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SBG STUDY

10/06/17

Method of Differentiation:

* Method of differentiation:

$$\frac{d}{dx}x^2 = 2x \quad , \quad \int 2x\, dx = x^2 + c$$

Q. diff $y = \sin x$. using first principle

$$f(x) = y = \sin x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(h+x) - \sin x}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{2}\cos(x + h/2)\, \sin h/2}{h/2 \cancel{\times 2}} = \cos x \cdot 1 = \cos x.$$

* $y = f(x)$

$$\frac{dy}{dx} = y_1 = y' = f'(x) = Dy$$

* Basic theorems:

1) Term by Term Differ.-

If $y = u(x) \pm v(x)$

then $\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$

OR

$$\boxed{y' = u' \pm v' \pm w'}$$

ii) If $y = k f(x)$

then $\frac{dy}{dx} = k f'(x)$

iii) Product Rule:

$y = f(x) \cdot g(x) = \underset{\mathrm{I}}{f} \cdot \underset{\mathrm{II}}{g}$

$\frac{dy}{dx} = f \cdot \frac{d}{dx} g + g \frac{df}{dx}$

$y' = fg' + gf'$

$y = \overset{\mathrm{I}}{e^x} \cdot \overset{\mathrm{II}}{\sin x}$

$= \cancel{e^x \cdot \frac{d}{dx}} \; e^x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} e^x$

$y' = e^x \cos x + \sin e^x$.

$y = \sin x - 3e^x + x \ln x$

$y' = \frac{d}{dx} \sin x - 3 \cdot \frac{d}{dx} e^x + x \cdot \frac{d}{dx} \ln$.

* $y = fgh$

$\cancel{y' = \frac{df}{dx} \cdot \frac{dg}{dx} \cdot \frac{dh}{dx}}$

$y' = f'gh + fg'h + fgh' \Rightarrow \boxed{y' = (fg)'}$

OR

$y' = \frac{(fg)'h + (fh)'g + (gh)'f}{2}$

co → -ve

~~Cosent~~ Quotient Rule:

$$y = \frac{f(x)}{g(x)}$$

$f \Rightarrow$ ~~f(x) - f g(x)~~ $= \dfrac{g\frac{d}{dx}f - f\cdot\frac{d}{dx}g}{g^2}$

$$= \frac{gf' - fg'}{g^2}$$

Q. If ~~f(x)~~ = $f(x) = \dfrac{\sin x}{x}$

$f'(x) = ?$ $\dfrac{\sin x}{x}$ $= \dfrac{x\frac{d}{dx}\cancel{\sin x}\ \cancel{\cos x}}{x^2}$

$$= \frac{x\frac{d}{dx}\sin x - \sin\frac{d}{dx}x}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

* Note: before Applying ~~Cosent~~ Quotient Rule Try to simplyfy ~~f(x)~~ if Possible

* $y = \sin x \Rightarrow \dfrac{dy}{dx} = \cos x$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} = \ = \cos\frac{\pi}{6} = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

* $\left.\begin{array}{l}\text{cos}\\ \text{cosec}\\ \text{cot}\end{array}\right\}$ -ve

$2\sin x\cos x$
$2\sin x\cos x$

Que!

i) $D\left(\frac{1}{\sin x}\right)$

Ans $\cancel{k}$ $-\operatorname{cosec} x \cdot \cot x$

iv) $D\left(\sin\frac{\pi}{3}\right) = 0$

ii) $D(\sin^{-1}x + \cos^{-1}x)^2 = \frac{\pi}{2}$

$= 0$

(v) $D\left(\frac{1-\cos 2x}{\sin 2x}\right)$

$= D\left(\frac{\cancel{1} - \cancel{1} + 2\sin^2 x}{2\sin x\cos x}\right)$

$= D(\tan x) = \sec^2 x$

iii) $D(\tan \tan^{-1}x) = 1.$

$D(\tan^{-1}\tan) = 1$

vi) If ~~De^x~~ $De^x = e^x$

then $De^{\pi} = 0$

$\log b = \frac{1}{\log_b a}$

formula

vii) $D\left(\frac{1}{\log_x e}\right) = D(\log_e x) = \frac{1}{x}$

Di~~ff not Possible~~.

(viii) $D\left(\cos^4\frac{x}{2} - \sin^4\frac{x}{2}\right)$

$= D\left(\left(\cos^2\frac{x}{2}\right)^2 - \left(\sin^2\frac{x}{2}\right)^2\right)$

$= D\left[\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right]\left[\cos^2\frac{x}{2} + \sin^2\frac{x}{2}\right]$

$D(\cos x) = -\sin x.$

ix) $D\left(e^{\ln_e \cot^{-1}x}\right)$

$= D(\cot^{-1}x) = -\frac{1}{1+x^2} \rightarrow \left[a^{\log_a N} = N\right]$

$1-s^2x$

$1-2\sin^2 x$

Q. If $y = 1 + x^2 + x^3 - \cdots x^{100}$

then $\left(\frac{dy}{dx}\right)_{x=1} = 1 + 2x + 3x^2 - \cdots 100x^{99}$

$\left(\frac{dy}{dx}\right)_{x=1} = 1 + 2 + 3 + \cdots - 100$

Here $= \frac{n(n+1)}{2} = n = 100 \longrightarrow \frac{100(101)}{2} = \frac{\cancel{100}^{50}(101)}{\cancel{2}} = 50 \times 101$

$= 5050$ A

Que! let $f(x) = f(x).(g(x).h(x)$ for some $x = \alpha$

$f'(\alpha) = 21\ f(\alpha)$

$f'(\alpha) = 4 f(\alpha)$

$g'(\alpha) = -7 g(\alpha)$ $\quad f(\alpha) = f(\alpha).g(\alpha).h(\alpha)$

$h'(\alpha) = kh\alpha$ $\quad (4 - 7 + k) = 21$

then find K. $\quad$ $k = 10$

$= f'(x) = f'(x)\, g(x)\, h(x) + f(x)\, g'(x) h(x) + f(x)\, g(x)\, h'x$

$f'(\alpha) = f'(\alpha)\, g(\alpha)\,(h(\alpha) + f(\alpha)\, g'(\alpha)\, h(\alpha) + f(\alpha) g(\alpha) h\alpha$

$21 f(\alpha) = 4.f(\alpha)\, g(\alpha)\, h(\alpha) - 7.f(\alpha)\, g(\alpha) h(\alpha) + K (f(\alpha) g(\alpha) h(\alpha))$

