P & C

* fundamental Principle of Counting:

① Principle of multiplyication:

Work A is done by 'm' ways. work B is done by 'n' ways ---. work is done only when work A as well as work B both are finished then no. of ways by which work can done by = m×n×p× ---

② Principle of Addition:

work A is Can be finised by n ways ---
" B " " "
!
!

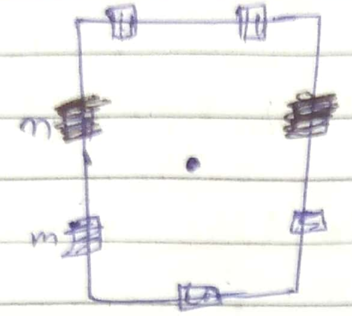
work is Consider as Complited if either of the work is finised. then it can done by m+n+p---

Ex: A work is done 3 ways
B " " " 2 way

$m \times n = 3 \times 2 = 6$ ways.

no of Sayyed wents kota to delhi by 3 ways & delhi to Azamgarh 2 ways

Ex

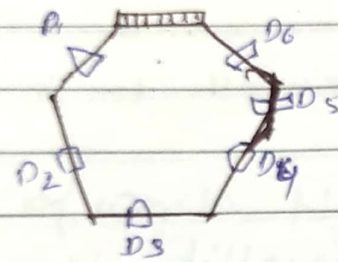


m = 5

n = 2

no. of ways = 5 + 2 = 7

Ex



① No. of ways franchi can inter leve the c.n by the Door

= 6×5 = 30

Ex franchi can inter and leave by any door

6 × 6 = 36

Ex franchi inter D1, D2 leave by D3, D4, D5, D6 ways

⇒ 2 × 4 = 8

Q. 10 true / false Qn

① find d total no. sequence of An

Q 2 Q1 2 × 2 = 4

Q2 T T

T F

F T

F F

3 Ques TTT

TTF

TFF

TFT

FTT

FFT

FFF

FTF

Q. 1, 2, 3, 4 given

(i) no. of 4 digit no.

(I) without replation

$4 \times 3 \times 2 \times 1 = n!$

(II) with repation

$4 \times 4 \times 4 \times 4 = 256$

Q. 0, 1, 2, 3, 4
no. of ways 3 digit no formed

(1) without repetation
without 2 + with zero

$4\times3\times2 + 2\times4\times3$

(2) with reputati
without zero + with zero

0.

$5\times3\times3 + 4\times4\times4$

Q. no. of words MIRACLE
(1) no. of different words form taken all
$7!$

(2) no. of word start with m finite with E

$1\times1\times5!$

(3) volel occupie evenposition

$(3\times4!$

(4) volet occupie odd position

$(4\times3\times2)\times4!$

(XI) DAUGHTER

(i) find total no. of 4 latter word formed   TEAR

8 7 6 5   8 7 _ _

(ii) no. of four latter word formed which included G.   first comes G. in every

~~8~~ ~~7~~ ~~6~~ 5   _ _ 4 _

$4 \times 7 \times 6 \times 5$

M-2 :

$8 \times 7 \times 6 \times 5 - \underline{7 \times 6 \times 5 \times 4}$

G नहीं थ.

इसमें सारे है
G included
G exclusion.

Dictionary Problems :

Tough

find rank of Tough.

GHOTU - ①

GHOUT - ②

TG _ _ _

TH _ _ _

TOU _ _

G से शुरू होने वाले word $= \lfloor 4 = 24$

H $= \lfloor 4 = 24$

O $= \lfloor 4 = 24$

TG $= \lfloor 3 = 6$

TH $= \lfloor 3 = 6$

TOG $= \lfloor 2 = 2$

TOH $= \lfloor 2 = 2$

TOU 1

89 An

$\lfloor 0 = 1$

* $\lfloor 3 = 3 \cdot 2 \cdot 1$
$\lfloor 4 = 4 \lfloor 3 = 4 \cdot 3 \cdot 2 \cdot 1$
$\lfloor n = n \cdot (n-1) \cdot (n-2) \text{ --- } 3 \cdot 2 \cdot 1.$

$\lfloor 2n = 2^n \lfloor n . [1 \cdot 3 \cdot 5 \cdot 7 \text{ -- } (2n-1)]$

* no. of Prumuttation of n distinct things taken are at a time is

$$\boxed{{}^nP_r = P(n,r) = A^n_r = \frac{\lfloor n}{\lfloor n-r}}$$

(2) no. of — — — — — — —
— taken all at a time.

$${}^nP_n = P(n,n) = A^n_n = \lfloor n \qquad \lfloor 0 = 1$$

(3) no. of Combination/selection of n distinct things taken are at a time

$$\boxed{{}^nC_r = C(n,r) = \binom{n}{r} = \frac{\lfloor n}{\lfloor r \lfloor n-r}}$$

* D K P $\quad {}^3P_2$

selection ${}^3C_2 = \frac{\lfloor 3}{\lfloor 2 \lfloor 3-2} = 3$ DK, DP, KP

D	K
K	D
D	P
P	D
P	K
K	P

Arranging ${}^3P_2 = {}^3C_2 \times \lfloor 2 = 3 \times 2 = 6$

* $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

* $^nC_r = {}^nC_{n-r}$

* if $^nC_x = {}^nC_y \Rightarrow$ either $x = y$
or $x + y = n$.

* $^nP_r = {}^nC_r \lfloor r$

Que: if a Birthday Party 101 People Present then find no. of hand Shake

101 full people Double people are hand shake

$= {}^{101}C_2$ Ans because

$\frac{\lfloor 101}{\lfloor 2 \lfloor 101-2} \quad \frac{\lfloor 101}{\lfloor 2 \lfloor 99} \quad \frac{101 \times 100}{2} = 101 \times 50 = 5050$

$101 \times 100 \times$
101
50
000
505
$\boxed{5050}$

Q. There n Point in Plane, no. Three are which Collinear
(i) no. of straight lines.
$= {}^nC_2$ Ans $\qquad {}^5C_2 = \quad {}^5C_2$.

