extrima : Local maxima
Local Minima

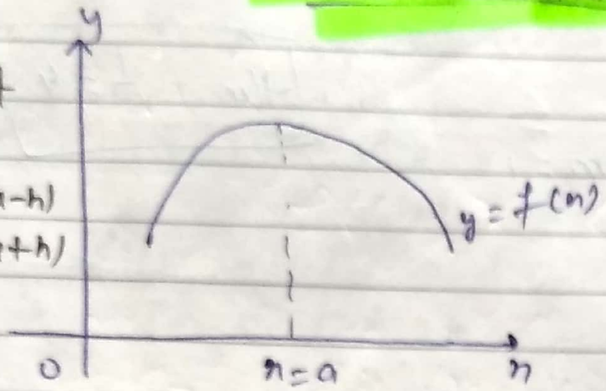
Maxima - Minima

28/07/17

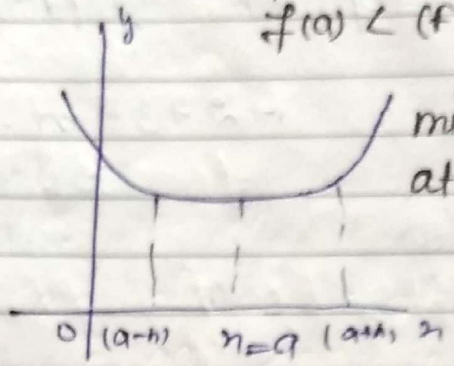
Maxima - minima.

maxima at $x = a$

$f(a) > f(a-h)$
$f(a) > f(a+h)$



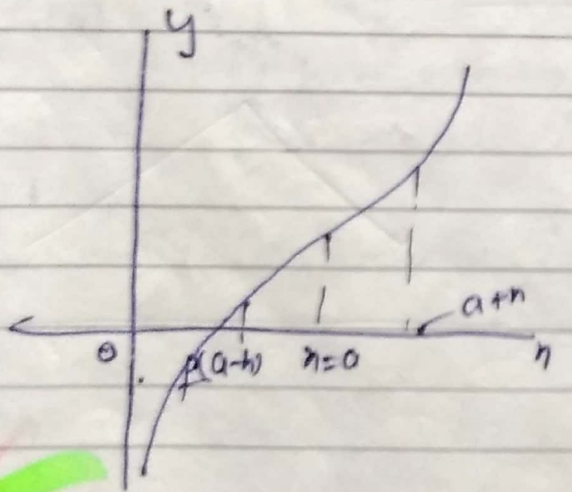
$f(a) < f(a-h)$
$f(a) < (f(a+h)$



minima at $x=a$

⇒ f^n $y = f(x)$ is said to have maximum at $x = a$ if $f(x)$ is greater then any other value assumed value in the neighbou hood value of $f(x) = a$

Here x is very small Positive small Real no.

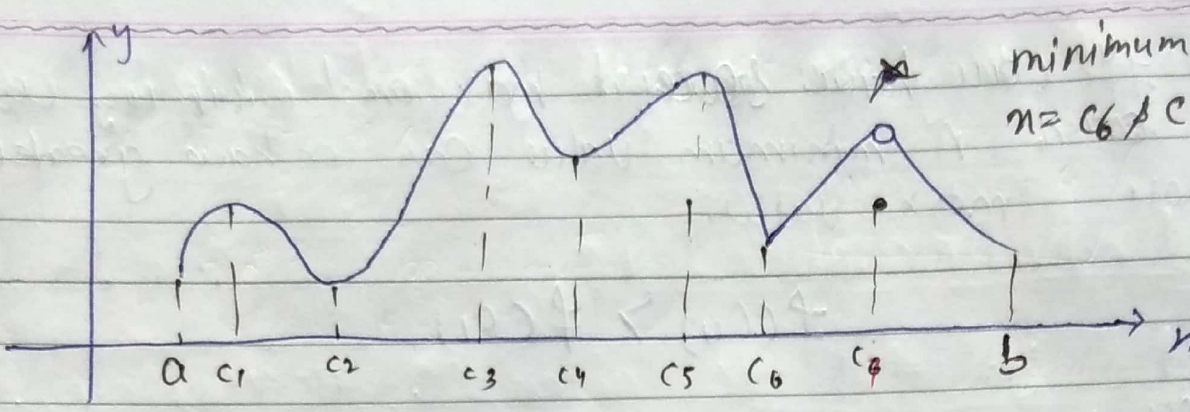


$f(a) < f(a+h)$
$f(a) > f(a-h)$

At $x = a$ f^n is monotonic

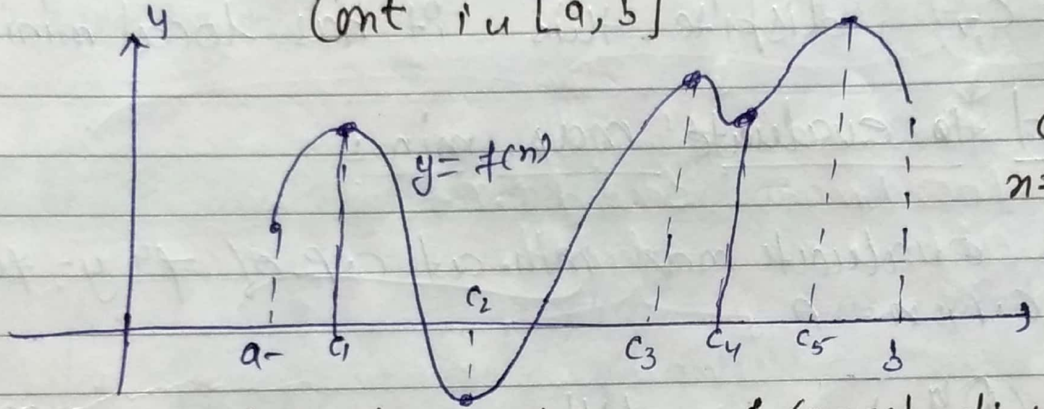
Note: 1) extrima means local maxima and local min. maxima / minima also known as local max / local min or Relative max / Relative min.

(2)



minimum at $x = c_6$ & c_7

Cont in [a, b]



C.P of f(x) is $x = c_1, c_2, c_3, c_4, c_5$

Local maximum at $x = c_1$, & corresponding local maximum value $f(x) = c$

(3) ~~A f^n co~~
maxima and minima of a cont. f^n occur alternatively & b/w two conjugative maximum value there is a minimum value and vice-versa.

(4) We will study maximum/minimum value of monotonicity or monotonic behaviour at critical point

(5) term Extremum or Extremal or turning value is used for maximum or a minimum value

(6) maximum/minimum value of a f^n may not be the greatest/least value of fn.

Study at only C.P

7) A f^n can have several max and minimum value and A minimum value can also have greater than max value

$$f(C_4) > f(C_1)$$

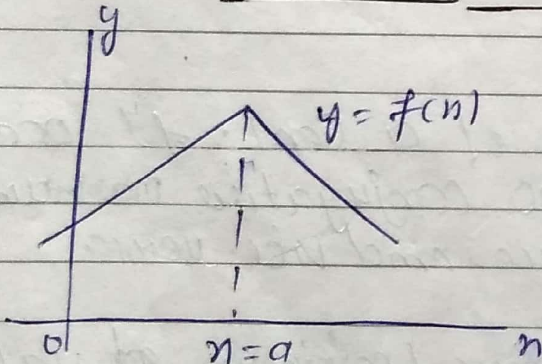
(I)
8) f^n is not diff at $x = C_6$ and dis cont at $x = C_7$ dispite this it has local minima.

* **Method to evaluate max - min :**

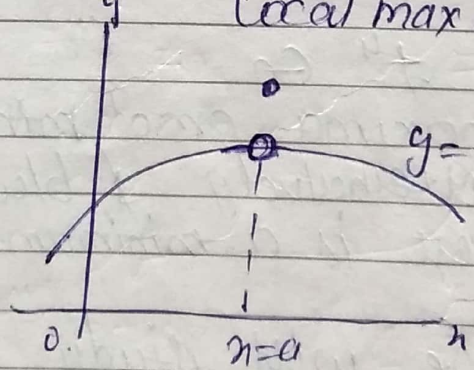
we will evaluate max min at C.P of f^n $y = f(x)$.
suppose C.P is $x = a$

Case : 1 (f^n is not diff at $x = a$ (may be dis. cont. also)) :

(i)



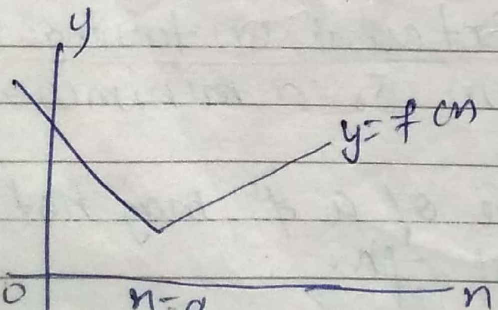
Local max at $x=a$



$f(a) > f(a-h)$
$f(a) > f(a+h)$

Same

(ii)



Local min at $x=a$
$f(a) < f(a-h)$
$f(a) < f(a+h)$

Case : 2 when f^n is diff. at $x=a$!

(B) diff. at $x=a$

(i) First D. T (~~detect~~

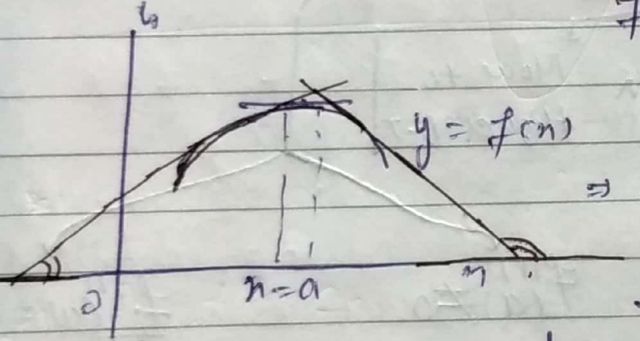
(ii) IInd D. T

(iii) Higher order D.T

(A) not. diff. $x=a$

Some time figure make is help full in the solving Question

(i) First order Derivative Test :

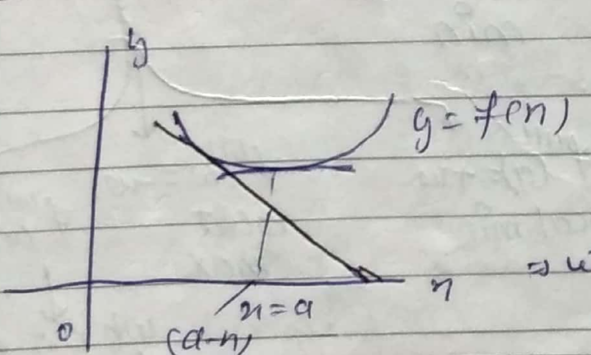


$f'(a) = 0$ (in all figure).

$f'(a^-) = +ve$

$f'(a^+) = -ve$

$\Rightarrow$ while travelling left to right about C.P slope changes from +ve to zero to negative then f^n has local maxima at $x=a$.



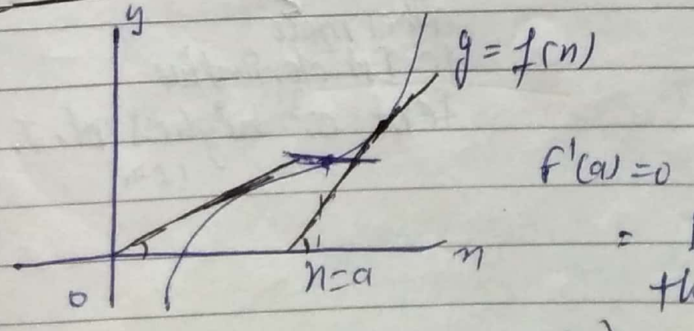
$f'(a^-) = -ve$

$f'(a) = 0$

$f'(a^+) = +ve.$

$\Rightarrow$ while tr

minima.



$f'(a) = 0$

$f'(a^-) = +ve$

$f'(a) = 0$

$f'(a^+) = +ve.$

= If $f'(x)$ does not change its sign then it is not ~~pos~~ point of Exterimum at $x=a$

(then that f^n is monotonic)

n^{th} d.T → odd d.T = 0 then

5 D.T = 0
the 6th D.T.

Here f^n may be increasing and decreasing at $x = a$.

★ IInd Deravative Test :

Let $y = f(x)$ is given f^n. $x = a$ (as c.p).

$$y = f''(x)$$

- $f''(a) = +ve$ → Local min at $x = a$
- $f''(a) = -ve$ → Local max at $x = a$
- $f''(a) = 0$ → Second D.T failed.
  - then move to Ist D.T **or** Move to Higher D.T
    - $f'''(a) \neq 0$ ∴ neither max, nor min
    - $f'''(a) = 0$
      - $f''''(a) = +ve$ → Local min
      - $f''''(a) = -ve$ → Local max
      - $f''''(a) = 0$ → 4th D.T failed, either move to Ist derivative test or higher d.T. (6th, - - - .)

