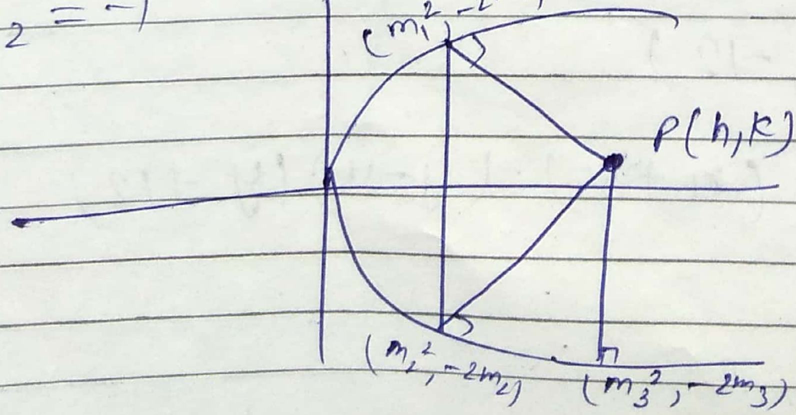
Q-1
(511)



Que. Normal are drawn at point A, B & C on the parabola $y^2 = 4x$ which intersect at P(h,k) find locus of P if:

(i) two of them are ⊥.

$m_1 m_2 = -1$



$m_1 m_2 m_3 = \frac{-k}{a}$

$-m_3 = \frac{-k}{a}$ $\boxed{m_3 = \frac{k}{a}}$

Put m_3 in eqⁿ — ①

$am^3 + (2a-h)m + k = 0$ — ①

m_1 m_2 m_3

$$a\left(\frac{k}{a}\right)^2 + (2a-h)\left(\frac{k}{a}\right) + k = 0$$

$h \to x$

$k \to y$

(ii) Product of slope of two normal is 3

$m_1 m_2 = 3$

find value of $m_3 = \cancel{}$ then Put in eqⁿ ①.

(iii) If slope of line joining feet of them = 2.

$$m_{AB} = \frac{-2m_1 + 2m_2}{m_1^2 - m_2^2}$$

$$2 = \frac{-2\cancel{(m_1 - m_2)}}{\cancel{(m_1 - m_2)}(m_1 + m_2)}$$

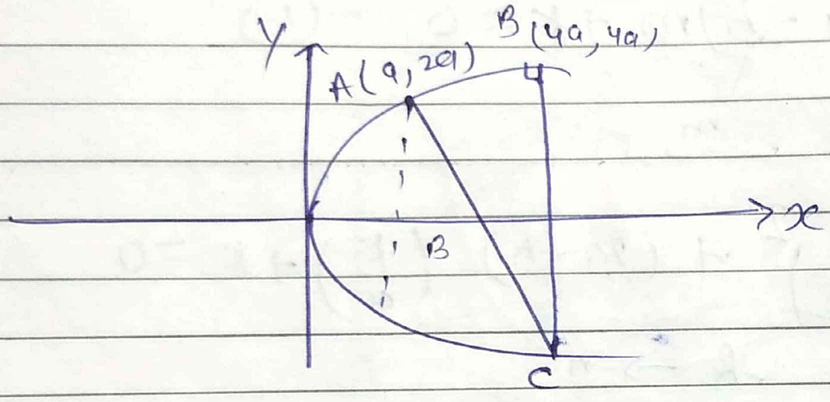
$m_1 m_2 = -1$

$-m_3 = -1$

$m_3 = 1$

Put $m_3 = 1$ in eqn (i) & then solve.