$21 f(\alpha)(g(\alpha) h(\alpha) = f(\alpha)\, g(\alpha) h(\alpha)(4 - 7 + K)$

$21 = 4 - 7 + K$

$K = 21 - 4 + 7 = 24$

Que! ~~2t~~ Diff. fn fgh are given such that

$f(0) = 1$, $g(0) = 2$, $h(0) = 3$ &

~~ffg~~ $(fg)'(0) = 6$, $(gh)'(0) = 4$, $(hf)'(0) = 1$

then find $(f.gh)'(0) =$

An $(fg)'(0) \cdot h(0) + (fh)'(0) + (gh)' \cdot f(0)$

$$= \frac{6\times 3 + 5\times 2 + 4\times 1}{2}$$

$$= \frac{18 + 10 + 4}{2} = \frac{32}{2} = 16$$

Que! $y = \frac{x^3 + 2^x}{e^x}$

Ans: $\frac{e^x \cdot [3x^2 + \cancel{2x}\, 2^x \log 2] - (x^3 + 2^x) \cdot e^x}{(e^x)^2}$

Q. $y = \frac{1 + \sin x}{\tan x}$

Ans! $y' = \frac{(\tan x) \cdot \cos x - (1 + \sin x) \cdot \sec^2 x}{\tan^2 x}$

Q. $y = \frac{1 - \ln x}{1 + \ln x}$

$y' = \frac{(1 + \ln x)\left(-\frac{1}{x}\right) - (1 - \ln x)\left(\frac{1}{x}\right)}{(1 + \ln x)^2}$

Que! Diff. if $y = \frac{x^4 + x^2 + 1}{x^2 + x + 1}$

then find y'.

$x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$

$= x^2 - x + 1$

$y' = 2x - 1$

Si

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

Que. $y = \dfrac{\sin^{-1}x - \cos^{-1}x}{\sin^{-1}x + \cos^{-1}x} = \frac{2}{\pi}[\sin^{-1} - \cos^{-1}x)$

Ans

$y' = \frac{2}{\pi}\left[\left(\frac{1}{\sqrt{1-x^2}}\right) - \left(-\frac{1}{\sqrt{1-x^2}}\right)\right]$

Boar

Que: $y = \dfrac{x^3 + x^2 + x}{1 + x^2}$

• Degree of $N^r \geq$ Deg of D^r $\quad \frac{x^3}{x^2}$

$y = \dfrac{x^3 + x^2 + x}{1 + x}$

if de of $N^r \geq$ Deg. of D^r

then Devide first

$y = \dfrac{x^3 + x^2 + x + 1 - 1}{1 + x^2}$

$\dfrac{x(x^2+1) + (x^2+1) - 1}{x^2+1} = \dfrac{(x^2+1)(x-1) - 1}{x^2+1}$

$= x + 1 - \dfrac{1}{x^2+1}$

Q: $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$
then find $\frac{dy}{dx}$

Ans $(\sec x - \tan x)(\sec x + \tan x) = 1$

$y = \frac{\sec x + \tan - 1}{\tan x - \sec x + (\sec x - \tan x)(\sec x + \tan x)}$

$= \frac{\sec x + \tan x - 1}{(\sec x - \tan x)[\sec x + \tan x - 1]}$

see

$= \sec x + \tan x.$

Diff using chain Rule:

if $y = f(u)$
& $u = g(x)$

then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= f'(u) \cdot g'(x)$

Q = $y = \sin x$
$\frac{dy}{dx} = \cos x$

$y = \sin x^2$
$\frac{dy}{dx} = \frac{d}{dx}(\sin x^2)$
$\frac{d}{dx^2}(\sin x^2) \cdot \frac{d(x^2)}{dx}$
$\cos x^2 \cdot 2x$

H.W: Q-1 1, 2, 3, 4,
S-2, 1,

$$\frac{dy}{dx} = \cos x^2 \cdot \frac{d}{dx} x^2$$

$$= 2x \cos x^2$$

Que: $y = \sqrt{\sin x^2}$

$$\frac{dy}{dx} = \frac{d}{dy} (\sin x^2)^{1/2}$$

$$= \frac{1}{2} (\sin^2 x)^{-1/2} \frac{d}{dx} \sin x^2$$

$$= \frac{1}{2\sqrt{\sin x^2}} \cdot \cos^2 x \cdot \frac{d}{dx} x^2$$

$$= \frac{1}{2\sqrt{\sin x^2}} \cdot \cos^2 x \cdot \cancel{d}\, 2x.$$

Que: $y = (\ln x^3)^4$

$$4 (\ln x^3) \cdot \frac{d}{dx} (\ln x^3)$$

$$\frac{1}{x^3} \frac{d}{dx} \cdot x$$

$$y' = 4 (\ln x^3)^3 \cdot \frac{1}{x^3} \cdot 3x^2$$

Que:

Note: $\boxed{f(x) = \exp. \sin x = e^{\sin x}}$

Q. $y = \left(\tan e^{(\sin x^2)}\right)^3$

$y = 3\left(\tan e^{(\sin x^2)}\right)^2 \cdot \left(\sec^2 e^{\sin x^2}\right) \cdot e^{\sin x^2} \cdot \cos x^2 \cdot 2x.$

Q. $y = e^{\sqrt{\sin(\ln(x^2+7))^5}}$

$$= \frac{\left(e^{\sqrt{\sin(\ln(x^2+7)}}\right) \cdot \cos \ln(x^2+7)^5 \quad 5(\ln x^2+7)^4 \cdot 2x}{(2)\left(\sin(\ln(x^2+7)^5\right) \ln(x^2+7)^3}$$

Ans

Logarithmic diff:

$$\boxed{a^{\log_a N} = N}$$ Imp

i) A fn which is the Product or Quotient of a no. of functions

ii) $f^n \; (f(x))^{g(x)}$ where f and g both are diff.

then in such situation log. diff is needed.

$$\Rightarrow f(x) = \frac{x^2 2^x \sin x}{\cos 2x}.$$

$$\log \frac{abc}{de} = \log a + \log b + \log c - \log d - \log e$$

$$\ln f(x) = \ln x^2 + \ln 2^x + \ln \sin x - \ln(\cos 2x)$$

$$\frac{1}{f(x)} \cdot f'(x) = \frac{1}{x^2} \cdot 2x + \frac{1}{2^x} \cdot 2^x \ln 2 + \frac{1}{\sin x} \cos x - \frac{1}{\cos 2x}(-2\sin 2x)$$

$$f'(x) = \frac{x^2 2^x \sin x}{\cos 2x}\left[\frac{2}{x} + \ln 2 + \cot x + \frac{2\sin 2x}{\cos 2x}\right)$$

Q $y = x^{\sin x}$

$\log y = \log x^{(\sin x)}$ $\qquad y = e^{\ln x^{(\sin x)}}$

$\ln y = \underset{I}{\sin x} \; \underset{II}{\ln x}$ $\qquad y = e^{\sin x \cdot \ln x}$