2nd ≠ 0
4th ≠ 0
6th ≠ 0
then ~~max~~
neither of max or nor min

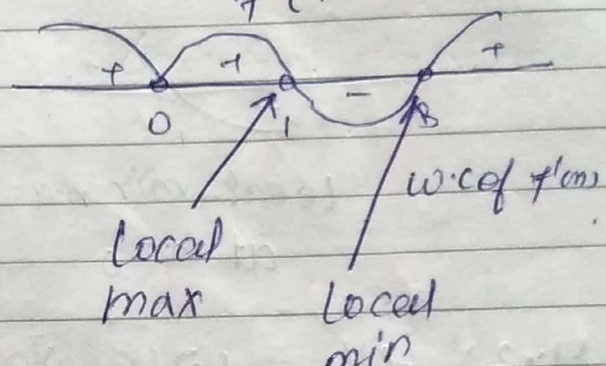
Que: find max min of f^n.

$$f(x) = x^5 + 5x^4 + 5x^3 - 10$$

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$
$$= 5x^2(x^2 - 4x + 3)$$
$$= 5x^2(x-3)(x-1)$$

$f'(1^-) = +ve$

$f'(1^+) = -ve.$



C.P of $f(x)$ is $x = 0, 3, 1$

$f'(0^-) = +ve$

$f'(0^+) = +ve$

neither max nor min at $x = 0$

$\therefore$ maxima at $x = 1$, & maximum value $= f(1)$

Method-2

IInd D.T:

$$f(x) = x^5 - 5x^4 + 5x^3 - 10$$
$$f'(x) = 5x^4 - 20x^3 + 15x^2 = 0, \quad x = 0, 1, 3$$

$$f'(x) = 20x^3 - 60x^2 + 30x.$$
$$= 10x(2x^2 - 6x + 3)$$

$f''(1) = -ve$ $\therefore$ Local max at $x = 1$	$f''(3) = 30[18 - 18 + 3] = +ve$ $\therefore$ Local min at $x = 3$

$f''(0) = 0$ $\therefore$ IInd D.T failed

move to Ist D.T

higher

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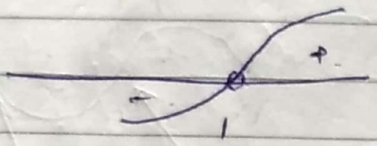
Que: $f(x) = (x-1)^4$

check maxima-minima or extremum

M-1

$f'(x) = 4(x-1)^3 = 0 \Rightarrow x = 1$ is C.P.



local minima at $x=0$

(M.2) $f''(x) = 12(x-1)^2$

$f''(1) = 0 = 2^{nd}$ D.T failed.

$f'''(x) = 24(x-1)$,

$f'''(1) = 0$

$f''''(x) = 2x$, $f''''(x) = +ve$

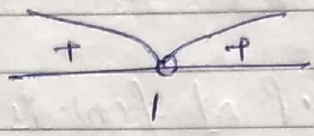
$\therefore$ local min at $x=1$

Q. $f(x) = (x-1)^5$

M-1)

$f'(x) = 5(x-1)^4 = 0$

$x=1$ is C.P



neither max or nor min at $x=0$

(M.2)

$f''(x) = 20(x-1)^3$,

$f''(1) = 0 = 2^4$ D.T failed

$f'''(x) = 60(x-1)^2$, $f'''(1) = 0$

$f''''(x) = 120(x-1)$

$f''''(1) = 0 \Rightarrow 4^{th}$ D.T fail

$f'''''(x) = 120 \neq 0$

neither max, min

Que: find max slope of curve

$y = -x^3 + 3x^2 + 2x - 27$

Ans $y = -x^3 + 3x^2 + 2x - 27$

$m = y' = -3x^2 + 6x + 2$.

$\frac{dm}{dx} = -6x + 6 = 0$ $x = 1$ (for ~~getting~~ getting value of m)

$\frac{d^2m}{dx^2} = -6$

$\therefore$ m is max at $x=1$

& maximum value of slope is

$m(x-1) = -3 + 6 + 2 = 5$ Ans

$3x^2 - 6x$

$-3x^2 + \ldots$ $+2$

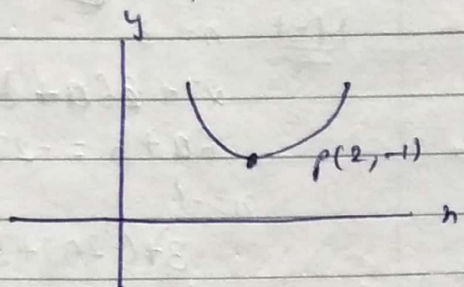
x^2

$(x-3)(x-2)$

$x^2 - 2x - 3x - 6$

$x^2 - 5x \ldots$

Que:- f^n $y = \frac{ax+b}{(x-1)(x-4)}$ has turning point at P. (2, -1) then find a and b



$x = 2,$

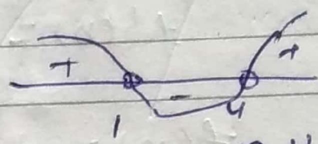
$\frac{2a+b}{(1)(-2)} = \frac{2a+b}{-2}$

$y' =$

$\frac{2a+b}{-2} = 0$

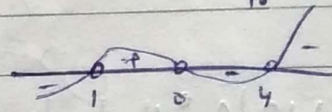
$a = \frac{-b}{2}$

$x = a, -1$



$x = -1$ $\frac{-a+b}{(-2)(-5)}$

$= \frac{-a+b}{10}$



$y = \frac{ax+b}{(x^2-5x+4)}$

$-1 = \frac{2a+b}{4-10+4}$ —(i)

$y' = \frac{(x^2-5x+4)a - (ax+b)(2x-5)}{(x^2-5x+4)^2}$

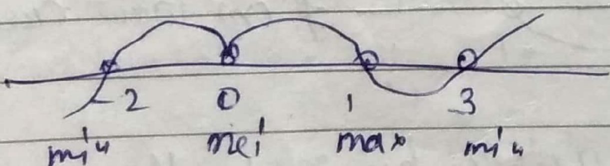
$(y')_{x=2} = 0$ —(ii)

Que:- $f(x) = \int_{-1}^{x} t(e^t-1)(t-1)(t+2)(t-3)^5 \, dt$

find point of local maxima/minima of fcn.

$= x(e^x-1)(x-1)(x+2)(x-3)^5 \, dt.$

$f'(x) =$

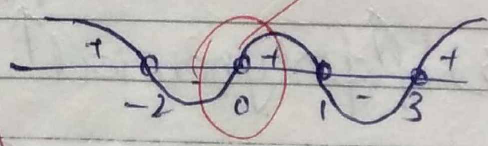


$\int_{-1}^{x} t(e^t-1)(t-1)(t+2)(t-3)^5 \, dt$

$= x(e^x-1)(x-1)(x+2)(x-3)^5 \, dt.$

$= x = 0, 1, -2, 3,$

mistake



max = -2, 1,

minima = 0, 3.

Least D.f = 2 power.

Que! $f^n(x) = x^3 + ax^2 + bx + c$ has Local min at $x=3$ max at $x=-1$ then find a, b, c. monic Polynomial

$f(x) = x^3 + ax^2 + bx + c$

$f'(x) = 3x^2 + 2ax + b$

M ⇒ 2 $f'(x) = \lambda(x-3)(x+1)$

$f'(x) = \lambda(x^2 - 2x - 3)$

$f(x) = \lambda\left(\frac{x^3}{3} - \frac{2x^2}{2} - 3x\right) + \cancel{x}$

$x = 3$

M-1 $x = 3$

$27 + 36a + b$

$6a + b = -27$ (i)

$x = -1$

$3 + (-2a) + b$

$-2a + b = -3$ (ii)

$= 6a + b = -27$

$-2a + b = -3$

(i) − (ii)

$6a + b - (-2a + b) =$

Note: 1) Polynomial f^n $f(x)$ has Extrema at $x = a$ and $x = b$

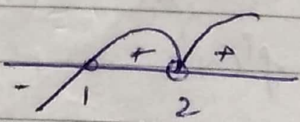
$f'(x) = (x-a)(x-b)\,g(x)$

Here $g(x)$ is any Constant

↑ Least degree polynomial.

(2) if $f(x)$ has neither max nor min at critical Point $x = 2$ and max at $x = 1$

$f'(x) = \lambda(x-2)^2(x-1)$



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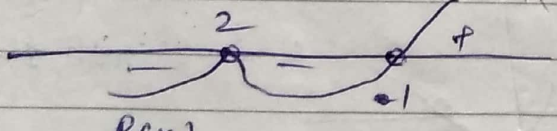

$f'(x)$ wave curve.

(3) if polynomial $P(x)$ is divisible by $x+1$ then

$3\overline{)15}(5$

$\underline{15}$

$\times$

$15 = 3\times5+0$

$P(x) = (x-1)\,g(x) + 0$

↑ Polynomial. ↓ another Polynomial. Remainder.

(4) f(n) is four degree polynomial having extremal at n = 1. if f'(n) is divisible by n^2+1 then

f   4

f'   — 3

$f'(n) = \lambda(n^2+1)(n-1)$

~~f(n) = (n^2+1)+(...)~~

~~f'(n) = (n^2+1)(...)~~

$P(n) = (n-1)n^2$

P is divisible by n^2.

$P'(n) = 2n(n-1) + n^2$.

P' is D by n.

$P''(n) = 2(n-1) + 2n + 2n.$

(~~4~~) ~~if f~~

If f is D by n^3
if P' D by n^2
P'' D by n.

(5) f is 4 degree polynomial. if f'(n) has exterima at n = 1 and 2 and f has exterima at n = 0. given that

$f(-1) = 1$ then write.

f — 4 degree. has exterima at n=0

f' — 3 degree

f'' — 2 degree

$f'' = 2$

f   — 4 $f' = 3$

$f' = \lambda(n-1)(n-2)$

$f(n) = n^2$

$f(-1) = 1$.

$f''(n) = \lambda(n-1)(n-2)$

$= \lambda(n^2-3n+2)$.

(O-1) ✗/ Bin & L.V

H.W: 1, 4, 5, 6, 7, 8, 9, 10, 11, 16, 19, 20, 21,

(S-1) ⇒ 1, 2, 3(a), 4, 15

$f'(x) = \lambda\left(\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right) + \mu.$

$f'(0) = \lambda 0 + \mu = 0$

$\mu = 0$

$f'(x) = \lambda\left(\frac{x^3}{3} - \frac{3}{2}x^2 + 2x\right)$

$f(x) = \lambda\left(\frac{x^4}{12} - \frac{3x^2}{2 \cdot 3} + \frac{2x^2}{2}\right) + \delta.$

Q. Polynomial $P(x)$ degree 4 having exterimum at $x = 1$ & $x = 2$

if $\lim_{x \to 0} f(x) = \left(1 + \frac{f(x)}{x^2}\right) = 3$

then find $f(2) = ?$

$P'(x) = \lambda(x-1)(x-2)(x-a)$

$P'(x) = (x-1)(x-2)(ax+b)$

$P(x) = ax^4 + bx^3 + cx^2 + dx + e$

$\lim_{x \to 0}\left[1 + \left(ax^2 + bx + c + \cancel{\frac{d}{x}} + \cancel{\frac{e}{x^2}}\right)\right]$

$P(x) = ax^4 + bx^3 + cx^2$

$1 + 0 + 0 + c = 3$

$c = 2$

$P(x) = ax^4 + bx^3 + 2x^2$

$P'(x) = 4ax^3 + 3bx^2 + 4x$

$P'(1) = 0$ — (i)

$P'(2) = 0$ — (ii)

$P(x) =$ $P(x)$ — 4 ✗

$P'(x)$ — 3

$P''(x)$ — 2.

$f''(x) = \lambda(x-1)(x-2)$.

$f\left(1 + \frac{f(x)}{x^2}\right) = 1(0) = 0$

$1 + \frac{0}{x^2} = 1$

$\lambda(x^3-$

$(x-1)(x-2)$ ✗

$1 \times 0 = 0$.

$x + \frac{3f(x)}{x^3}$

$= \frac{x^2}{2} +$

$(\lambda(x-1)(x-2))$

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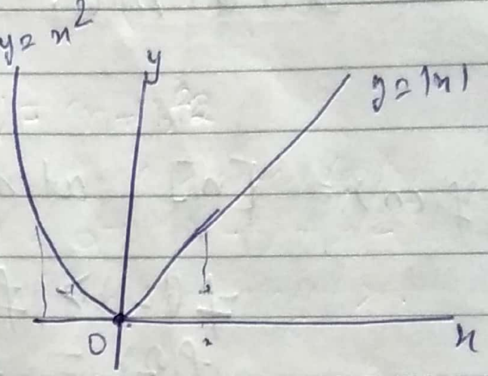
Q. ~~find~~ check exterima, if exist of the following f^n at the mentioned point.