(ii) no. of triangle formed
${}^nC_3$ Ans

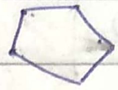
(iii) No. of

Qu

$^6C_2 - 6$

$3 \cancel{\frac{6\times5\times4\times3\times2\times1}{2\times4\times3\times2}} = 15-6 = 9$

Q. n point ~~Q in a line~~ in a plane, no three are of them collinear then find no. of diagonal in the polygon



$^nC_{n-2}$

$^5C_2 - 5 = \cancel{\frac{5\times4\times3\times2\times1}{2\times3\times2}} = 10$, $10-5=5$

$^5C_2$ → side + diagonal

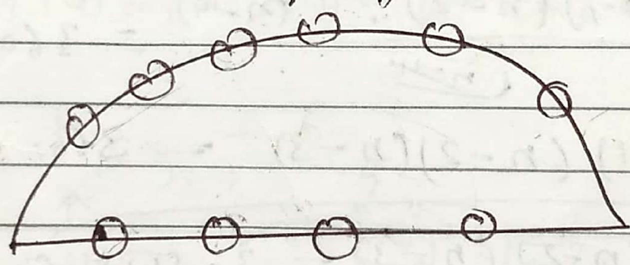
$^nC_2 - n$.

all side are substract from full:

find full $^nC_2$ Pair − sides

* Circle $^5C_3$ because 3 Point are not collinear make circle.

Qu 10 Point in a plane of which no three lines collinear except four.



24

(i) no. of straight line formed. $^6C_2 + 1$

$\frac{6\times5}{2} = 15 + 1 = 16$.

$^4C_2 + {}^6C_2 + 1$

$^{10}C_2 + 1$

M-1 $^{10}C_2 - {}^4C_2 + 1$ because four are collinear

$\frac{10\times9}{2} - \frac{4\times3}{2} + 1$

$45 - 6 + 1 = 40$

M-2 $^6C_2 + {}^6C_1 \cdot {}^4C_1 + 1$. $15 + 6\times4 + 1 = 25 + 15 = 40$

Q. no. of triangle formed $= {}^6C_3 + {}^4C_3$

$^6C_3 + {}^4C_2 + {}^6C_2 \, {}^4C_1$

Ans (M-1) $^{10}C_3 - {}^4C_3$

(M-2) $^6C_3 + {}^6C_2 \cdot {}^4C_1 + {}^6C_1 \cdot {}^4C_2$

Six possible and one other

*Q. if $^nP_4 = 360$

then find n.

$\frac{\lfloor n}{\lfloor n-4} = 360$

$\frac{n \cdot (n-1)(n-2) \ldots (n-4)}{\cancel{\lfloor n-4}} = 360$

$n(n-1)(n-2)(n-3) = 360$

$n(n-1)(n-2)\cdot(n-3) = 3 \cdot 4 \cdot 5 \cdot 6$

$n = 6$ Ans

$\frac{\lfloor n}{\lfloor n-4} = 360$

$\lfloor n = 360 n - 1440$

$n \cdot (n-1) \cdot (n-2)$

Q. In How many ways 8 I, 4 Am., 4 Ch. man be setted together in a row such that

(i) All Person of same noday well sett together,

$\lfloor 3$ I C A

$\lfloor 3 \lfloor 8 \lfloor 4 \lfloor 4$

firstly we combined combined and then Do factorial for 8.

(ii) No. two indians are together.

Create Gap.

Eight Chines and American are sitted and creat 9 gap and sitting Indi—

$\lfloor 8 \quad {}^{9}C_{8} \quad \lfloor 8$ Ans.

Q. Out of 6 men and 4 women a Commetty of 5 to be formed which include at least one lady in Commetty. find no. of ways.

$${}^{6}C_{4}\,{}^{4}C_{1} + {}^{6}C_{3}\,{}^{4}C_{2} + {}^{6}C_{2}\,{}^{4}C_{3} + {}^{6}C_{1}\cdot{}^{4}C_{4}$$

(4M + 1W) or (3m + 2W) or (2m + 3W) or (1m + 4W)

M-2

$${}^{10}C_{5} - {}^{6}C_{5}$$

can full of the ~~to~~ full

↑ full Case selection.

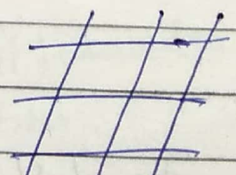
↑ and ~~...~~ not for all men because not selected 5 men

Q. m Parallel lines are cut by another set of n parallel lines find no. of ||gm formed

Ans $\;{}^{m}C_{2} * {}^{n}C_{2}$ =

m=3
n=4

2 lines of m and 2 lines of n are made of the Parallogram then formed.

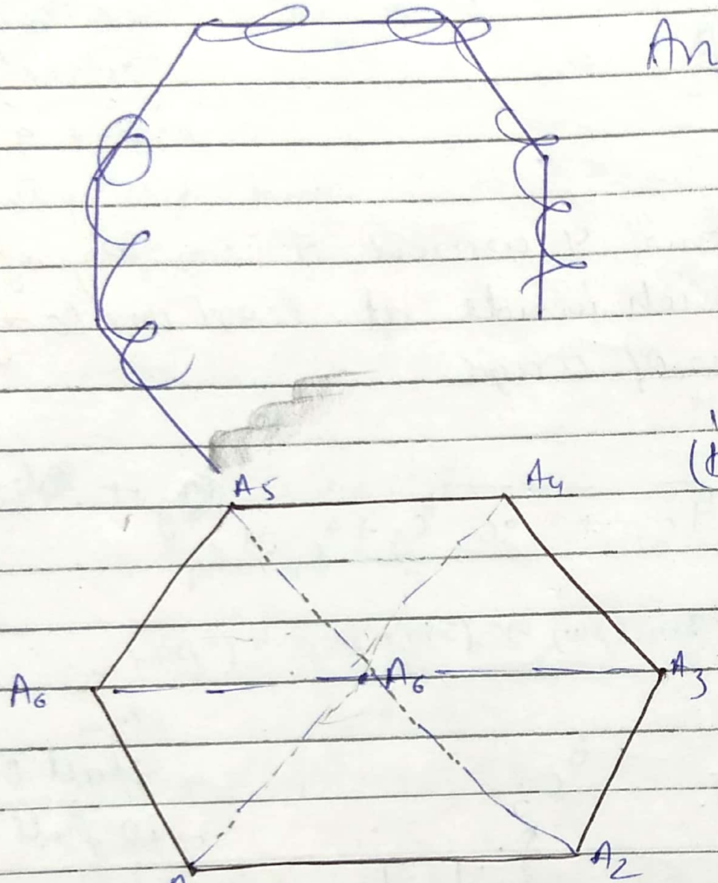


${}^{3}C_{2} \cdot {}^{4}C_{2}$

7×6×

Q. consider 6 vertices of regular hexagon and it's centre

(i) find no. of straight line formed



Ans: $^7C_2 - 2 - 2 - 2$

$= \frac{\lfloor 7}{\lfloor 5 \lfloor 2} - 6$

$= 15$

Two sides and one side

A_1A_4, A_1A_6, A_4A_6 are in same line

(ii)

(ii) no. of triangles

$^7C_3 - 3$ Ans

because Three are colinear so they not count.

