22/08/17

MATRICES

* multiplication

(1) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}_{2\times 2} \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 6 & 1 \\ -10 & -9 \end{bmatrix}_{2\times 2}$

Que:

(2) $\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}_{2\times 3}$

$= \begin{bmatrix} -1 & 4 & 5 \\ 9 & 8 & -1 \end{bmatrix}_{2\times 3}$

* **Properties:**

if $AB = C$ where A, B are sq. matrix,

$\det. A \cdot \det. B = \det C$ or $|A||B| = |C|$

Q. Let $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$

$B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$

if $(A+B)^2 = A^2 + B^2$ find a, and b

$\therefore (A+B)^2 = A^2 + B^2$ $\qquad AB = -BA$.

$AB = \begin{bmatrix} \cancel{1} & -1 \\ 2 & -1 \end{bmatrix} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$= \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix}$.

$=$

$BA = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} a+2 & -a+1 \\ b-2 & -b+1 \end{bmatrix}$

$\begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} = -\begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix}$

$= \begin{bmatrix} -a-2 & a+1 \\ -b+2 & b-1 \end{bmatrix}$

Q. if $X_{m\times n} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$

then find X.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

let $X = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}_{2\times 3}$$

$= [$

let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$n = 2 \qquad m = 2$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & \end{bmatrix} = \begin{bmatrix} & & \end{bmatrix}$$

$a + 4b = -7$

$2a + 5b = -8$

$c + 4d = 2$

$2c + 1d = 4.$

AB+BA = Sym
AB−BA = Skew

Q:1 1, 2, 3, 4, 5, 6, 7, 8, 9
10, 13, 14,
S-1 = 1, 2, 3, ●,

Q. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ =

$4+3 = 7$

$f(x) = x^2 - 4x + 7$

then show that $f(A) = 0$

$= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$f(A) \Rightarrow A^2 - 4A + 7$

$$\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7I$$

$$\begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{-6} & \cancel{-12} \\ 0 & \cancel{-7} \end{bmatrix} \quad \begin{bmatrix} \cancel{0} & \cancel{12} \\ \cancel{0} & \cancel{0} \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{null}.$$

Q. Expressed as sum of two matrix. one symmetric and other one is skew sym.

$$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \quad A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\frac{1}{2}(A + A') = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\frac{1}{2}(A - A') = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$A = \frac{A + A'}{2} + \frac{A - A'}{2}$$

A. J.M and S.A.

Q. If Matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$

is sym matrix then find x, y, z, t

$AA' = I$

$\frac{A + A'}{2} = A + A'$

$A = \begin{bmatrix} 5 & 2 & x \\ y & 2 & -3 \\ 4 & t & -7 \end{bmatrix} + \begin{bmatrix} 5 & y & 4 \\ 2 & z & t \\ x & -3 & -7 \end{bmatrix}$

$= \frac{1}{2}\begin{bmatrix} 10 & 2+y & x+4 \\ y+2 & 2z & -3+t \\ 4+x & t-3 & -14 \end{bmatrix}$

$= x, y, z, t$

$x \to 4$, $y \to 2$, $z \to 1$, $t \to -3$

5

9

Q. if $|A| = 2$ where A is sq matrix of order 3.
then find ~~ag~~. adj A

(i) $|\text{adj } A| = 2^{3-1} = 2^2 = 4$
(ii) $|\text{adj adj } A| = 2^{(3-1)^2} = 2^{(2)^2} = 2^4 = 16$
(iii) $|\text{adj adj adj } A| = 2^{(3-1)^3} = 2^{(2)^3} = 2^8 =$

Que! Construct 2×3 matrix

$a_{ij} = \left|\frac{i-2j}{3}\right| =$

$= \left| \cancel{a_{11} = \frac{-1}{3} \quad a_{12} = \frac{-3}{3} = -1 \quad a_{13} = \frac{-5}{3}} \quad a_{21} \right.$

$= \begin{vmatrix} -\frac{1}{3} & -1 & -5/3 \\ 0 & -\frac{2}{3} & -4/3 \\ \frac{1}{3} & \frac{1}{3} & \frac{-3}{3} = -1 \end{vmatrix} = \begin{pmatrix} \frac{1}{2} & 3/2 & 5/2 \\ 0 & 1 & 2 \end{pmatrix}$