(i) $f(x) = \begin{cases} x^2 & x \le 0 \\ |x|, & x > 0 \end{cases}$ at $x=0$



$f(0) < (0+h)$
$f(0) < (0-h)$
Local min at $x=0$

$(a-h) < 0$
$(a+h) < 1$

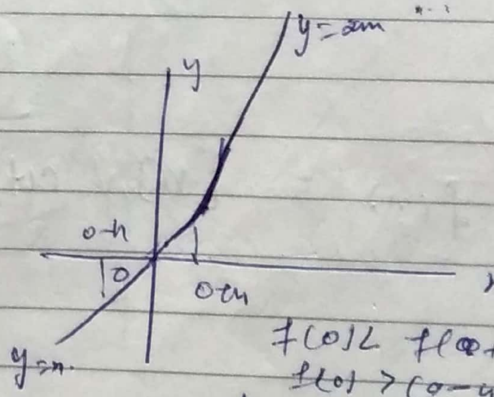
min at $x=0$

(2)

$f(x) = f(x) = \begin{cases} x & x < 0 \\ 2x & x \ge 0 \end{cases}$ at $x=0$

$f(0) < f(0+h)$
$f(0) > f(0-h)$ MI at $x=0$

~~Local~~

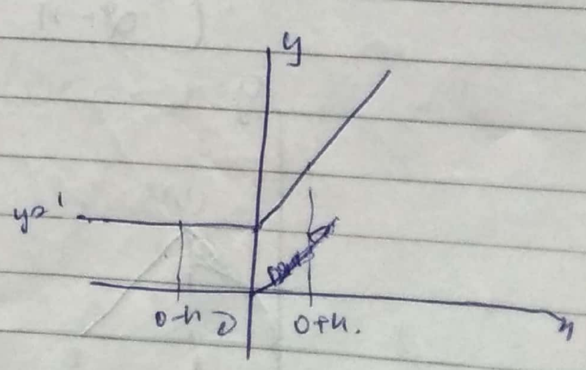


$f(0) < f(0+h)$
$f(0) > (0-h)$
MI at $x=0$

(3) $f(x) = \begin{cases} 1 & x \le 0 \\ x+1 & x > 0 \end{cases}$ at $x=0$

$f(0) < f(0+h)$
$f(0) = f(0-h)$

nei extrima nor monotonic at $x=0$

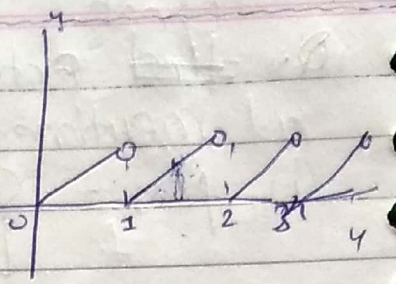


nei. min nor max

Compage

$\rightarrow \{I\} = 0.$

Que:-(1) $f(x) = \{x\}$ at $x = 1$



$f(1) < f(1+a)$

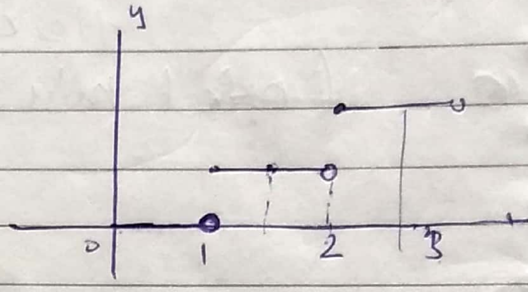
local min. $\quad f(1) < f(1-a)$

(2) $f(x) = [x]$ at $x = 2$

$f(2) > f(2^-)$

$f(2) = f(2^+)$

neither Ext. or not at $x = 2$



(3) $f(x) = \{x\}$ at $x = \frac{3}{2}, \frac{5}{2}$

$f(\frac{3}{2}) > (\frac{3}{2} - h)$

$f(\frac{3}{2}) < (\frac{3}{2} + h)$

M I ~~not at max~~

$f(\frac{3}{2}) = (\frac{3}{2} + h)$

$f(\frac{3}{2}) = (\frac{3}{2} - h)$

nei: M. and Ext.

(4) $f(x) = [x]$ at $x = \frac{5}{2}$ $\Rightarrow$ $f(\frac{5}{2}) = (\frac{5}{2} + h)$

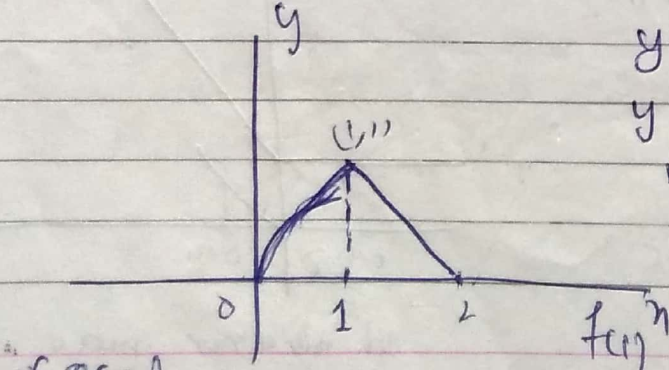
$f(\frac{5}{2}) = (\frac{5}{2} - h)$

nei: M. and Ex.

at $x = 5/2$.

Que: $f(x) = \begin{cases} \sin \frac{\pi x}{2}, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$ at $x = 1$

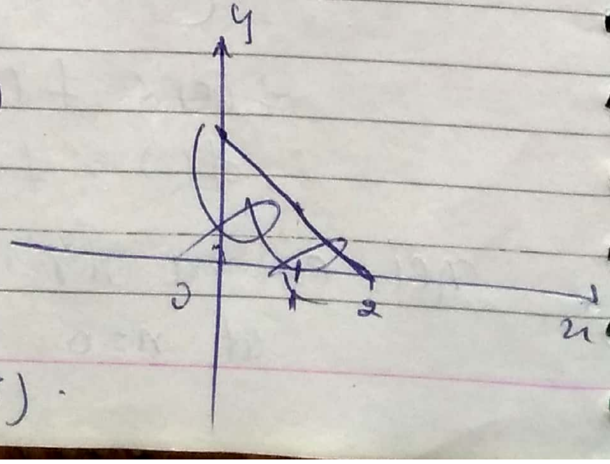
x=1 x=2 x=3
y=1 y=0 -1



$y = mx + c$

$y = 2 - x$

$m = -1$



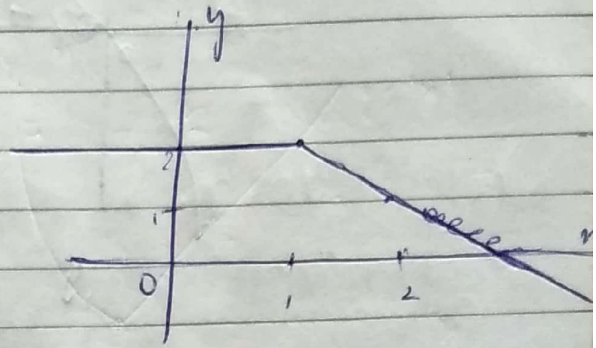
Local max

$f(1) > f(1^-)$

$f(1) > f(1^+)$.

$y = 3-x$
$x=1 \quad y=2$
$x=2 \quad y=1$
$x=3 \quad y=0$

Que: $f(x) = \begin{cases} 2 & x \leq 1 \\ 3-x & 1 < x < 2 \end{cases}$ at $x = 1$



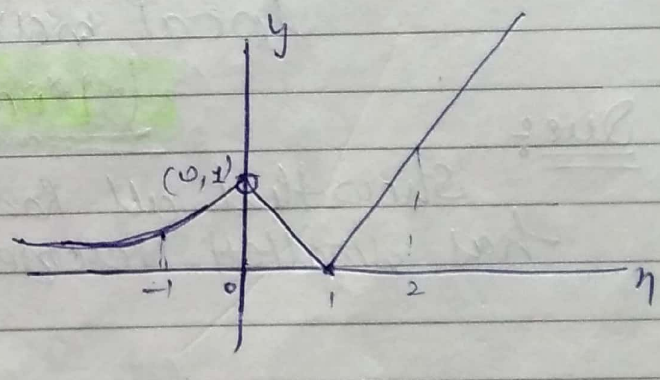
$f(1) = f(1^-)$
$f(1) > f(1^+)$
neither max ~~nor m~~
nor min

Que: $f(x) = \begin{cases} e^{x} & x < 0 \\ |1-x| & x \geq 0 \end{cases}$

$(x-1)$
$x > 1$

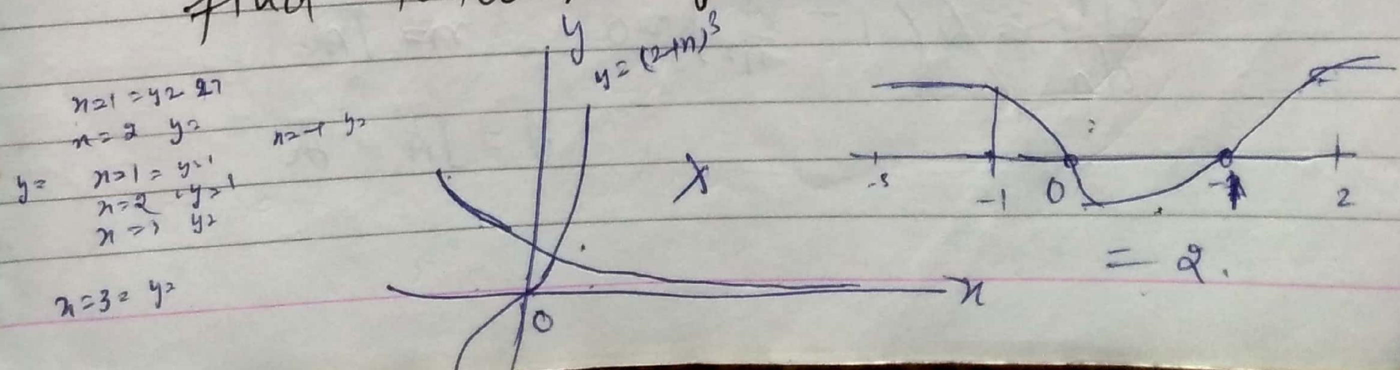
check max min in $(-1, 2)$

max:
local maxima at $x=0$
local minima at $x = 1$

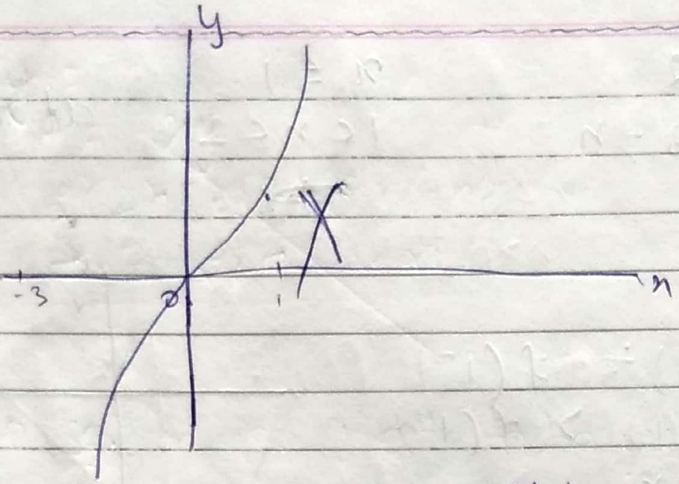


Que: $f(x) = \begin{cases} (2+x)^3 & , -3 < x \leq -1 \\ x^{2/3} & -1 < x < 2 \end{cases}$

find total no. of local max or min



$= 2.$

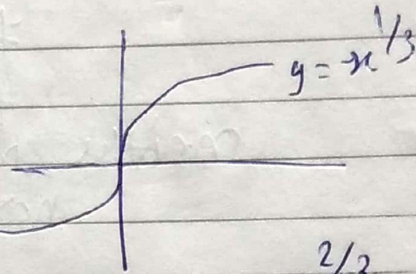


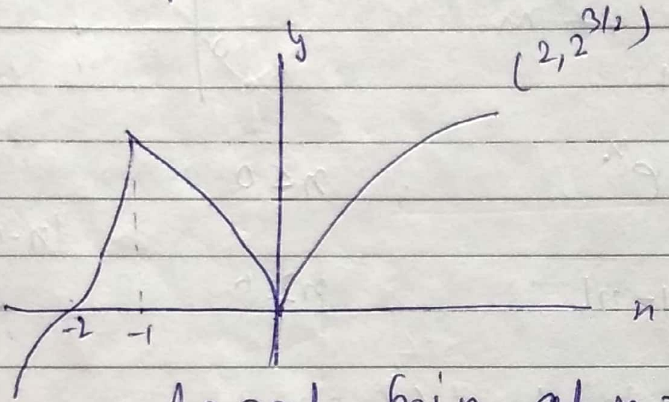
$y = x^{2/3}$ $\quad 3^2$

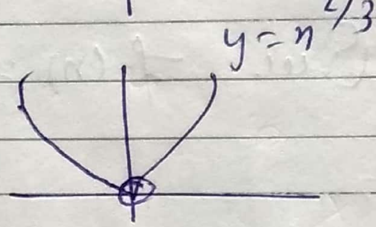
$x=1 \quad y=1$

$x=2 \quad y=1$

$x=3 \quad y=4$







local min at $x = 0$

local max at $x = -1$

Geometrical Problems:

Que:

Show that all Rectangles of given area the square has smallest Perimeter. $A = xy$

