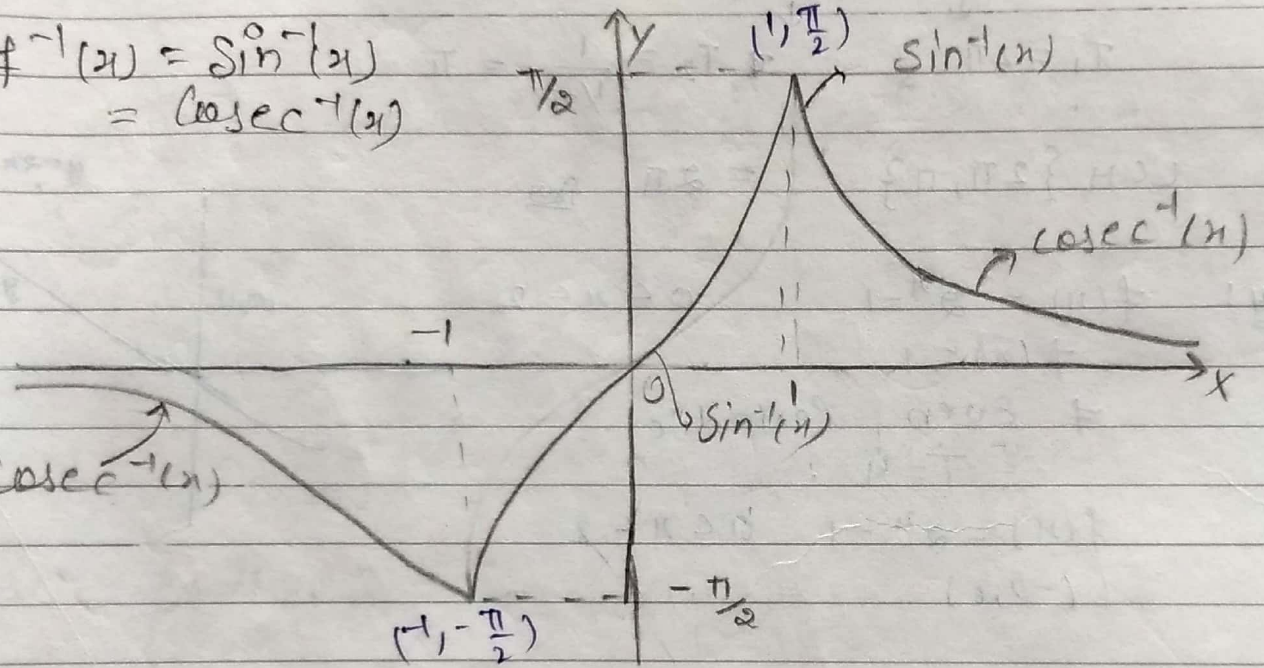
SBG STUDY

18/05/17

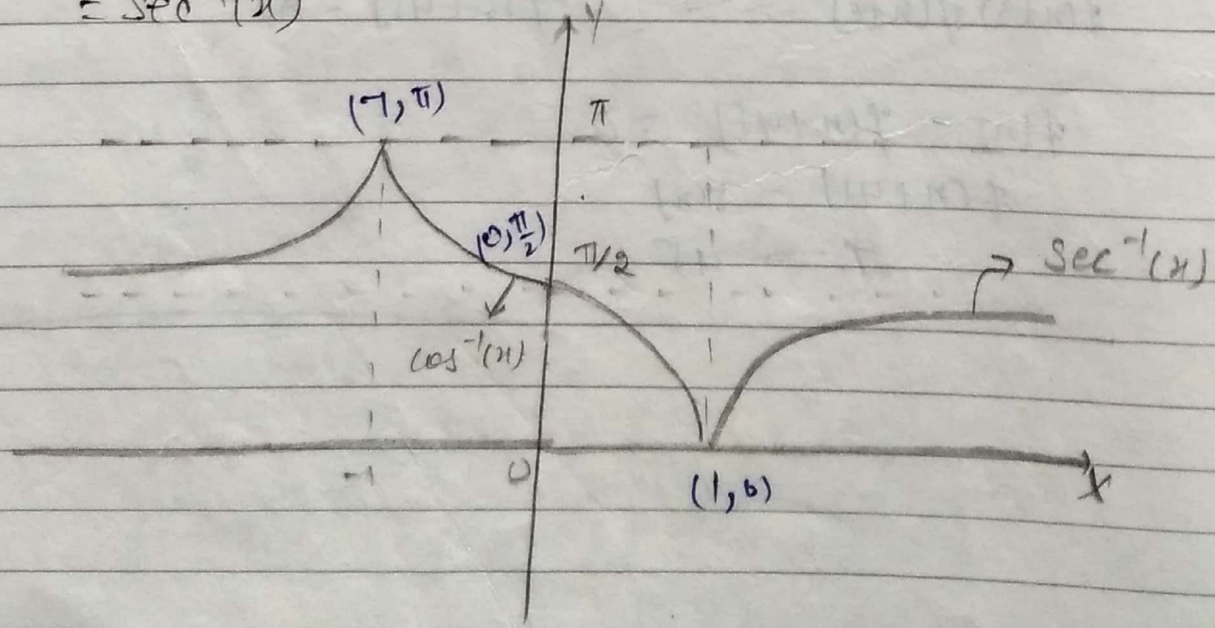
Inverse Trigonometric function :