Q. for $\theta = \frac{3\pi}{5}$

let $B = [b_{ij}]$ B is a sq. matrix of order 2
such that

$b_{ij} = \begin{cases} \cos\theta & i=j \\ \cos j\left(\frac{\pi}{2}+\theta\right) & i>j \\ \sin\left(\frac{j\pi}{2}-\theta\right) & i<j \end{cases}$

$i > 2$
a_{12}
$2 \quad 1$
$i < 2$

then find $Tr(B)^8$

$A =$

$$B = \begin{bmatrix} \cos\theta & \\ & \cos\theta & \\ & & \cos\theta \end{bmatrix}$$

$$B = \begin{bmatrix} \cos\theta & \cancel{\cos\left(\frac{j\pi}{2}+\theta\right)} \ \sin\left(\frac{j\pi}{2}-\theta\right) \\ \cos\left(\frac{j\pi}{2}+\theta\right) & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\frac{3\pi}{5} & \sin\left(\frac{\pi}{2}-\theta\right) \\ \cos\left(\frac{\pi}{2}+\theta\right) & \cos\frac{3\pi}{5} \end{bmatrix}$$

$\cos$
$\pi - \frac{2\pi}{5}$
$\pi - \frac{2\pi}{5}$

$$= \begin{bmatrix} \cos\frac{2\pi}{5} & \sin\frac{3\pi}{5} \\ -\sin\frac{3\pi}{5} & \cos\frac{3\pi}{5} \end{bmatrix}$$

$$= \cos\frac{2\pi}{5} \times \cos\frac{3\pi}{5}$$

$$B^2 = B \cdot B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$B^5 = \begin{bmatrix} \cos 3\theta & \sin 5\theta \\ -\sin 5\theta & \cos 5\theta \end{bmatrix}$$

$$Tr(B^5) = 2\cos 5\theta$$
$$= 2 \cdot \cos 5 \frac{3\pi}{5}$$
$$= 2\cos(3\pi)$$
$$= -2.$$

Que:- Solve

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & -1 \\ 0 & 4 \end{bmatrix}$$

$$2A - B + 3I =$$

$$\begin{bmatrix} 2 & 4 \\ -2 & 6 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} =$$

$$\begin{bmatrix} 7 & 5 \\ -2 & 5 \end{bmatrix} =$$

Que:- Solve the eqn. $[x \quad 2y \quad 3z)$

$[x \quad 2y$

$[x \quad 2y \quad 3z] - 2[y \quad z \quad -x] + 3[-z \quad x \quad y]$

$= [-12 \quad 1 \quad 17]$

$[x \quad 2y \quad 3z] - [2y \quad 2z \quad +2x] + [-3z \quad 3x \quad 3y]$

$$\left[\begin{array}{l} x - 2y - 3z = -12 \quad -i, \\ 2y - 2z + 3x = 1 \\ 3z - 2x + 3y = 17. \end{array}\right]$$

$x - 2y - 3z = -12$

$3x + 2y - 2z = 1$

$= 4x - 5z = -11 \quad -i.$

$-2x + 3y + 3z = 17$

$D_1 =$

D, D_1, D_2, D_3

$x = \frac{D_1}{D}.$

Q. A matrix has 12 elements. find no. of possible order it can have

1×12
2×6
3×4
~~6×2~~
4×3
6×2
12×1

= 6

Q. if $A + 2B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

and $2A - B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 5 & 7 \\ 4 & -1 & 1 \end{bmatrix}$ ×2

then find ~~trace A and~~

$3Tr(A) + 5Tr(B) \cdot 2$

~~$A + 2B =$~~ $A + 2B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

$4A - 2B = \begin{bmatrix} 4 & 0 & 2 \\ 6 & 10 & 14 \\ 8 & -2 & 2 \end{bmatrix}$

$= 5A = \begin{bmatrix} 5 & 2 & 5 \\ 5 & 14 & 19 \\ 8 & -2 & 3 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2/5 & 1 \\ 1 & 14/5 & 19/5 \\ 8/5 & -2/5 & 3/5 \end{bmatrix}$.