P

$\frac{dP}{d\,\text{variable}}$

$\frac{dP}{dx}$ $\quad y = \frac{A}{x}$

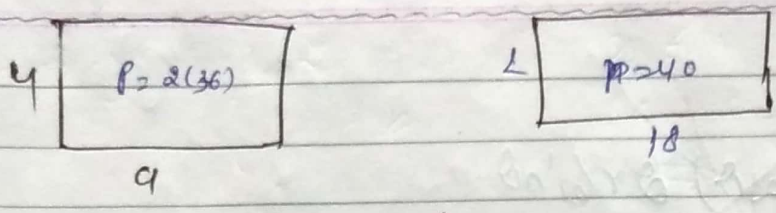
$P = 2(x+y)$

$P = 2(x + \frac{A}{x})$

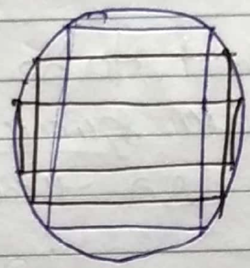
$\frac{dP}{dx} = 2(1 - \frac{A}{x^2}) = 0 \quad , \quad x = \sqrt{A}$

$y = \sqrt{A}$

0+ = 2, 3, 6, 12, 13, 14, 15, 17, 22,
J.M: 1, 2, 3, 4, 5, 6, 7, 8, 9,



Q. Show that maximum Area of Rectangular ~~that~~ that can be inscribe in a circle



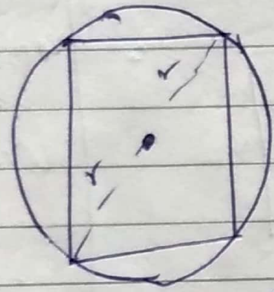
$A = xy$: $y = \frac{A}{x}$

$= \pi r^2$

$\frac{dP}{dx} = 2\pi r$ $\sqrt{a^2+b^2}$ $= (a^2+b^2)^{1/2}$

$A = \frac{a^2+b^2}{2}$

$= \frac{dA}{dx} =$



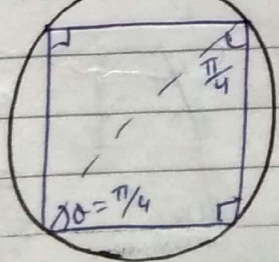
$$x^2 + y^2 = 4r^2$$

$$A = xy$$
$$= x\sqrt{4r^2 - x^2}$$

$$\frac{dA}{dx} = \sqrt{4r^2 - x^2} + \frac{x \cdot 1 \times (-2x)}{2\sqrt{4r^2 - x^2}} = 0$$

$$4b^2 - x^2 - x^2 = 0$$
$$2x^2 = 4r^2$$
$$x = \sqrt{2}r, \quad y = \sqrt{2}r.$$

(M-2)



$$\cos\theta = \frac{x}{2r}$$
$$x = 2r\cos\theta$$
$$\sin\theta = \frac{y}{2r}$$
$$y = 2r\sin\theta$$

$y = 0$

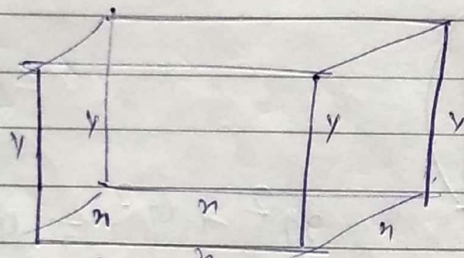
$A = xy$

$2(2r\cos\theta)\, 2r\sin\theta$

$A = 2r^2(\sin 2\theta)$ $\quad A_{max} = 2r^2$

$\theta = \frac{\pi}{4}$

Que: 40 sq. m of sheet to be used in construction of an open tank with square base, then find dimensions of tank so that its capacity is greatest possible



$4xy + x^2 = 40$

$V = x^2 y$

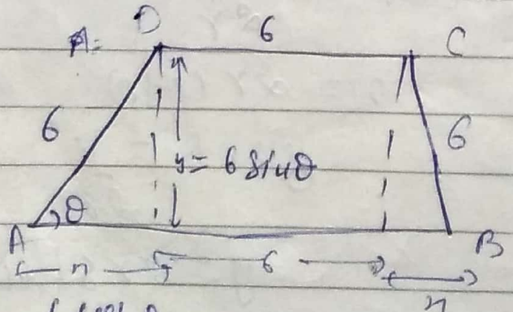
$= x^2 \cdot \frac{40 - x^2}{4x}$

$V = \frac{1}{4}(40x - x^3)$

$\frac{dV}{dx} = 0$

Q. The three sides of a trapezium are equal each being 6 cm long. Find area of trapezium when it is maximum.

Ans:



$4xy$

$\frac{1}{2} \times (a+b)$

$\frac{dA}{d\theta}$

$6 \times \frac{1}{2}$

Point minimize
Draw normal
and meet curve

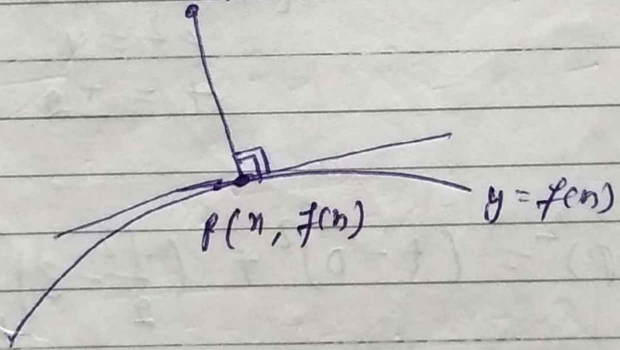
$A = \frac{(2x+6+6)\,xy}{2}$ $\quad y = \sqrt{36-x^2}$

$= (x+y)\,y.$

$A = (x+6)\sqrt{36-x^2}$

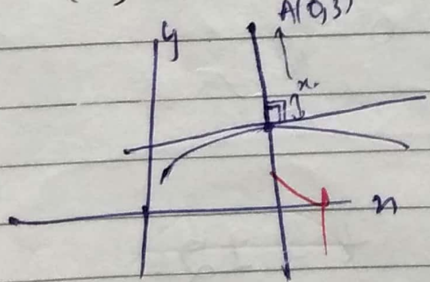
$\frac{dA}{dx} = 0$

<u>Note</u>:- Given fixed point A (a,b) and moving point P x, f(x) on the curve of y = f(x) then A.P will be maximum or minimum if it is normal to the curve from point

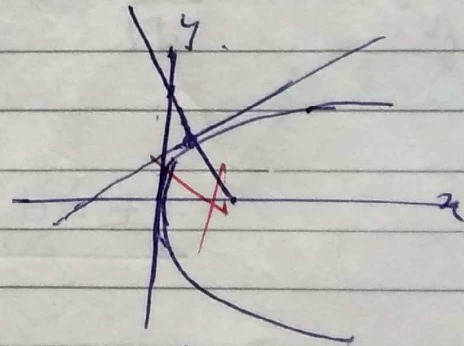


$\frac{d\ \text{distance}}{d\ \text{variable}} = 0$

Que: Find point on the Parabola which is nearest to A (0,3) = $x^2 = 2y$



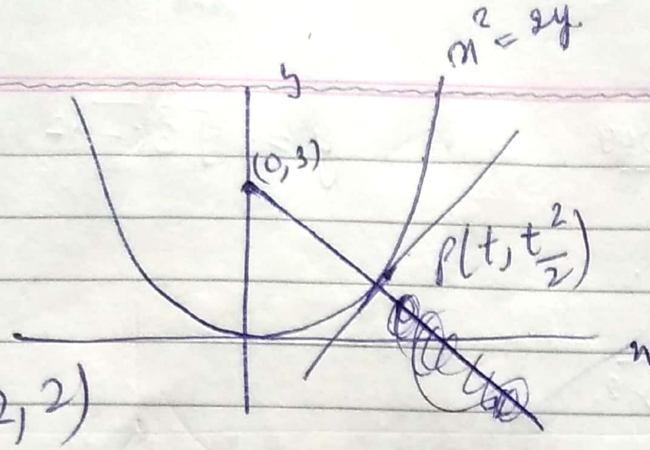
$\frac{dx}{d\theta} = 0$



$x^2 = 2y$
$y = \frac{x^2}{2}$
$y' = \frac{2x}{2}$

$x^2 = 2y$
$2x = 2y'$
$y' = \frac{2x}{2}$
$y' = x.$
$2\theta = 4$
$x =$

Ans:



$P(-2,2)$

(M-1)

$2x = 2y'$

$y' = x$

$(y')_P = t$

$\therefore$ Slope of tangent at $P = t = m_1$

Slope of line $AP = \dfrac{3 - \frac{t^2}{2}}{0 - t} = \dfrac{t^2 - 6}{2t}$

$m_{PA} \cdot m_1 = -1$

$\dfrac{t^2 - 6}{2t} \times t = -1$

$t^2 - 6 = -2$

$t^2 = 4$

$t = \pm 2$

(M-2)

$(AP)^2 = (t-0)^2 + \left(\frac{t^2}{2} - 3\right)^2$ If AP is min then AP^2 is min.

Common

$\dfrac{dd}{dt} = \cancel{2}t + \cancel{2}\left(\frac{t^2}{2} - 3\right) \times \frac{2t}{\cancel{2}} = 0$

$t\left(1 + \frac{t^2}{2} - 3\right) = 0$

$t = 0, \quad \frac{t^2}{2} = 2$

$t^2 = 4 \qquad t = \pm 2.$

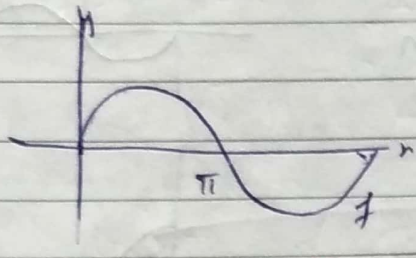
Que: $f(x) = \begin{cases} x(x+3) & -1 \le x < 0 \\ -\sin x & 0 \le x < \frac{\pi}{2} \\ -(1+\cos x) & \frac{\pi}{2} \le x \le \pi \end{cases}$