$y' = e^{\sin x \ln x} \cdot \frac{d}{dx}\{\underset{I}{\sin x} \cdot \underset{II}{\ln x}\}$

$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{1}{x} + (\ln x) \cdot (\cos x)$

$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \cos x(\ln x)\right]$

Que: $y = f(x) = x^x$

$\log f(x) = \log x^x$

$\frac{1}{f} f' = x \cdot \log x.$

$= x \cdot \frac{1}{x} \cdot \log x \cdot 1$

$\boxed{= f' = x^x(1 + \ln x)}$ Remember

Que: $y = x^{\sin x} + x^x$

$y = u + w$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dw}{dx}$

$= x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \ln x\right) + x^x(1 + \ln x)$

Que:- $y = x^{x^x} + \tan x^{(\sin x)}$

$\ln y = \ln x^{x^x} + \ln \tan x^{(\sin x)}$

$= x^x \ln x + \sin x \ln \tan x$

$x^x \cdot \frac{1}{x} + x^{x}{}' \ln x (1+\ln x) \quad \cos x \cdot \ln \tan x + \sin x \cdot \frac{1}{\tan x} \sec^2 x$

$x^{x^x} \left[x^x \frac{1}{x} + \ln x (1+\ln x) \right] + \tan^{(\sin x)} ($

Sol:-

$y = \underset{v}{x^{x^x}} + \underset{u}{\tan x^{(\sin x)}} \longrightarrow v = x^{x^x}$

$v = (\tan x)^{\sin x}$

$\log v = \sin \ln(\tan x)$

$\frac{1}{u} \log u = (\cos x \cdot \ln(\tan x) + \sin \frac{1}{\tan x} \sec^2 x)$

$\tan x (\cos x \cdot \ln(\tan x) + \sin \frac{1}{\tan x} \sec^2 x)$

$\ln v = \ln x^{x^x}$

$= x^x \ln x$

$\frac{1}{u} \cdot u' = x^2 \cdot \frac{1}{x} + \log(x^x)(1+\ln x)$

$= x^{x^x} (x^2 \cdot \frac{1}{x} + \log(x^x)(1+\ln x))$

$\sqrt{x} = x^{1/2}$ $\frac{1}{2}$

Que: $y = \sin x \cdot \sin 2x \cdot \sin 3x \cdots \sin nx$.

Ans $\ln y = \ln \sin x + \ln \sin 2x \cdots \ln \sin nx$

$$\frac{1}{y} y' = \frac{\cos x}{\sin x} + \frac{2\cos 2x}{\sin 2x} \cdots \frac{n(\cos nx)}{\sin nx}.$$

$$y' = (\sin x \sin 2x \cdots \sin nx)\left[\cot x + 2\cot 2x + \cdots n\cot nx\right)$$

Imp

Que: $y = \sqrt{x}^{\sqrt{x}} \cdot e^{x^2}$

then find $y'(1)$

Ans

$\ln y = \ln \sqrt{x}^{\sqrt{x}} \cdot \ln e^{x^2}$

$\sqrt{x} \ln \sqrt{x}$ $\qquad x^2 \ln e$

$\sqrt{x} \cdot \frac{1}{\sqrt{x}} + \frac{1}{2\sqrt{x}} \ln \sqrt{x}$. $\qquad 2x \ln e + x^2 \frac{1}{e}$

$\frac{\sqrt{x}}{\sqrt{x}} + \frac{1}{2\sqrt{x}} \ln \sqrt{x}$

Ans: $\ln y =$

$= \sqrt{x} \ln x^{1/2} + x^2$

$$\frac{\sqrt{x}}{2} \cdot \frac{1}{x} + \frac{\ln x}{2} \times \frac{1}{2\sqrt{x}} + 2x$$

$$\frac{y'(1)}{e} = \left(\frac{1}{2} + 2\right) \Rightarrow y'(1) = \frac{5e}{2}$$

$\frac{398}{4}$ 3^2

$$\boxed{\prod_{r=1}^{3} \sin r}$$

Imp

Que: if $f(x) = \prod_{n=1}^{100} (x-n)^{n(101-n)}$

find $\frac{f(101)}{f'(101)}$

Ans $\prod_{n=1}^{100} (x-1)^{1(101-1)} \times (x-2)^{2(101-2)} \cdots\cdots (x-n)^{n(101-n)}$

$(x-1)^{100} \times (x-2)^{2(99)} \times \cdots\cdots (x-100)^{100(101-100)}$

$= (x-1)^{100} \cdot (x-2)^{198} \times (x-3)^{294} \cdots\cdots (x-100)^{100(101-100)}$

$\ln f(x) = 1 \cdot 100 \ln(x-1) + 2 \cdot 99 \ln(x-2) + 3 \cdot 98 (\ln(x-3) +$
$100 \cdot 1 \ln|x-100|$

$\frac{1}{f(x)} \cdot f'(x) = \frac{1 \cdot 100}{x-1} + \frac{2 \cdot 99}{x-2} + \frac{3 \cdot 98}{x-3} + \cdots \frac{100 \cdot 1}{x-100}$

$\frac{f'(101)}{f(101)} = 1+2+3+\cdots\cdots 100 = 5050$

find $\frac{f(101)}{f'(101)} = \frac{1}{5050}$

* Pera metric diff.

1)
$$y = f(x) \times$$

$$y = f(t) \ \& \ x = g(t)$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{f'(t)}{g'(t)}$$

* Diff of one fn $f(x)$ with respect to another fn $g(x)$.

$$\frac{d f(x)}{dg(x)} = \frac{\frac{d f(x)}{dx}}{\frac{dg(x)}{dx}} = \frac{f'(x)}{g'(x)}$$

Que Diff different sin x with respect to log x

$$\frac{d f(\sin x)}{d(lo} \quad \frac{d(\sin x)}{d(\ln x)} =$$

$$= d \frac{\frac{d \sin x}{dx}}{\frac{d}{dx} \ln x} = \frac{\cos x}{1/x} = x \cos x.$$

Q. find $\frac{dy}{dx}$ if $x = a(\cos t + t \sin t)$

$$y = a(\sin t - t \cos t).$$

$$\frac{\frac{dx}{dx}}{\frac{dx}{dt}} = \frac{d \cdot a(\cos t + t \sin t)}{d(a(\sin t + t \cos t)}$$

Ans: $\frac{dx}{dt} = a\left(-\sin t + \sin t + t\cos t\right)$

$\frac{dy}{dt} = a\left(\cos t - \cos t + t\sin t\right)$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \tan t$

