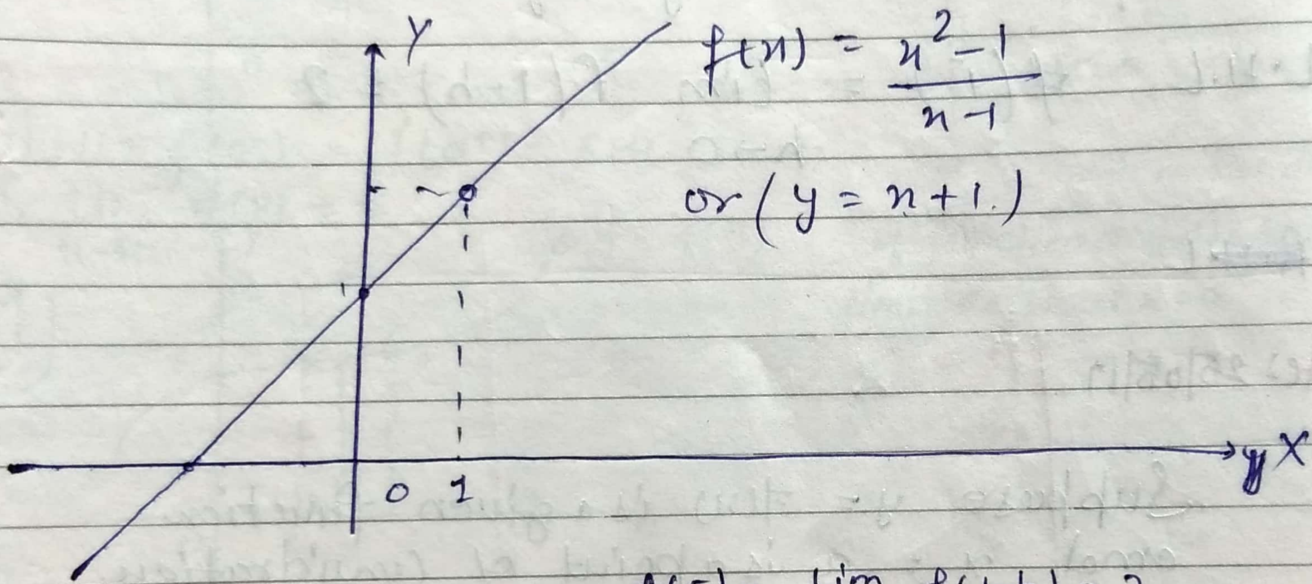
24/05/17 Chapter :- SBG STUDY

* Limit

$f(x) = \frac{x^2-1}{x-1}$ Domain = $R - \{1\}$

$= \frac{(x-1)(x+1)}{(x-1)} = x+1$ $\because x \neq 1$



$f(x) = \frac{x^2-1}{x-1}$

or $(y = x+1.)$

$f(1^-) = \lim_{h \to 0} f(1-h) = 2$

L.H.L :-

x	0.9	0.9999	0.99999 - -
$f(x)$	1.9	1.9999	1.9999999

Hence LHL aproches towards to '2'

R.H.L			
x	1.1	1.0001	1.000001
$f(x)$	2.1	2.0001	2.000001

Hence R.H.L aproches towards to '2'

R.H.L $f(1^+) = \lim_{h \to 0} f(1+h)$.

* Because L.HL and RHL both aproches towards to 2 (finite quanity). Hence limit exist and equal to '2'

i.e lim

Concept of neutrality

i.e, $\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$ $\frac{(x-1)(x+1)}{\cancel{(x-1)}} = x+1 = 1+1 = 2$

$\boxed{x \to 1 \neq x = 1}$

But $x \to 1 \Rightarrow x = 1 \pm h$

where h is very very small positive no.

L.H.L $f(1^-) = \lim_{h \to 0} f(1-h) = 2$

~~R.H.L~~

Date:- 25/08/17

Suppose $y = f(x)$ is a given function and $x = a$ is a point of considration.

function $f(x)$ is defined in the neighbourhood $x=0$

$f(x) = \frac{x^2-1}{x-1}$

L.H.L $= f(a^-) = \lim_{h \to a} f(a-h) = l$

R.H.L $= f(a^+) = \lim_{h \to 0} f(a+h) = l$

finite quantity

Here h is very small positive no.
If L.H.L = RHL = finite quantity then limit of function $f(x)$ at $x = a$ exist.

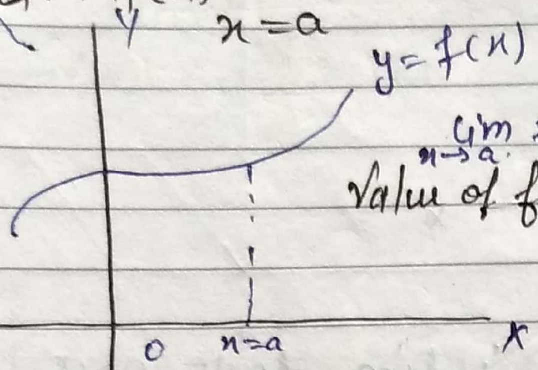
$x=a$ lie in the domain in the function but
Value of function $f(a)=m$
despite the limit exist. $L \neq m$

$$\lim_{x \to a} f(x) = L$$

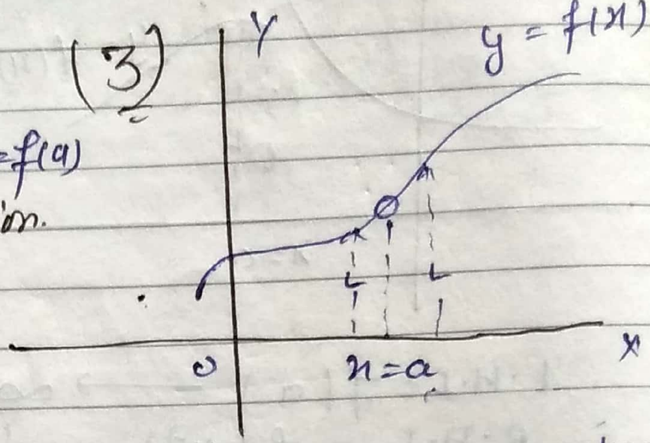
$LHL = f(a^-) = L$
$RHL = f(a^+) = L$

$\lim f(x) = L$

(2)



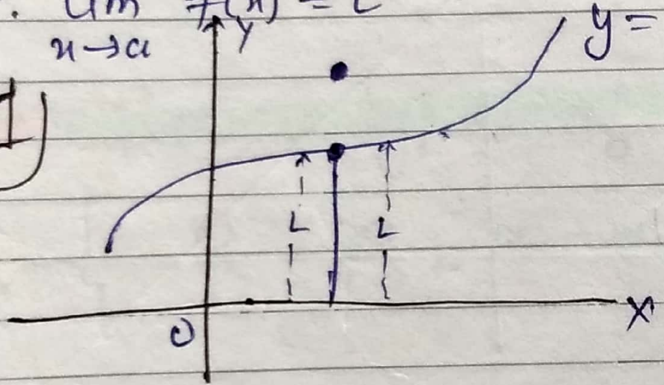
(3)



$x=a$ not in the domain of function despite this limit exist at $x=a$

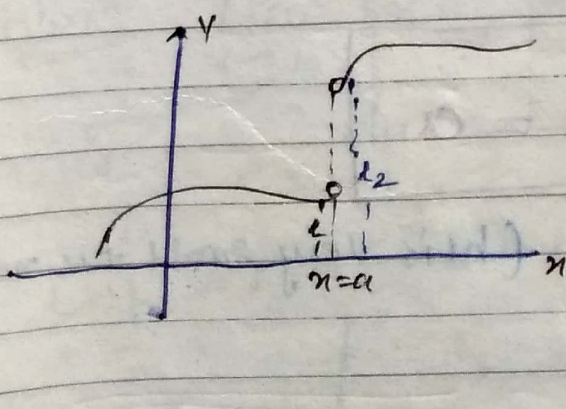
$LHL = f(a^-) = f(a^+) = RHL$
$\therefore \lim_{x \to a} f(x) = L$

(1)



because 2nd example continous there its limit will also exist.

(1) and last figure are discontinious despite this limit exist at $x=a$.

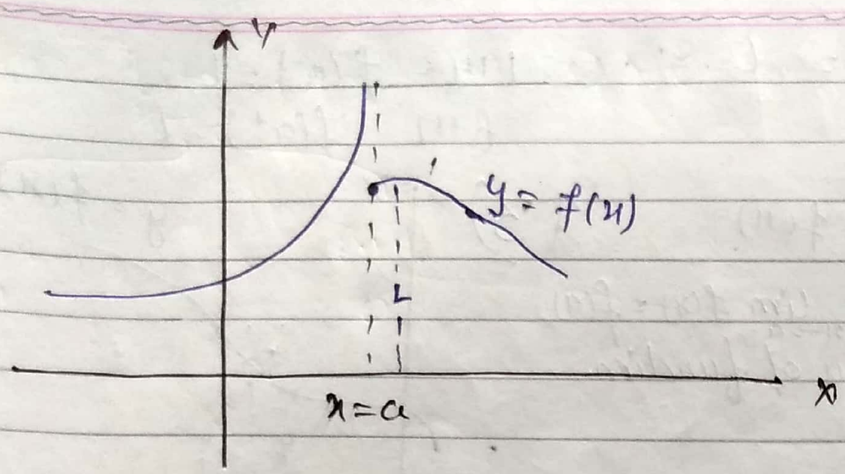


$f(a^-) = l_1$ (finite)

$f(a^+) = l_2$ (finite)

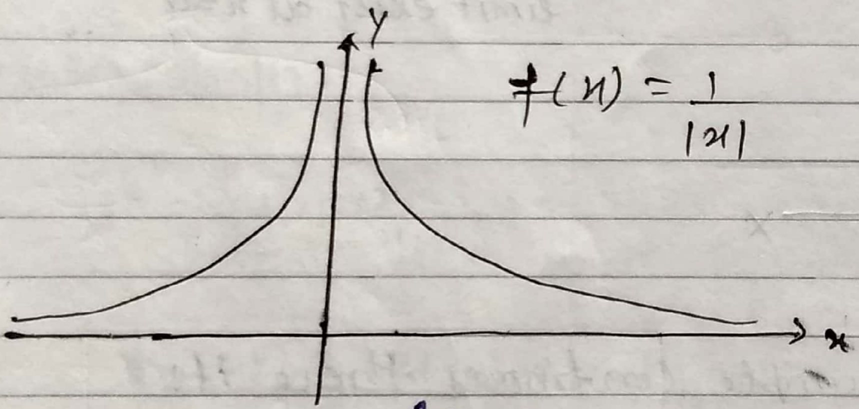
$\therefore LHL \neq RHL$

$\therefore \lim_{x \to a} f(x) = DNE$



$L.H.L = f(a^-) \longrightarrow \infty$ $\quad \therefore \lim_{x \to a} f(x) = DNE$

$R.H.L = f(a^+) \longrightarrow l$



$f(a^+) = \infty$

$f(a^-) = \infty$

$\therefore \lim_{x \to 0} f(x) = DNE$

$$\boxed{x \longrightarrow a \neq x = a}$$

$\downarrow$

$x = a \pm h$ (h is very small +ve no.)