Draw and Point out Local max / local min

In the given interval $(-1, \pi]$

Ans:-

$-\sin x.$



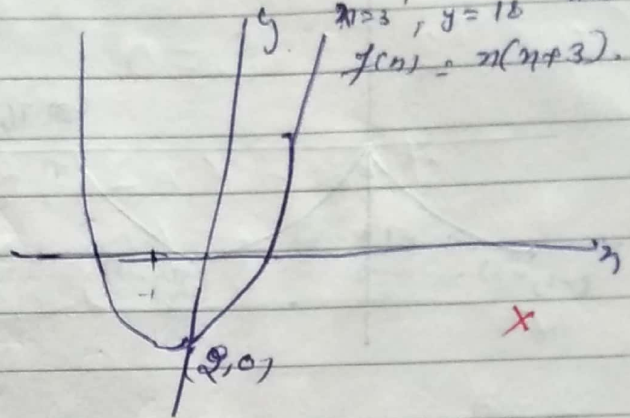
$y = x(x+3)$, $x=-1$, $y=-2$

$x=1$, $y=4$, $x=-2$, $y=-2$

$x=2$, $y=10$

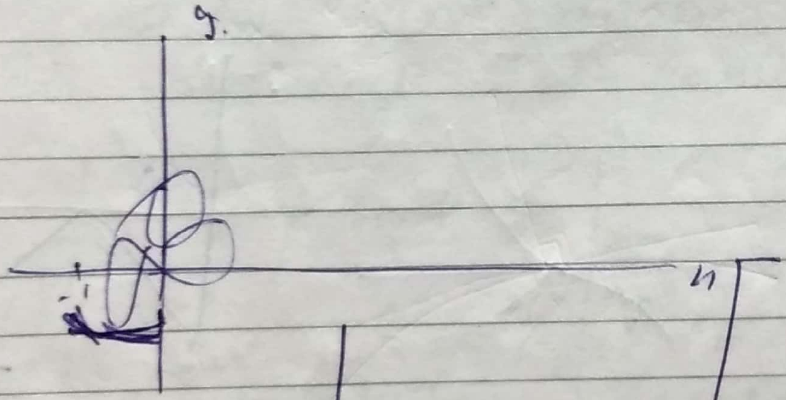
$x=3$, $y=18$, $x=0$

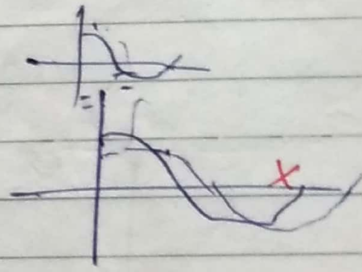
$f(x) = x(x+3).$

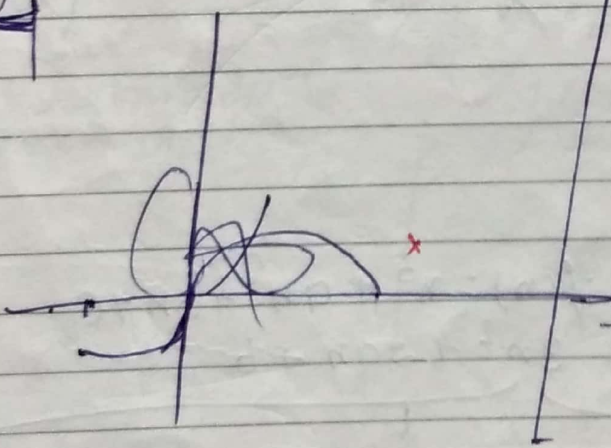


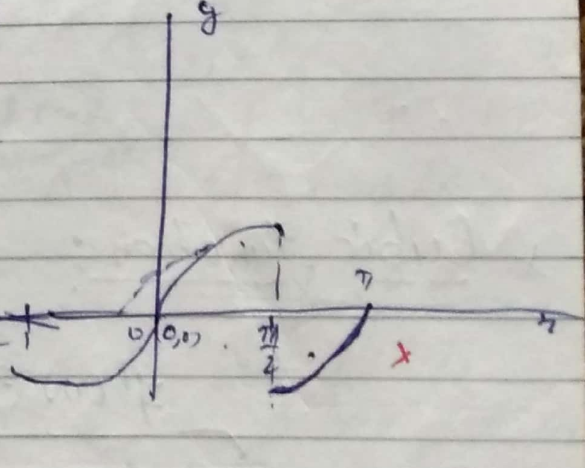
$-(1+\cos x)$

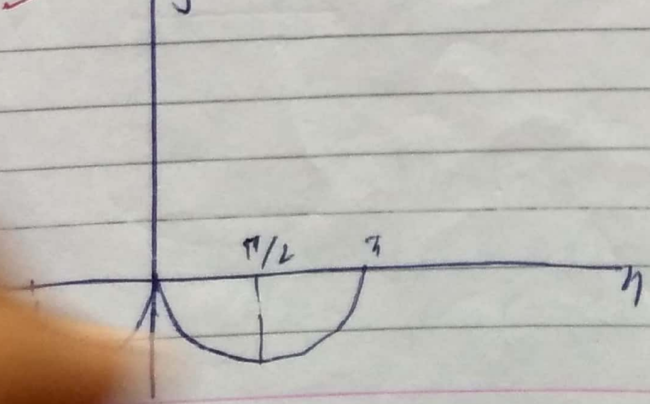
$1+\cos x$





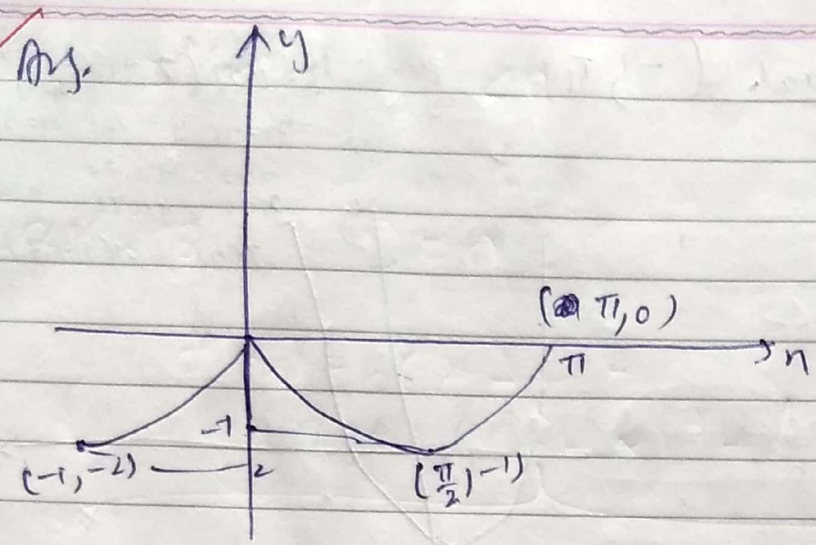


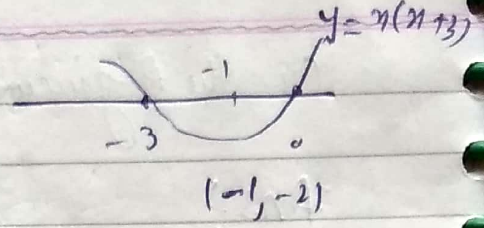


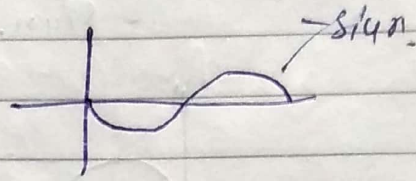


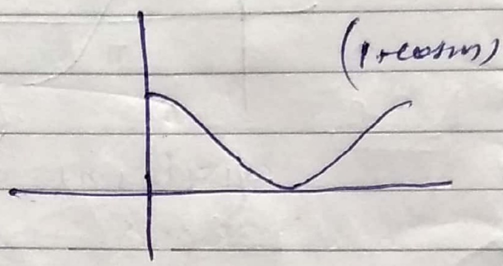
Right fig. in next page

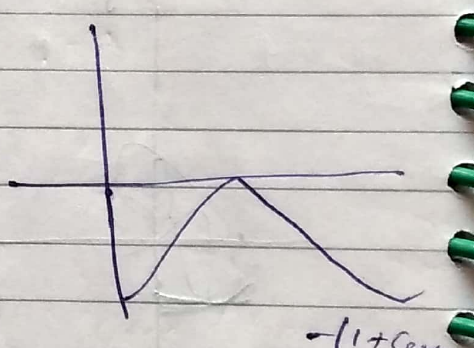
Ans.











Cubic function:

$$f(x) = x^3 + ax^2 + bx + c$$

$$f'(x) = 3x^2 + 2ax + b$$

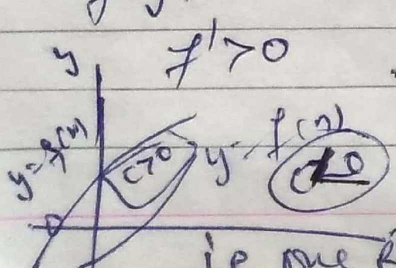
Case (1) $D < 0$

$a > 0$

$D < 0$

fig: III

$f' > 0$



i.e one Real root

~~M~~ : ~~f~~ is ~~monotonic~~ function is monotonic increase.
MI

Remaining two are
Imaginary Roots.

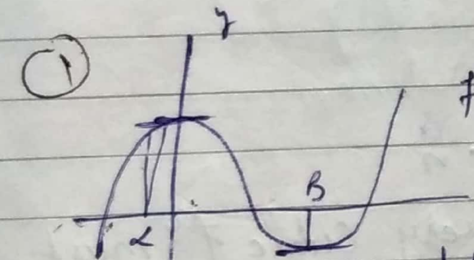
Real Root having
c opposite Root

Case : 2 $D > 0$

$$f'(x) = 3(x-\alpha)(x-\beta)$$

In these case five possible figures of $y = f(x)$

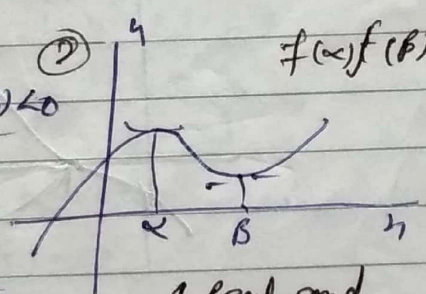
(1)



$f(\alpha)f(\beta) < 0$

3 real and distinct roots

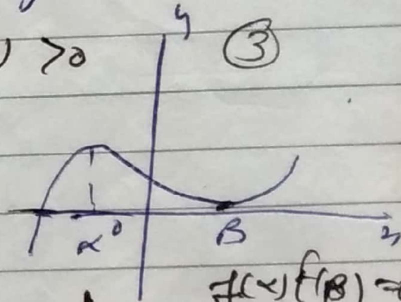
(2)



$f(\alpha)f(\beta) > 0$

1 Real and 2 Imaginary Roots

(3)

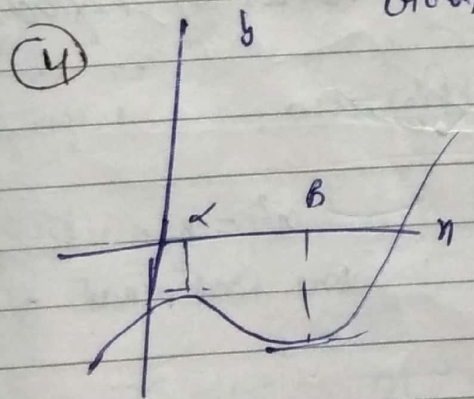


$f(\alpha)f(\beta) = 0$

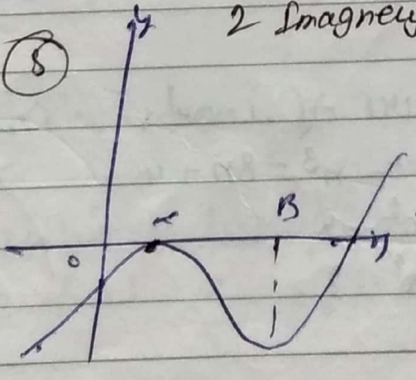
3 ~~Real~~ Roots

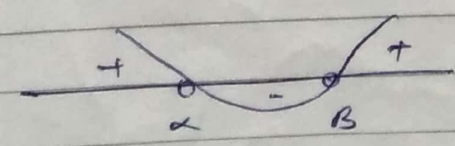
3 Distinct roots
Out of this
2 are coincident

(4)



(5)





f

sign curve of f'

$f' > 0$ is $(-\infty, \alpha) \cup (\beta, \infty)$
$f' < 0$ in (α, β)

H.W: J.A: 1, 2, 3, 4, 5, 6(A), 7, 8, 9, 10

01/08/17

$f(\alpha) f(\beta) < 0$ Real: ! Distincts :

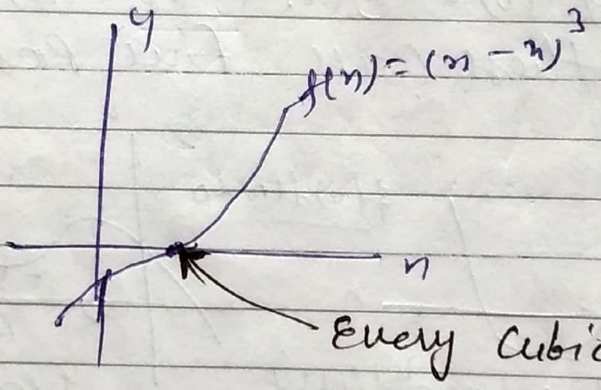
Case : 3 $D = 0$

$f'(x) = 3(x-\alpha)^2$

$= (x-\alpha)^3 + d$

$d = 0$ Three real & concided root

$d \neq 0$ One real root



Every cubic f^n must have Point of Inflection

Que: Find nature of roots. Draw fig also.

$f(x) = x^3 - 8x + 4$

$f'(x) = 3x^2 - 8$

$x^2 = \frac{8}{3}$

27 - 24 =

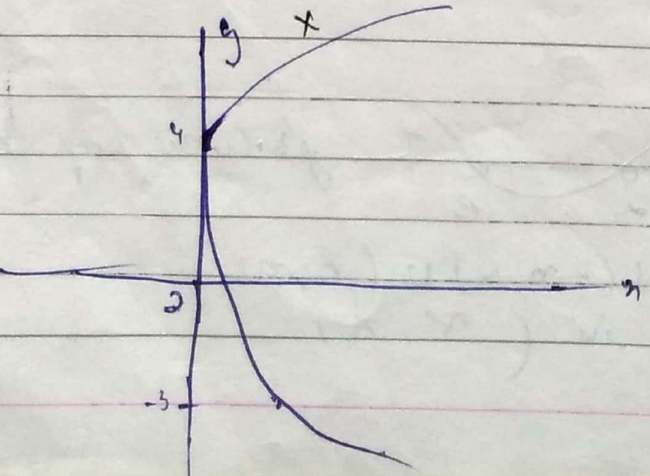
8 - 16 + 4

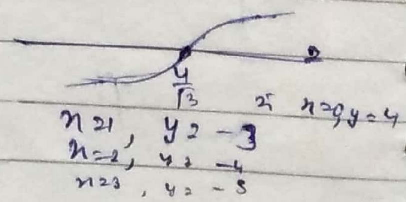
-4

$x^2 = \frac{8}{3} = \pm\frac{2\sqrt{2}}{\sqrt{3}}$

One Real Root

Three real roots.