Que: $x = \frac{3t}{1+t^3}$; $y = \frac{3t^2}{1+t^3}$

$\frac{dx}{dx} = \frac{\frac{3t}{\cancel{1+t^3}}}{\frac{\cancel{3}t^2}{\cancel{1+t^3}}} = \frac{3t}{\cancel{3}t^2} = \frac{1}{\cancel{3t}}$

$\frac{dy}{dx}\ \frac{(1+t^3)\cdot 6t - 3t^2\cdot 3t^2)}{(1+t^3)^2}$

Sol: $\frac{dx}{dt} = \frac{(1+t^3)\,3 - 3t \times 3t^2)}{(1+t^3)^2}$

Que: Diff $(\ln x)^{\tan x}$ ~~w~~ w.r.t x^x :

$\frac{d}{dx}(\ln x)^{\tan x}$
$\frac{d}{dx} x^x$

$\frac{\ln x}{\cancel{x \ln x}}$
$\ln x + x\cdot\frac{1}{x}$
$x^4\ \ln x + 1$

$\frac{d}{dx}$ ~~tan x ln x~~

$y = (\ln x)^{\tan x}$

$\ln y = \tan x \cdot \ln(\ln x)$

$\frac{1}{y}\cdot y' = \sec^2 x \cdot \ln(\ln x) + \tan x \frac{1}{\ln x}\cdot\frac{1}{x}$

$= \frac{(\ln x)^{\tan x}\left[\sec^2 x\cdot \ln(\ln x) + \frac{\tan x}{x \ln x}\right]}{x^x(1+\ln x)}$

Sol → 2(a), b, 3, 4, 5, 6, 7, 8, 10
Ex → 10

Que: $e^{\sin^{-1}}$ w.r.t $e^{-\cos^{-1}x}$

$\ln e^{\sin^{-1}}$
$\sin^{-1} \ln e$
$\frac{1}{\sqrt{1-x^2}} \cdot \ln e + \sin^{-1} \frac{1}{e}$

$\ln e^{-\cos^{-1}x}$
$-\cos^{-1}x \cdot \ln e$

$\frac{1}{\sqrt{1-x^2}} \cdot \ln e + -\cos^{-1}x \frac{1}{e}$

$$\Rightarrow \frac{\frac{d}{dx} e^{\sin^{-1}x}}{\frac{d}{dx} e^{-\cos^{-1}x}} = \frac{e^{\sin^{-1}x} \cdot \frac{1}{\cancel{\sqrt{1-x^2}}}}{e^{-\cos^{-1}x} \cdot + \frac{1}{\cancel{\sqrt{1-x^2}}}}$$

$$= e^{\sin^{-1}} \cdot e^{\cos x^{-1}} = e^{\sin^{-1}+\cos^{-1}x} = e^{\pi/2}$$

Que: $x = a\sqrt{\cos 2t}\cos t$ $\quad y = a\sqrt{\cos 2t}\sin t$
then find $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{6}}$

$\frac{dy}{dx} = a\sqrt{\cos 2t}\cos t + \frac{1}{2\sqrt{\cos 2t}}\sin t$

Sol: $x = a\sqrt{\cos 2t}$
$= \frac{a1}{2\sqrt{\cos 2t}} + a\sqrt{\cos 2t}$
$a\sqrt{\cos 2t} \quad -\sin t + \frac{1}{2\sqrt{\cos 2t}}\cot t$

$(-\sqrt{3})$

$= a\sqrt{\cos 2\frac{\pi}{63}} - \sin\pi/6 + \frac{1\cot \frac{\pi}{6}}{2\sqrt{\cos 2\frac{\pi}{63}}}$

$a\sqrt{\cos\frac{\pi}{3}} - \sin\frac{\pi}{6} + \frac{\cot\pi/6}{\sqrt{\cos 2\pi}}$

Sol:- $\frac{dx}{dt} = a\cos t \frac{1}{2\sqrt{\cos 2t}} \times -2\sin 2t + a\sqrt{\cos 2t}(-\sin t)$

$= \frac{a}{\sqrt{\cos 2t}}[\cos t \sin 2t + \sin t \cos 2t]$

$= -\frac{a}{\sqrt{\cos 2t}}(\sin 3t)$

$y = a\sqrt{\cos 2t} \sin t$

$\frac{dy}{dt} = a\sqrt{\cos 2t} \cdot \cos t + a\sin t \frac{1}{2\sqrt{\cos 2t}} \times -2\sin 2t.$

$= \frac{a}{\sqrt{\cos 2t}}[\cos 2t \cdot \cos t - \sin 2t \sin t)$

$= \frac{a}{\sqrt{\cos 2t}} \cdot \cos(2t + t)$

$\left(\frac{dy}{dx}\right) = \frac{\frac{a}{\sqrt{\cos 2t}} \cdot \cos 3t}{-\frac{a}{\sqrt{\cos 2t}} \cdot \sin 3t} = \cot 3t$

$\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{6}} = \cot\frac{\pi}{2} = 0$

* Important Substitution:

Expression	Substitution
$a^2 + x^2$	$x = a\tan\theta$
$a^2 - x^2$	$x = a\sin\theta$ or $x = a\cos\theta$
$x^2 - a^2$	$x = a\sec\theta$ or $x = a\operatorname{cosec}\theta$
$\sqrt{\frac{a+x}{a-x}}$ or $\frac{a-x}{a+x}$	$x = a\cos\theta$
$\frac{2x}{1+x^2}, \frac{2x}{1-x^2}, \frac{1-x^2}{1+x^2}$	$x = \tan\theta$ or $\cos 2\theta$

$$\boxed{\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}}$$

Q.
(5) $y = \sqrt{\frac{1-x}{1+x}}$ $\qquad x = \cos\theta \quad \frac{dx}{d\theta} = -\sin\theta$ — (1)

$x = 2\cos^2\frac{\theta}{2} - 1$

$\frac{(x+1)}{2} = \cos^2\frac{\theta}{2}$

$= \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$= \sqrt{\frac{\cancel{1}-(\cancel{1}-2\sin^2\theta/2)}{1+(2\cos^2\frac{\theta}{2}-1)}}$

$= \sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\theta/2}}$

$y = \tan\frac{\theta}{2}$

$\frac{dy}{dx} = \frac{d}{dx}\left(\tan\frac{\theta}{2}\right)$

$= \frac{d}{d\theta}\left(\tan\frac{\theta}{2}\right)\frac{d\theta}{dx}$

$= \sec^2\frac{\theta}{2}\cdot\left(-\frac{1}{\sin\theta}\right)$ — from (1)

$= \frac{1}{\cos^2\frac{\theta}{2}}\left(-\frac{1}{\sin\theta}\right)$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-x^2}$

6. $y = \ln\left\{\frac{x + \sqrt{(a^2 + x^2)}}{a}\right\} = \ln(\sec\theta + \alpha)$

$$\boxed{\begin{aligned} y &= \sin^{-1}\frac{2x}{1+x^2} \\ y &= \cos^{-1}\frac{1-x^2}{1+x^2} \\ y &= \tan^{-1}\frac{2x}{1-x^2} \end{aligned}}$$

Que. $y = \tan^{-1}\left(\frac{\sqrt{x} - x}{1 + x^{3/2}}\right)$ Diff.