$2A - B = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 5 & 7 \\ 4 & -1 & 1 \end{bmatrix}$

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Ans: $Tr(A+2B) = Tr\begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

$Tr A + 2\, Tr B = 6$. — (i)

$Tr(2A - B) = Tr\begin{pmatrix} 2 & 0 & 1 \\ 3 & 5 & 7 \\ 4 & -1 & 1 \end{pmatrix}$

$2x - y = 8$ — (ii)

$x + 2y = 6$ $\qquad$ $4x - 2y = 16$

$5x = 22$

$x = 22/5$

$y = 2x - 8 = 2 \cdot \frac{22}{5} - 8 = \frac{44 - 40}{5} = \frac{4}{5}$

Q.

$\alpha, \beta, \gamma \in R$ and

$A = \begin{bmatrix} \alpha^2 & 6 & 8 \\ 3 & \beta^2 & 9 \\ 4 & 5 & \gamma^2 \end{bmatrix}$ $\qquad$ $B = \begin{bmatrix} 2\alpha & 3 & 5 \\ 2 & 2\beta & 6 \\ 1 & 4 & 2\gamma - 3 \end{bmatrix}$

if $Tr(A) = Tr(B)$ then find $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

$\{ \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 2 \}$

$Tr(A) = \alpha^2 + \beta^2 + \gamma^2$ $\qquad$ $Tr(B) = 2\alpha + 2\beta + 2\gamma - 3$

$\alpha^2 + \beta^2 + \gamma^2 = 2\alpha + 2\beta + 2\gamma - 3$

m > order

$\alpha^2 + \beta^2 + \gamma^2 = 2\alpha + 2\beta + 2\gamma - 3$

$\alpha^2 - 2\alpha + \beta^2 - 2\beta + \gamma^2 - 2\gamma = -3$

$\alpha^2 - 2\alpha + 1 + \beta^2 - 2\beta + 1 + \gamma^2 - 2\gamma + 1 = 0$

$(\alpha - 1)^2 + (\beta - 1)^2 + (\gamma - 1)^2 = 0$

$\alpha = 1,\ \beta - 1 = 0,\ \gamma - 1 = 0$

$\alpha = 1 = \beta = \gamma$

$A^2 = A$

Q. if A is an idempotent non zero matrix and I is an identity matrix of same order and find $n \in N$ such that

$(A + I)^n = I + 127A$

$(A + I)^n = {}^nC_0 A^n + {}^nC_1 A^{n-1} + \dots {}^nC_n A^n$

$(A + I)^n = I + 127$

$I + (2^n - 1)A =$

$2^n - 1 = 127$

$2^n = 127$

$= 2^7$

$n = 7$ Ans

$2^n - 1 = 127$

$^nC_0 = 1$

$^nC_1 = n$.

Q. (i) Show that $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ can be decomposed as sum of a unit and a nilpotent matrix.

(ii) and hence find the value of $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{2007}$ ①.

$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} =$ $A + A' =$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} =$

$A = I + M$

$M^2 = \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = I$.

m is nilpotent.

$= \cancel{\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{2007}} = \cancel{(1)^{2007}} = \cancel{A}$ $\cancel{\begin{bmatrix} 1^{2007} & 0^{2007} \\ 2^{2007} & 1^{2007} \end{bmatrix}}$ $= \cancel{\begin{bmatrix} 1 & 0 \\ 2^{2007} & 1 \end{bmatrix}}$

$A^{2007} = (I + m)^{2007}$

$= {}^{2007}C_0 \, I^{2007} + {}^{2007}C_1 \, I^{2006} M + {}^{2007}C_2 \, I^{2005} M^2 \ldots$ — high power o/m

$= 1 \cdot I + 2007$

$A^{207} = I + 2007M$.

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 4014 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4014 & 1 \end{bmatrix}$

Q. A, B, C are given Sq. matrix such that

$AB = O$ & $BC = I$

then Proof that $(A+B)^2 (A+C)^2 = I$

$AB = 0$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$BC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C(AB) =$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$ABC = OC = O$

$A(BC) = 0$ (Associative Property).