* Graph :

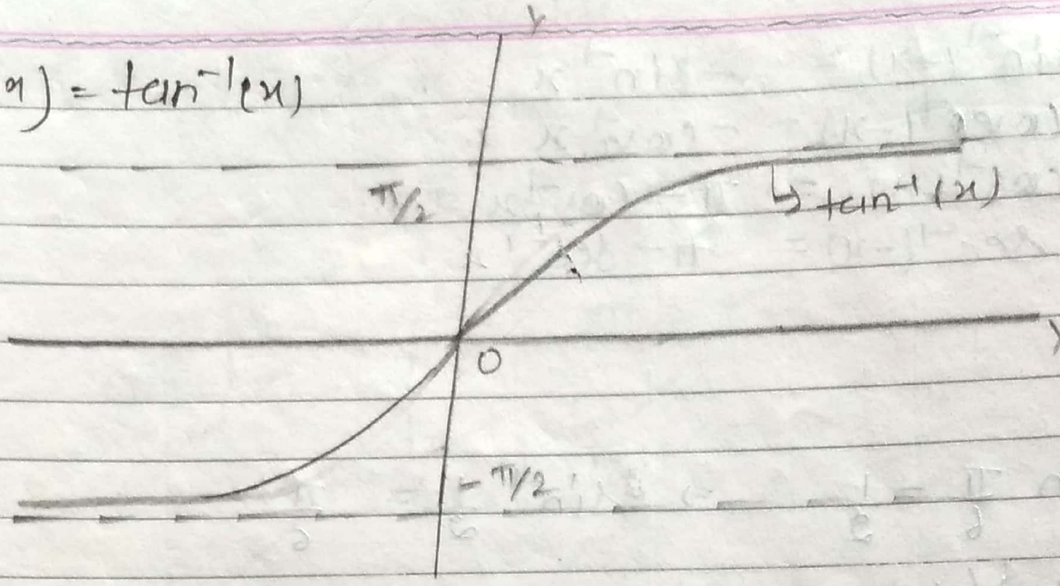
$f^{-1}(x) = \sin^{-1}(x)$
$= \operatorname{cosec}^{-1}(x)$