$\tan^{-1}\frac{\sqrt{x} - x}{1 + x \cdot x^{1/2}}$

$y = \tan^{-1}\sqrt{x} - \tan^{-1}x$

$y' = \frac{1}{1+x}\frac{d\sqrt{x}}{dx} - \frac{1}{1+x^2}$ A

Que: Diff $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ $\div b\cos x$, $\div b\cos x$ divide both are $b\cos x$.

Ans:

$\tan^{-1}\frac{\overset{x}{\frac{a}{b}} - \overset{y}{\tan x}}{1 + \frac{a}{b}\tan x}$

$y = \tan^{-1}\frac{a}{b} - \tan^{-1}\tan x$

Q. $\tan^{-1}\left(\frac{a\cos x - b\sin x}{a\cos x + b\sin x}\right)$ Both divided by $a\cos x$

$= \tan^{-1}\left(\frac{1 - \frac{b}{a}\tan x}{1 + \frac{b}{a}\tan x}\right)$

$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right)$

$= \tan^{-1}\tan\frac{\pi}{4} - \tan^{-1}\tan\theta$

$= \frac{\pi}{4} - \tan^{-1}\tan\theta.$

$$\boxed{\sqrt{x^2} = |x|}$$

* ~~Deu~~ Derivative of Implisit fn :

$x + y = 1$ Implisit

$y = 1 - x$ Explisit

* Diff. term by term and Collect $\frac{dy}{dx}$ together of one side one equal side find its value.

Q. Diff. $x^y = e^{x-y}$ P.T $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$

$y \ln x = x - y$

$\frac{y}{x} + \ln x \cdot y' = 1 - y'$

$y'[1 + \ln x] = \left(1 - \frac{y}{x}\right)$ $y' = \frac{1 - \frac{y}{x}}{(1 + \ln x)}$

$y \ln x = x - y$

$y(1+\ln x) = x$ $\qquad dx^2$

$y' = \dfrac{1-\frac{y}{x}}{(1+\ln x)}$ $\qquad 2x\dfrac{dx}{dx} = 2x$

$= \dfrac{1-\frac{1}{1+\ln x}}{1+\ln x} = \dfrac{\ln x}{(1+\ln x)^2}$

Que: $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x \ldots \infty}}}$

$y = \sqrt{\sin x + y}$

$\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{\sin x}} y + \sqrt{\sin x} \cdot \dfrac{1}{y} y'$

Sol: $y^2 = \sin x + y$

$2yy' = \cos x + y'$

$y' = \dfrac{\cos x}{2y-1}$

∞

$y = x^{x^{x^{x}}} \to y$

Q: $y = x^{x}$

$= y = x^{y}$

$\ln y = y \ln x$

$\dfrac{1}{y} y' = \dfrac{y}{x} + \ln x \, y'$

$y' = \ldots$

$$Q.\ y = (\ln x)^{(\ln x)^{\cdots\infty}}$$

$$\cancel{\ln} y = \cancel{\ln x}^{y}$$

$$\ln y = y \ln (\ln x)$$

$$y' = \frac{1}{\ln x}(\ln x) + \ln \frac{1}{x}$$

$$y = (\ln x)^y$$

$$\ln y = y \ln (\ln x)$$

$$\frac{1}{y} \cdot y' = y'(\ln(\ln x)) + y \cdot \frac{1}{x \ln x}$$

$$= y'\left(\frac{1}{y} \cdot y'\right) = 0$$

$$y'\left(\frac{1}{y} \cdot y'\right) = (\ln x)^y (\quad " \quad)$$

Que: $y = \frac{x}{1+} \frac{x}{2+} \frac{x}{1+} \frac{x}{2+} \cdots \infty$

$$= \cfrac{x}{1 + \cfrac{x}{2 + \cfrac{x}{1 + \cfrac{x}{2 + \cdots \infty}}}}$$

$$y = \cfrac{x}{1 + \cfrac{x}{2+y}}$$

$$y = \frac{x(2+y)}{x+y+2} \rightarrow (x+y+2)y = x(2+y)$$

$$xy + y^2 + 2y = 2x + xy$$

$$= 2yy' + 2y' = 2$$

$$y' =$$

Que: if $\sin y = x \sin(a+y)$

then Proff that $\frac{d}{dx} : \frac{\sin^2(a+y)}{\sin a}$

Sol: $x = \frac{\sin y}{\sin(a+y)}$

$$\frac{dx}{dy} = \frac{\sin(a+y) \cdot \cos y - \sin y \cdot \cos(a+y)}{\sin^2(a+y)}$$

$$= \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

Que: if $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ find $\frac{dy}{dx} = ?$

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$ $\quad \boxed{x = \sin\theta,\ y = \sin\phi}$

$$2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} = a\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$$\cos\left(\frac{\theta+\phi}{2}\right)\cdot\left[2\cos\frac{\theta-\phi}{2} - a\sin\frac{\theta-\phi}{2}\right]$$

$\cos\frac{\theta+\phi}{2} = 0$

$\cos\frac{\theta+\phi}{2} = 0$

$\frac{\theta+\phi}{2} = (2n+1)\frac{\pi}{2}$

$\sin^{-1}x - \sin^{-1}y = Cot$

$$\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$2\cos\frac{\theta-\phi}{2} = a\frac{\theta-\phi}{2}$

$\tan\left(\frac{\theta-\phi}{2}\right) = \frac{2}{a} = Cota$

$\theta - \phi = Cot$

$\sin^{-1}x - \sin^{-1}y = Cot.$

ITF = curve leorn.