$AI = 0$

$\boxed{A = 0}$

LHS: $(0+B)^2 (0+C)^2$

$= (BC)^2 = I^2 = I$ Wrong

$= BBCC$

$= B(BC)C$

$= BIC$

$= (BI)C$

$= BC = I$ An

~~O-1~~ : 11, 12, 13, 14, 17, 18, 19, 20

Q-I : 4, 5, 6, 9, 11, 12, 13, 15, 17,

J.M : 1, 2, 3, 4, 5, 6.

Que: A is sq. matrix of order 3. then value of

$|(A - A')^{2017}| = 0$ T/F.

True.

order is odd

$(A - A')^{2018} = 0$ $|A - A'||(A - A')| \cdots |(A - A')| = 0$

② Use $\therefore |A| \neq 0$

Que: if $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ then P.T $5A^{-1} = A^2 + A - 5I$.

$|A - \lambda I| = 0$

$= (A - \lambda I) = 0$

$= \left(\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right) = 0$

$1 - \lambda = 0$
$\lambda = 1$

$\begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & -1-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix}$

$= \cancel{1 - \lambda\{(-1-\lambda) - (-1-\lambda)\}}$

$= \cancel{1 - \lambda(-1 - \lambda + 1 + \lambda)}$

$= \lambda^3 + \lambda^2 - 5\lambda - 5 = 0$ satisfied by matrix A^3

$$(A^{-1}A)A + (A^{-1}A) - 5(A^{-1}A) - 5(A^{-1}I) = 0$$

$$A^2 + A - 5I - 5A^{-1} = 0$$

* $\alpha, \beta > 0$

$AM = \frac{\alpha + \beta}{2}$

$GM = (\alpha\beta)^{1/2}$

$\frac{2}{H} = \frac{1}{\alpha} + \frac{1}{\beta}$

$A \geq G \geq H$

* $\alpha, \beta, \gamma > 0$

$A = \frac{\alpha + \beta + \gamma}{3}$

$G = (\alpha\beta\gamma)^{1/3}$

$\frac{3}{H} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$A \geq G \geq H$

If any two mean (it means $\alpha = \beta (= \gamma)$) same

Q. if A and B are sq. matrix of order 3 such that $|A| = -1$

$|B| = 4$

then find $|3AB|$

$= 3|AB|$

$= |3AB| = \cancel{3|AB|} \cdot 3|A| \times |B| = 3 \times (-1) \times 4$

$= \cancel{-12}$

$|3A| \cdot |B|$

$= 3^3 |A| |B|$

$= 3^3 (-1) 4$

$= -27 \times 4$

$$\boxed{|KA| = K^n |A|}$$

A, B, and C are 3rd order de

$|A| = -1, \quad |B| = 4, \quad |C| = 2$

then find $|3AB^2C^{-1}| =$

$|3A| \; |B^2| \; |C^{-1}| =$

$3^3 |A| \, |B^2| \, |C^{-1}|$

$= 27 |A| \, |B|^2 \, |\frac{1}{C}|$

$= 27 \times -1 \times 4^2 \times \frac{1}{2}$

$$\boxed{C^{-1} = \frac{1}{|C|}}$$

Qu: if Diag. element Diag(α, β, γ) of a non singular diag m. of order 3 are root of the eq.

$$x^3 - 9x^2 + kx - 27 = 0 \qquad k \in R.$$

where $\alpha, \beta, \gamma > 0$. then find such Matrix.