Q. 4 B and 4 G are to be sitted in a line

(i) no. of ways they can be sitted

$\lfloor 8$

~~$\lfloor 2$~~

Phone E.

(ii) No. two girls are together.

o — o — o — o — o full and select gap.

$\lfloor 4 \; {}^5C_4 \; \lfloor 4$

(iii) Boys are and Girls are alternate

$G_1B_1\; G_1B_2\; G_1B_3\; G_1B_4$. no girls are

$B_1 G_1\; B_2 G_1\; B_3 G_1\; B_4$.

Both are diff

$2 \cdot \lfloor 4 \lfloor 4$

(iv) All girls are sitted together

B B B B (G) G line

$\lfloor 5 \lfloor 4$ Ans

Because Boys (4) girl and are sitted and total are make in 5 the $\lfloor 5$

(v) B and G are Alternate and a perticular B and girl are never adjecent to each other.

$2\lfloor 4 \lfloor 4 - {}^7C_1 \, 2$ $(B_1 G_1)$

— — — — — — (2)

$2\lfloor 4 \lfloor 4 - \underbrace{2\lfloor 3 \lfloor 3}_{\text{Two are inside}} \; \underbrace{{}^7C_1 \cdot 1}_{\text{Both are in Alternate and sitting.}}$

Q. in How many ways Can 4 Passenger be approached in 3 Car in each car can accomudate any no. of Passenger.

$3 \times 3 \times 3 \times 3 = 3^4$ Ans.

— — —

1 Passenger Sitting any car

Next Race

NCERT

$^5C_0 : \frac{5}{...}$

Shoes Problem :

10 Persons (5 married Couples) are present in a party and 4 are them Randomly selected

(a) find no. of ways

H_1 H_2 H_3 H_4 H_5

W_1 W_2 W_3 W_4 W_5

(i) excately 2 couples :- $^5C_2$

(ii) " 1 couple selected $^5C_1 = {}^5C_1 \cdot {}^4C_2 \cdot {}^2C_1 \cdot {}^2C_1$

full — couple not to be sel

only one couple selected

(iii) at least 1 cou. selected

(iii) $^5C_2 + {}^5C_1 \cdot {}^4C_2 \cdot {}^2C_1 \cdot {}^2C_1$

① + ②

(iv) no couple selected

$^5C_4 \cdot {}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1 \cdot {}^2C_1$

Q. BT Express move from Jaunpur to delhi stem at 12 intermediat stations

Q. A train having 12 stations in its root hence stop at 4 stations. find no. of ways it can be stop if no too stopping station are conjugating.

K A B C D E F G H I J K L NDL

$12 - 4 = 8$ ∴ gap $8+1=9$

$^9C_4$.

A B C D E F G H I J K L

Q. find no. of 3 digit no. if each successive digit from left to right are in their decending order.

_ _ _ $\{0,1,2,3,4,5,6,7,8,9\}$ 5 3 2

$\lfloor 10$

$\lfloor 3 \lfloor 7$. $\lfloor 9 \lfloor 8 \lfloor 7$

(ii) in ascending 1 3 9.

(i) ${}^{10}C_3$ Ans $\{0,1,2,3,4,5,6,7,8,9\}$

(ii) ${}^{9}C_3$ $\{1,2,3,4,5,6,7,8,9\}$ $\neq 0$

formation of group:

Distribution of different thing among person by formation of gp.

Suppose $m+n$ $(m \neq n)$ thing are ~~distributed int~~ divided into two gps. the no. of ways it can be done

$$= \frac{\lfloor m+n}{\lfloor m \lfloor n}$$

no. of ways it can be distributed in two person $(m \neq n)$ $= \frac{\lfloor m+n}{\lfloor m \lfloor n} \times \lfloor 2$

* $m = n$

if gp size are same then ~~two~~ 2m distinct thing divided into two gps $= \frac{\lfloor 2m}{\lfloor m \lfloor m \lfloor 2}$

2m — — distributed in — — person $= \frac{\lfloor 2m}{\lfloor m \lfloor m \lfloor 2} . \lfloor 2$

Ex: 1 केला + 1 Apple + 1 पेड़ा

$m = 2 \quad n = 1$

Division in to gp $\frac{\lfloor 3}{\lfloor 2 \lfloor 1} = 3$

Ist Bag	IInd Bag
B	AS
A	BS
S	AB

Distribution $\frac{\lfloor 3}{\lfloor 2 \lfloor 1} \cdot \lfloor 2$

① पहला	② दूसरा
B	AS
AS	B
A	BS
BS	A
S	AB
AB	S

Ex: 1 मैंगो + 1 केला + 1 A + 1 पेड़ा

$m = 2 \quad n = 2$

Division into gp $= \frac{\lfloor 4}{\lfloor 2 \lfloor 2 \lfloor 2} = \frac{6}{\lfloor 2} = 3$

when same size gp. all present

①	②
BM	AS
AM	BS
SM	AB

Distribution $= \frac{\lfloor 4}{\lfloor 2 \lfloor 2 \lfloor 2} \cdot \lfloor 2$

involve persons

* $m+n+p$ thing $(m \neq n \neq p)$

no. of ways it can be divide into three gp $= \dfrac{\lfloor m+n+p}{\lfloor m \lfloor n \lfloor p}$

" " " " " distribution -- " person :

$$= \frac{\lfloor m+n+p}{\lfloor m \lfloor n \lfloor p} \lfloor 3$$

$m+m+m$ distinct thing divided into three gp $= \dfrac{\lfloor 3m}{\lfloor m \lfloor m \lfloor m \lfloor 3}$

" " " distributed " " person $= \dfrac{\lfloor 3m}{\lfloor m \lfloor m \lfloor m \lfloor 3} \cdot \lfloor 3$

ex: 10 distinct thing are divided in three gp. such that $\left\langle \begin{matrix} 5 \\ 3 \\ 2 \end{matrix} \right.$

$\therefore$ no. of ways $\dfrac{\lfloor 5+3+2}{\lfloor 5 \lfloor 3 \lfloor 2}$

(ii) 10 things are distributed in among three person such that $\left\langle \begin{matrix} 5 \\ 3 \\ 2 \end{matrix} \right.$

no. of ways $= \dfrac{\lfloor 10}{\lfloor 5 \lfloor 3 \lfloor 2} \lfloor 3$

(iii) Distributed " " " " $\left\langle \begin{matrix} 4 \\ 4 \\ 2 \end{matrix} \right.$

$$= \left(\frac{\lfloor 10}{\lfloor 4 \lfloor 4 \lfloor 2 \lfloor 2} \right) \lfloor 3$$