S-1
Q.1



$y^2 = 4ax$

$4a = 4$

$a = 1$

$at_1 = a$ $t_1 = 1$

$at_2^2 = 4a$ $\quad 2at_2 = 4a$

$t_2 = 2$ $\quad t_2 = 2$

$t_1 t_2 = 2.$

Q. find common tangent $y^2 = 4x$ & $x^2 = -32y$

$y = mx + 1/m$

$x^2 = -32mx - \frac{32}{m}$

$$x^2 + 32mx + \frac{32}{m} = 0$$

$$D = b^2 - 4ac$$

$$(32m)^2 - 4 \times 1 \times \frac{32}{m} = 0$$

$$1024m^2 = \frac{128}{m}$$

$$m^3 = \frac{128}{1024}$$

$$m = 1/2$$

$$y = \frac{1}{2}m + 2$$

M-2

$$y = mx + \frac{1}{m}$$

$$x^2 = 4ay$$

$$y = mx + 8m^2 \qquad y = mx - am^2$$

Identical $\frac{1}{1} = \frac{m}{m} = \frac{1/m}{8m^2}$

$$8m^3 = 1$$

$$m = 1/2.$$

Que:- find Common tangent $x^2+y^2=2$

$$y^2=8x$$

$$y=mx+\frac{2}{m} \rightarrow y=mx+\sqrt{2}\sqrt{1+m^2}$$

$$\left(mx+\frac{2}{m}\right)^2=8x$$

$$m^2x^2+\frac{4}{m^2}+4x=8x$$

$$m^2x^2+\frac{4}{m^2}=4x$$

$$m^2x^2+\frac{4}{m^2}-4x=0$$

find value of m &

$D=0$

get value of y.

M-2 $\quad y=mx+2/m$

$$\frac{1}{1}+\frac{m}{n}=\frac{\sqrt{2}\sqrt{1+m^2}}{2/m}$$

$$\frac{2}{m}=\sqrt{2}\sqrt{1+m^2}$$

$$\frac{\cancel{4}^2}{m^2}=2(1+m^2)$$

* O-1 : 20, 21, 28, 52,
S-1 : 3, 4, 7, 8,

Q. write eq^n of tangent drawn from Point P to the Parabola shown

~~$(y - y_1)^2 = 4a(x - x_1)$~~ .

tangent eq^n

$$y = mx + \frac{4}{m}$$

$$y = m(-3) + \frac{4}{m}$$

$$4m = -3m^2 + 4$$

$$3m^2 + 4m - 4 = 0$$

$$3m^2 + 6m - 2m - 4 = 0$$

$$(3m - 2)(m + 2) = 0$$

* Homogenisation :

Q. P.T chord Passing through $(4a, 0)$ to the Par. $y^2 = 4ax$ Su)

(11)

$$\Rightarrow m^4 + m^2 - 2 = 0$$

$$m^4 + 2m^2 - m^2 - 2 = 0$$

$$(m^2 + 2)(m^2 - 1) = 0$$

$$m = \pm 1$$

$$y = x + 2$$

$$y = -x - 2$$

$$y^2 = 8x \Rightarrow y = mx + \frac{2}{m}$$

$$x^2 + y^2 = 2$$

$$mx - y + \frac{2}{m} = 0$$

$$C(0,0), \; r = \sqrt{2}$$

$$P = r$$

$$\left| \frac{0 - 0 + 2/m}{1 + m_2} \right| = \sqrt{2}$$

2 Day
~~Test~~ Tenolives

* Rules of Transformation :

$y^2 = 4an$ | ① $y = mx + \frac{a}{m}$ | $y = mx - 2am - am^3$

$x^2 = 4an$ | $y = mx - am^2$ | $y = mx + 2a + a/m^2$

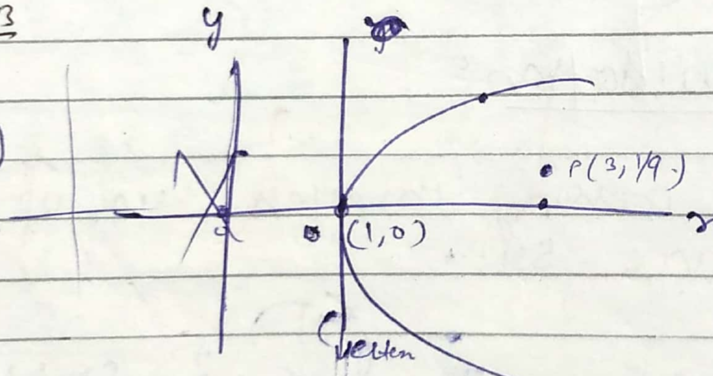
Que: find No. of Normals drawn from $P(3, \alpha)$ to the Parabola $y^2 = 4x$