Que! find no. of Real roots and Draw fig. also

$$f(n) = 2n^3 - 15n^2 + 36n + 1$$

$$f'(n) = \cancel{6n^2 - 30n + 36}$$

$$= 6(\cancel{n^2 - 5n + 6})$$

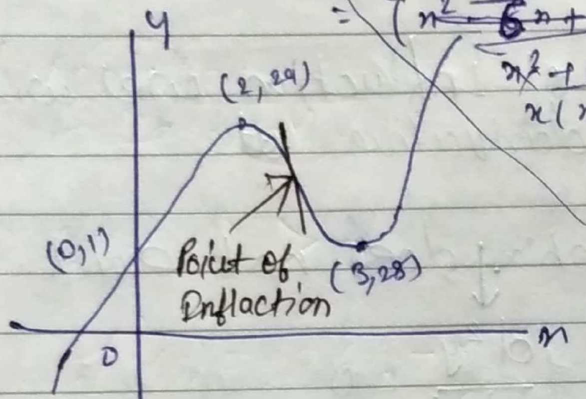
$$= (\cancel{n^2 - 6n + 1n + 6})$$

$$\cancel{n^2 + 1n - 6n + 6}$$

$$\cancel{n(n+1) - 6(n+1)}$$

$$\cancel{(n+1)(n-6)}$$

$$\cancel{n = -1} \quad n = 6.$$



$$f(n) = 2n^3 - 15n^2 + 36n + 1$$

$$f'(n) = 6n^2 - 30n + 36$$

$$= 6(n-2)(n-3)$$

$$f(2) = 29$$

$$f(3) = 28$$

$f(2)\, f(3) > 0$

one Real Root

Que! find all possible value of paramet~~er~~ is so that eq has 3 real and distints roots

$$f(n) = n^3 - 3n + a = 0$$

$$f'(n) = 3n^2 - 3$$

$$n^2 = 1$$

$$n = \pm 1$$

$$f(1) = 1 - 3 + a = 0$$

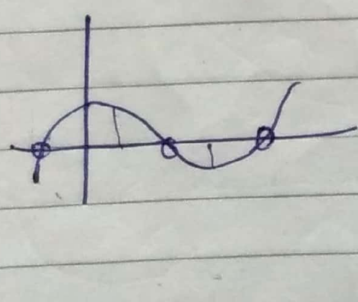
$$-2 + a = 0$$

$$a = +2$$

$$f(-1) = -1 + 3 + a = 0$$

$$2 + a = 0$$

$$a = -2$$



$$\Rightarrow -2 < n < 2.$$

Asimtote is a line tangent which touches to the curve at Infinity.

Point of Influction:

It is point where graph of f^n have tangent line and where Concavity changes is called Point of Influction.

For finding Point of Influction we will evaluate double derivative

At point of inflaction does not exist

$$\frac{d^2y}{dx^2} = 0$$

and y'' changes sign about point of Influction where tangent cross the line

Ex:

$f(x) = x^3$

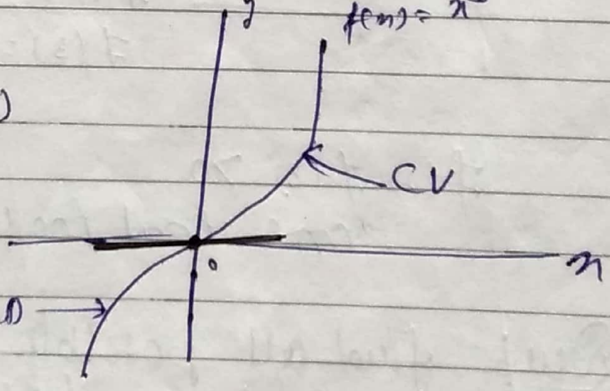
$f'(x) = 3x^2$

$f''(x) = 6(x - 0)$

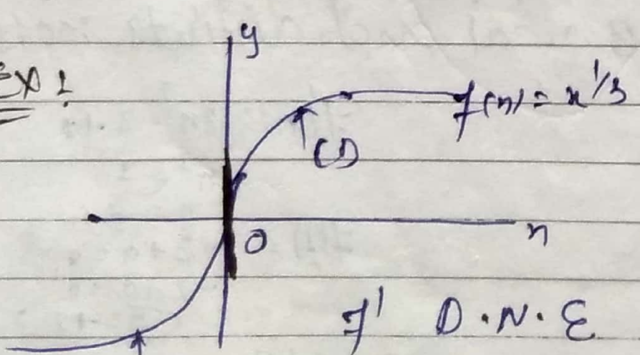
$f''(x) = 0$

$f''(0^-) = -ve$

$f''(0^+) = +ve$ (CV)

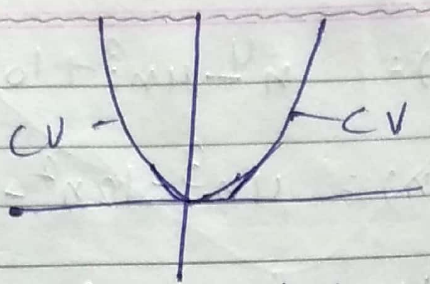


Ex:



f' D.N.E

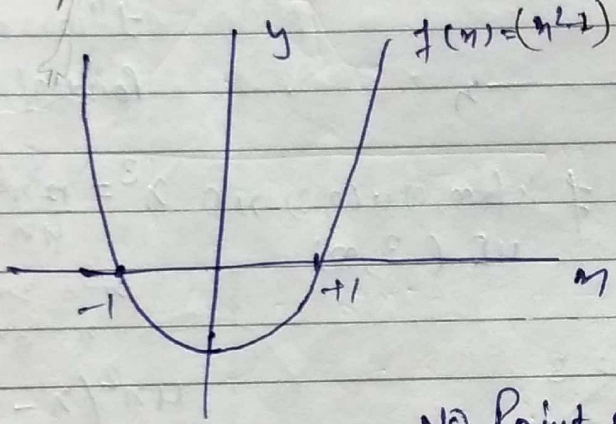
Ex: $f(x) = x^4$



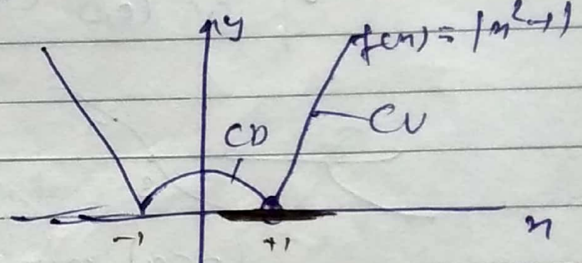
$f'(x) = 4x^3$
$f''(x) = 12x^2 = 0$
$x = 0$

~~not~~ 1
NO Point of Inflection

Ex: $f(x) = |x^2 - 1|$ at $x = \pm 1$



$x^2 = 1$
$x = \pm 1$



NO Point of I because No tangent cross the line.

Que: Stationary point: (~~Not Imp~~)

Points are domain where $f'(x) = 0$

Que: Interval of D/I, Point of Extr, Pt of Inflection

J.M = 10 – complete
J.A = ~~10 – 15~~.
11 – 13.

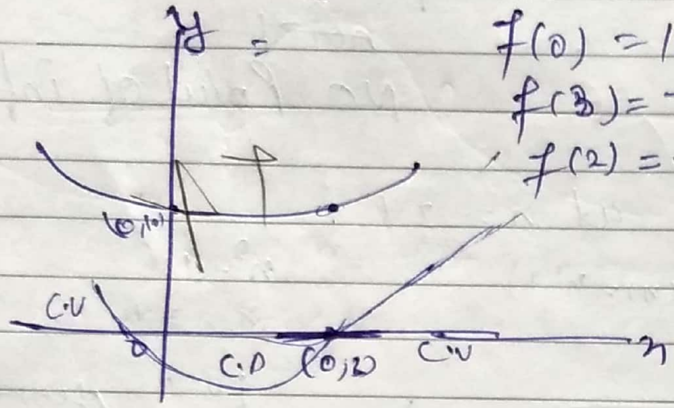
Revise. MI
~~class~~

Qus $f(x) = x^4 - 4x^3 + 10$

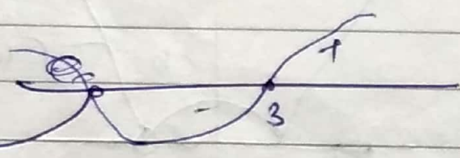
$f'(x) = 4x^3 - 12x^2$.

$f''(x) = 12x^2 - 24x$.
$x^2 = \frac{24x}{12}$
$x^2 = 2x$
$x = 2$

$f''(2) = 48 - 48$
$= 0$.
$f''(2) = 0$
Point of I = 2.



$f(0) = 10$
$f(3) = -17$
$f(2) = -6$



f is $(-\infty, 0) \cup (0, 3)$ MD
MI $(3, \infty)$

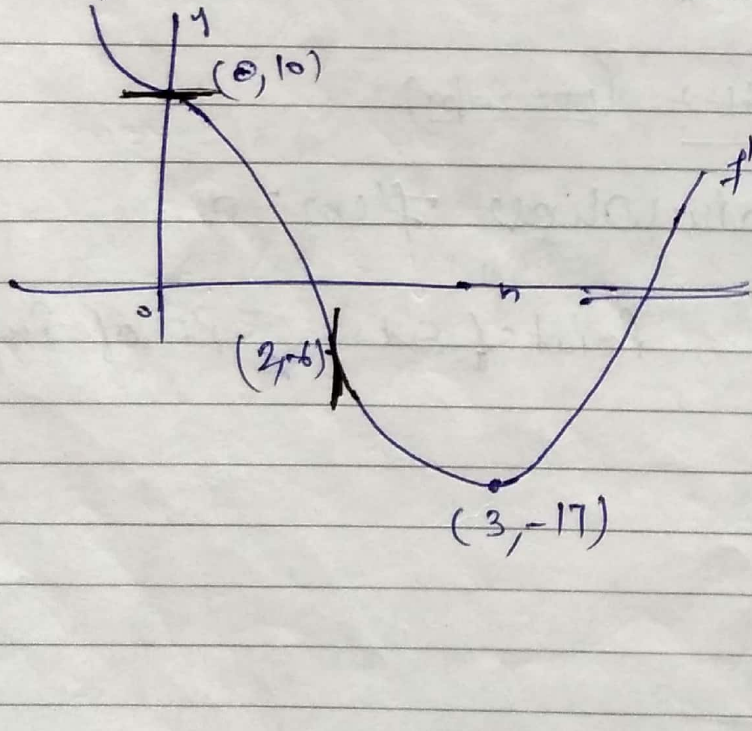
$x^3 = \frac{12x^2}{4x}$
$x^3 = 3x$.
$x = 3x^{1/3}$

Local min $x = 3$
Point of I = $x = 2$.
N.max, min at $x = 0$

$4x^2(x-3)$
$x = 3$

27
4
108

$f'(x) = 108 - 10$
$f'(x) = 0$.