$\frac{4}{0}$ - N.D

$\frac{0}{4} \Rightarrow 0$

* $\lim_{x \to 1} \frac{x^2-1}{x-1}$ $\begin{matrix} \to 0 \\ \to 0 \end{matrix}$ [aproching value of 0 neumator or denomitor]

~~Lim~~ because both aproching toward zero i.e ~~intet~~ Indeterminant form of $\frac{0}{0}$

* $\lim_{x \to 1} (x+1) = 1+1 = 2$

Because continious.

*** Seven types of Indeterminant form:-**

$\left\{ \frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \cdot \infty, 1^{\infty}, 0^{0}, \infty^{0} \right\}$

{these value are aproches that}

*

Que:- $\lim_{x \to 0} \frac{2-2}{x} = \frac{\text{Ext. } 0}{\text{Non-Zero}}$

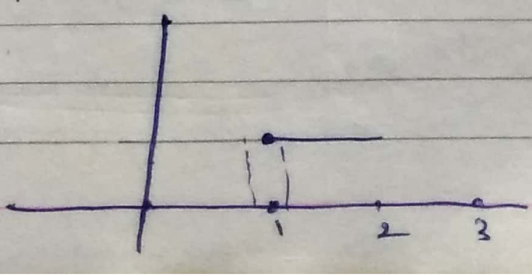
(This is not Indeterminant form)

Que: $\lim_{x \to 1} [x]$ (ii) $\lim_{x \to 0} \frac{|x|}{x}$

L.H.L $= 0$

R.H.L $= 1$

$LHL \neq RHL$

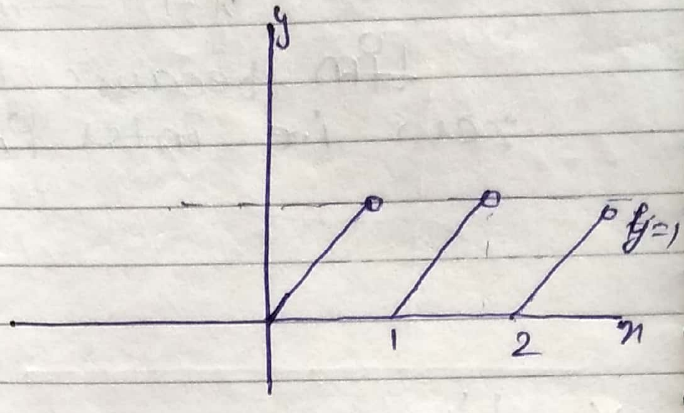


(iii) $\lim_{x \to 2} \{x\} \Rightarrow$ DNE

$0 < \{x\} < 1$

L.H.L $= 1$

R.H.L 0



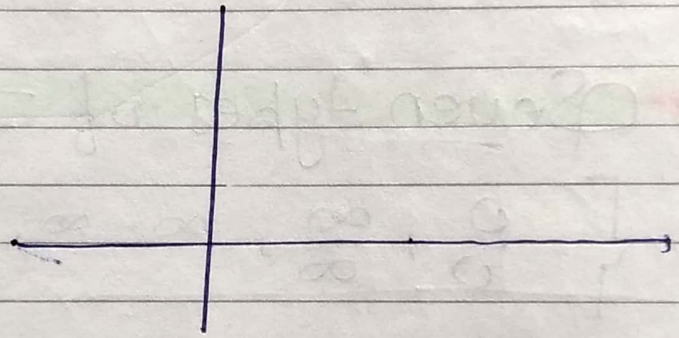
(ii) $\lim_{x \to 0} \frac{|x|}{x} =$ DNE

L.H.L

$-\frac{\cancel{x}}{\cancel{x}} = -1$

R.H.L

$\frac{x}{x} = 1$



$$\boxed{|x| = \begin{cases} x, & x \geq 0 \\ -x & x < 0 \end{cases}}$$

Que:- i) $\lim_{x \to 0} \tan^{-1} \frac{1}{x} =$ DNE

L.H.L

$-\infty$

$-\frac{\pi}{2}$

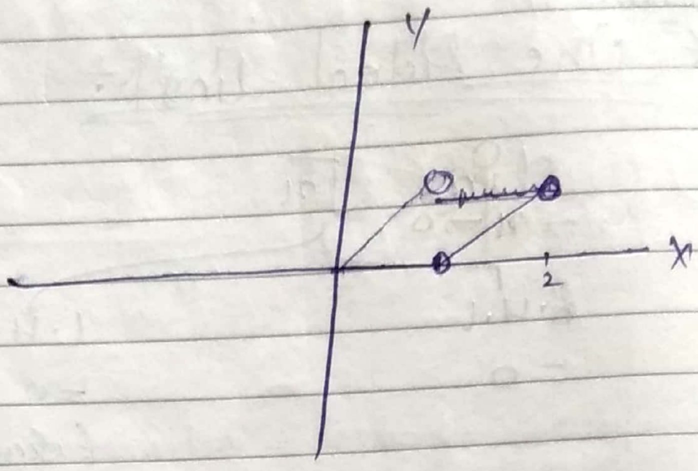
$\neq$

R.H.L

∞

$\frac{\pi}{2}$

(ii) $\lim_{x\to 1} [x] + \sqrt{\{x\}} = 1$



L.H.L

$0 + \sqrt{1}$

$= 1$

R.H.L

$1 - \sqrt{0}$

$= 1$

(iii) $\lim_{x\to 1} \frac{[x]}{x}$

L.H.L

0

R.H.L.

1

(iv) $\lim_{x\to \frac{\pi}{2}} [\sin x] = 0$

L.H.L

$[1^-]$

$= 0$

RHL

$[1^-]$

$= 0$

L.H.L = RHL

Que!

* One sided limit ÷

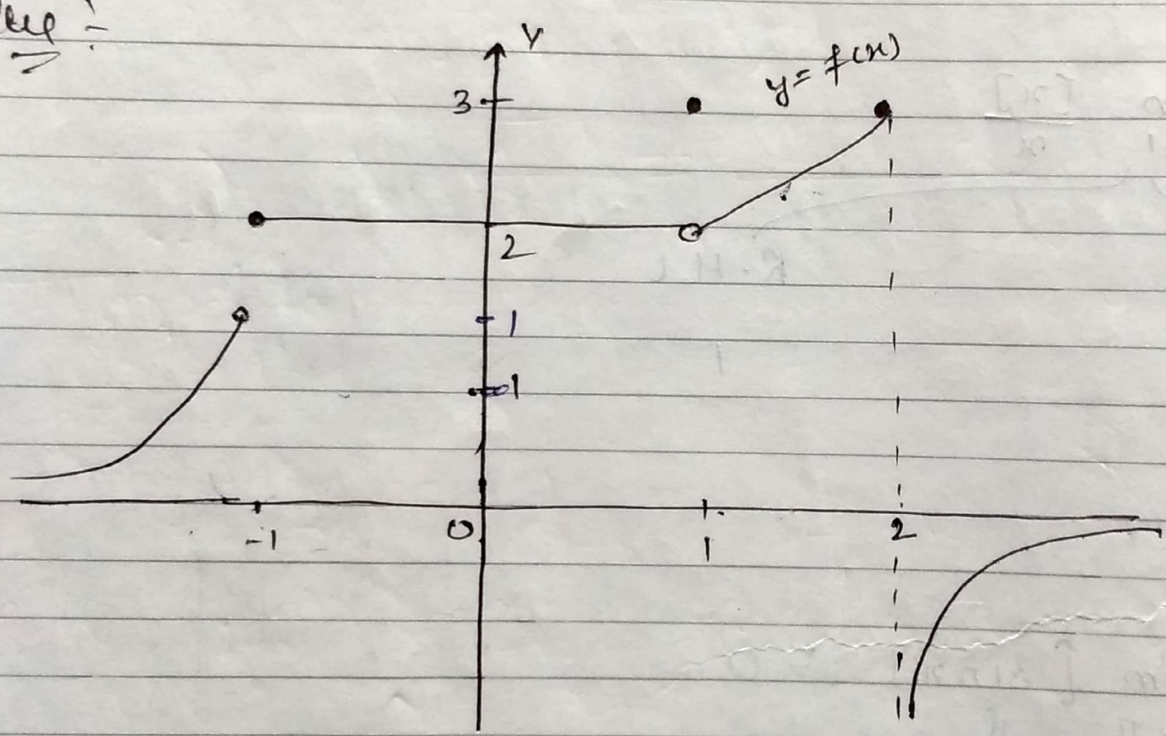
$$\lim_{x \to 0} \sqrt{x}$$

R.H.L $= 0$

L.H.L $= 0$ (= Same as R.H.L)

Can not evaluated

Que ÷



then evaluate:

a) $\lim_{x \to -1^+} f(x) \Rightarrow 2$

b) $\lim_{x \to -1^-} f(x) \to 1$

c) $\lim_{x \to -1} f(x) =$ DNE

d) $\lim_{x \to 1^+} (fx) = 2$

e) $\lim_{x \to 1^-} f(x) = 2$

f) $\lim_{x \to 1} f(x) = 2$

g) $\lim_{x \to 2^+} f(x) = -\infty$

h) $\lim_{x \to 2^-} f(x) = 3$

(i) $\lim_{x \to 2} f(x) = DNE$

(j) $\lim_{x \to 1^-} f(x) = 2$

(k) $\lim_{x \to 0^+} f(x) = 2$

(l) $\lim_{x \to 1} f(x) = 2$

m) $f(1) = 3$

~~Five~~ Fundamentals Principle/Theorems:

if $\lim_{x \to a} f(x) = L$ & $\lim_{x \to a} g(x) = M$

1) $\lim_{x \to a} (f(x) + g(x)) = \lim_{x \to a} f(x) + \lim_{x \to a} (g(x)) = L + M$

2) $\lim_{x \to a} (f(x) - g(x)) = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M$

iii) $\lim_{x \to a} \left(f(x) \cdot g(x) \right) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} (g(x)) = L \cdot M$

iv) $\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M}$ (Here $M \neq 0$)

v) $\lim_{x \to a} (\lambda f(x)) = \lambda \lim_{x \to a}$

(λ is constant multiple)

$$\lim_{x \to 1} \frac{x^2-1}{x-1} \neq \frac{\lim_{x \to 1} (x^2-1)}{\lim_{x \to 1} (x-1)}$$

Important

* <u>Observation</u>:

1) If $\lim_{x \to 0}$ exist, and $\lim_{x \to}$ $g(x)$

$\lim_{x \to a} f(x)$	$\lim_{x \to a} g(x)$	$\lim_{x \to a} (f(x) \pm g(x))$	$\lim_{x \to a} (f(x) \cdot g(x))$	$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right)$
E	E	E	E	E
E	DNE	DNE	CNS	CNS
DNE	E	DNE	CNS	CNS
DNE	DNE	$\underbrace{\text{CNS}}_{\text{Can't say}}$	CNS	CNS

* $\lim_{x \to 1} [x]$ - DNE

* $\lim_{x \to 1} \{x\}$ - DNE

* $\lim_{x \to 1} ([x] + \{x\})$

* $\lim_{x \to 1} x$ = ~~Ex~~ exist.