i) $\alpha = \frac{1}{9}$ $y^2 = 4x$ $a = 1$

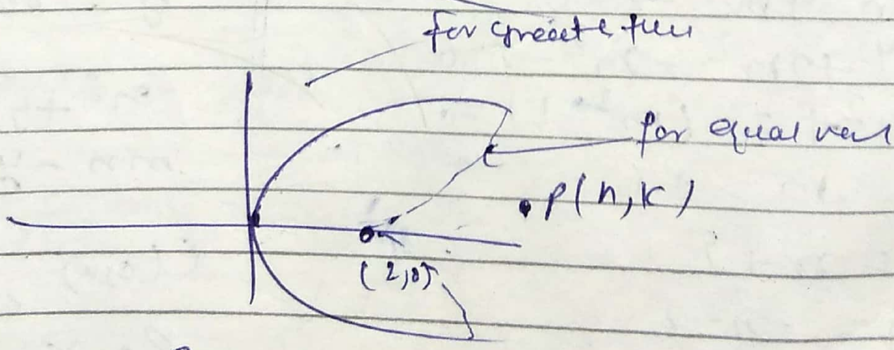
$(at^2, 2at)$

ii) $\alpha = \frac{2\sqrt{3}}{9}$

$P(3, 1/9)$



(i)

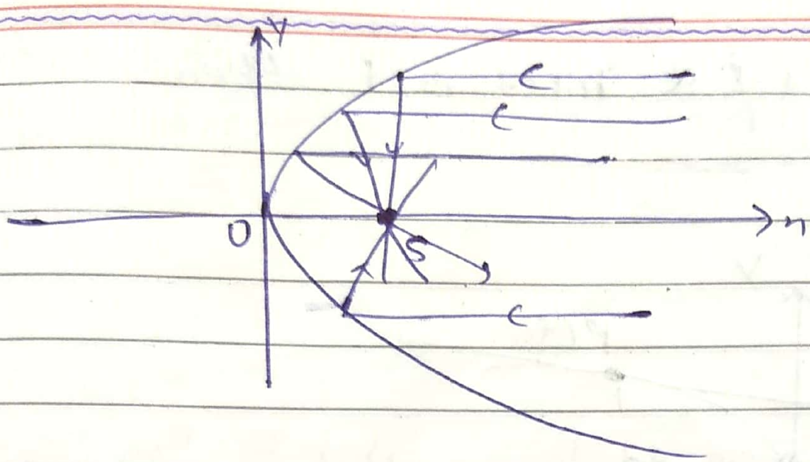


$27k^2 < 4(h-2)^3$

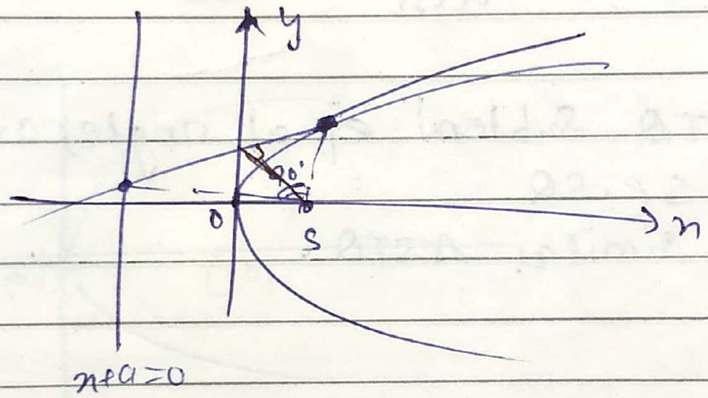
i) $h = 3$ $27 \cdot \frac{1}{81} < 4(1)$

$k = \frac{1}{9}$

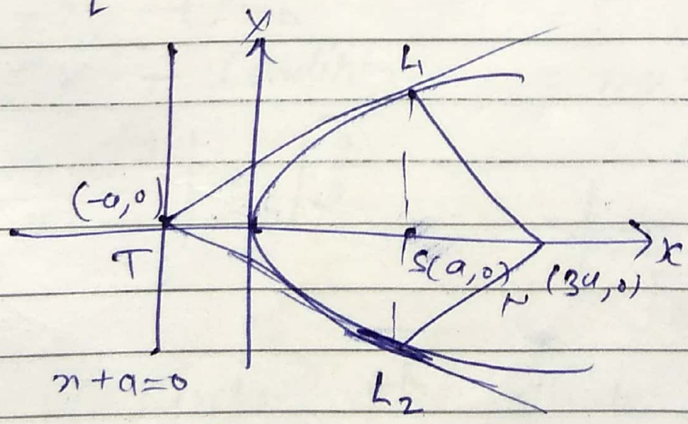
$\frac{1}{3} < 4 \Rightarrow$ Three Normal (distinct)