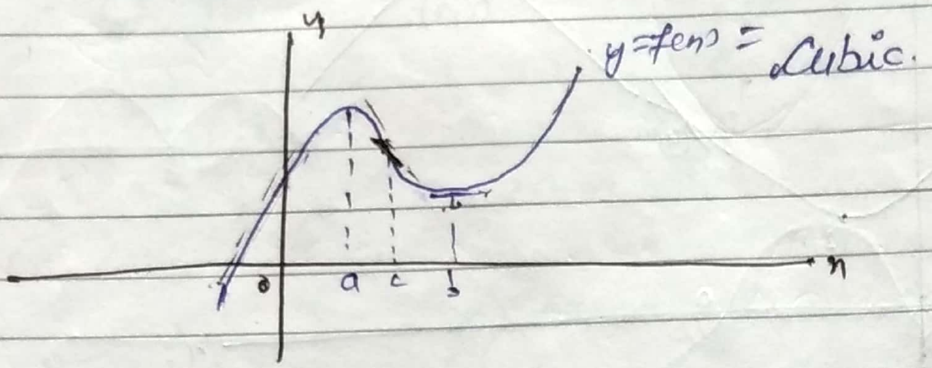


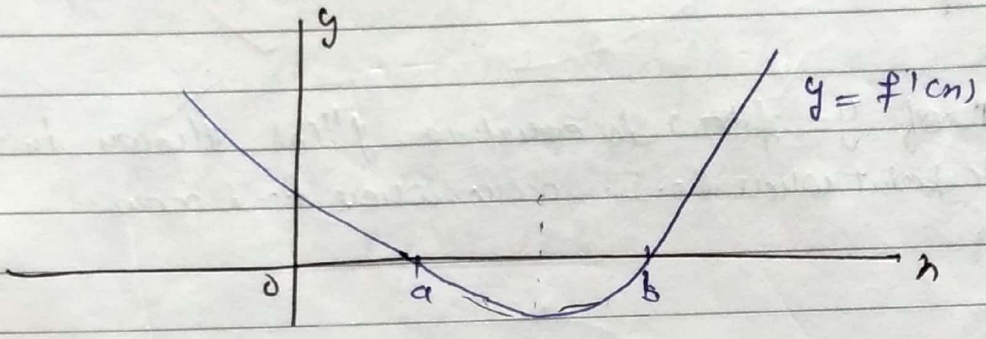
$f'(x) = 0$

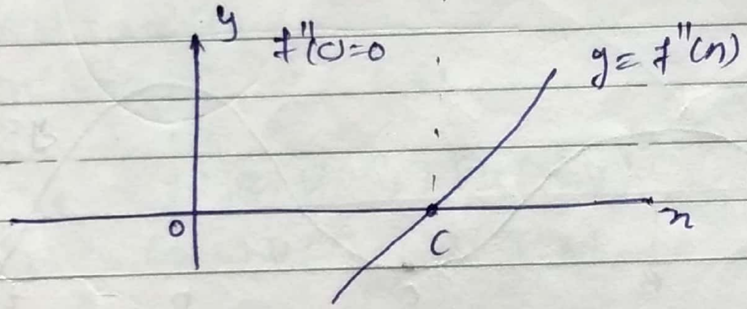
$= x = 1, y = 4$
=

02/08/17

*

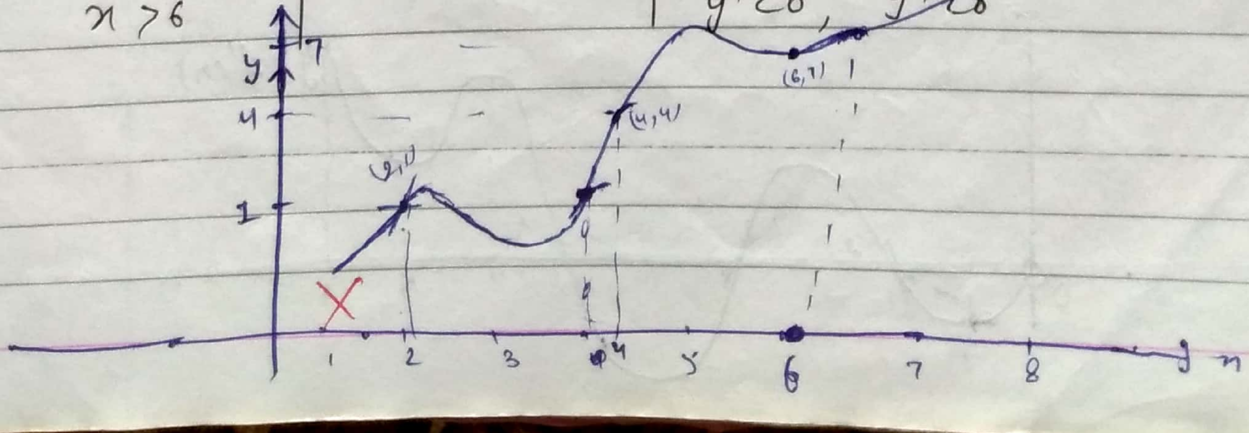




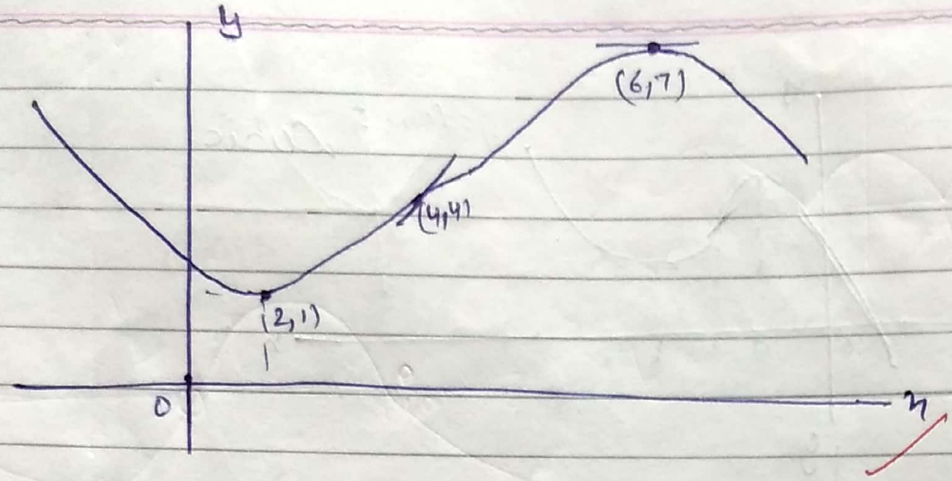


Q1 Draw y = f(x) given that

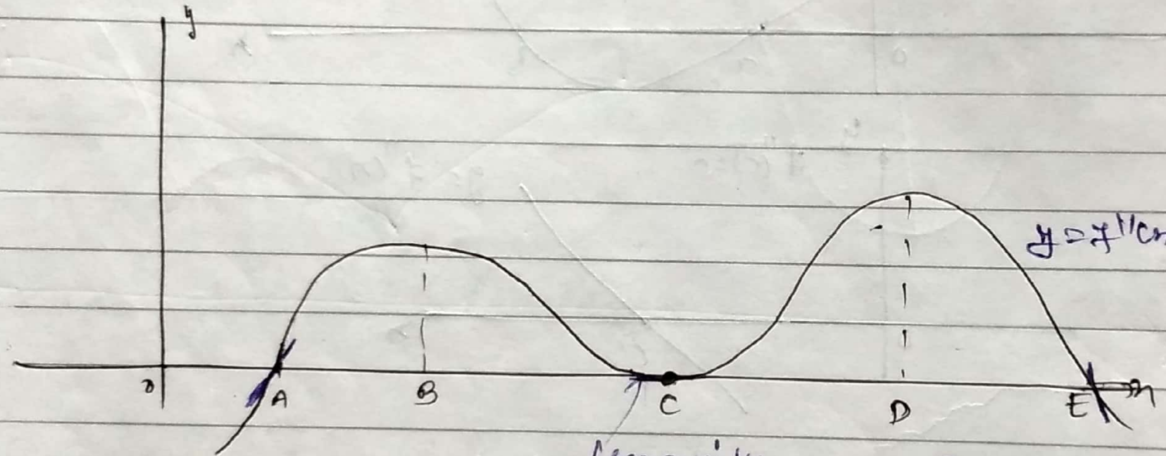
x	y	Derivative
$x<2$		$y'<0$, $y''>0$ d.f" concave up
$x=2$	1	$y'=0$, $y''>0$
$2<x<4$		$y'>0$, $y''>0$
$x=4$	4	$y'>0$, $y''=0$
$4<x<6$		$y'>0$, $y''<0$
$x=6$	7	$y'=0$, $y''<0$
$x>6$		$y'<0$, $y''<0$



Ans:



Q. The Graph of $y = f(x)$ is equal to $f''(x)$ shown in the figure. Locate point where P.o.I. occur where P.o.f occur



Concavity not change

$x \Rightarrow A, \cancel{C}, E$

P.o.f ~~A~~, E

~~f is max at x = B, D.~~

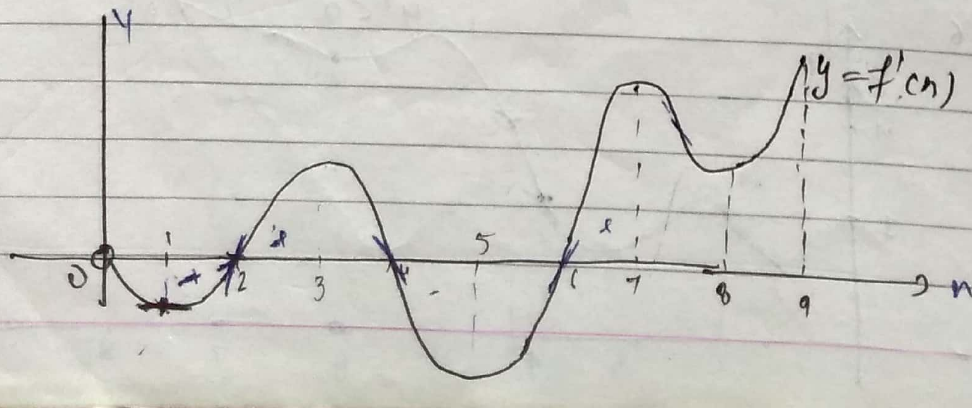
~~min at x = C.~~

$f''(A) < 0 \Rightarrow y = f(x)$ show pt of Inflection

$f''(A) > 0$

$f''(A) = 0$

Q. Graph of $y = f'(x)$ is shown below.



Now the following,

Decrease : [illegible]

(1) On what interval $y = f(x)$ is D / Increase : [illegible]

(2) at what value of x where local max = [illegible]

local min occur = [illegible]

(3) On what interval is concave up or = [illegible]

concave down = [illegible]

(4) at what value of x =

$y = f(x)$ shows point of inflection = 2, 4, 6.

(1) $y = f(x)$ = $D = (0,2) \cup (4,6)$

$I = (2,4) \cup (6,9)$

(ii) max $x = 4$

minima = $x = 2, 6$

(iii) $CU = (1,3) \cup (5,7) \cup (8,9)$

$CD = (0,1) \cup (3,5] \cup (7,8)$

(iv) $x = 1, 3, 5, 7, 8$

Cont. Dont Break
Diff = No Sharp Corner
No diff = Sharp Corner

Q. Draw $y = f(x)$ using following Information.

(i) f is Cont & define $\forall x$.

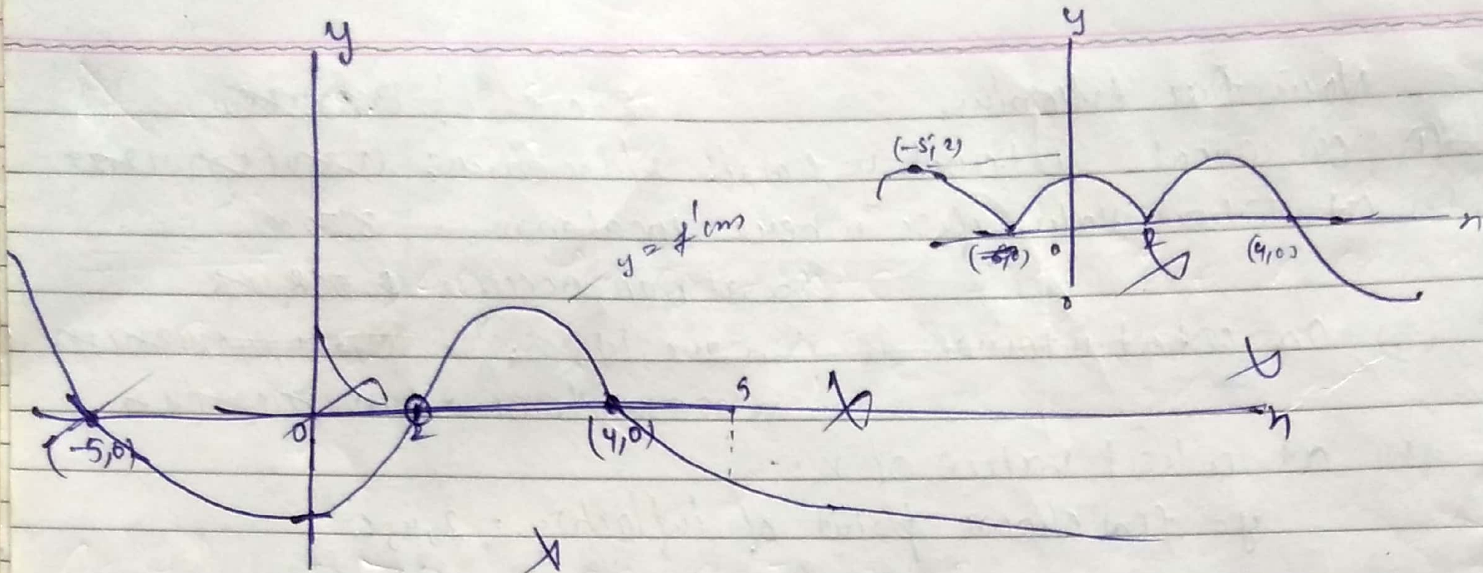
(ii) $f'(-5) = 0$, $f'(2)$ = Not define $f'(4) = 0$