H.W: S-1 11, 12, 13, 15, 19, 25
SJ-M: 1, 2, 3, 5,
J.A: 5

Que: If $y^5 + xy^2 + x^3 = 4x + 3$ find $\left(\frac{dy}{dx}\right)_{(2,1)} = ?$

$$5y^4 y' + y^2 + 2xyy' + 3x^2 = 4$$

$$y'(5y^4 + 2xy) = 4 - 3x^2 - y^2$$

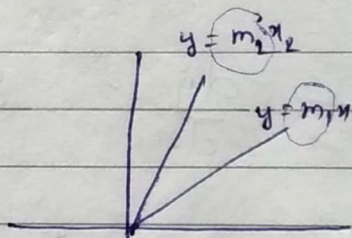
$$(y')_{(2,1)} = \frac{4 - 3\cdot 4 - 1}{5\cdot 1 + 2\cdot 2\cdot 1} = \frac{-9}{9} = -1$$

Que: if $ax^2 + 2hxy + by^2 = 0$

P.T $\frac{dy}{dx} = \frac{ax + hx}{hx + by} = \left(\frac{y}{x}\right)$

$$2ax + 2h(xy' + y) + 2byy' = 0$$

$$(ax + hy) + y'(hx + by) = 0$$



Que: diff. $y = \tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1+x}}\right)$ $x \in [-1, 1]$

Ans:

$\sqrt{1 - \cos\theta} = \sqrt{1 - 1 + 2\sin^2\frac{\theta}{2}}$ $x \in (-1, 1)$

$\sqrt{1 + \cos\theta} = \sqrt{1 + 2\cos^2\frac{\theta}{2} - 1}$ $\cos\theta \in [-1, 1]$

$\theta \in [0, \pi)$

$\tan^{-1} \frac{|\cos\frac{\theta}{2}| - |\sin\frac{\theta}{2}|}{|\cos\frac{\theta}{2}| + |\sin\frac{\theta}{2}|}$ $\frac{\theta}{2} \in \left(0, \frac{\pi}{2}\right)$

$$\tan^{-1}\left(\frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right)$$

$$\tan^{-1}\left(\frac{1 - \tan\theta/2}{1 + \tan\theta/2}\right) = \tan^{-1}\left(\frac{\tan\pi/4 - \tan\theta/2}{1 + \tan\frac{\pi}{4}\tan}\right)$$

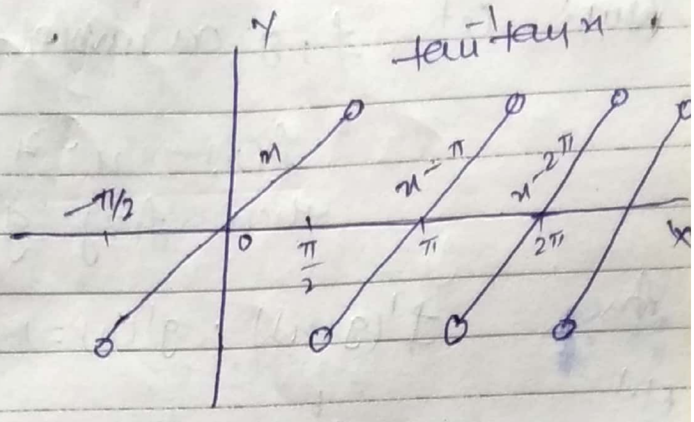
$0 \le \frac{\theta}{2} \le \frac{\pi}{2}$ $\qquad \frac{\pi}{4}+0 \ge \frac{\pi}{4}-\frac{\theta}{2} \ge \frac{\pi}{4}-\frac{\pi}{2}$

$$-\frac{\pi}{4} \le \frac{\pi}{4}-\frac{\pi}{2} \le \frac{\pi}{4}$$

$= \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

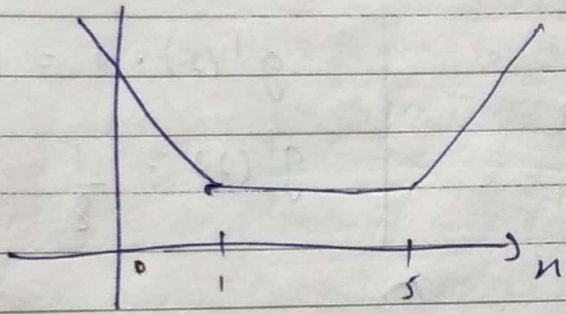
$=$

$\frac{\pi}{4}-\frac{\theta}{2} = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$



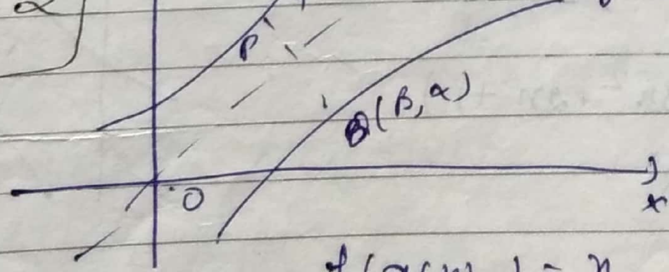
Que: $y = |x-1| + |x-5|$ $\qquad \left(\frac{dy}{dx}\right)_{x=2}$

$= 0$



Derivative of an inverse function:
If f and g are inverse fn of each other

$f(\alpha) = \beta$
$f(\beta) = \alpha$



$f(g(x)) = x \quad \& \quad g f(x) = x$

$f'(g(x)) \cdot g'(x) = 1 \quad \Big| \quad g'(f(x)) \cdot f'(x) = 1$

Que: Let f, g are inverse of each other

$f(x) = x^3 + x^5$ $\qquad f(1) = 2$

then find $g'(2)$

Ans $\quad f'(g(x)) \cdot g'(x) = 1$

M.1 $\quad x = 2$

$f'(g(2)) \cdot g'(2) = 1$

$f'(1) \cdot g'(2) = 1$

$f'(1) = 3 + 5 = 8$

$g'(2) = \frac{1}{8}$

M.2

$g'(f(x)) \cdot f'(x) = 1$

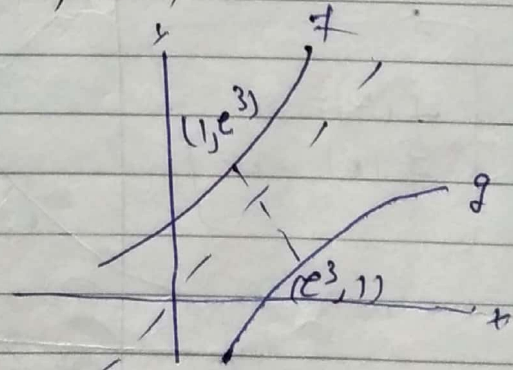
$x \to 1$ $\quad g'(f(1)) \cdot f'(1) = 1$

$g'(2) \cdot 8 = 1$

$g'(2) = \frac{1}{8}$

Que: $f(x) = \exp(\sin x) = e^{\sin x}$.