* Portion of tangent lie b/w curve and Directrix, Subtend $90°$ at its focial



* T/N at extremity of L.R of Parabola $y^2=4x$ Constitute a sq. and their pt of intersection

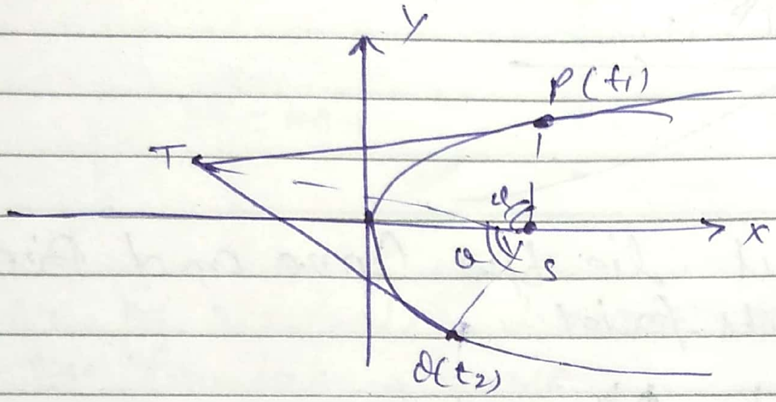


TL_1NL_2 — Square

Side length $= 2\sqrt{2}a$.

Properties
I.A Attempted

* If tangent at P & Q meet in t then



NT

(i) TP & TQ subtend equal angles at its ~~focus~~ focus

(ii) $(ST)^2 = SP \cdot SQ$

(iii) ΔSPT similar ΔSTQ.

(iii) $h = 3$

$k = \frac{2\sqrt{3}}{9}$ $\qquad \frac{27 \cdot 2\sqrt{3} \cdot 2\sqrt{3}}{9 \cdot 9} = 4(3-2)^2$

$4 = 4.$

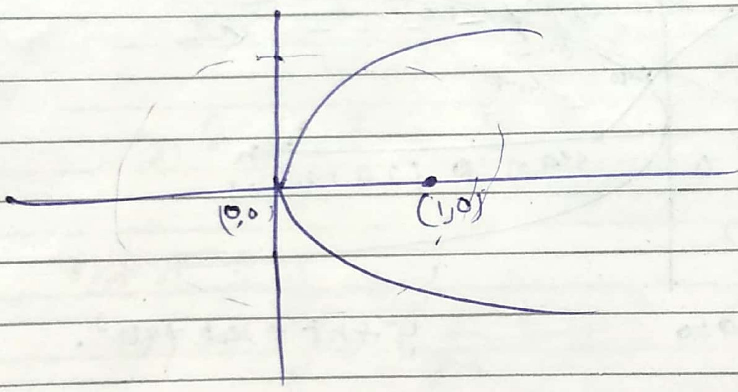
Que: find eqⁿ of line which is normal to the P $y^2 = 4x$ & and touches circle $(x^2 + y^2 = \frac{9}{2}$

$y = \sqrt{4x}$

$x^2 + 4x = \frac{9}{2}$

$2x^2 + 8x = 9$

$2x^2$



$y + xt = 2t + t^3$

$y + xt - 2t - t^3 = 0$

$p = r$ — Condition

$\left| \frac{0 + 0 - 2t - t^3}{\sqrt{1 + t^2}} \right| = \sqrt{\frac{9}{2}}$

$y = mx - 2m - m^3$

$mx - y - 2m - m^3 = 0$

$p = r$

$\left| \frac{-2m - m^3}{\sqrt{1 + m^2}} \right| = \sqrt{\frac{9}{2}}$

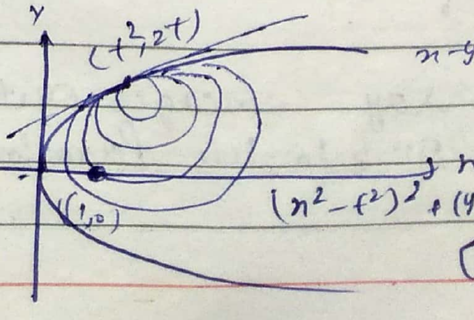
. find eqⁿ of circle which touches P ($y^2 = 4x$) and passes through its focus.