* Method to evaluates limit :

~~Factorization~~ : Important factor !

a) $x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + \cdots + a^{n-1})$, $n \in N$

2) $x^n + a^n = (x+a)(x^{n-1} - ax^{n-2} + \cdots + a^{n-1})$, n is an odd no.

Note : $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

b) Rationalization or double rationalization.

c) lim when $x \to 0$

i) Divide by greatest power of x in numerator and denomi

ii) Put $x = 1/y$ and apply $y \to 0$

d) Squeeze play theorem (Sandwich theorem).

If $f(x) \le g(x) \le h(x)$; $\forall x$ in the neighbourhood at $x = a$ and

$\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$ then $\lim_{x \to a} g(x) = l$.

* Factorization, rationalisation, Double rationalisation, Series Expension, Alzabric identity. use of Binomal theorem, law of love.

$$a^2-b^2=(a-b)(a+b)$$

* $$a^{1/2}-b^{1/2}=\frac{a-b}{\sqrt{a}+\sqrt{b}}$$

$$a^3-b^3=(a-b)(a^2+b^2+ab)$$

* $$a^{1/3}-b^{1/3}=\frac{a-b}{a^{2/3}+a^{1/3}b^{1/3}+b^{2/3}}$$

Que!:

$$\lim_{x\to1}\frac{x^2-1}{x-1}\quad\left(\frac{0}{0}\right)$$

$$=\lim_{x\to1}\frac{(x-1)(x+1)}{(x-1)}$$

$$=\lim_{x\to1}(x+1)=1+1=2$$

Que!

$$\lim_{x\to9}\frac{3-\sqrt{x}}{4-\sqrt{2x-2}}\quad\left(\frac{0}{0}\right)$$

$$\lim_{x\to9}\frac{(3-\sqrt{x})(3+\sqrt{x})\,[4+\sqrt{2x-2}\,]}{(3+\sqrt{x})\,[4-\sqrt{2x-2}\,][4+\sqrt{2x-2}\,]}$$

$$\lim_{x\to9}\frac{(9-x)\cdot[4+\sqrt{2x-2}\,]}{[3+\sqrt{x}\,]\,[16-(2x-2)]}=\lim_{x\to9}\frac{(9-x)[4+\sqrt{2x-2}\,]}{(3+\sqrt{x})\,[9-x]\cdot2}$$

$$=\frac{4+4}{6\cdot2}$$

Higher

Que: $\lim_{x\to\infty} (x - \sqrt{x^2+x})$ $(\infty - \infty)$

$\lim_{x\to\infty} \frac{(x - \sqrt{x^2+x}) \times (x + \sqrt{x^2+x})}{(x + \sqrt{x^2+x})}$

$\lim_{x\to\infty} \frac{\cancel{x^2} - \cancel{x^2} - x}{x + \sqrt{x^2+x}}$

$\lim_{x\to\infty} \frac{-x}{x + \sqrt{x^2+x}} \left(\frac{\infty}{\infty}\right) = \lim_{x\to\infty} \frac{-1}{1 + \sqrt{1 + \frac{1}{x}}}$

$= \frac{-1}{1+1} = \frac{-1}{2}$

Que: if $\lim_{x\to 1} \frac{x^2 + a}{x - 1} = l$ (Exist & finite)

then find a and l.

$N^r \neq 0$

$\lim_{x\to 1} \frac{\text{Non zero}}{0} = \infty$

$N^r = 0$

$= 1^2 + a = 0$

$a = -1$

$\lim_{x\to 1} \frac{x^2 - 1}{x - 1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}$

$= 2 = l.$

Que → $\lim_{x\to\frac{\pi}{4}} \frac{-1 + \cot^3 x}{-2 + \cot x + \cot^3 x}$ $\left(\frac{0}{0}\right)$

$\cot x = y$

when $x \to \frac{\pi}{4}$, $y \to 1$

$\lim_{x\to 1} \frac{\cancel{y^3}}{y^3 \cancel{+ y - 2}}$

$\lim_{x\to 1} \frac{y^3 - 1}{y^3 + y - 2}$

$ax^n = nax^{n-1}$

$$\lim_{y\to 1} \frac{(y-1)(y^2+y+1)}{(y-1)(y^2+y+2)} = \frac{3}{2}$$

$\frac{y}{2}$ $\quad y^3+y-2 = (y-1)(y^2+y+2)$

$= (y-1)(y^2+y+2)$

$y^2(y-1)+y(y-1)+2(y-1)$

$\cancel{y^2+y}(y-1)(y^2+y+2)$

$y^2 \Rightarrow \cancel{(y^2-1)}$ $\quad y^3+y^2$

$0 = -1+b \Rightarrow b=1$

$$\lim_{y\to 1} \frac{y^3-1}{y^3+y-2} \quad \left(\frac{0}{0}\right)$$

* law of love :

Que: $\lim_{x\to\infty} ((x+1)(x+2)(x+3))^{1/3} - x \quad (\infty, \infty)$

$$\lim_{x\to\infty} \left((x+1)(x+2)(x+3)\right)^{1/3} - (x^3)^{1/3}$$

$$\lim_{x\to\infty} \frac{(x+1)(x+2)(x+3) - x^3}{[(x+1)(x+2)(x+3)]^{2/3} + (x^3)^{2/3} + [(x+1)(x+2)(x+3)]^{1/3} \cdot (x^3)^{1/3}}$$

$$\lim_{x\to\infty} \frac{(\cancel{x^3}+6x^2+11x+6)-\cancel{x^3}}{} \quad \left(\frac{\infty}{\infty}\right)$$

Higher Coefficient

$$\lim_{x\to\infty} \frac{6x^2+11x+6}{\left[\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\right]^{2/3}+1+\left[\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\right]^{1/3}}$$

[take common x^2] →

$$\lim_{x\to\infty} \left[\frac{6+\frac{11}{x}+\frac{6}{x^2}}{\left[\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\right]^{2/3}+1+\left[\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)\left(1+\frac{3}{x}\right)\right]^{1/3}}\right]$$

$$= \frac{6}{1+1+1} = 2 \text{ Ans}$$

$$\lim_{x\to\infty} \left((x+a)(x+b)(x+c)(x+d)\right)^{1/4} \qquad \left(\frac{a+b+c\ldots}{n}\right)$$

$$= \cancel{x+b}\ \frac{a+b+c+d}{4} \text{ Ans}$$

* Law of love:

$$\lim_{x\to\infty} \frac{a_0x^n+a_1x^{n-1}+a_2x^{n-2} - - a_n}{b_0x^m+b_1x^{m-1}+ - - - b_m} \quad \left(\frac{\infty}{\infty}\right)$$

$$\lim_{x\to\infty} \frac{x^n\left[a_0+\frac{a_1}{x} - - - \frac{a_n}{x^n}\right]}{x^m\left[b_0+\frac{b_1}{x} - - \frac{b_m}{x^m}\right]}$$

$\boxed{n>m}$: $a_0b_0>0 = \infty$; $a_0b_0<0 = -\infty$

$\boxed{m=n}$: $\frac{a_0}{b_0}$

$\boxed{n<m}$: 0

H.W.!

Que! $\lim_{x\to\infty} \dfrac{4x^3+3x^2-7}{9x^3-25x}$

$= \dfrac{4}{9}$

(2) $\lim_{x\to\infty} \dfrac{x^{99}-32x+12}{12x^{100}-72x^2+1}$

$\Rightarrow = 0$ $\quad n = 99, m = 100$

$n < m$

(3) $\lim_{x\to\infty} \dfrac{12x^{75}-76x+77}{-19x^{74}+79} = -\infty$ Ans.

Que! If $\lim_{x\to\infty}\left(\dfrac{x^2+1}{x+1} - ax - b\right) = 0$
then find a and b (I.C.T)

Ans! $\lim_{x\to\infty} \dfrac{(x^2+1) - (ax+b)(x+1)}{x+1}$

$\lim_{x\to\infty} \dfrac{x^2(1-a) - x(a+b) + (1-b)}{x+1} = 0$

$1-a=0 \Rightarrow a=1$

&

$a+b=0 \qquad b=-1$

H.W. : O-1 → 1, 2, 3, 4, 5, 8, 10, 11, 19, 36, 38

O-2 : 15,

S-1 → 1, 2, 10(i).

Que | $\lim_{x\to\infty} \cancel{x^2(x+1)+(ax+b)} = 4$

$\frac{(x^2+1)-(ax+b)(x+1)}{(x+1)}$

$\lim_{x\to\infty} \frac{x^2(1-a) - x(a+b) + (1-b)}{x+1} = 4$

Ans $1 - a = 0$ —(i)

$\frac{-(a+b)}{1} = 4$ —(ii)

✗ limit using Series Expansion :

a) $a^x = 1 + \frac{x \ln a}{1!} + \frac{x^2 \ln^2 a}{2!} + \frac{x^3 \ln^3 a}{3!} + \cdots + a > 0$

b) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

c) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ for $-1 < x \le 1$

d) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$

e) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

f) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots$

g) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$

h) $\sin^{-1} x = x + \frac{1^2}{3!}x^3 + \frac{1^2 \cdot 3^2}{5!}x^5 + \frac{1^2 \cdot 3^2 \cdot 5^2}{7!}x^7 + \cdots$

(i) $\sec^{-1} x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \frac{61x^6}{6!} + \cdots$

(j) $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots \quad n \in Q.$

$\boxed{\text{Ex. } 0-1}$

(4) $\lim_{x \to 1} \frac{x^{\frac{1}{n}} - 1}{x^{1/m} - 1}$

$\lim_{x \to 1} \frac{\frac{1}{n} x^{\frac{1}{n}-1}}{\frac{1}{m} x^{1/m-1}} = \frac{m}{n}$

(5) $\lim_{x \to a} \frac{(2x - \sqrt{x^2+3a^2})(2x + \sqrt{x^2+3a})(\sqrt{x+a} + \sqrt{2a})}{(\sqrt{x+a} - \sqrt{2a})(\sqrt{x+a} + \sqrt{2a})(2x + \sqrt{x^2+}}$

$\frac{4x^2 - x^2 - 3a^2}{(x+a-2a)} \cdot \frac{\sqrt{x+a} + \sqrt{2a}}{2x + \sqrt{x^2+2a^2}}$

$\Rightarrow \lim_{x \to 0} \frac{3(x-a)(x+a)}{(x-a)} = \frac{1}{\sqrt{2}}$

$\{ x^n - 1 = (x-1)(x^{n-1} - x^{n-2} \ldots 1)$

(S-1)

(49) $\lim_{x \to 0} \frac{(1 + x + x^2 - \cdots - x^{100}) - 1}{x-1}$ L Hospital Differenciate.