(4) 20 thing -- - Distributed among 7 person $\left\langle \begin{matrix} 2 \\ 2 \\ 2 \\ 4 \\ 4 \\ 5 \\ 1 \end{matrix} \right.$

$$= \left(\frac{\lfloor 20}{\lfloor 2 \lfloor 2 \lfloor 2 \lfloor 4 \lfloor 4 \lfloor 5 \lfloor 1 \lfloor 3 \lfloor 2} \right) \lfloor 7$$

Ex! in how many ways a pack of 52 card be equal dist. among 4 player

$$= \left(\frac{52!}{13!\,13!\,13!\,13!\,4!} \right) 4!$$

Note: जितने gp same हैं उतने factorial से divide करना है।

Q. in How many ways 12 distinct items are distributed among 3 Person $12 \to 7, 4, 1$ such that Younger person recieve 7 items

$$\left(\frac{12!}{7!\,4!\,1!} \right) 1 \cdot 2!$$

Q. in How many ways 4 Passanger can be accumadated in 2 roome if each room accomadate any no. of Passanger

(M-1) $2 \cdot 2 \cdot 2 \cdot 2 = 16$.

(M-2) $(4+0) + (1+3) + (2+2)$

$$\frac{4}{4!\,0!} 2! + \frac{4}{1!\,3!} 2! + \frac{4}{2!\,2!\,2!} 2! = 16 \text{ Ans}$$

432

Q. 6 different Books to will distributed among K/D/P such that each Person gets at least one Book

M-2 $= \left(\frac{6!}{2!\,2!\,2!\,3!} \right) 3!$

$(4,1,1) + (3,2,1) + (2,2,2)$

$$\left(\frac{6!}{4!\,1!\,1!} + \frac{6!}{3!\,2!\,1!} + \frac{6!}{2!\,2!\,2!\,3!} \right) 3!$$

30 + 60 + 20

110 × 6

660 Ans

Q In how many ways 13 cards to each of the four player be distributed from a pack of 52 card
Such that each may have
(i) A/K/Q/J ~~such that~~ of same suit

$$\frac{\lfloor 52}{\lfloor 13 \lfloor 13 \lfloor 13 \lfloor 13}$$

$$\lfloor 4 \left(\frac{\lfloor 36}{\lfloor 9 \lfloor 9 \lfloor 9 \lfloor 9 \lfloor 4} \right) \lfloor 4$$

* Permutation of Alike objects:

no. of Permutation of n things of which p all of one kind q are of 2nd kind rest are distinct.
then no. of permutation

$$= \frac{\lfloor n}{\lfloor p \lfloor q}$$

* 1 fan, + 1 Bulb + 1 Pen + 1 Copy no. of Per $= \lfloor 4$

* 2 fan + 1 Pen + 1 Copy = no. of per $= \frac{\lfloor 4}{\lfloor 2} = \frac{4 \cdot 3 \cdot 2}{2} = 12.$

Q. no. of Permutation, ~~taken all~~

MAHABHARAT

1 4 2 1 1 1
M A H B R T

no. of Perm. $\frac{\lfloor 10}{\lfloor 4 \lfloor 2}$

(2) no. of Re-arrangement of MAHABHARAT

$$\text{no. of Per} = \frac{\lfloor 10}{\lfloor 4 \lfloor 2} - 1$$

Q. Seven Identical balls are Present then

(i) no. of ways of selecting seven balls = 1 Ans

" " " " Six " = 1 "

Five = 1

Four = 1

3 = 1

2 = 1

1 = 1

Q. No. of Possible arrangement of Car SUCCESS

SUCCESS $\overset{1}{S}\overset{1}{U}\overset{2}{C}\overset{3}{S}$

6 $\lfloor 2$ $\frac{\lfloor 7}{\lfloor 2 \lfloor 3}$

Q. There are 5 Positive sign and 4 (−)ve sign How many ways these nine sign can be arrange in a line

$$= \frac{\lfloor 9}{\lfloor 5 \lfloor 4}$$

Q. 9 7 w 3 Red. 3 Red Rem

Q. 21 w and 19 Black Ball are arranged in a line (Balls of the same colour are alike). find no. of arrangem if black balls are ~~separated~~ seperatedly together.

$$1 \times \lfloor 21 \cdot {}^{22}C_{19} \lfloor 19 \Rightarrow 1 \times {}^{22}C_{19} \times 1$$

white balls are same.

black ball are sep

~~Director~~ AEIOU 6

Q. DIRECTOR

DIRECTOR

$\boxed{E\ I\ O}$

(i) no. of Permutation taken all

$$\frac{\lfloor 8}{\lfloor 2}$$

(ii) no. of Per if vowel are together

$\lfloor 6 \lfloor 3 \lfloor 5$ D R C T (...)

$$\frac{\lfloor 6 \ \lfloor 3}{\lfloor 2}$$

$D\overset{2}{R}CTR$

(iii) no. two vowels are together $\overset{1}{D}\ \overset{2}{R}\overset{1}{C}\overset{1}{T}$

without vowels $\rightarrow \frac{\lfloor 5}{\lfloor 2} \cdot {}^6C_3 \cdot \lfloor 3$ = three vowels are selected

same R 2

Q. ASSASSINATION

$A^3 S^4 I^2 N^2 T^1 O^1$ (6)

(i) no. of Permutation taken all

$$\frac{\lfloor 13}{\lfloor 3 \lfloor 4 \lfloor 2 \lfloor 2}$$

(ii) no. of Permutation such that no. two vowels are conjugated.

$$\frac{\lfloor 7}{\lfloor 4 \lfloor 2} \cdot {}^8C_6 \cdot \frac{\lfloor 6}{\lfloor 3 \lfloor 2}$$

no. of gap b/w the seven non-vowel

clear

(iii) No two S ~~are separated~~ ~~to~~ occur together.

$$\frac{\lfloor 9}{\lfloor 3 \lfloor 2 \lfloor 2} \cdot {}^{10}C_4 \cdot 1$$

(iv) no. of Permutation taken all without the changing the order of Viowel.

ASSASSINATION 13

$A^3 S^4 I^2 N^2 T^1 O$

$\overset{6}{X}\ \overset{4}{S}\ \overset{2}{N}\ \overset{1}{T}$

AAIAIO

$$\frac{13!}{6!\,4!\,2!}$$

Replace all Viowels in identical.

M-2 $\;{}^{13}C_6 \cdot \dfrac{7!}{4!\,2!}$

Position.

(v) keeping the each Viowel fixed.