consider point circle at pt. P

$(x - t^2)^2 + (y - 2t)^2 = 0$

$y \cdot 2t = 2(x + t^2)$

$x - yt + t^2 = 0$



$(x - t^2)^2 + (y - 2t)^2 + \lambda(x - yt + t^2) = 0$

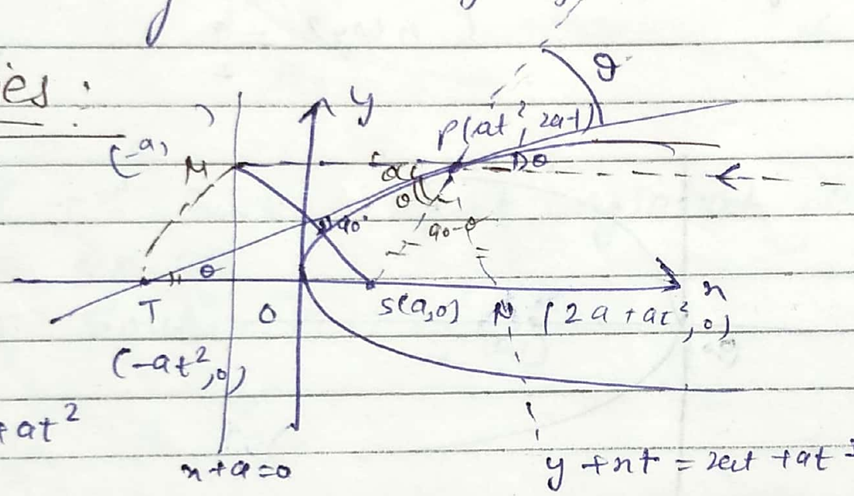
Hence family of circle touches the given line at Pt. P is
$S + \lambda L = 0$

$\because$ circle passes through focus
$S(1, 0)$
$(1-t^2) + (0-2t)^2 + \lambda(1+t^2) = 0$

Same Que as
Ex. Que
$\Rightarrow$ (S-1 $\Rightarrow$ 13)

By this way we get get the value of λ.

* **Properties :**



$yt = x + at^2$

$x + a = 0$

$y + xt = 2at + at^3$.

$TS = at^2 + a = SN = PS = PM = TM$

Tangent and Normal at Pt P on the Parabola are the bisector of angle b/w the focal radial SP and $\perp$ to from P ~~the~~ on the directrix

* TSPM is Rhombus

* Centre of Circle Circumscribe triangle of PTN is focus of Parabola and its Diameter is TN

* All ray emerges out from focus will become $\parallel$ to the axis to the Parabola After reflection.