Ans

$= (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 9$

$\alpha\beta + \beta\gamma + \gamma\alpha = k$

$\alpha\beta\gamma = 27$

$A = \frac{\alpha + \beta + \gamma}{3} = 3$

$G = (\alpha\beta\gamma)^{1/3}$

$= (27)^{1/3} = 3$

$\because A = G$

$\alpha = \beta = \gamma = 3$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\begin{vmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{vmatrix} \neq 0.$

$x^3 - 9x^2 + kx - 27.$

$x^3 + 9x^2 + kx = 27$

$x(x^2 - 9x + k) = 27.$

$x(x^2 - 3x - 6x + k$

$AM = \frac{\alpha + \beta}{2},$

$\Rightarrow A = \frac{\alpha + \beta + \gamma}{3}$

Q. A is sq. mat orde ≤ 4.

Q. A is sq. matrix of order ≤ 4
Such that $|A - A'| \neq 0$
and $B = \text{adj } A$.
if $|A| = 3$ then find $Tr(\text{adj}(AB))$

$$B = \underline{\text{adj } A} \qquad |A| = 3.$$

$$Tr(\text{adj}(AB)) = Tr((\text{adj } B)(\text{adj } A)).$$

$$\cancel{Tr((B))} \; Tr((\text{adj } B)(B)).$$

$$Tr((\text{adj } B)(\text{adj } B)).$$

$$Tr((\text{adj } B)^2) = |B| I_n^2$$
$$= 3^2 = 9.$$

Sol: $A = 2$
$n = 2$.

$|A| = 3$
$Tr(\text{adj}(AB))$

$= Tr(\text{adj}(A \text{ adj } A))$
$= Tr \text{ adj}(|A| I)$
$= Tr(|A|^{n-1} \text{adj } I)$
$= Tr(3I) = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 6.$

$$\boxed{\text{adj}(kA) = k^{n-1}(\text{adj } A)}$$

J.M = 7, 8, 10, 11, 12, 13, 14, 15, 16,

J.A ⇒ 1, 4(B), 8, ~~9~~, 10, 15, 16.

Q4 skew = ( )'.

matrix

Q. C is skew symmetric matrix of order 3.
X is 3×1 column matrix
then Proof that

$X'CX$ is singular

$|\ | = 0$

C = skew sym.

$X = 3\times1$

$X'CX = 0.$

P.T $\underset{1\times3}{X'}\ \underset{3\times3}{C}\ \underset{3\times1}{X} = [K]_{1\times1}$

$(X'CX)' = [K]$

$X'C'(X')' = [K]$

$X'(-C)X = [K]$

$X'CX = [K]$

$-[K] = [K]$

$[-K] = (K)$

$-K = K$

$K = 0$

Q. Sol

$$x + 2y + 3z = 2$$
$$2x + 4y + 5z = 3$$
$$3x + 5y + 6z = 4$$

~~A + XB~~ $\quad AX = B.$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \quad \Big[\; = \begin{matrix} -1 - 2(-3) + 3(-2) \\ -1 + 6 - 6 = -1 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\boxed{\begin{matrix} x + 2y + 3z = 2 \\ 2x + 4y + 5z = 3 \\ 3x + 5y + 6z = 4 \end{matrix}}$$

$$\boxed{X = A^{-1}B}$$

A (adj) =

$$Adj\,A = \begin{bmatrix} -1 & +3 & -2 \\ +3 & -3 & +1 \\ -2 & +1 & 0 \end{bmatrix}$$

$$= \quad A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 3 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$X = -\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} +1 \\ -3 \\ 1 \end{bmatrix}$$

$x = -1$
$y = -3$
$z = 1$

Determinant

J.M

Q.6

$x + \alpha y + \alpha^2 z = 1$

$\alpha x + y + \alpha z = -1$

$\alpha^2 x + \alpha y + z = 1.$

$$D = \begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$= x + \cancel{\alpha y} + \alpha^2 \quad D = (\alpha^2 - 1)^2 = 0 \qquad \alpha \Rightarrow \pm 1$

$x+$

$$D_1 = \begin{vmatrix} 1 & \alpha & \alpha^2 \\ -1 & 1 & \alpha \\ 1 & \alpha & 1 \end{vmatrix} = \cancel{0}\, 1 - \alpha^2 - \alpha(-1$$

$= \alpha^3 + \alpha^2 - \alpha - 1$

$\alpha = 1, \; -1 + 1 + 1 - 1 = 0$

$$D_2 = \begin{vmatrix} 1 & 1 & \alpha^2 \\ \alpha & -1 & \alpha \\ \alpha^2 & 1 & 1 \end{vmatrix} = 2\alpha^3 + \alpha^2 - 2\alpha - 1$$