Voice fixed

$$1 \cdot \frac{7!}{4!\,2!\,1!} \text{ Ans}$$

(vi) without changing the relative order of Viowel and Consonant

Vowel की जगह Viowel Comes and Consonat की जगह Consonat

$$\frac{7!}{4!\,2!\,1!} \cdot \frac{6!}{3!\,2!} \text{ Ans}$$

Advanced

A* AP

* Total Per/C taken some at a time ⇒

Qu. INDEPENDENCE ($E^4 N^3 D^2 I^1 P^1 C^1$)

Find ~~toto~~ P/C of 5 letter word $\quad {}^{12}C_5 \lfloor 5$

Type	Combination	Perm.
(1) 4 alike + 1 distinct	$1 \cdot {}^5C_1$	$1 \cdot {}^5C_1 \cdot \frac{\lfloor 5}{\lfloor 4}$ clear same no.
(2) 3 alike + 2 alike	${}^2C_1 \ {}^2C_1$	${}^2C_1 \ {}^2C_1 \left(\frac{\lfloor 5}{\lfloor 3 \lfloor 2}\right)$
(3) 3 alike + 2 dist	${}^2C_1 \ {}^5C_2$	${}^2C_1 \ {}^5C_2 \ \frac{\lfloor 5}{\lfloor 3}$
(4) 2 alike + 2 alike + 1	${}^3C_2 \ {}^4C_1$	${}^3C_2 \ {}^4C_1 \ \frac{\lfloor 5}{\lfloor 2 \lfloor 2}$
(5) 2 alike + 3 distinct	${}^3C_1 \ {}^5C_3$	${}^3C_1 \ {}^5C_3 \ \frac{\lfloor 5}{\lfloor 2}$
(6) 5 distinct	${}^6C_5$	${}^6C_5 \lfloor 5$

* Circular Permutation :-

Circular Perm. of n distinct things, taken all at ~~a~~ a time = $\lfloor n-1$

(clockwise & anticlockwise both included & different)

$\frac{1}{2} \lfloor n-1$ (only clockwise & anti clockwise are same)

* Out of n distinct things circular Permutation of are distinct object.

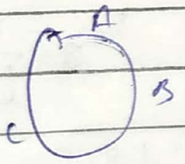
$$= \begin{cases} {}^{n}C_{r}\,\lfloor r-1, & (\text{↻↺}) \text{ are distinct} \\ {}^{n}C_{r}\left(\frac{1}{2}\lfloor r-1\right), & (\text{↻↺}) \text{ are same.} \end{cases}$$

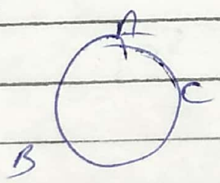
Ex: A, B, C ' P = $\lfloor 3 = 6$

ABC
ACB
BAC
BCA
CAB
CBA

C.P of 3 thing (A, B, C) taken all

$= \lfloor 3-1 = \lfloor 2 = 2$





Q. How many ways 10 Person can be sitted around circular table

$= \lfloor 10-1 = \lfloor 9$

Q. How many ways 7 out 10 Person can be sitted at C. tab

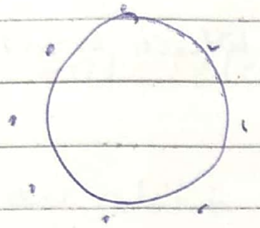
${}^{10}C_7\,\lfloor 7-1 = {}^{10}C_7\,\lfloor 6$

Q. Out of 10 flower of diff. colour, How many diff. can be made if each garl consist of 6 flower of different colour

${}^{10}C_6\left(\frac{1}{2}\lfloor 5\right)$

Q. no. of ways in which 7 boys and 7 girls can be sitted on a round table no two girls are conjugated.

Ans) 7B + 7G



$\lfloor 7-1 \quad \lfloor 7$

first sitted boys and then numberin Girls sitting is fixed

Q. No. of Ci Per- of 10 Person if 2 specified Person are never together.

Ans 1. $\lfloor 9 - {}^{9}C_2 \ldots$

$$= \boxed{\frac{{}^{20}C_1 \, {}^{(20-3-4)+1}C_2}{5}}$$

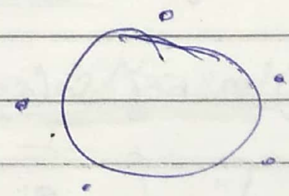
same as Q. 5

$$\lfloor 10-1 - \lfloor 9-1 \, \lfloor 2$$

साथ साथ है + साथ साथ नहीं

Q. No. of ways in which 5 G and 5 B can be sitted on a circular table alternatevely if Ram and Jeet are never adjecent to each other in any arrangment

$$\lfloor 10-1 - \lfloor 9-1$$



$$\lfloor 5-1 \; {}^{3}C_1 \cdot \lfloor 4$$

because

Ques 5) 15 Person are sitting on a circular table. no. of ways 3 Person can be selected if no two are of them conjugated

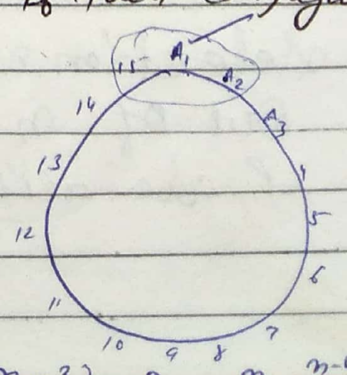
$$15_{\bigcirc}$$

$$^{15}C_1 \left((15-3) - 2 \right) + 1 \, C_2$$

for gap

select 1 out of 15

$$= {}^{15}C_1 \; {}^{11}C_2$$



$$^{n}C_1 (n-3) - 2 = {}^{n}C_1 \; {}^{n-4}C_2$$

⇒ fixed one point on circle . $^{15}C_1$ (say A_1). leave 2 neighbour A_1 and A_{n-1} (or A_2 and A_{14}) there for $n-3$ left.
I have to pic Two person from $n-3$ candidate so ~~(n-3)~~-2

So $(n-3)-2$ person sitting on a line which creat $n-4$ gaps

now chose two gap by $^{n-4}C_2$ ways.

Hence our outcome will be $(^nC_1\ ^{n-4}C_2)$

$= \frac{^nC_1 \cdot ^{n-4}C_2}{3}$ or $\left(\frac{^{15}C_1\ ^{11}C_2}{3}\right)$

this is the aquarate Ans of the Que ~~but~~ the three person ~~can~~

* Total no. of Combination

① no. of selection of zero or more thing out of n identical thing = $n+1$

② no. of selection of one or more thing from n identical thing = n.

{ in case ① we can add to ~~at~~ zero for the ways | in case ② ~~add~~ subs zero }

③ Selection of zero or more thing of which 'p' out of n different thing which contain ~~p~~ t of P are alike, Q are alike of different kind.