Que Let $f(x) = \exp(x^3 + x^2 + x)$ and $g(b)$ its inverse then $g'(e^3) = ?$



Ans:

$= e^{x^3 + x^2 + x} \cdot (3x^2 + 2x + 1)$

$x = 1, \quad f(1) = e^3$

$f'(g(e^3)) \cdot g'(e^3) = 1$

$f'(1) \cdot g'(e^3) = 1$

$6e^3 \cdot g'(e^3) = 1$

$$\boxed{g'(e^3) = \frac{1}{6e^3}}$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Que:

$$y = \tan^{-1}\frac{1}{x^2+x+1} + \tan^{-1}\frac{1}{x^2+3x+3} + \tan^{-1}\frac{1}{x^2+5x+7} + \dots n\ \text{te}$$

$\frac{dy}{dx}$. $T_1 = \tan^{-1}\frac{(x+1)-x}{1+x(x+1)}$ $\qquad T_2\ \tan^{-1}\frac{1}{1+(x^2+3x+2)}$

$$t_2 = \frac{(x+2)-(x+1)}{1+(x+1)(x+2)}$$

$T_1 \quad = \tan^{-1}(x+1) - \tan^{-1}x$

$$t_2 = \tan^{-1}(x+2) - \tan^{-1}(x+1)$$

$$T_1 = \tan^{-1}(x+1) - \tan^{-1}x$$
$$T_2 = \tan^{-1}(x+2) - \tan^{-1}(x+1)$$
$$T_3 = \tan^{-1}(x+3) - \tan^{-1}(x+2)$$

$$T_n = \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$$
$$y = \tan^{-1}(x+n) - \tan^{-1}x.$$

✗ Successive diff. :- $y = f(x)$

$$\frac{dy}{dx} = f'(x) = y' = Dy$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = f''(x) = y'' = y_2 = D^2y$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = f'''(x) = y''' = y_3 = D^3$$

$y = \sec^2 x.$

$$\frac{dy}{dx} = 2\sec x \cdot \sec x \tan x$$

$$= 2\sec^2 x \tan x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}.(2\sec^2 x \tan x).$$

O-1 ⇒ ap - 18th. Que.
O-2 ⇒ 3,
S-1 ⇒ 14, 15, 16, 17
J.M = comp.

Que: $y = \sin(\sin x)$

P.T $y_2 + y_1(\tan x) + y\cos^2 x = 0$

Ans: $y_1 = \cos(\sin x)\cdot\cos x$

$\cos x\left[-\sin(\sin x)\right] + \cos(\sin x)(-\sin x)$

$= -\cos^2 x\,\sin(\sin x) - \cos(\sin x)\cdot\sin x$

$y = -\cos^2 x\,\sin(\sin x) - \cos(\sin x)\cdot\sin x + \cos(\sin x)\,\cos x\cdot\frac{\sin x}{\cos x} + \sin\cdot\cos^2 x$

$= 0$

Que: $e^{x+y} = y^2$ P.T $y'' = \frac{2y}{(2-y)^3}$

$e^{x+y}\cdot\left[1 + \frac{dy}{dx}\right] = 2y\frac{dy}{dx}$

$y^2\left[1 + \frac{dy}{dx}\right) = 2y\frac{dy}{dx}$

$y' = \frac{y}{2-y}$

$y'' = \frac{d}{dx}(y') = \frac{d}{dx}\left(\frac{y}{2-y}\right) = \frac{dx}{dy}\left(\frac{y}{2-y}\right)\frac{dy}{dx}$

$\frac{(2-y)\cdot 1 - y(-1)}{(2-y)^2} \times \frac{y}{2-y}$

$= \frac{2y}{(2-y)^3}$

Que: $x = a(t + \sin t)$

$y = a(1 - \cos t)$

$\dfrac{d^2y}{dx^2}$

$$\frac{dx}{dt} = a(1 + \cos t) \qquad\qquad x = a(t + \sin t)$$

$$\frac{dy}{dt} = a \sin t \qquad\qquad y = a(1 - \cos t)$$

$$\frac{d^2y}{dx^2}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sin t}{1+\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{\sin t}{1+\cos t}\right)$$

$$\frac{d}{dt}\left(\frac{\sin t}{1+\cos t}\right) \cdot \frac{dt}{dx}$$

$$= \frac{(1+\cos t)\cos t - \sin t(-\sin t)}{(1+\cos t)^2} \times \frac{1}{a(1+\cos t)}$$

$$= \frac{(1+\cos t)}{a(1+\cos t)^3} = \frac{1}{a(1+\cos t)^2}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx} \cdot \left(\frac{d^2y}{dx^2}\right)$$

$$\frac{d}{dt}\left(\frac{1}{a(1+\cos t)^3}\right) \cdot \frac{dt}{dx}$$

$$= \frac{1}{a} \cdot \frac{-2}{(1+\cos t)^2} \times (-\sin t) \times \frac{1}{a(1+\cos t}$$

$$= \frac{2\sin t}{a^2(1+\cos t)^4}$$

$e^{xy} = (1)$

Que: $e^{xy} + y\cos x = 2$

(i) find $y'(0)$

$$e^{xy}[xy' + y] + y'\cos x - y\sin x = 0$$

$$y'(xe^{x} + \cos x) = y(\sin x - e^{xy})$$

$$y' = \frac{y(\sin - e^{xy})}{xe^{xy} + \cos x}$$

at $x = 0$

$e^{0} + y\cos x = 2$

$1 + y = 2$

$\therefore y = 1$

$(0, 1)$

$$y'(0) = (y')_{(0,1)} = \frac{1(0-1)}{0+1} = -1$$

2) (a)

(ii) find $y''(0)$

Imp

$\cancel{1(0-1)}$

$$y'' = \frac{(xe^{xy} + \cos x)\cdot\left[y'(\sin x - e^{xy}) + y(\cos x - e^{xy}(y + xy'))\right] - y(\sin x - e^{xy})\cdot\left[e^{xy} + xe^{xy}\{y + xy'\} - \sin x\right]}{(xe^{2y} + \cos x)^2}$$

$$= \frac{(0+1)\left[-1(0-1) + 1(1-1(1+0)\right] - 1(0-1)\cdot[1+0-0]}{(0+1)^2}$$

$$= \frac{1(1+0] + 1}{1} = 2$$

Que: find value of expression on the ellips

$y^3 \frac{d^2y}{dx^2}$ $\quad 3 \cdot x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$x = 2\cos\theta, \; y = \sqrt{3}\sin\theta$$

$$\frac{dx}{d\theta} = -2\sin\theta, \; \frac{dy}{d\theta} = \sqrt{3}\,\cancel{\sin\theta}\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\sqrt{3}}{2}\cot\theta$$

$$\frac{d^2y}{dx^2} = -\frac{d}{d\theta}\left(\frac{\sqrt{3}}{2}\cot\theta\right)\cdot\frac{d\theta}{dx} = -\frac{\sqrt{3}}{2}(-\operatorname{cosec}^2\theta) \times \frac{1}{-2\sin\theta}$$

$$\cancel{x^2} \; y^3 \cdot \frac{d^2y}{dx^2} = -3\sqrt{3}\sin^3\theta \times -\frac{\sqrt{3}}{2\cdot 2} \; \frac{1}{\sin^3\theta} = -\frac{9}{4}$$