$m^4 + m^2 - 2 = 0$

$m^4 + 2m^2 - m^2 - 2 = 0$

$(m^2 + 2)(m^2 - 1) = 0$

$m = \pm 1$

$y = x + 2$

$y = -x - 2$

III

$y^2 = 8x \rightarrow y = mx + \frac{2}{m}$

$x^2 + y^2 = 2$

$\cancel{mx +}$

$mx - y + \frac{2}{m} = 0$

$C(0,0)$, $r = \sqrt{2}$

$P = r$

$$\left| \frac{0 - 0 + 2/m}{1 + m_2} \right| = \sqrt{2}$$

70

30/09/17

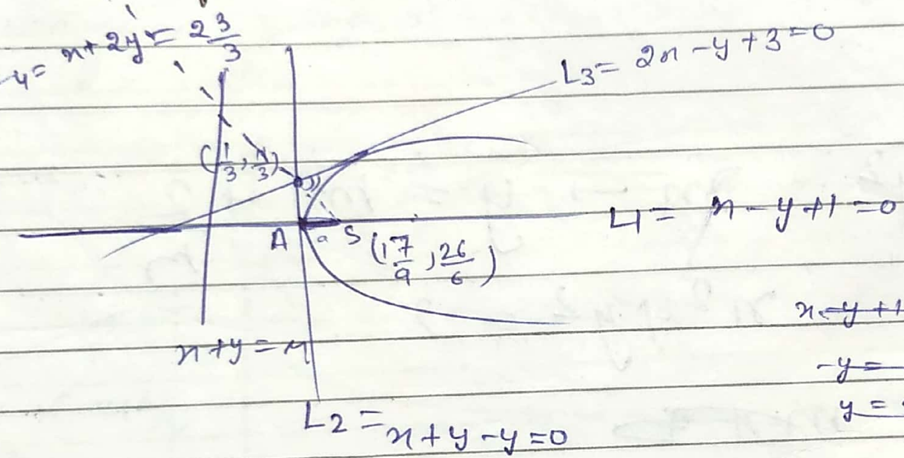
Q $L_1 \equiv x - y + 1 = 0$

$L_2 = x + y - 4 = 0$

$L_3 = 2x - y + 3 = 0$

Q L_1 be the axis of Parabola, L_2 is tangent of same Parabola at it's vertex and L_3 is one of its tangent

(i) find coordinate of focus of Parabola



$x - y + 1 = 0$

$-y = -x - 1 = 0$

$y = x + 1$

$y^2 = 4ax$

$(x+1)^2 = 4ax$

$x^2 + 2x + 1 = 4ax$

$x^2 - 2x + 1 = 0$

$x + 2y + \lambda = 0$

$\frac{1}{3} + 2 \cdot \frac{11}{3} + \lambda = 0$

$\lambda = -\frac{23}{3}$

Solve L_1 and L_4.

(ii) Length of L.R.

$4a = \frac{17}{9}$

$a = \frac{17}{4 \cdot 9}$

$AS = a$

$l \cdot (L \cdot R) = 4a$

Que: $L_1 = 5x - y - 3 = 0$

$L_2 = x + 5y - 11 = 0$

are tangent to a Parabola which read Parabola at a A and B.

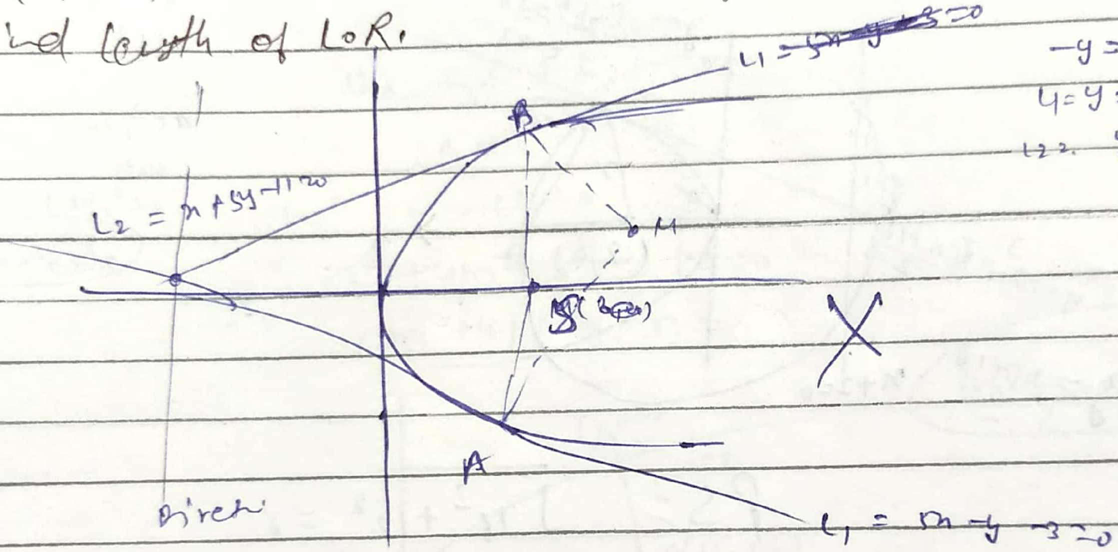
also normal at A and B intersect at Point $M(3,4)$, on the the axis of Parabola.