Ex $n = 10$ — p are alike, q are alike

$$n = 10 \to 3 \text{ fan } (p),\ 7 \text{ bulb } (q)$$

total selection $= (p+1)(q+1)$

⇒ r are alike of third kind.

$$n = 10 \to 3 \text{ fan } (p),\ 7 \text{ bulb } (q),\ 4 \text{ pen } (r)$$

total selection $(~~p+1~~)(q+1)(r+1)$

$\Rightarrow \quad p(q+1)(r+1)$

p means

$\boxed{\text{atleast 1 fan is compulsory in the selection.}}$

~~because no~~

⇒ Selection of at least one thing

$(p+1)(q+1)(r+1) - 1$ (खाली हाथ ना

⇒ pqr ⇒ in selection ~~...~~ 3 में से एक जरूरी है always

* selection of at least one thing Out of **n different** thing

$$= \boxed{{}^nC_1 + {}^nC_2 + \cdots {}^nC_n = 2^n - 1}$$

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n = 2^n}$$

1.

OR

$$= \underbrace{(1+1)(1+1)(1+1) \cdots (1+1)}_{n} - 1$$

$$= 2^n - 1$$

Q. ~~AAAAA, BBBB, CCC, DD,~~

* P are alike of one kind
Q " " " 2nd "
and Remaining r items are all distinct

$$\boxed{\text{Total selection} = (P+1)(q+1)(2^r)}$$

Q. AAAAA, BBBB, CCC, DD, EFG.
(i) find total no. of selection = $(5+1)(4+1)(3+1)(2+1)2^3$
(ii) Selection of at least one thing = $(5+1)(4+1)(3+1)(2+1)2^3 - 1$
(iii) " " " " one 1A and at least 1B =

$$(5)(4)(3+1)(2+1)2^3$$

(iv) Selection at least 2A and 1E

~~$^5C_2 - 2(5)(1)(4+1)(3+1)(2+1)2^2$~~

2A — 1
3A — 1
4 — 1
5A — 1

$$4\cdot(4+1)(3+1)(2+1)(1+0)(1+1)(1+1)$$

Q. Out of three diff. maths Book ~~Calculus~~, $(M_1+M_2+M_3)$
four diff Phy Book $(P_1+P_2+P_3+P_4)$
5 diff chem " $(C_1+C_2+C_3+C_4+C_5)$
find no. of selection so that
if each selection consist of

(i) exectly one Book on each Sub. ~~(3)(4)(5)~~

$$^3C_1 \cdot {}^4C_1 \cdot {}^5C_1$$

(ii) at least one book on each Sub.

~~(3)(4)(5)~~.

$$(2^3-1)(2^4-1)(2^5-1)$$

(iii) taking at least one Book.

$$(2^3)(2^4)(2^5) - 1$$

* T arrangement N latters are to be capped in n envalaped. no. of ways they can be placed if non of latter goes to correct envalap.

$$D(n) = \lfloor n \left(1 - \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} - \frac{1}{\lfloor 3} - - - (-1)^n \frac{1}{\lfloor n}\right)$$

$$D(4) = \lfloor 4 \left(1 - \frac{1}{\lfloor 4} + \frac{1}{\lfloor 2} + \frac{1}{\lfloor 3} + \frac{1}{\lfloor 4}\right)$$

Excatly one is way in correct

find no. of ways. $= {}^4C_1\, D(3)$

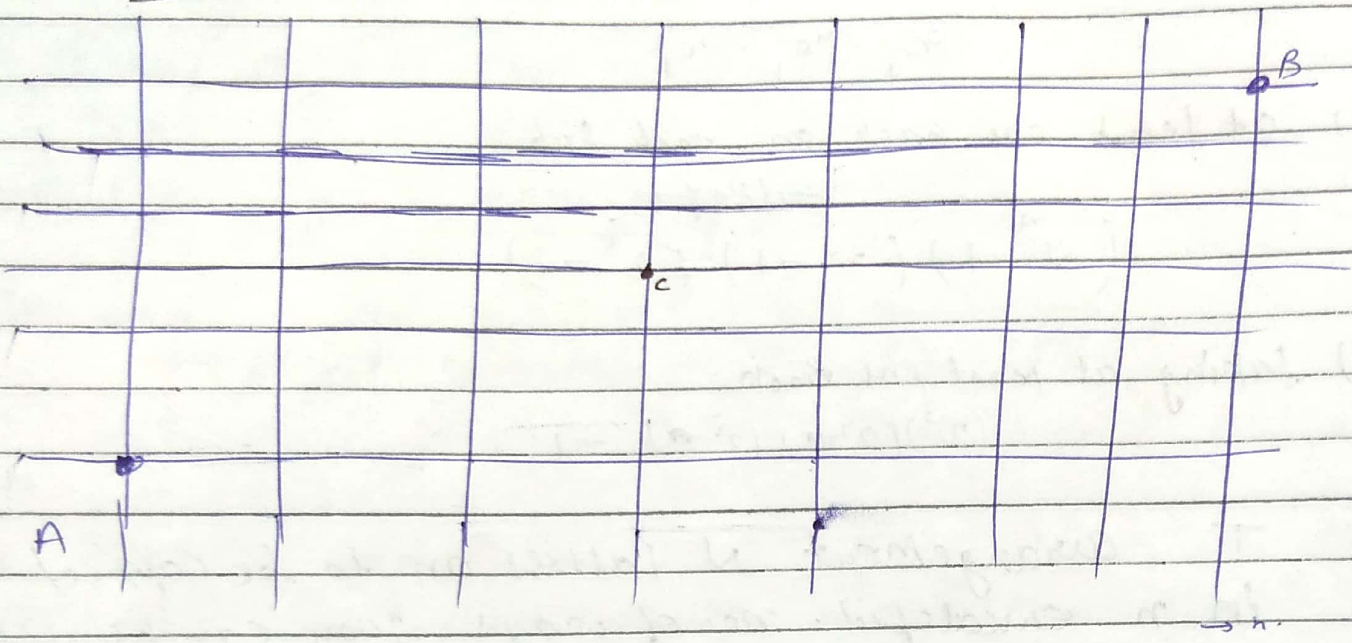
* max value of $\overset{\rightarrow \text{given}}{{}^{n}C_{r}}$ (r unknown) $= \begin{cases} r = \frac{n}{2} & , n = \text{Even} \\ r = \frac{n-1}{2} \text{ or } \frac{n+1}{2} & , n = \text{odd} \end{cases}$

Ex: find max value of ${}^{24}C_r$ $= r = 12$

$${}^{13}C_r = r = 6 \text{ or } 7$$

$$r = \frac{13-1}{2}, \frac{13+1}{2}$$

* Grread Problem :



6 horizontal or 8 virtical

① Shortest Path.

for Shortest Path move to the given dir^n.

or

$\xrightarrow{R}$ or ↑ up.