~~Que~~: $\frac{dx}{dy} = \frac{1}{\left(\frac{dy}{dx}\right)}$

$\times$

Starting with here

P.T $\frac{d^2x}{dy^2} = -\frac{d^2y/dx^2}{\left(\frac{dy}{dx}\right)^3}$

$\cancel{\frac{d^2x}{}}$ $\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)$

$$\frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)^{-1} \; \frac{dx}{dy}$$

$$= -1\left(\frac{dy}{dx}\right)^{-2} \times \frac{d^2y}{dx^2} \times \frac{1}{\frac{dy}{dx}}$$

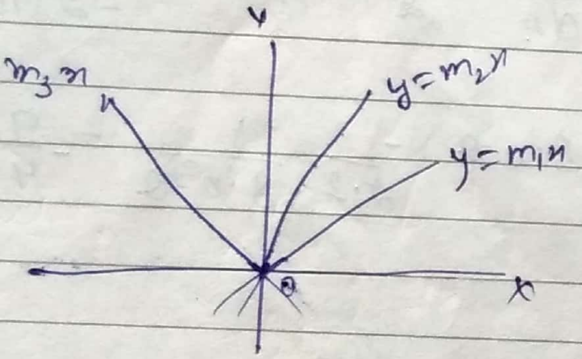
$$= \frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3} \quad \text{Ans}$$

Que: $x^3 + 3x^2y - 6xy^2 + 2y^3 = 0$
find $\left(\frac{d^2y}{dx^2}\right)_{(1,1)}$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(\text{slope})$$

$$= 0$$



$$\frac{dy}{dx} = m_1$$

L Hospital Rule:

Let f and g are diff fn for n times
and $\lim_{x \to a} \frac{f(x)}{g(x)}$ $\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$ form.
then according to L Hospital Rule it will be equal to $\frac{f'(x)}{g'(x)}$

$\left(\frac{0}{0}\right)$ or $\left(\frac{\infty}{\infty}\right)$

Indeterminate form is removed
Put $x = a$

$$\lim_{x \to a} \frac{f''(x)}{g''(x)}$$

Q. $\lim_{x\to 0} \frac{x - \sin x}{x^3}$ (can keep on above steps)

So

$+ = \cos x$

$\frac{x - (x - \frac{x^3}{\underline{|3}} + \ldots)}{x^3} = \frac{1}{\underline{|3}}$

(L.H) Rule

$\lim_{x\to 0} \frac{1-\cos x}{3x^2}$ (L.H Rule)

(LH)

$\lim_{x\to 0} \frac{1-\cos x}{3x^2} \left(\frac{0}{0}\right)$

$\frac{1-\cos x}{3x^2} = \frac{1}{2\cdot 3} = \frac{1}{6}$

$\lim_{x\to 0} \frac{\sin x}{6x}$

(L.H Rule)

$\lim_{x\to 0} \frac{\sin x}{6x} = \frac{1}{6}$

$\lim_{x\to 0} \frac{\sin x}{6x}\left(\frac{0}{0}\right) = \lim_{x\to 0} \frac{\cos x}{6}$

$= \frac{1}{6}$

* Method to handel to determine 0^0 and ∞^0 indetermenant:-

$A = \lim_{x\to 0} (\operatorname{cosec} x)^x$

$\ln A = \lim_{x\to 0} x \ln(\operatorname{cosec} x)$

$\lim_{x\to 0} \frac{\ln(\operatorname{cosec} x)}{\frac{1}{x}} \left(\frac{\infty}{\infty}\right)$

$= \lim \frac{\frac{1}{\operatorname{cosec} x} \times -\operatorname{cosec} x \cdot \cot x}{-\frac{1}{x^2}} = \lim_{x\to 0} \frac{(\cos x)}{\left(\frac{\sin x}{x}\right)} \cdot x$

$(A = 1)$. $= 0$

Q: $\lim_{x\to 0} \sin^{\tan x}$

Ans $\ln \sin_x^{\tan x}$

$\lim_{x\to 0} \tan(\ln \sin x)$

$$\boxed{\begin{aligned}&\lim_{x\to 0} \sec^2 x \ln \cancel{\sin x} + \tan x \, (\cancel{\cos x}) \frac{1}{\sin x} \cdot \cos x \\ &\lim_{x\to 0} \cancel{\sec} \quad \times \\ &\quad \sec x \tan x \ln \sin x\end{aligned}}$$

$\lim_{x\to 0} \cancel{\lim} \dfrac{\frac{1}{\sin x} \cdot \cos x}{-\cot \cancel{\sec}^2 x} \leftarrow \lim_{x\to 0}(-\sin x \cos x) = 0$

$\lim_{x\to 0} (-\sin x \cdot \cos x) = 0$

$A = e^0 = 1$

Que: $\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x}$

$\ln A = \lim_{x\to 0} \ln\left(\frac{1}{x}\right)^{\tan x}$

$\lim_{x\to 0} \tan x \ln\left(\frac{1}{x}\right) =$ $\quad$ $\sec^2 x \cdot \ln\frac{1}{x} + \tan x$ $\quad$ $\cancel{\sec^2 0} \ln\frac{1}{0} + \tan 0$

$\lim_{x\to 0} \dfrac{-\ln x}{\cot x}$

$\lim_{x\to 0} \dfrac{-1/x}{-\operatorname{cosec}^2 x} = \lim_{x\to 0}\left(\dfrac{\sin x}{x}\right) \cdot \sin x = 1 \cdot 0 = 0$

Que: $\lim_{x\to 0} \frac{e^x \sin x - x - x^2}{x^2 + x - \ln(1-x)}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{e^x \sin x - x - x^2}{x^2 + x - \ln(1-x)}$

$\lim_{x\to 0} \frac{e^x \sin x + e^x \cos x - 1 - 2x}{2x + 1 + \frac{1}{1-x}}$ Ar $\frac{0}{2} = 0$ Ans

$\cancel{\lim_{x\to 0} \frac{e^x(\sin x) - 2}{2}}$

* **Diff. of Determinant:**

$$f(x) = \begin{vmatrix} f & g & h \\ u & v & w \\ l & m & n \end{vmatrix}$$

Here $f, g, h, u, v, w, l, m, n$ are either f^n of x are constant

Rule: $f'(x) \begin{vmatrix} f' & g' & h' \\ u & v & w \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u' & v' & w' \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u & v & w \\ l' & m' & n' \end{vmatrix}$

Same diff Aplicable to column wise also.