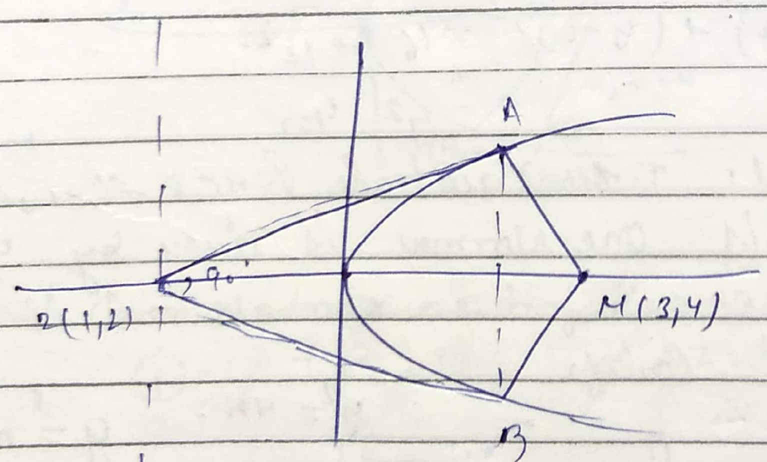
i) find length of LR.

$-y = -5x + 3$

$L_1 = y = 5x - 3$

$L_2: \; y = \frac{-x}{5} + \frac{11}{5}$



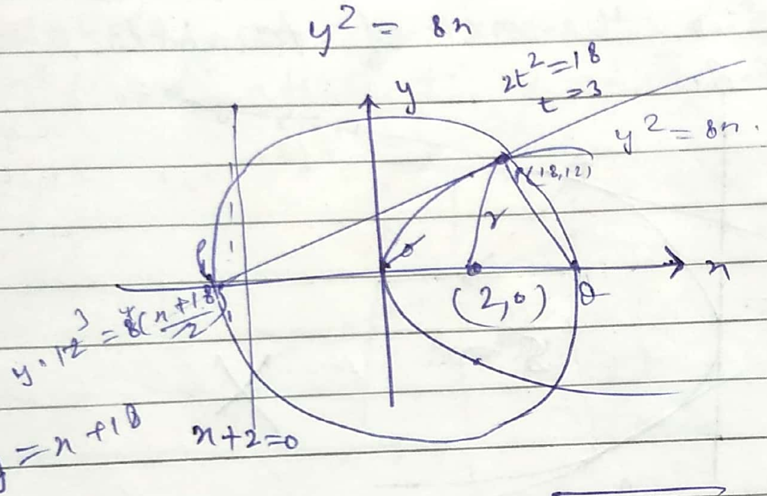


Length of (AB) $= l(2M) = l(L\cdot R) = 2\sqrt{2}$

$M_{2M} = 1$

Direct $y - 2 = -1(x - 1)$

Q. If T & N at Pt $P(18, 12)$ of the Parabola $y^2 = 8x$ intersect x axis at Points Q & R respectively find eqn of circle circumscribing ΔPQR.



$4a = 8$

$y^2 = 8x$

$at^2, 2at$

$(2t^2, 4t)$

$x^2 + y^2 = r^2$

$$PS = \sqrt{16^2 + 12^2} = r$$

$$(x-2)^2 + (y-0)^2 = 16^2 + 12^2.$$

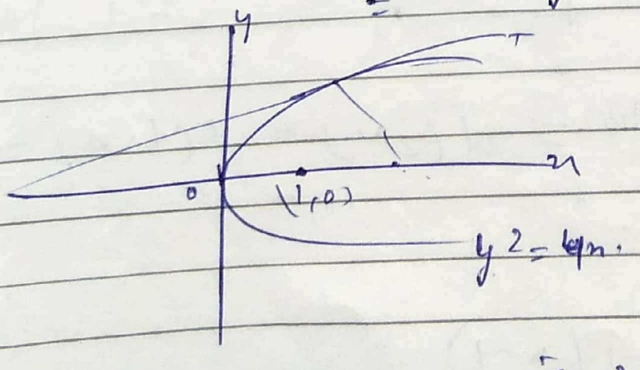
$y^2 = 4x$.

Que: Three Normals To the Parabola Passes through $P(15, 12)$ if one Normal is given by $y = x - 2$ then find remaining two normals and coordinate of co-normal points.