$(x-1) + (x^2-1) - \cdots (x^{100} - 1)$

$= \frac{(x-1)[1 + (x+1) + (x^2+x+1) \cdots (x^{99} + x^{98} + \cdots)}{}$

$1 + 2 + 3 + \cdots 100 = 5050.$

(6)

$$\prod_{r=1}^{4}$$

Ans $0{-}1$

(8) (ii) $x = -y$

$y \to \infty$

$$\lim_{y \to \infty} \left(\sqrt{y^2+2y-1} - \sqrt{y^2+7y+3} \right)$$

$$\frac{-5y-4}{\sqrt{1+\frac{2}{y}-\frac{1}{y^2}}+\sqrt{1+\frac{7}{y}+\frac{3}{y}}}$$

$$\frac{-5-\frac{4}{y}}{\sqrt{1+\frac{2}{y}-\frac{1}{y^2}}+\sqrt{1+\frac{7}{y}+\frac{3}{y}}} = \frac{-5}{2}$$

Que: Let $S_n = 1+2+\ldots n$ & $P_n = \prod_{n=2}^{n} \frac{S_n}{S_{n-1}}$,

Find $\lim_{n \to \infty} P_n$.

Sol:

$$= \frac{n(n+1)}{2}$$

$$S_n - 1 = \frac{n(n+1)}{2} - 1$$

$$= \frac{n(n+1)-2}{2}$$

$$= \frac{n^2+n-2}{2} = \frac{n^2+2n-n-2}{2} = \frac{(n+2)(n-1)}{2}$$

$$\frac{S_n}{S_n-1} = \frac{\frac{n(n-1)}{2}}{\frac{(n+2)(n-1)}{2}} = \frac{n(n+1)}{(n+2)(n-1)}$$

$$\boxed{\prod_{r=1}^{4} r = 1.2.3.4}$$

$$P_n = \prod_{n=2}^{n} \frac{S_n}{S_{n-1}}$$

$$= \prod_{n=2}^{n} \frac{n(n+1)}{(n-1)(n+2)}$$

$$= \lim_{n\to\infty} \left(\prod_{n=2}^{n} \frac{n}{n-1}\right) \cdot \prod_{n=2}^{n} \left(\frac{n+1}{n+2}\right)$$

$$\lim_{n\to\infty} \left[\left(\frac{2}{1}\cdot\frac{3}{2}\cdot\frac{4}{3} \cdots \frac{n}{n-1}\right) \times \left(\frac{3}{4}\cdot\frac{4}{5} \cdots \frac{n+1}{n+2}\right)\right]$$

$$\lim_{n\to\infty} \left(\frac{n}{1} \times \frac{3}{n-2}\right)$$

$$= \frac{3}{1} \text{ Ans}$$

Que: Find a, b, c such that

$$\lim_{x\to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x^2}$$

$$\lim_{x\to 0} \frac{a\left(1 + x + \frac{x^2}{\lfloor 2} + \cdots\right) - b\left(1 - \frac{x^2}{\lfloor 2} + \cdots\right) + c\left(1 - x + \frac{x^2}{\lfloor 2} \cdots\right)}{x^2} = 2$$

$$\lim_{x\to 0} \frac{\cancel{(a-b+c)} + (a+c) + x^2\left(\frac{a}{\lfloor 2} + \frac{b}{\lfloor 2} + \frac{c}{\lfloor 2}\right) + x^3(\quad) + x^4(x) + \cdots}{x^2} = 2$$

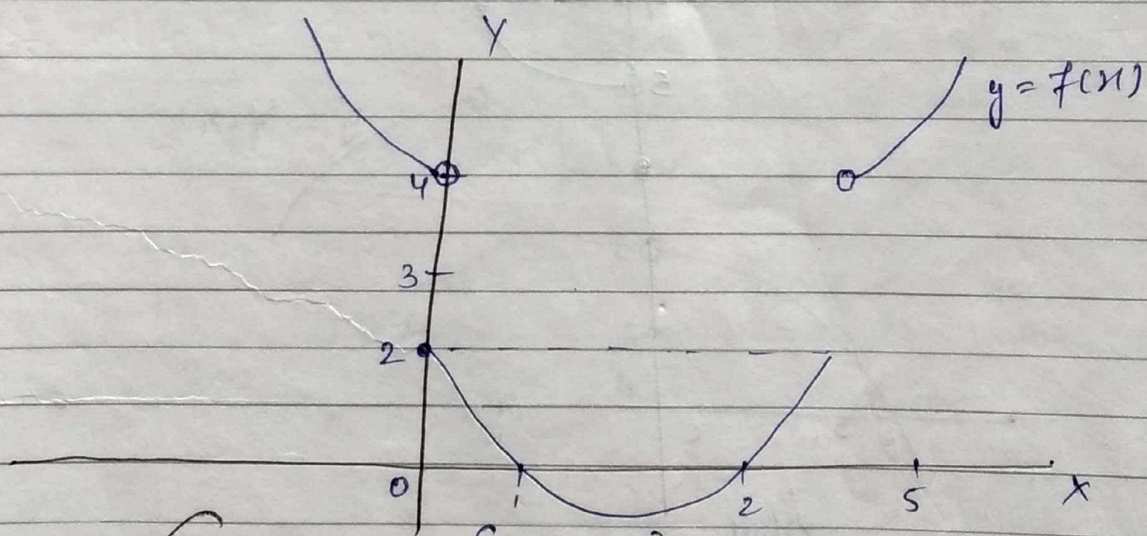
$=$

$$a-b+c=0 \quad \text{—(i)}$$
$$a+c=0 \quad \text{—(ii)}$$

$$\lim_{x\to0} \frac{\cancel{(a-b+c)}^{0} + x\cancel{(a+c)}^{0} + x^2\left[\left(\frac{a}{\lfloor2}+\frac{b}{\lfloor2}+\frac{c}{\lfloor2}\right)+x(\ )+x^2(\ )+ \ldots\right]}{x^2}$$

$$= \frac{a+b+c}{\lfloor2} = 2 \quad \text{—(iii)} \quad Ag$$

$\Rightarrow$ $$\boxed{(1+x)^{1/x} = e\left(1-\frac{x}{2}+\frac{11}{24}x - \ldots\right)}$$



$$g(x) = \begin{cases} (x-2)^2, & x<2 \\ 7-x, & x\geq 2 \end{cases}$$

(i) $\lim_{x\to2} f(g(x))$

L.H.L
$f(g(2^-))$
$f(0^+)$
$=2$

R.H.L
$f(g^-)$
$=2$

(ii) $\lim_{x \to 0} gf(x)$

L.H.L $gf(0^-)$
$f(2^-)$
0

R.H.L $gf(0^+)$
$f(4^+)$
3

~~f(g~~

$gf(0^-)$
$g(4^+)$
$= 3$

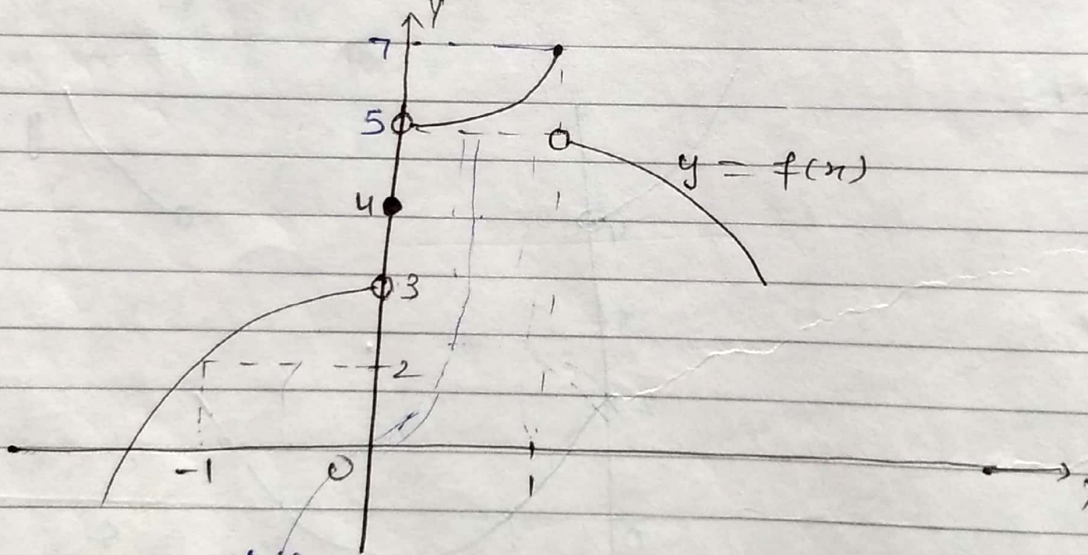
$gf(0^+)$
$g(4^+)$
0

(iii) $\lim_{x \to 5} g(f(x))$

L.H.L
$\cancel{g(f(x))}$
$g(f(x^-)$
$g(f(5^-)$
$g(0)$

R.H.L
$g(f(x^+)$
$g(f(5^+)$
$g(4)$

Que:



1) $\lim_{x \to 0^+} f[x + \sin x]]$
$= \cancel{[5^+]} = \cancel{f[5^+]} = \cancel{[0 + \sin x]} = 5$
$f[5] =$

$= \lim_{x \to 0^+} f([x - \sin x]]$
$[x \to 0^+ | x - \sin x) = +ve]$ $f(0) = 4$

(ii) $\lim_{x \to 0^-} f([x - \tan x]])$

ii) $\cancel{f \, 0 \, 3}$ $f(0) = 4$
Ans

(iii) $\lim_{x \to 0^+} f\left\{\frac{1}{\tan x}\right\}$

$= \cancel{3, 4}$
$\{1^-\} = 1^-$
$\{1^+\} = 0^+$
$= 7$ Ans

(iv) $\lim_{x \to \frac{\pi}{2}^+} f(\{\cos x\}) = f\{0^-\} = f(1) = 7$ Ans

= Ans = Ans

* Standard Limit :

→ Where All x is in radian.