Q. In the situation find no. of shortest path to arrive at Pt P.

Total no. of arrangement of 7H & 5V = $\frac{\lfloor 12}{\lfloor 7 \lfloor 5}$

7H + 5V

in this case we move to shortest

and 7H & 5V all Possible arrangement.

Q. find no. of shortest Path A to B v/a C.

~~3H+8~~ 3H + 2V + 4H + 3V. AC + CB

$\frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \frac{\lfloor 7}{\lfloor 4 \lfloor 3}$

* Exponential ~~in~~ Prime factrization of $\lfloor n$:

$$\lfloor 3 = 2 \cdot 3 = 2^1 3^1$$

$$\lfloor 4 = 2 \cdot 3 \cdot 4 = 8 \cdot 3 = 2^3 \cdot 3^1$$

Prime factrization =

$$= \lfloor n = 2^a \, 3^b \cdot 5^c \cdot 7^d \cdots$$

where $a = \left[\frac{n}{2}\right] + \left[\frac{n}{2^2}\right] + \left[\frac{n}{2^3}\right] \cdots$

$$b = \left[\frac{n}{3}\right] + \left[\frac{n}{3^2}\right] + \left[\frac{n}{3^3}\right] \cdots$$

Here $[\cdot]$ is G.I.F.

Q. $\lfloor 12 = \cancel{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4} \cdot 3 \cdot 2$ $= \lfloor 12 = 2^{10} \cdot 3^5 \cdot 5 \cdot 7^1 \cdot 11^1$

$= \cancel{12 \cdot 2^3 \cdot 2 \cdot 2 \cdot 11 \cdot 10 \cdot 3^2 \cdot 7 \cdot 5 \cdot 3}$

$$a = \left[\frac{12}{2}\right] + \left[\frac{12}{2^2}\right] + \left[\frac{12}{2^3}\right] + \cancel{\left[\frac{12}{2^4}\right]}^0$$

$$= 6 + 3 + 1$$

$$b = \left[\frac{12}{3}\right] + \left[\frac{12}{3^2}\right] + \cancel{\left[\frac{12}{27}\right]}^0 = \cancel{4 + \frac{4}{3} = \frac{16}{3}} = 4 + 1 = 5$$

$b = \frac{16}{3}$ $\quad b = 5$

$$c = \left[\frac{12}{5}\right] + \left[\frac{12}{25}\right]$$

~~N = 7560 = $2^4 3^3$~~

$N = 12 = 2^2 \cdot 3^1$

No. of Divi $= (2+1)(1+1) = 6$.

Proper division $= 6 - 2$

-6 1 को हटाना तक or खुद को

no. of odd division =

= no 2 must be taken.

$= (0+1)(1+1) = 2$

Q. Sum of all divisors $= 2^0 3^0 + 2^1 3^0 + 2^2 3^0 + 2^0 3^1 + 2^1 3^1 + 2^2 \cdot 3^1$

$= 3^0(2^0 + 2^1 + 2^2) + 3^1(2^0 + 2^1 + 2^2)$

$= (3^0 + 3^1)(2^0 + 2^1 + 2^2)$

no. of ways an N can be Resolved can be Product of Perfect sq.

$a = 2, \ b = 1$

N can be dissolve as Product of two D $= \frac{(2+1)(1+1)}{2} = 3$

Ex: $12 = 1 \times 12$

2×6

3×4

$\Rightarrow$ dissolve in two relative Prime

- no. of prime fact. $= 2^{n-1}$

$n = 2$

$= 2^{2-1} = 2.$

$12 = 1 \times 12$

2×6

3×4

$12 = 3 \times 4$

$\frac{1}{2\cdot2\cdot2\cdot2}$ $\left(\frac{75600}{2}\right)+\left(\frac{75600}{4}\right)+\left(\frac{75600}{8}\right)$

Q. Consider no.

$N = 75600 = 2^4 3^3 5^2 7^1$

i) no. of divisions

$= (4+1)(3+1)(2+1)(1+1)$

$=$

Q (ii) no. of proper divisors = Total no. of division -2

$= (4+1)(3+1)(2+1)(1+1)-2$

(iii) no. of odd divisors =

$(0+1)(3+1)(2+1)(1+1)$

Q. no. of even division =

$\cancel{(4+1)(0+1)(0+1)(0+1)}$

at least one 2 must be taken

$= (4+0)(3+1)(2+1)(1+1)$

no 2

(v) no. of division divisible by 6. $= 2^1 3^1$

$\cancel{(0+1)(3+1)(0+1)(0+1)}$

at least one 2 and one 3 must be taken

$(4+0)(3+0)(1+1)(1+1)$

Q (vi) no. of division divisible by 100 = 18

$2^4 3^3 5^2 7^1$

$100 = 2^2 5^2$

22 — 1

222 — 1

2222 — 1

$(1+1+1)(3+1)(1)(1+1)$

$3 \cdot 4 \cdot 1 \cdot 2$

Q. Sum of all division =

xvii Sum of all division

$= \cancel{2^0 3^3 5^2 7^1 + 2^0 3^3 5^4 7^1 + 2^0 3^3 5^0 7^1}$

$$\left(2^0+2^1+2^2+2^3+2^4\right)\left(3^0+3^1+3^2+3^3\right)\left(5^0+5^1+5^2\right)\left(7^0+7\right.$$

Q. no. of ways N can be resolved Product of division

$N = p^a q^b r^c \ldots \quad (p, q, r \text{ — are prime})$

(one included)

$$= \begin{cases} \frac{1}{2}(a+1)(b+1), & \text{if } N \text{ is not Perfect Square} \\ \frac{(a+1)(b+1)(c+1) \ldots + 1}{2}, & N \text{ is PS} \end{cases}$$

no. of dissolve

$\Rightarrow \frac{(4+1)(3+1)(2+1)(1+1)}{2}$ = N can be resolve of Product of two.

$\Rightarrow$ no. of ways N can be resolved as a product of two division which are relatively Prime

$= 2^{n-1}$

where n is the no. of primes involved in the Prime factorisation of N

$n = 4 \qquad = 2^{4-1} = 2^3 = 8.$

* <u>Distribution of alike objects</u> :- (Beggers method)

no. of ways $n = 10$ identical ~~points~~ coin is distributed among $P = 4$ beggers.