$\alpha = -1, \; \cancel{\quad\quad} = 0$

$-2 + 1 + 2 - 1 = 0$

$$D_3 = \begin{vmatrix} 1 & \alpha & 1 \\ \alpha & 1 & -1 \\ \alpha^2 & \alpha & 1 \end{vmatrix} = (1+\alpha)^2 (1-\alpha)$$

Q. A Sq. matrix of order 4. such that $|A| = 2$
then find $|Adj\ A|$

(i) $|Adj\ A| = 2^{4-1} = 2^3$

(ii) $|adj(adj\ A)| = 2^{(4-1)^2}$

(iii) $|adj\ adj\ (adj\ A)| = 2^{(4-1)^3}$

Q. find $|adj(K I_n)| = |K^{n-1} adj I|$

$= |\cancel{K^{-1}}\ \cancel{adj\ I_n}|$

$=$

$\cancel{|K^{n-1}|}\ \cancel{|I|}^{n-1}$

$|K^{n-1} I_n| = \begin{vmatrix} \lambda & 0 & 0 & \cdots \\ 0 & \lambda & \cdots & \\ 0 & 0 & \cdots & \lambda \end{vmatrix}$

$= \lambda^n |I_n|$

$\neq (K^{n-1})^n$

$K^{n(n-1)}$

$\cancel{|K^{-1}|}$

$|K^{-1}|\ |adj\ I_n|$

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Q. If P is orthogonal matrix and A is involutory ($A^2 = I$).
if Q is PAP' and $X = P'Q^3P$ then find X inverse.

$PP' = P'P = I$

$A^2 = I$

$Q = PAP'$

$X = I^3\ X = P'P^3A^3(P')^3P\cancel{X}$

$X =$

$= P'QQQP$

$= P'PAP'.PAPQ\ PAP'P.$

$= (IA)(IA)(IA)I = A^3 = A^2A = IA = A.$

$X = P'Q^3P.$

$X = Q^3I.$

$X = Q.$

$X = PAP'$

$X = AI$

$X = A.$

$X^{-1} = A^{-1}$

$(A+B)^2 = (A+B)(A+B)$

$A^2 + B^2 + 2AB - X = A$

Q. $A = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix} = 2$

$\begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix} = \begin{vmatrix} 36+22 & 66+44 \\ 12+8 & 22+16 \end{vmatrix}$

then find $A = 20$

$|A^{2005} - 6A^{2004}|$

$A^{2004}(A^1 - 6A)$

$A^{2004} |A^{-1}|$

$= A^{2004} |A - 6|$

$A |A - 6|$

$A^{2004} |A - 6|$

$|A|^{2004} |A - 6I|$

$= 2^{2004}$

$A - 6I = \begin{bmatrix} 6 & 11 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

$= \begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix}$

$2^{2004} \begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix} = -22 \times 2^{2004}$.