* $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (from lower side)

* $\lim_{x \to 0} \frac{x}{\sin x} = 1$ (from higher side)

* $\lim_{x \to 0} \frac{\tan x}{x} = 1$ (from higher side)

* $\lim_{x \to 0} \frac{x}{\tan x} = 1$ (from lower side)

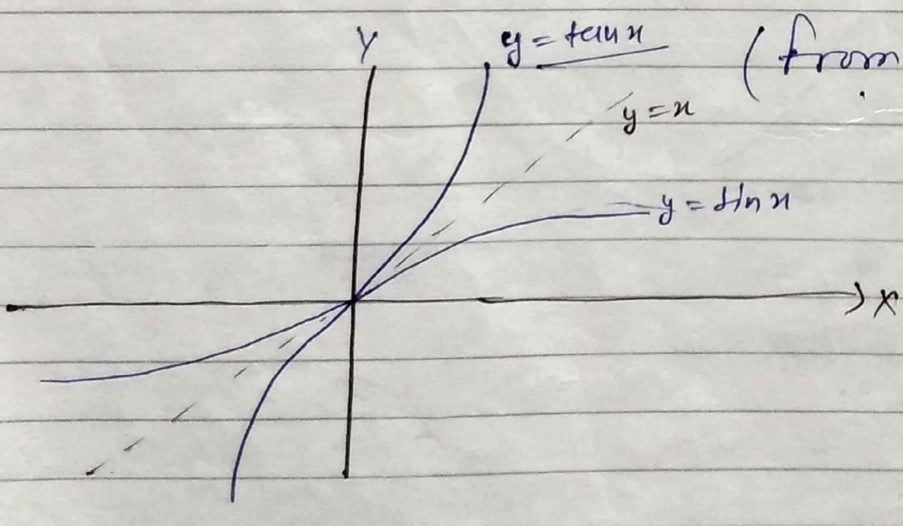
* $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$ (higher side,

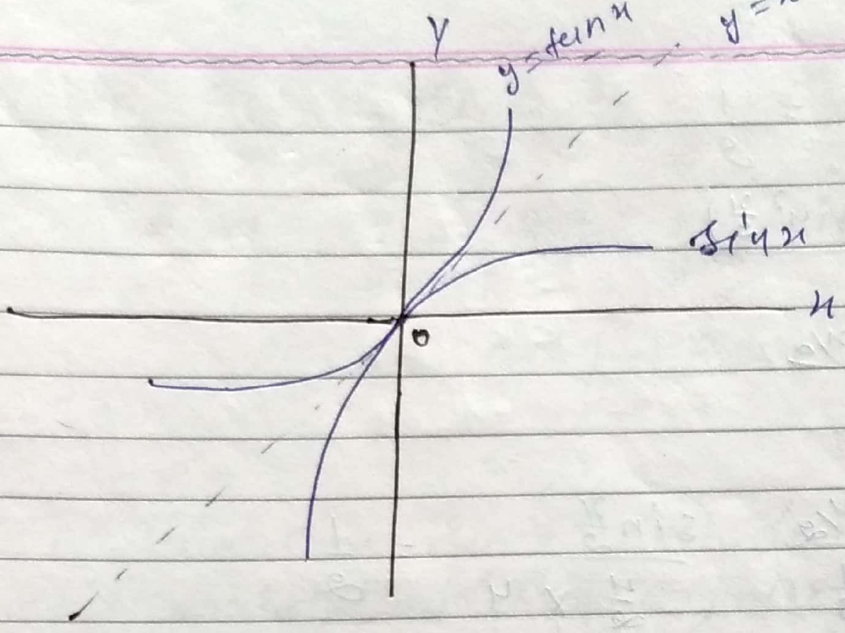
* $\lim_{x \to 0} \frac{\tan^{-1} x}{x} = 1$ (lower side)

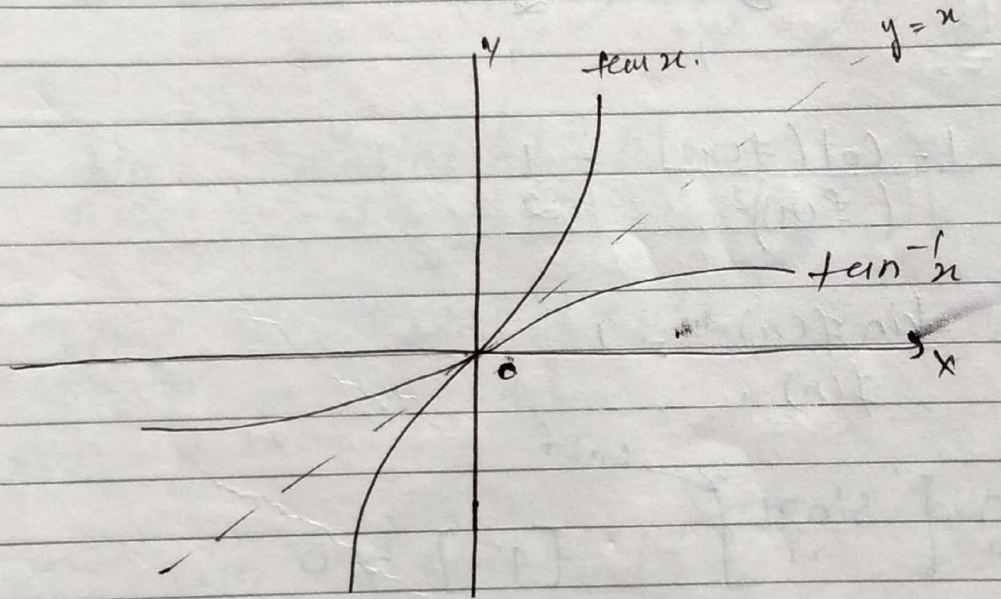
$$\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{x + \frac{x^3}{3} + \frac{2x^5}{5} + \cdots}{x}$$

$$\lim_{x \to 0} \left(1 + \frac{x^2}{9} + \frac{2}{11} x^4 - - - \right) = 1$$

(from higher side)







Note:- $\lim_{x \to a} \frac{\sin f(x)}{f(x)} = 1$ where $\lim_{x \to a} f(x) \to 0$

$\lim_{x \to 0} \frac{\sin 2x}{2x} \cdot 2$

$= \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot \lim_{x \to 0} 2$

$= 1 \times 2 = 2$ (Lower Hole).

* $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$

$\lim_{x \to 0} \frac{1-(1-2\sin^2\frac{x}{2})}{x^2}$

$\lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$

$\Rightarrow \lim_{x \to 0} 2 \cdot \frac{\sin x/2}{\frac{x}{2}} \frac{\sin\frac{x}{2}}{\frac{x}{2} \times 4} = \frac{1}{2}$

1   1

* $\lim_{x \to a} \frac{1-\cot(f(x))}{(f(x))^2} = \frac{1}{2}$

* $\lim_{x \to a} \frac{\tan f(x)}{f(x)} = 1$

G.I.F

Que: $\lim_{x \to 0} \left[\frac{\sin x}{x}\right]$ $[1^-] = 0$

* $\left[\lim_{x \to 0} \frac{\sin x}{x}\right] = [1] = 1$ Ans

* $\lim_{x \to 0} \left[\frac{\tan x}{x}\right] = [1^+] = 1$ Ans

$180^\circ = \pi^c = \pi$

* $\lim_{x \to 0} \frac{\sin x^\circ}{x}$ $1^\circ = \frac{\pi}{180}$

$\lim_{x \to 0} \frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}} \times \frac{\pi}{180}$ $x^\circ = \frac{\pi}{180} \times x$

$= 1 \times \frac{\pi}{180} = \frac{\pi}{180}$ Ans

Σ

H.W : Q-1 : 9, 13, 15, 17, 18, 19, 21, 35
Same   Int.   42,
Q-1 : 8,

Que: (a) $\lim_{x \to \infty} \frac{\sin x}{x}$:

$x = \frac{1}{y}$ $\lim_{y \to 0} y \sin \frac{1}{y}$

$= 0 \times [-1 \times 1]$

$= 0$

In this case $\sin \frac{1}{y}$ will oscillate b/w -1 to 1 and multiplied by 0 will gives us 0.

Que: $\lim_{x \to \infty} \frac{x^4 \sin(\frac{1}{x}) + x^2}{1 + |x|^3}$

$\lim_{x \to -\infty} \frac{x^4 \sin(\frac{1}{x}) + x^2}{1 - x^3}$ $(y = x)$

$x = -y$ $(y = \frac{1}{t})$

$\lim_{x \to \infty} \frac{-y^4 \sin(-\frac{1}{y}) + y^2}{1 + y^3}$

$\lim_{y \to \infty} \frac{-y^4 \sin \frac{1}{y} + y^2}{1 + y^3} =$

$y = \frac{1}{t}$ $\lim_{t \to 0}$ $\lim_{t \to 0} \frac{-\frac{1}{t^4} \sin t + \frac{1}{t^2}}{1 + \frac{1}{t^3}}$

$\lim_{t \to 0} \frac{-\frac{\sin t}{t} + t}{t^3 + 1} = \frac{-1 + 0}{0 + 1} = -1$ Ans

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ex-0-1

15) $^{k+1}C_2 = \frac{\lfloor k+1}{\lfloor 2 \lfloor k-1} = \frac{(k+1)k}{2}$

$1 - \frac{1}{^{n+1}C_2} = 1 - \frac{2}{k(k+1)}$

$\frac{k^2+k-2}{k(k+1)} = k^2+2k-k-2 = \frac{(k+2)(k-1)}{k(k+1)}$

$$\lim_{n\to\infty} \prod_{k=2}^{n} \frac{(k+2)(k-1)}{k(k+1)}$$

$$\prod_{k=2}^{n} \frac{k+2}{k+1} \prod_{k=2}^{n} \frac{k-1}{k}$$

$$\left(\cancel{\frac{4}{3}} \cdot \cancel{\frac{5}{4}} \cdot \frac{n+2}{n+1}\right)\left(\frac{1}{\cancel{2}} \cdot \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{4}} \cdots \frac{\cancel{n-1}}{n}\right)$$

$$\lim_{n\to\infty} \frac{(n+2)}{3n} = \frac{1}{3} \text{ Ans}$$

18) $\lim_{x\to 0} \left[\frac{\sin x}{x}\right] + \left[\frac{2\sin 2x}{x}\right] + \cdots \left[\frac{10\sin 10x}{x}\right]$

$[1^2-1] + [2^2-] + [3^2-] \cdots [10^2-]$

$= [1^2-1] + (2^2-1) + 10^2-1$

$= (1^2+2^2+ \cdots 10^2) - (10)$

$= \frac{10\times(10+1)(2\cdot 10+1)}{6} - 10$

$$\frac{A(R^n-1)}{R-1}$$

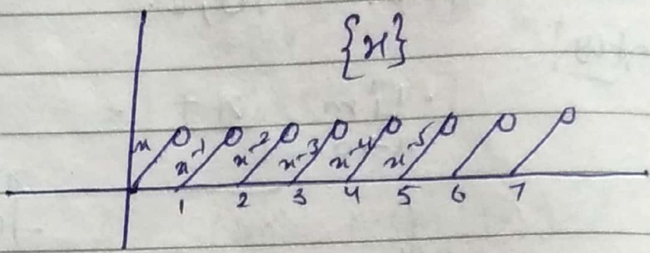
Sol (19)

$$f(x) = \frac{\sin\{x\}}{x^2+ax+b}$$

$f(3^+)$, $f(5^+)$ Exist, finite, non-zero

$$\lim_{x\to 3} \frac{\sin(x-3)}{(x-3)(x-5)}$$

$$= \frac{1}{3-5} = \frac{1}{-2}$$



$x^2+ax+b = (x-3)(x-5)$

3   5

$x^2-8x+15$

$a = -8$

$b = 15$ Ans

(21)

$a = (x+1)^2+2$ $b = \frac{1}{2}$

$(a)_{min} = 2$

$$\sum_{r=0}^{n} a^r b^{n-r} = b^n + a^1 b^{n-1} + a^2 b^{n-2} + \dots a^n$$

$$= \frac{b^n \cdot \left[\left(\frac{a}{b}\right)^{n+1} - 1\right]}{\frac{a}{b}-1}$$ Ans

$\sin^2$ $\sin x^2 = 1$

29/05/17

Que! $\dfrac{4 + \sin 2x + A\sin x + B\cos x}{x\tan x} = L$ (finite)

find A, B, L.