=

o o | o o | o o o | o o o o $\qquad P-1 = 4-1 = 3$

o | o o o | o o o o o | o o

no. of ways of distribution $= (10 \text{ coin}) + (\text{Beeg} - 1)$

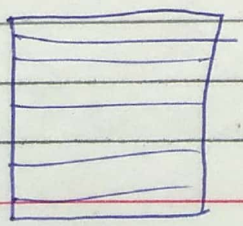
$= \dfrac{\lfloor n+P-1}{\lfloor n \, \lfloor P-1}$

$= {}^{n+P-1}C_{P-1}$

$$\boxed{= {}^{\text{सिक्का} + \text{भिखारी} - 1}C_{\text{भिखारी} - 1}}$$

Q. a almira contain 6 seprate compartmen. find no. of ways in which

i) find no. of ways in which 12 identical like marble is placed in the compartment.



$n = 12, \quad P = 6.$

${}^{12+6-1}C_{6-1} = {}^{17}C_5$

no. ${}^{12+5}C_5$

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 12$

(ii) no. of ways of distri. Such that no compartment is empty.

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 6.$$

$n = 6 \qquad p = 6$

$$^{6+6-1}C_{6-1} = {}^{11}C_5$$

(iii) in how many ways 30 marks be alloted to 8 question if at ~~x~~ 2 mark are to be given to each que. (Assuming marks are alloted is non negative integral value

30 (14) n ~~p~~ 8 $^{14+8-1}C_{8-1} = {}^{21}C_7 =$

Que: no. of natural Solution ~~n.~~ to this eqn $x + y = 100$

OR

no. of ways 100 coin is distributed among two person Such that get at least one ~~Rs.~~ Coin.

$x' + y' = 98$, Coin $= 98$

Begger $= 2$

$$^{98+2-1}C_{2-1} = {}^{99}C_1 = 99.$$

$x' + y' = 98$

98	0
97	1
96	2
⋮	
⋮	
1	97
0	98

~~11:15~~ ~~12:30~~

ii) no. of ways of dist. of 100 coin. b/w two person such that any person get any no. of pt.

$x + y = 100$

$n = 100$

$B = 2$

$^{100+2-1}C_{2-1} = {}^{101}C_1 = 101$ m

x	y
100	0
99	1
.	
.	
1	99
0	100

total 101

Q. no. of non-negative Integral sol^n $x + y \le 100$

100 + 0 —

~~1~~ + 99 —

1 + 2 —

$x + y + w = 100$

100 0 0 = 0

Here w is dummy Variable

Coin = 100

$B \to 3$

$^{100+3-1}C_{3-1} = {}^{102}C_2$

Q $x + y < 100$. Find no. of whole no. solution

$x, y \ge 0$ & integer

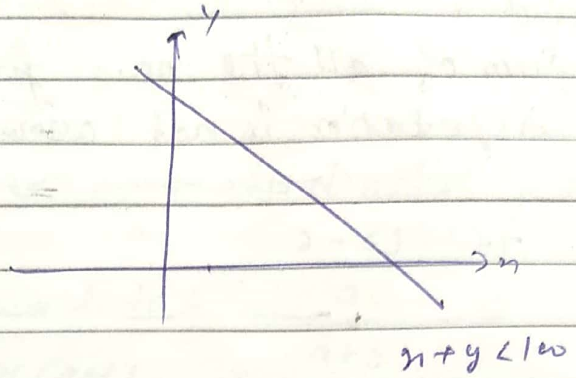
$x + y \le 99$

$x + y + w = 99$

ways $= {}^{99+3-1}C_{3-1}$

Que: no. of Integral pt's lie in the first quad. and satisfy eq?

$x+y<100$



$x>0$ & $x \in N$

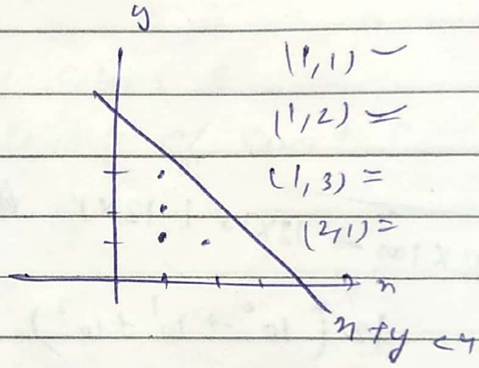
$y>0$ & $y \in N$

$x_1+y_1<98$

$x_1+y_1 \le 97$

$x_1+y_1+w \le 97$

$\text{ways} = {}^{97+3-1}C_{3-1}$



Ex: $x+y<4$

$x, y>0$ & both $\in N$

$x_1+y_1<2$

$x_1+y_1 \le 1$

$x_1+y_1+w=1$ small

$\text{ways} = {}^{1+3-1}C_{3-1}$

$= {}^3C_2 = 3$

* Sum of no. form

If $a_1, a_2, \ldots a_n$ are the digits (non-zero) and no. formed from all of them taken all at a time then

(1) Sum of digit in unit place taken all at a time $= \underline{(n-1)!\,(a_1+a_2+\ldots a_n)}$ repetation is not allowed

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(ii) Sum of all digits formed from all = $\lfloor n-1 \lfloor (a_1 + a_2 + \dots a_n)(10^0 + 10^1 + \dots 10^{n-1})$

Q. ~~Sum~~ find Sum of all the no.s formed the ~~no~~ digit 1, 2, 3 repetation is not allowed

1, 2, 3 Sum of the unit place in ~~the~~ unit in the Sum.

Total = $\lfloor 3 = 6$

___ ___ $\underline{2}$ $\lfloor 2$

$= \lfloor 3-1 \, (2+1+3) = 2 \cdot 6 = 12$

$7(10^1 + 10^1$

231
321
132
312
123
213

$12 \times 100 + 12 \times 10 + 12 \times 1$ An.

$12(10^0 + 10^1 + 10^2)$

Q. find sum of all Possible four no. 1, 2, 3, 5

Sum of all no. formed = $\lfloor 4-1 \, (1+2+3+5) \, (10^0 + 10^1 + 10^2 + 10^3) =$

~~$66 \times 100 + 66 \times 10 + 66 \times 1$~~

Q. Sum of all the no. greater then 10000 formed by digit 0, 1, 2, 4, 5, no. digit being Reapeated

$\lfloor 5-1 \, (0+1+2+4+5)(10^0 + 10^1 + 10^2 + 10^3 + 10^4)$ -

$\lfloor 5-1 (0+1+2+4+5)$ - ~~$\lfloor 5-1 \, (0+1+2+4+5)$~~

$\lfloor 5-1 \, (0+1+2+4+5)(10^0 + 10^1 + 10^2 + 10^3 + 10^4) - \lfloor 4-1 \, (1+2+4+5)(10^0 + 10^1 + 10^2 + 10^3)$

Ans