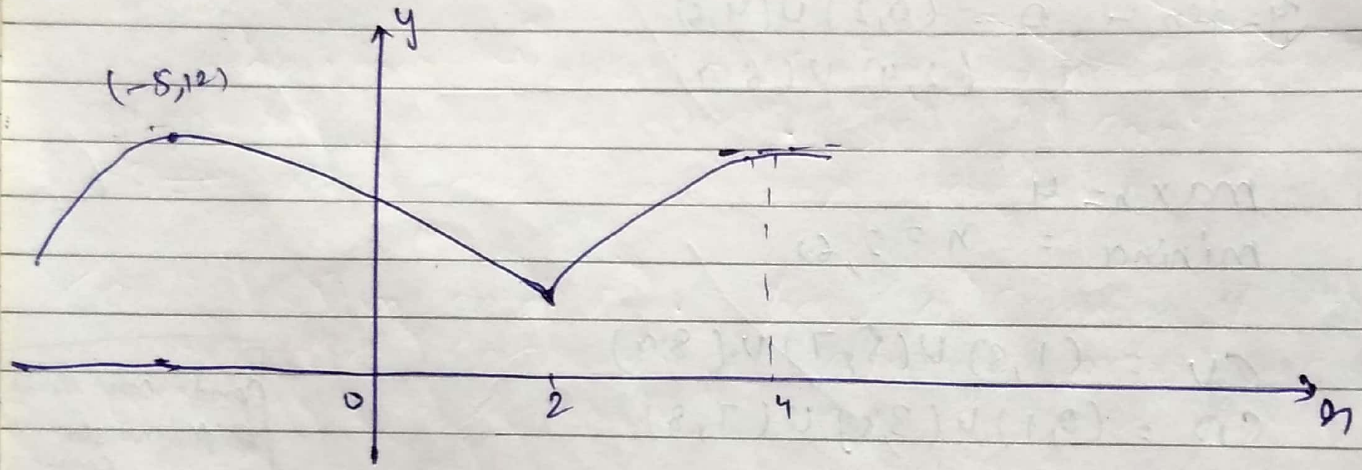
(iii) $f(-5) = 12$

(iv) $f''(2)$ = Not define but $-ve$ every where

(v) $f'(x) < 0$ in $(-5,2) \cup (4,\infty)$

$f'(x) > 0$ $(-\infty,-5] \cup (2,4)$





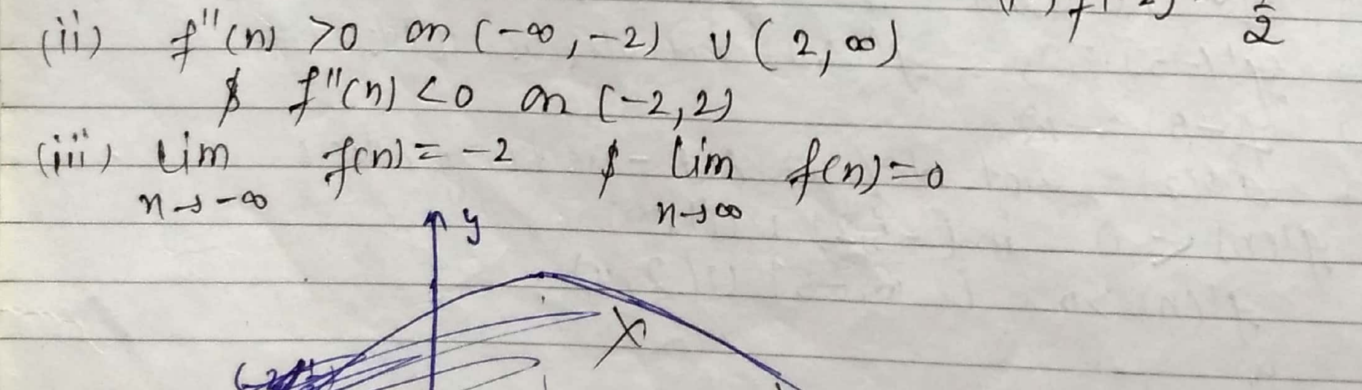
Q. Draw $y = f(x)$ satisfy following condition.

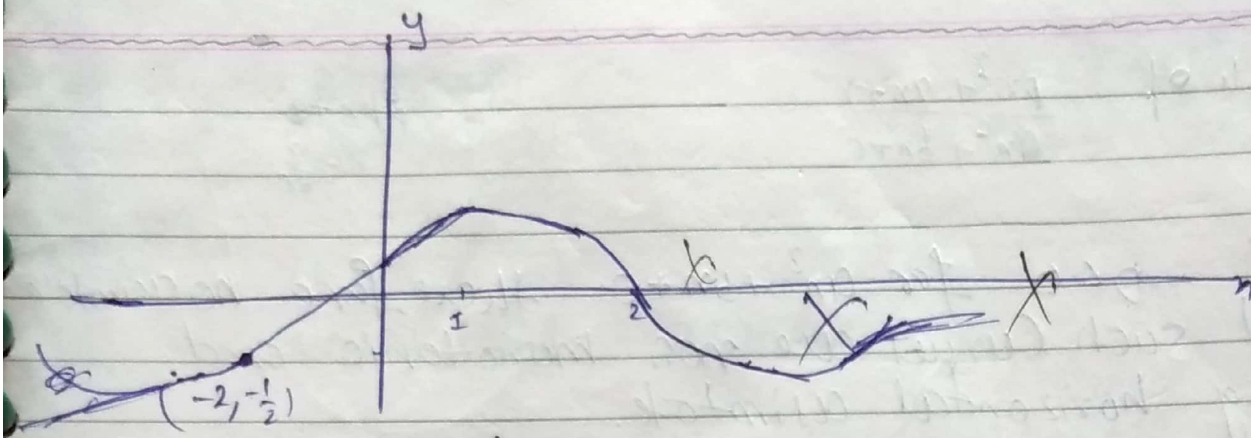
(i) $f'(x) > 0$ on $(-\infty, 1)$
$f'(x) < 0$ on $(1, \infty)$

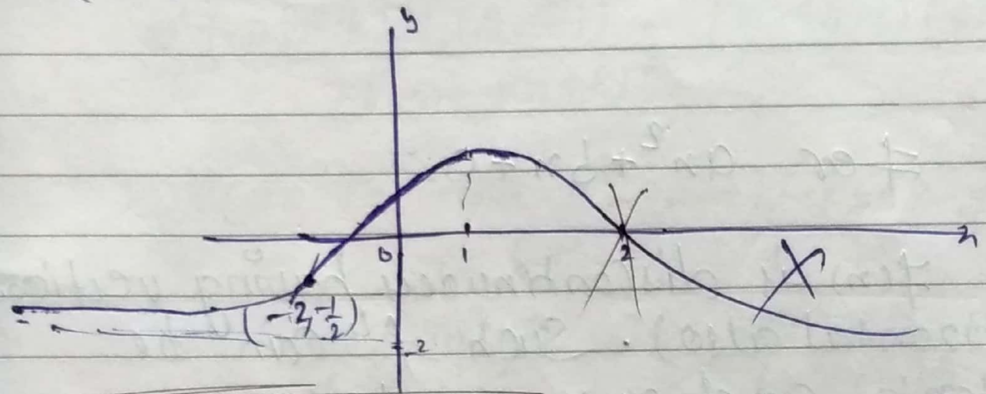
(iv) $f(-2) = \frac{1}{2}$

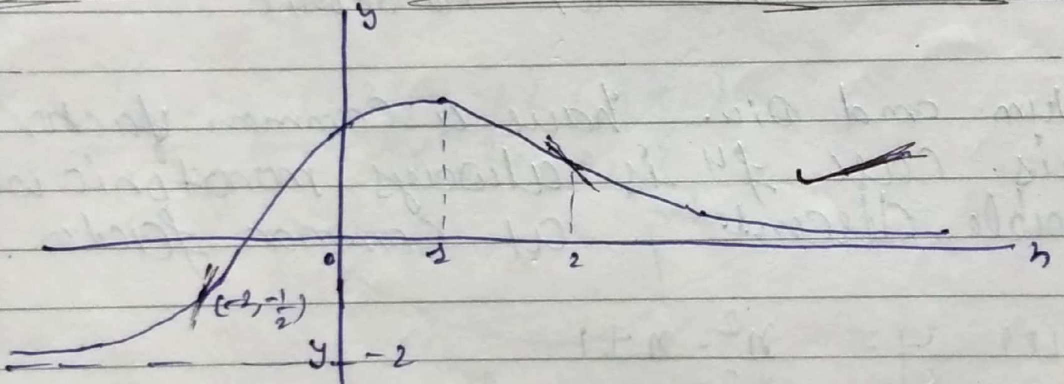
(ii) $f''(x) > 0$ on $(-\infty, -2) \cup (2, \infty)$
& $f''(x) < 0$ on $(-2, 2)$

(iii) $\lim_{x \to -\infty} f(x) = -2$ & $\lim_{x \to \infty} f(x) = 0$









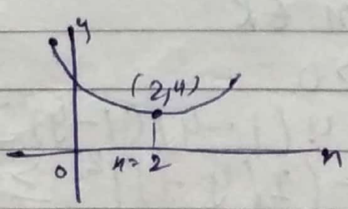
Que: y is ~~diff~~ multiple diff. f^n $y = f(x)$ such that $f'(2) = f''(2) = f'''(2) = f''''(2) = 0$ & min value of $f(x)$, $f(x) = 4$ at $x = 2$ then find $f(4) = ?$

Ans

$f'(2) = 0$

~~$f'(2) = k$~~

$f(4) = 4$



$f'(2) = 0$

~~$f(4) = 2$~~

~~$f(2) = 1$~~

$= f' = 4$

$f(x) = 4x$

$f'''''(2) \neq 0$

$= +ve$

$$\boxed{f(x) = (x-2)^6 + 4}$$

~~A~~ Graph of $\frac{px^2+qx+r}{ax^2+bx+c}$ $2px+q$, $x=\frac{q}{2p}$

Case 1:

$D<0$ for ax^2+bx+c therefore no asimtote and such curves are non monotonic and having horizontal asimtote.

(C-2) $D>0$ for ax^2+bx+c

therefore $f(x)$ is discontinuous having vertical asimtote (horizontal also). Such f^n can be ~~non~~ monotonic and non monotonic

Case 3 Num. and Din. have a common factor in this case f^n is always monotonic with removable discont at common factor

Que: Draw $y=\frac{x^2-x+1}{x^2+x+1}$

$$x^2-x+1=yx^2+yx+y$$
$$x^2(1-y)-x(1+y)+(1-y)=0$$
$$\because x\in R$$
$$D\geq 0$$
$$(1+y)^2-4(1-y)(1-y)\geq 0$$
$$(1+y)^2-[2(y-1)]^2\geq 0$$
$$[1+y+2(y-1)][1+y-2(y-1)]\geq 0$$
$$(3y-1)(-y+3)\geq 0$$
$$(3y-1)(y-3)\leq 0$$
$$\frac{1}{3}\leq y\leq 3$$

Complete JM and JA

(2) $y = \dfrac{\cancel{x^2-2x-1}}{\cancel{(x+1)}(x-2)}$ (3) $y = \dfrac{(\cancel{x-1})(\cancel{x-2})}{(\cancel{x-2})(x+3)}$

(3)

$=$ $D < 0$

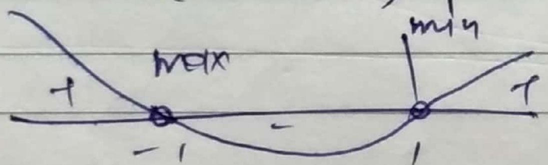
NO verticle asimtote.

$$y' = \frac{(x^2+x+1)(2x-1) - (x^2-x+1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{(2x^3+2x^2+2x-x^2-x-1) - (2x^3-2x^2+2x+x^2-x+1)}{(x^2+x+1)^2}$$

$$y' \; \frac{4x^2-2x^2-2}{(x^2+x+1)^2} = \frac{2x^2-2}{(x^2+x+1)^2}$$

$$\frac{2(x-1)(x+1)}{(x^2+x+1)^2}$$

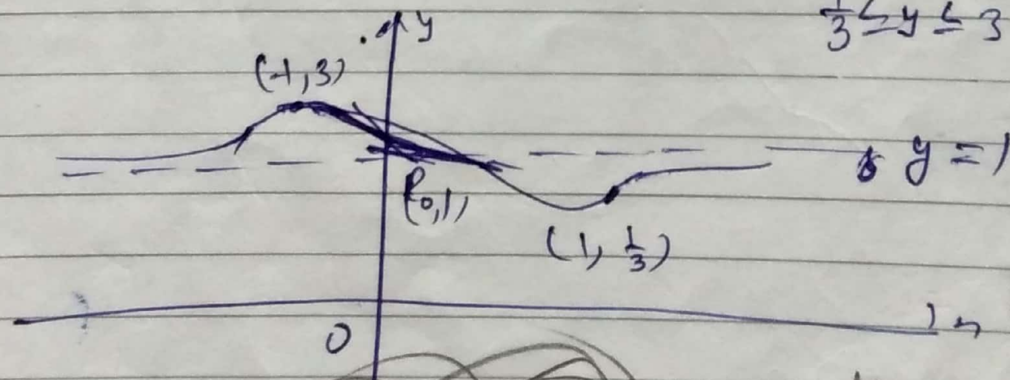


f is MD $(-1, 1)$

f is MI in

$y(x=-1)$
$= 1+1+1$
$= 3$

$\frac{1}{3} \le y \le 3$



$\int \frac{d}{dx} \ne 1$

$x^4 = 3x^3$

x^4

$4x^3$

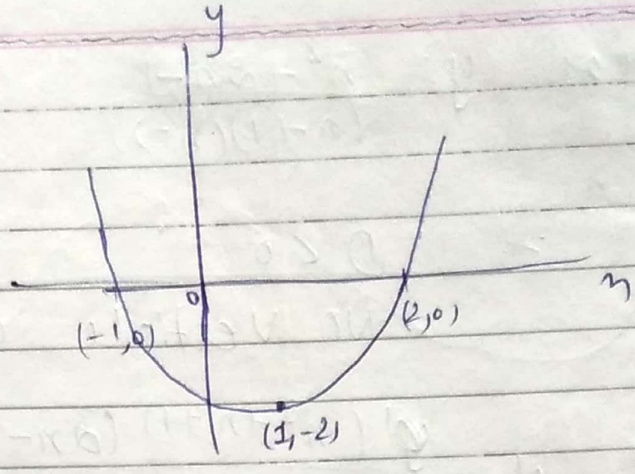
$x^3 = \frac{x^4}{4}$

$4x^3$

$4x^4$

$-x^4$

Q.2 $\dfrac{x^2-2x-1}{(x+1)(x-2)}$



Q.3 $y = \dfrac{(x-1)(x-2)}{(x-2)\ (x+3)}$