Ans!

$$\lim_{x \to 0} \frac{\frac{x + \text{---} - B\cos x}{x^2}}{\frac{\tan x}{x}}$$

$\downarrow$

$x \to 0$

$$\lim_{x \to 0} \frac{4 + \sin 2x + A\sin x + B\cos x}{x^2}$$

$$= \frac{\cancel{\sin\ 4 + \sin 2x + A\sin + B\cos x}}{x^2}$$

$$= \cancel{\sin 2x \quad 1 + \sin 2x}$$

$$\frac{4 + \sin 2x}{x^2} + \frac{A\sin x}{x^2} + \frac{B\cos x}{x^2}.$$

$$\cancel{A\sin x +} \quad \frac{4}{1} + \frac{1}{2} = \text{---} (1)$$

$$L = \lim_{x \to 0} \frac{x + \left((2x) - \left(\frac{(2x)^3}{\lfloor 3} + \ldots\right)\right) A\left(x - \frac{x^3}{\lfloor 3} + \ldots\right) + B\left(1 - \frac{x^2}{\lfloor 2} + \ldots\right)}{x^2}$$

$$\Rightarrow L = \lim_{x \to 0} \frac{\overset{0}{(4 + B)} + x\overset{0}{(2 + A)} + x^2 - \left(\frac{-B}{\lfloor 2}\right) + x^3(\ ) + x^4(\ ) + \ldots}{x^2}$$

$$\lim_{x \to 0} \frac{4 + \sin 2x + A\sin x + B\cos x}{x^2}$$ $4 + B = 0$

$A + 2 = 0$

Ans $\Rightarrow$ [$A = -2$, $B = -4$, $L = 2$ A]

$1-\frac{\cos x}{\sin x} = \frac{1}{2}$ $\boxed{\sin^4 x \Rightarrow 4}$

Left - Right = New Variable

used formula $\boxed{\frac{1-\cos x}{x^2} = 1}$

Que: $\lim_{x\to 0} \sin 8x \cdot \cot 3x$

Ans $\frac{\sin 8x}{8x} \cdot 8x \times \frac{\cot 3x}{3x} \cdot 3x$

$= 1 \times 8x \times 1 \times 3x$

$= 24x$

Ans: $\lim_{x\to 0} \frac{\sin 8x}{8x} \frac{8x}{3x} \cos 3x \frac{3x}{\sin 3x}$

$= 1 \cdot \frac{8}{3} \cdot 1 \cdot 1 = \frac{8}{3}$

(2) $\lim_{x\to 0} \frac{1-\cos 5x}{3x^2}$

$\frac{1-\cos 5x}{3x^2}$

$\frac{1-\cos 5}{3x}$

$\frac{\cos}{\sin}$

Ans: $\lim_{x\to 0} \frac{1-\cos 5x}{(5x)^2} \cdot \frac{(5x)^2}{3x^2}$

$= \frac{1}{2} \cdot \frac{25}{3} = \frac{25}{6}$

3) $\lim_{x\to 0} \frac{\cos 7x - \cos 9x}{\cos x - \cos 5x}$

$\frac{7x-9x}{x-5x} = \frac{-2x}{-4x}$

$\frac{1}{2}$ Ans

Ans: $\lim_{x\to 0} \frac{2\sin 8x \sin x}{2\sin 3x \sin 2x}$

$x\to 0 \quad \frac{\sin 8x}{8x} \cdot \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{3x}{\sin 3x} \cdot \frac{2x}{\sin 2x}$

$= 1 \cdot \frac{8}{6} = \frac{4}{3}$ Ans

Substitution

Que: $\lim_{x\to 1} (1-x) \tan \frac{\pi x}{2}$

$x-1 = y$

$(1-x)$

2) $\lim_{x\to 0} \frac{1-\cos x(1-\cos x)}{\sin^4 x}$

$\frac{1-\cos x(1-\cos x)}{4}$

$= 1-\cos^2 x - \cos x$

$1 - \frac{\sin x}{(1)} \; 1 - \frac{\cos x}{x} x$

$1-\left(\frac{x}{x}\right)$

$\frac{1-1\left(1-\cos x\right)}{4} = \frac{1+1+1+1}{+3-1} \over 4$

$= \frac{2}{4} = \frac{1}{2}$ Ans

3) $\lim_{x\to 0} \dfrac{x^2 \cos\frac{1}{x}}{\sin x}$

$\dfrac{x^2\cos\frac{1}{x}}{}$

$= 1 . 0 [-1 \text{ to } 1)$

$= 0$

(4) $\lim_{x\to \frac{\pi}{2}} \dfrac{\tan x}{\tan 3x}$ $= x - \frac{\pi}{2} = y$

$x \to \frac{\pi}{2},\ y \to 0$

$= -\cot y$

$\dfrac{\tan x}{\tan 3x} = \dfrac{\tan(\frac{\pi}{2}+y)}{\tan(\frac{3\pi}{2}+3y)}$

$= \dfrac{-\cot y}{-\cot 3y} = \dfrac{\cot y}{\cot 3y}$

$\lim_{y\to 0} \dfrac{\cot y}{\cot 3y}$

$\lim_{y\to 0} = 3\dfrac{\tan 3y}{\tan y} = \lim_{x\to 3}$

$\left(\dfrac{\tan 3y}{3y}\right) \cdot \dfrac{3y}{y} \cdot \dfrac{y}{\tan} = 3$

(1) Ans:

$x - 1 = y$ $\lim_{x\to 0} -y\tan\left(\frac{\pi}{2}(y+1)\right]$

$\lim_{y\to 0} -y\tan\left(\frac{\pi}{2} + \frac{\pi y}{2}\right)$

$\lim_{y\to 0} -y \times -\cot\frac{\pi y}{2}$

$\lim_{y\to 0} \dfrac{\frac{\pi}{2}y}{\sin\frac{\pi y}{2}} \quad \dfrac{\cos\frac{\pi y}{2}}{\pi/2}$

$1 . 1 . \frac{2}{\pi} = \frac{2}{\pi}$

(2) $\lim_{x \to 0} \frac{1-\cos x(1-\cos x)}{(1-\cos x)^2} \times \frac{(1-\cos x)^2}{x^4}$

$\frac{1-\cos x(1-\cos x)}{1-(\cos)^2} \cdot \left(\frac{1-\cos x}{x^2}\right)^2$

$= \frac{1}{2} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{8}$ Ans

* Standard limit !

* $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \; (a > 0)$

* $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

* $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$

$\Longrightarrow$ for Apply to all $\Rightarrow$ $\frac{a^{f(x)} - 1}{f(x)} = \ln a, \; \lim_{x \to a} f(x) \to 0.$

$\lim_{x \to 0} \frac{\ln(1+\sin x)}{\sin x} \cdot \frac{\sin x}{x} \cdot \frac{x}{x} \cdot \frac{x}{\tan x} = 1$

* $\lim_{x \to 0} \frac{e^{\tan x} - e^x}{\tan x - x}$

$= \frac{e^x\left[e^{(\tan x - x)} - 1\right]}{(\tan x - x)} = e^x \cdot 1$

$= e^0 \cdot 1$

$= 1$ Ans

Date | 30/05/17

Que: $\lim_{x\to 0} \frac{\ln(1+\sin x)}{3^x-1}$

Sol! $\frac{\ln(1+\sin x)\cdot x}{(3^x-1)x} = \frac{1+\sin x}{x} \quad \frac{x}{(3^x-1)} = \frac{\cancel{xxxx}}{3^x-1}$

$= \frac{\cancel{2x}\,(2x)}{3^x-1}$

Ans: $\frac{\ln(1+\sin x)}{\frac{3^x-1}{x}\ \sin x} \quad \frac{x}{\sin x} = \frac{1}{\ln 3 \cdot 1} = \frac{1}{\ln 3}$

2) $\lim_{x\to e} \frac{\ln x - 1}{x-e}$

$x - e = y$. $\quad \frac{\ln y - 1}{x-e} = \ln(\quad \frac{\ln y - 1}{y} = \frac{y(\ln - \frac{1}{y})}{y}$

$\ln - \frac{1}{y}$

Ans $\lim_{y\to 0} \frac{\ln(y+e)-1}{y} = \lim_{y\to 0} \frac{1+\ln(1+\frac{y}{e})-1}{\frac{y}{e}\times e}$

$= \frac{1}{e}$

(3) $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$

Ans: $\frac{e^{x^2} - \cos x}{x^2 - 1} = \frac{e^{x^2} - \cos x}{2 - 1} =$

$$\lim_{x \to 0} \left[\frac{(e^{x^2} - 1)}{x^2} + \frac{(1 - \cos x)}{x^2} \right]$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$= 1 + \frac{1}{2} = \frac{3}{2} \text{ Ans}$$

Que: If $\lim_{x \to 0} \frac{x^2 + ax + b}{\ln^2(2 - x)} = l$ (exist & finite).

then find $a + b + l = ?$

$\left[(\ln x)^2 = \ln^2 x \right]$

Ans:

$$\frac{x^2 + ax + b}{(x-1)^2}$$

$\frac{(x-3)(x-4)}{(x-1)^2}$ X

$\frac{(x-1)(x-4)}{(x-1)^2}$ X

$$\lim_{x \to 1} \frac{(x-1)^2}{(x-1)^2} = 1$$

$x^2 + ax + b = x^2 + 1 - 2x$

$b = 1, a = -2, l = 1$

$\therefore a + b + l = 0$ Ans

$\frac{1}{\infty} = \infty$

Que: $\lim_{x\to 0} \left(\ln(1+\sin^2 x)\right) \cdot \cot\left(\ln^2(1+x)\right)$

$$, \left(\frac{\ln(1+\sin^2 x)}{\sin^2 x}\right) \cdot \left(\frac{\cos(\ln^2(1+x))}{\frac{\sin(\ln^2(1+x))}{\ln^2(1+x)}}\right) \cdot \sin^2 x$$

$$\times \left(\frac{\ln(1+x)}{x}\right)^2 \cdot x^2$$

$1 \cdot \frac{1}{1} \cdot \frac{1}{1} = 1$ Ans

$*$ (1^∞)

$$\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x = e$$

$x = \frac{1}{y}$

$$\lim_{x\to 0} (1+x)^{1/x} = e$$

$\boxed{2.7 < e < 2.8}$

$\lim_{x\to a} f(x) = 1$ & $\lim_{x\to a} g(x) \to \infty$

$$\lim_{x\to a} \left(f(x)\right)^{g(x)} \quad (1^\infty)$$

$\boxed{\text{Identity: } a^{\log_a N} = N}$

Imp: $\frac{\infty}{\infty}, \frac{0}{0}, \underline{(1)^{\infty}}$

1 की ओर $f(x)$ approach
∞ " " $g(x)$

$$\lim_{x\to a} \log_e (f^g) = \lim_{x\to a} e^{g \ln f}$$

$$\lim_{x\to a} e^{g \ln(1+(f-1))} = \lim_{x\to a} e$$

$$\lim_{x\to a} g \cdot \underbrace{\frac{\ln(1+(f(x)-1))}{f(x)-1}}_{1} \times f(x)-1$$

$$\boxed{\lim_{x\to a} e^{g(x)\cdot(f(x)-1)}}$$

Very Important.