Ans

$y^2 = 4x$. (1)

$y = x - 2$



$y - 12 = m(x - 15)$

$= y = (-2) - y = x - 2$

$(x-2)(12) =$

$y^2 = 4x$

$(y - 12) = \frac{1}{m}(x - 15)$

$y = x - 2$

$= P(15, 12)$

$(y - 12) =$

Ans $y = mx - 2m - m^3$

$12 = 15m - 2m - m^3$

$m^3 - 13m + 12 = 0$

$m^3 - 1$

$m^3 - 13m + 12 = (m-1)(m^2 + \lambda m - 12)$

$0 = -1 + b$

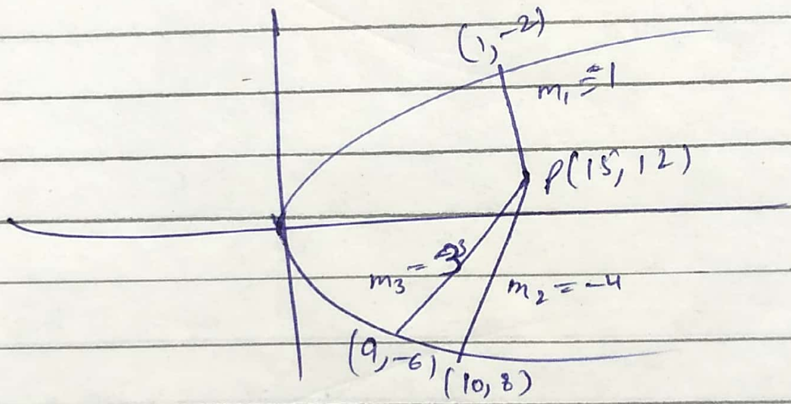
$+\lambda = 1$

$m^2 + m - 12 = 0$

$m^2 + 4m - 3m - 12 = 0$

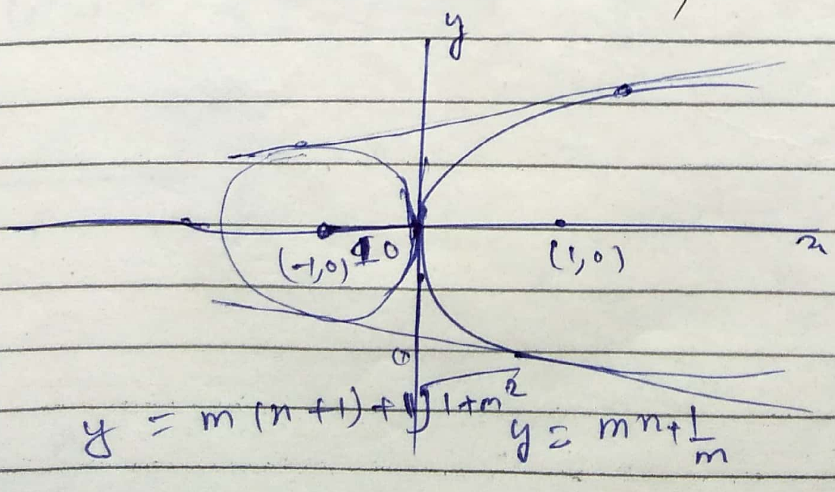
$(m+4)(m-3) = 0$

$(m^2, -2am)$



Que: find Common tangent to the circle and Parabola

$(x+1)^2 + y^2 = 1$ & $y^2 = 4x$



$y = m(x+1) + \sqrt{1+m^2}$

$y = mx + \frac{1}{m}$

$x = -1,\ y = 0$

$(-1, 0)\ \ r = \sqrt{1}$

$(x+1)^2 + y^2 - y^2 = 1 - 4x$

$x^2 + 2x + 1 = 1 - 4x$

$x^2 + 6x = 0$

$x(x+6)$

$x = 0$ | $x = -6$

$x + 6 =$

$y = mx + \frac{1}{x}$

$y = m(x+1) + 1\sqrt{1+x^2}$

$y = mx + x + \sqrt{1+m^2}$

$\frac{1}{1} + \frac{m}{m} = \frac{\frac{1}{m}}{m + \sqrt{1+x^2}}$

$\frac{1}{m} = m + \sqrt{1+m^2}$

$\left(\frac{1}{m} - m\right)^2 = (1+m^2)$

$m = 1$