Q. A and B are sq. matrix of order ~~three~~ 3.

$|A| = 2$, $|B| = 1$

then find $|A^{-1}(\text{adj}\, B^{-1})(\text{adj}\, 2A^{-1})|$

$|kA| = k^n |A|$

$|(\text{adj } A^{-1})| = |(\text{adj})^{-1}|$

Ans $|A| = 2$ $\quad |B| = 1$ $\quad AA^{-1} = I$

$|A^{-1} (\text{adj } B^{-1}) (\text{adj } 2A^{-1})|$

$=$

$|A|^{-1} |\text{adj } B^{-1}| \, |\text{adj } 2A^{-1}|$

$\frac{1}{|A|} |\text{adj}(\text{adj } B)| \, |2^2 (\text{adj } A^{-1})|$

~~$|A|A^{-1}(\text{adj } B^{-1})(\text{adj}$~~

$|A|^{-1} |(\text{adj } B^{-1}) (\text{adj } 2A^{-1})|$

$|A|^{-1}$ $\text{adj } \frac{1}{|B|}$ $\text{adj } \frac{1}{2A}$

$\frac{1}{2} \times 1 \times \frac{1}{4} = = \frac{1}{8}.$

$= \frac{1}{|A|} \cdot |B|^{(n-1)^2} \cdot 4^3 \, |\text{adj } A^{-1}|$

$B^{-1} = \frac{\text{adj } B}{|B|} = \text{adj } B.$

$= \frac{1}{|A|} \cdot |B|^{(n-1)^2} \cdot 4^3 \, |(\text{adj } A)^{-1}|$

$\frac{1}{|A|} \cdot |B|^{(n-1)^2} \cdot 4^3 \, \frac{1}{|\text{adj } A|}$

$= \frac{1}{|A|^3} \cdot |B|^4 \cdot 64 = \frac{1}{8} \times 1 \times 64 = 8$ Ans

Que: Find matrix A. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times2} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times2} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

$X \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} & \\ & \end{bmatrix}_{2\times2} \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}.$

$A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 1 & -1/2 \end{bmatrix}$ $\quad A^{-1} = X$

$A = \begin{bmatrix} 1/3 & 2/4 \\ 5 & -6 \end{bmatrix}$

Revise Vector. H.W $\left\{ \frac{J-A}{8-1} \right\}$

$\cancel{X} A Y = M$

$|X| = 1$

$|Y| = -1$

$(X^{-1}X) A (Y Y^{-1}) = X^{-1} M Y^{-1}$

$A = \underbrace{\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}}_{1} \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} \underbrace{\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}}_{-1}$

Q. Cofactor of elements of Diagonal matrix A of order 3 are Roots of eq.

$x^9 + kx^8 - 16x^6 = 0 \quad k \in R.$

$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ find $|A| = ?$

$a \to bc$

$b \to ac$

$c \to ab.$

$\begin{vmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{vmatrix}$

$bc(ab - ac)$

$acb^2 - abc^2.$

$x^6(x^3 + kx^2 - 16) = 0$

$x^9 + kx^8 - 16x^6 = 0,$

$x^6(x^3 + kx^2 - 16) = 0.$

root $6 = 0$

$\begin{bmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{bmatrix}$

$x^3 + kx^2 - 16 = 0$ — ab, bc, ca,

$ab \cdot bc \cdot ca = 16$

$abc = 4.$

$Adj(adj A) = |A|^{n-2} A.$

formula

Q.

*

(1) Work A is done by m ways.
" B " " n ways.
" C " " r ways.

Work is finished only when work A, B, C or all of them is Completed.
then it Can be done by
$= m \times n \times r$ ways

Que: A is sq. matrix of order 3 whose element are Real no. and

$$adj(adj(adj A)) = \begin{bmatrix} 16 & 0 & -3 \\ 0 & 4 & 0 \\ 0 & 3 & 4 \end{bmatrix} = 16(16) - 0 =$$

16
16
96
16
(256)
(256)

then find Adjoint A. or A^{-1}

$adj(adj(adj A)) = x.$

$A \longrightarrow adj A.$

$adj(adj(adj(adj A))) \to x$

$adj(adj(\underbrace{|A|^{n-2} A}_{A})) = x$

$\left| |A|^{n-2} A \right|^{n-2} A^{n-2} A.$ $= \cancel{|A^2|^2} \cdot A^2 =$

$A^4 = A$ $A^4 = I$ I

$= ,$

A

J - Advanced.

$$\left|\quad\right| = \begin{vmatrix} 16 & 0 & -3 \\ 0 & 4 & 0 \\ 0 & 3 & 4 \end{vmatrix}$$

$$|A|^{(n-1)^3} = 16 \cdot 4 \cdot 4$$

$$|A|^8 = 2^4 \cdot 2^2 \cdot 2^2 = 2^8.$$

$$|A| = 2.$$

$$adj(adj\, A) = |A|^{n-2} A.$$

$$adj\ adj\ adj\ A = |adj\, A|^1 (adj\, A)$$

$$= |A|^2\ adj\ A$$

(Given) $= 4\ adj\ A$

$$adj = \begin{bmatrix} 4 & 0 & -3/4 \\ 0 & 1 & 0 \\ 0 & \frac{2}{4} & 1 \end{bmatrix}$$