Que:

$\left(1 \text{ से बड़ा}\right)^{\infty} \to \infty \quad \left(1 \text{ से छोटा}\right)^{\infty} = 0$

$(1.000001)^{\infty} \to \infty \quad$ i.e. $\left(\frac{1}{2}\right)^{\infty} \to 0$

$$\lim_{x\to a} \frac{f'(x)}{g'(x)} = \lim_{x\to a} \frac{f''(x)}{g''(x)}$$

Que:

$$\lim_{x\to a} \frac{f(x)}{g(x)} \left(\frac{0}{0} \text{ or } \frac{\infty}{\infty}\right)$$

Que:

$$\lim_{x\to 0} \left(\underbrace{\tan\left(\frac{\pi}{4}+x\right)}_{f(x)}\right)^{\overbrace{1/x}^{g(x)}}$$

$\tan\frac{\pi}{4}$ $\frac{180}{4}$

$\tan\left(\frac{\pi}{4}+x\right)$
$f(x)$

$$\lim_{x\to a} e^{\frac{1}{x}\cdot\left(\tan\frac{\pi}{4}-1\right)}$$

$\left[\tan\frac{\pi}{2} = \infty\right]$ $\left(\frac{1}{0}\right) = \infty$

(1^{∞})

Ans $e^{\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)-1\right)\cdot\frac{1}{x}}$

$\frac{\pi}{2} = 90$

$e^{\lim_{x\to 0}\left(\frac{1+\tan x}{1-\tan x}-1\right)\cdot\frac{1}{x}}$

$\lim_{x\to 0} \frac{2}{1-\tan x}\frac{\tan x}{x}\cdot$ $\left(\frac{0}{0}\right)$

$= e$

$= e^{2}$ Ans

Que: $\lim_{x\to 1} (2-x)^{\tan\frac{\pi x}{2}}$ ∞ $g(x)$

$\tan\frac{\pi}{2} = \infty$

Ans: (1)

$$\boxed{\lim_{x\to 1} \quad e^{\lim_{x\to a}(f(x)-1)\,g(x)} \quad = \left(\tan\frac{\pi x}{2}-1\right)^{(2-x)}}$$

$(0-1)^{1}$ $(-1)^{1}$

$=$

$e^{\lim_{x\to 1}(2-x-1)\tan\frac{\pi x}{2}}$

$\lim_{x\to 1} (1-x)\tan\frac{\pi}{2}x$ $(0\cdot\infty)$

L'Hospital $\to$ $e^{\lim_{x\to 1}\frac{1-x}{\cot\frac{x\pi}{2}}}$ $\left(\frac{0}{0}\right)$

$e^{\lim_{y\to 0}\frac{-y\,\frac{\pi}{2}}{-\tan\frac{\pi y}{2}}} = \frac{\quad}{\pi/2}$

$= e^{2/\pi}$ Ans

$x \to \infty$

$x \to \frac{1}{y}$

$\lim_{y \to 0}$

Que: $\lim_{x \to \infty} \left(\sin \frac{1}{x} + \cos \frac{1}{x}\right)^{x}$

$x = \frac{1}{y}$

$$e^{\lim_{x \to \infty} \left(\sin \frac{1}{x} + \cos \frac{1}{x} - 1\right) x}$$

$$= e^{\lim_{y \to 0} \left(\frac{\sin y + \cos y - 1}{y}\right)}$$

$$= e^{\lim_{y \to 0} \left(y - \frac{y^3}{\lfloor 3} + \cdots\right) + \left(1 - \frac{y^2}{\lfloor 2} + \cdots\right)}$$

$$= e^{1}$$

$$\left\{ \begin{array}{l} \lim_{y \to 0} \frac{\sin y}{y} - \left(\frac{1 - \cos y}{y^2} \cdot y\right) \\ = e^{1 - \frac{1}{2} \cdot 0} = e^{1} \end{array} \right.$$

Que: $\lim_{x \to 1} \left(\frac{1 + x}{2 + x}\right)^{\frac{1 - \sqrt{x}}{1 - x}}$

$$\lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} = \left(\frac{0}{0}\right)$$

$$\lim_{x \to 1} \frac{(1 - \sqrt{x})}{(1 - \sqrt{x})(1 + \sqrt{x})} = \frac{1}{2}$$

$$= \left(\frac{2}{3}\right)^{1/2}$$

Que: $\lim_{x \to \infty} \left(\frac{x + 6}{x + 1}\right)^{x + 4}$ (1)

(2) $\lim_{x \to \infty} \left(\frac{x^2 + 2x - 1}{4x^2 - 3x - 2}\right)^{\frac{2x + 1}{x - 1}}$

Ans

$$e^{\lim_{x \to \infty} \left(\frac{x + 6}{x + 1} - 1\right)(x + 4)}$$

$$= e^{\lim_{x \to \infty} \frac{5}{(x + 1)}(x + 4)}$$

$$= e^{5} \text{ Ans}$$

Ans

$$\left(\frac{1}{2}\right)^{2} = \frac{1}{4}$$

સૌથી મોટાં વડે common લેવાનું

Que: $\lim_{x\to\infty}\left(5^{\frac{1}{n}}+3^{\frac{1}{n}}-1\right)^{n}$ $\left[5^{\frac{1}{n}}\left(1+\left(\frac{3}{5}\right)^{1/n}-\left(\frac{1}{5}\right)^{n}\right)\right]^{n}$

$(ab)^n = a^n \cdot b^n$

$$e^{\lim_{x\to\infty} 5\cdot\underbrace{\left[1+\left(\frac{3}{8}\right)^{\frac{1}{n}}-\left(\frac{1}{5}\right)^{1/n}\right]}_{f}^{n}} \text{ — } g$$

$$= e^{\lim_{x\to\infty} 5\cdot\left[\left(\frac{3}{5}\right)^{\frac{1}{n}}-\left(\frac{1}{5}\right)^{\frac{1}{n}}\right]\cdot n} \qquad n=\frac{1}{y}$$

standard limit $\lim_{x\to 0}\frac{a^n-1}{n}=\ln a.$

$$e^{\lim_{y\to 0}\frac{\left[\left(\frac{3}{5}\right)^{y}-\left(\frac{1}{5}\right)^{y}\right]}{y}}$$

$$e^{\lim_{y\to 0}\left(\frac{\left(\frac{3}{5}\right)^{y}-1}{y}-\frac{\left(\frac{1}{5}\right)^{y}-1}{y}\right)5}$$

$$= 5\ln\frac{3}{5}-\ln\frac{1}{5}$$

$= e$ Ans

* <u>Sandwich theorem</u>: (SE

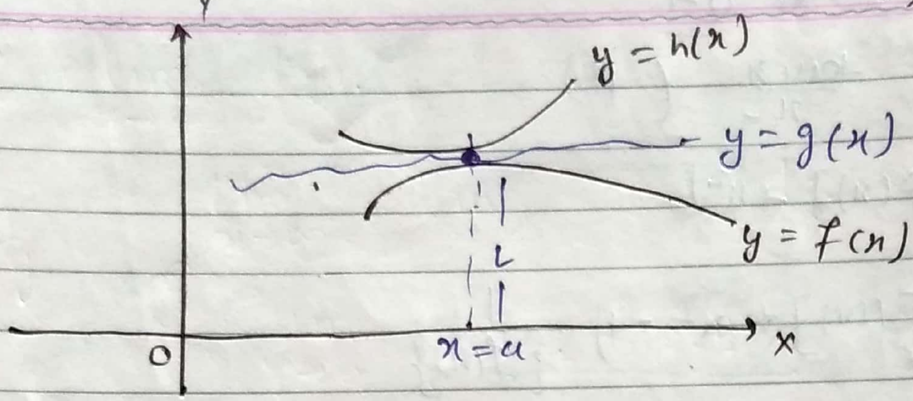
If f, g, h are three function such that

$f(x) \le g(x) \le h(x)\ \forall x$ Containing point $x=a$

$(1)^{\infty}$

H.W :- S-1, 3, 4, 5, 6, 7, 12, 16, 17, 18, 19, 20,

O-1 :- 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34.



* if $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = l$

then $\lim_{x \to a} g(x) = l$.

- Here $x = a$ may or may not in the domain of function.
- a may be infinite or finite
- (l) May be finite or infinite (l).

Date: 31/05/17

Ex' 0-1

(32) $f(x) = \frac{\tan x}{x}$ (1^+)

fup

$\lim_{x\to 0}\left([f(x)] + x^2\right)$

$\lim_{x\to 0}\left([f(x)] + x^2 - 1\right) \frac{1}{\{f(x)\}}$

$e^{\lim_{x\to 0}\left(\frac{x^2}{\frac{\tan x}{x} - 1}\right)} = e^{\lim_{x\to 0} \frac{x^3}{\tan x - x}}$

fup2

(33)

$\log_a a^N = N$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^2}{3} - \cdots$

Ans: $\log_{x\to\infty} \frac{e^n}{(1+\frac{1}{n})^{n^2}} = \lim_{x\to\infty} \frac{e^n}{e^{n^2 \ln(1+\frac{1}{n})}}$

$\ln\left(1+\frac{1}{n}\right)^{n^2} = x$

$e^{n^2 \ln(1+\frac{1}{n})}$

$= e^{n^2\left(\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} - \cdots\right)}$

$= e^{\left(n - \frac{1}{2n} + \frac{1}{3n^2} - \cdots\right)}$

$\lim_{n\to\infty} \frac{e^n}{e^n \cdot e^{\left(-\frac{1}{2} + \frac{1}{3n} + \cdots\right)}} = \frac{1}{e^{-\frac{1}{2}}} = \sqrt{e}$

(42) $l = \lim_{x\to\infty} \frac{a^{2x}-1}{a^{2x}+1}$

$0 < a < 1$

$l = \frac{0-1}{0+1}$

$= -1$ Ans

$a > 1$ $\left(\frac{\infty}{\infty}\right)$

$l = \lim_{x\to\infty} \frac{a^{2x}(1-a^{-2x})}{a^{2x}(1+a^{-2x})}$

$= 1$ Ans

Ex δ-1

Series
n-Hospital
2 Time differen

(7) $\lim_{x\to\infty} x^2\left[\left(1+\frac{2}{x}\right)^{1/2} - \left(1+\frac{3}{x}\right)^{1/3}\right]$

$\lim_{x\to 0} \frac{(1+2y)^{1/2} - (1+3y)^{1/3}}{y^2}$ $\left(\frac{0}{0}\right)$

$= (1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2} x^2$

$\lim_{y\to 0} \frac{(1+2y)^{1/2} - (1+3y)^{1/3}}{}$

$\frac{\left[1 + \frac{1}{2}\cdot 2y + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}(2y)^2 + \cdots\right] - \left[1 + \frac{1}{3}(3y) + \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{\lfloor 2}\cdot(3y)^2 + \cdots\right]}{y^2}$

$= \lim_{y\to 0} \frac{y^2\left[\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)2^2}{\lfloor 2} - \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)\cdot 3^2}{\lfloor 2}\right] + y^3(\ ) + y^4(\ ) + \cdots}{y^2}$

$= \frac{\frac{1}{2}\left(\frac{1}{3}-1\right)}{\lfloor 2}\cdot 2^2 - \frac{\frac{1}{3}\left(\frac{1}{3}-1\right)3^2}{\lfloor 2}$ Ans.

(12) $\sqrt{2} - \sqrt{1+\cos x}$

$\sqrt{2} - \sqrt{1+2\cos^2\frac{x}{2} - 1}$

$= \sqrt{2} - \sqrt{2}\cos\frac{x}{2}$

$= \sqrt{2}\cdot\left(1-\cos\frac{x}{2}\right)$

$27^x = (3\cdot 9)^x = 3^x\cdot 9^x$

$27^x - 9^x - 3^x + 1 = 3^x\cdot 9^x - 9^x - 3^x + 1$

$= (3^x-1)(9^x-1)$

$$\lim_{x\to 0} \frac{(3^x-1)(9^x-1)}{\sqrt{2}\left(\frac{1-\cos\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\left(\frac{x}{2}\right)^2}$$

$\lim_{x\to 0}$

$$\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)\cdot\left(\frac{9^x-1}{x}\right)\cdot\frac{x^2}{\sqrt{2}\cdot\frac{1}{2}\cdot\frac{x^2}{4}}$$

$$= \ln 3\cdot\ln 9\cdot\frac{1}{\sqrt{2}/8}.$$

(6) $\sin\left(\frac{\pi}{3}+2h\right) - 4\sin\left(\frac{\pi}{3}+3h\right) + 8\sin\left(\frac{\pi}{3}+2h\right) - \text{ ---}$

$\sin^2 C + \sin D = 2\sin\frac{C+D}{2}\cdot\cos\frac{C-D}{2}$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$$

(20) $a_n = 2^2 + 4^2 + 6^2$ —— n term.

$T_r = (2r)^2$

$\therefore a_n = \sum_{r=1}^{n} T_r = 4\sum_{r=1}^{n} r^2 = 4(1^2+2^2+\cdots n)$

$= \frac{4\cdot n(n+1)(2n+1)}{6}$

$T_r = A + (r-1)D$ 1, 3, 5

$= 1+(r-1)2$

$b_n = \sum_{r=1}^{n} (2r-1)^2$

H.W $\Rightarrow$ J.H and J. Adlauer

$= \sum 4r^2 + 1 + 4r$

$4\sum r^2 - 4\sum r + \sum 1$

$= 4 \cdot \frac{n(n+1)(2n+1)}{6} - 4 \cdot \frac{n(n+1)}{2} + n$

Ex! $4x - 9 \le f(x) \le x^2 - 4x + 7 \quad \forall\ x \ge 0$

find $\lim_{x \to 4} f(x)$

$\lim_{x\to 4} (4x-9) \le \lim_{x\to 4} f(x) \le \lim_{x \to 4} (x^2 - 4x + 7)$

$\downarrow \qquad \qquad \downarrow$

$7 \qquad \qquad 7$

$\therefore \lim_{x \to 4} f(x) = 7$

Que:

Evaluate $\lim_{n\to\infty} \left(\frac{n}{n^2+1} + \frac{n}{n^2+2} + \frac{n}{n^2+3} + \cdots \overbrace{\frac{n}{n^2+n}}^{\text{Very small}} \right)$

Ans!

$\frac{n}{n^2+n} < \frac{n}{n^2+1} = \frac{n}{n^2+1}$

$\frac{n}{n^2+n} < \frac{n}{n^2+2} < \frac{n}{n^2+1}$

$\frac{n}{n^2+n} < \frac{n}{n^2+3} < \frac{n}{n^2+1}$

$\frac{n}{n^2+1} = \frac{n}{n^2+n} < \frac{n}{n^2+1}$

$n \cdot \frac{n}{n^2+n} < \frac{n}{n^2+1} + \frac{n}{n^2+2} + \frac{n}{n^2+3} \cdots \frac{n}{n^2+n} < \frac{n \cdot n}{n^2+1}$

$$\lim_{n\to\infty} \frac{n \cdot n}{n^2+n} < \lim_{n\to\infty}\left(\frac{n}{n^2+1} + \frac{n}{n^2+2} + \cdots \frac{n}{n^2+n}\right) < \lim_{n\to\infty} \frac{n \cdot n}{n^2+1}$$

$$1 = \; ? \; = 1$$

$$\text{Ans} = 1$$

Que: let $f(1,\infty) \to (0,\infty)$ a continious decreasing function with

$$\lim_{n\to\infty} \frac{f(4n)}{f(8n)} = 1 \text{ then find } \frac{f(6n)}{f(8n)} = ?$$

$= 1$ Ans

$$8n > 6n > 4n$$

$$f(8n) < f(6n) < f(4n)$$

$$1 < \frac{f(6n)}{f(8n)} < \frac{f(4n)}{f(8n)}.$$

take $\lim_{n\to\infty}$

$$\lim_{n\to 0} 1 < \lim_{n\to\infty} \frac{f(6n)}{f(8n)} < \lim_{n\to\infty} \frac{f(4n)}{f(8n)}$$

$$1 = 1 = 1$$

Ans

Que: $\lim_{n\to 0} \dfrac{e^n - 1 - n}{n^2} = ?$

$= \cancel{Ef}\ \cancel{f}\ (e^{n^2}$

$$\lim_{n\to 0} \frac{1 + n + \frac{n^2}{L2} + \cdots) - 1 - n}{n^2} = \frac{1}{L2}$$

(ii) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

$\frac{x - \sin x}{x^3} \quad x - \left(x - \frac{x^3}{3x^3}\right.$

$\lim \frac{x - \left(x - \frac{x^3}{\lfloor 3} + - - \right).}{x^3}$

(iii) $\lim_{x \to \infty} \frac{x - \tan x}{x^3}$

$\frac{x - \tan x}{x^3} = \frac{x - x - x^3}{x^3} =$

$\lim_{x \to \infty} \left(x - x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{3x^7}{21}\right)$

Ans: $= -\frac{1}{3}$

Que: $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$

$e\left(\cancel{\quad}\right) =$

$\lim_{x \to 0} \frac{e\left(1 - \frac{x}{2} + \frac{11}{24}x^2 - - -\right) - e}{x}$

$= -\frac{e}{2}$

$N = a^{\log_a N} = \lim_{x \to 0} (1+x)^{1/x}$

$\frac{1}{x}\log(1+x)$

4 हो cot(a) − ∞ → 0

$N = a^{\log_a N}$

Method-1

$(1+x)^{\frac{1}{x}} = e^{\ln(1+x)^{1/x}} = e^{\frac{1}{x}(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots)}$

$= e^{(1 - \frac{x}{2} + \frac{x^2}{3} - \dots)}$

~~Built-in-limit~~

M-2

$$\lim_{x \to 0} \frac{e^{(1 - \frac{x}{2} + \frac{x^2}{3} \dots)} - e}{x}$$

$$= \lim_{x \to 0} e \frac{\left[e^{\left(-\frac{x}{2} + \frac{x^2}{3} - \dots\right)} - 1\right]}{\left(-\frac{x}{2} + \frac{x^3}{3} - \dots\right)} \cdot \frac{\left(-\frac{x}{2} + \frac{x^3}{3} - \dots\right)}{x}$$

$$= e \cdot 1 \cdot \left(-\frac{1}{2}\right] = -\frac{e}{2} \text{ Ans}$$

Built-in-limit:

$$f(x) = \lim_{n \to \infty} \frac{\tan \pi x^2 + (x+1)^n \sin x}{x^2 + (x+1)^n}$$

then find $\lim_{x \to 0} f(x)$.

$x < 0$

$\frac{\tan \pi x^2}{x^2}$

$x > 0$

$\frac{(x+1)^n \sin x}{(x+1)^n}$

$$\begin{cases} \sin x & x > 0 \\ \frac{\tan \pi x^2}{x^2} & x < 0 \end{cases}$$

R.H.L $= f(0)^+ = 0$

L.H.L $= f(0^-) = \lim_{x \to 0} \frac{\tan \pi x^2}{\pi x^2} \cdot \pi$

$a \to 0, \; t \to 1$

01/06/17

Ex: J.M.

(2) $x - 2 = \theta$

$$\lim_{\theta \to 0} \frac{\sqrt{1-\cos 2\theta}}{\theta} = \lim_{\theta \to 0} \frac{\sqrt{1-1+2\sin^2\theta}}{\theta}$$

$$\lim_{\theta \to 0} \frac{2|\sin\theta|}{\theta}$$

$\theta = 0^+ = 2$ Ans

$\theta \to 0^- = -2$ Ans

(J.A)

(4) $\left((1+a)^{1/3}-1\right)x^2 + \left((1+a)^{1/2}-1\right)x + \left((1+a)^{1/6}-1\right) = 0$

Let $(a+1)^6 = t^6$

$(t^2-1)x^2 + (t^3-1)x + (t-1) = 0$ $\boxed{a \to 0 \Rightarrow t \to 1}$

$(t-1)\cdot\left[(t+1)x^2 + (t^2+t+1)x + 1\right] = 0$

$2x^2 + 3x + 1 = 0$

$2x^2 + 2x + x + 1 = 0$

$(2x+1)(x+1) = 0$

$-\frac{1}{2}$, -1

(6) $$\lim_{x \to 0} \frac{x^2 \sin\beta \cdot (\beta x)}{(\beta x)\,(\alpha x - \sin x)}$$

$$\lim_{x \to 0} \frac{\beta x^3}{\alpha x - \left(x - \frac{x^2}{\lfloor 3} + \cdots\right)}$$

$\alpha - 1 = 0$ $$\frac{\beta x^3}{x(\alpha-1) + \frac{x^2}{\lfloor 3} + \cdots} = 1$$

$\beta = 1$

(1) $(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}$

$\frac{1}{2a} - \frac{1}{4} = 0$ $\quad L = \lim_{x\to 0} \frac{a - a\left(1 - \frac{x^2}{a^2}\right)^{1/2} - \frac{x^2}{4}}{x^4}$

$= \lim_{x\to 0} \frac{a - a\left[1 - \frac{1}{2}\cdot\frac{x^2}{a^2} + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}\cdot\frac{x^4}{a^4} - \cdots\right] - \frac{x^2}{4}}{x^4}$

$L = \lim_{x\to 0} \frac{x^2\cdot\left[\cancel{\frac{1}{2a} - \frac{1}{4}}\right]^{0} - \left[\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2\, a^3}\right] x^4 + \cdots}{x^4}$

$= \frac{-\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2\, a^3}$ $\quad \alpha - 1 = 0$

5) $\lim_{x\to 1} \left[\frac{\sin(x-1) - a(x-1)}{(x-1) + \sin(x-1)}\right]^{\frac{\cancel{(1-\sqrt{x})}\,(1+\sqrt{x})}{\cancel{1-\sqrt{x}}}}$

$\left[\frac{\frac{\sin(x-1)}{x-1} - a}{1 + \frac{\sin(x-1)}{(x-1)}}\right]^{(1+\sqrt{x})} = \left(\frac{1-a}{1+1}\right)^2 = \frac{1}{4}$ A

Solve it

SBG STUDY