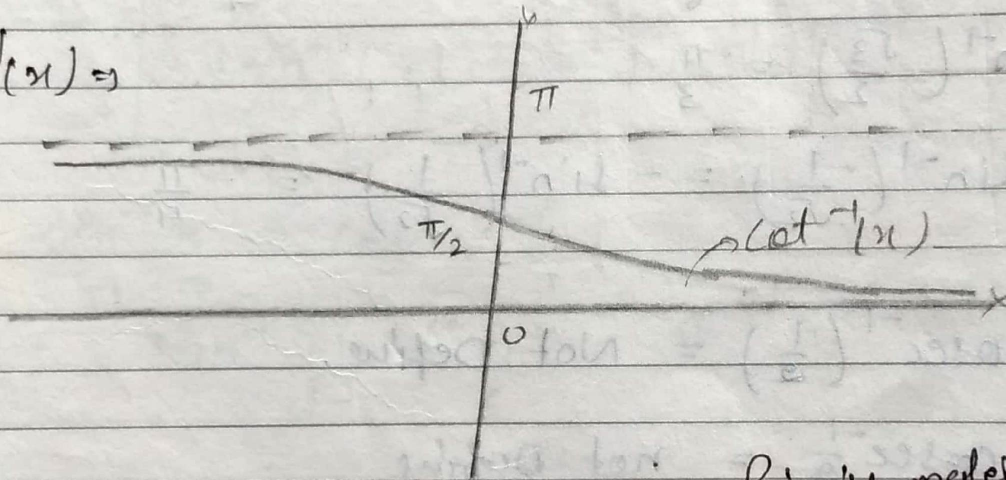
$f^{-1}(x) = \cos^{-1}(x)$
$= \sec^{-1}(x)$



$f^{-1}(x) = \tan^{-1}(x)$



$\cot^{-1}(x) \Rightarrow$



	Domain	Principle angles Range	Asymtot	Nature
$\sin^{-1}$	$[-1, 1)$ $\left\{ \begin{matrix} -1 \le x \le 1 \\ \text{or} \\ \lvert x \rvert \le 1 \end{matrix} \right\}$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	×	odd
$\operatorname{cosec}^{-1}(x)$	$(-\infty, -1] \cup [1, \infty]$ $[\lvert x \rvert \ge 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$y = 0$	odd
$\cos^{-1}(x)$	$[-1, 1]$	$[0, \pi$	×	Nei. E nor O
$\sec^{-1} x$	$[-\infty, -1] \cup [1, \infty]$ $[\lvert x \rvert \ge 1]$	$[0, \pi] - \frac{\pi}{2}$	$y = \frac{\pi}{2}$	Neither.
$\tan^{-1}$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$y = \pm \frac{\pi}{2}$	odd
$\cot^{-1}$	R	$(0, \pi)$	$y = 0, y = \pi$	Neithe

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\sin^{-1}(-x) = -\sin^{-1} x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\sin \frac{\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$

$\operatorname{cosec}^{-1}\left(\frac{1}{2}\right)$ = Not Define.

$\operatorname{cosec}^{-1} \frac{1}{2}$ = not Define

$= \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$ not define

$\sin \frac{\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$

$\boxed{\sin \frac{5\pi}{6} = \frac{1}{2} \Rightarrow \sin^{-1} \frac{1}{2} = \frac{5\pi}{6} \times} \times$

$\boxed{\text{arc } \tan x = \tan^{-1} x}$

कौ सैक

$\pi -$ cos sec cot

max value $\pi/2$

* $\sin^{-1} x$

min value $-\frac{\pi}{2}$

max π

* $\cos^{-1}(x)$

min 0

$$\boxed{\begin{aligned} \cot^{-1}(-x) &= \pi - \cot^{-1} x \\ \tan^{-1}(-x) &= -\tan^{-1} x \end{aligned}}$$

$\sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}$

$\tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$

$\cot^{-1}(1) = \frac{\pi}{4}$

$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3})$

$= \pi - \tan^{-1} \frac{1}{\sqrt{3}}$

$= \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Note

$$\boxed{\begin{aligned} \sin x &= \frac{1}{\operatorname{cosec}} \\ \sin^{-1} x &= \frac{1}{\cos^{-1} x} \end{aligned}}$$

X

$= \operatorname{cosec}^{-1} \frac{1}{x}$

Que: find $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + \cot^{-1}\sqrt{3} - \tan^{-1}(1) + \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}\frac{2}{\sqrt{3}} + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Ans:

$$\cancel{\sin\frac{1}{\sqrt{2}} + \cot} \quad + \cancel{\sin \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)} + \cancel{\cos^{-1}\frac{\sqrt{3}}{2}} + \cancel{\tan^{-1}\left(\sqrt{3}\right)}$$

Ans: $\Rightarrow$

$$\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) + \tan^{-1}\frac{1}{\sqrt{3}} - \frac{\pi}{4} + \frac{\pi}{3} + \cos^{-1}\frac{1}{2} - \sin^{-1}\frac{\sqrt{3}}{2} + \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$= \pi - \frac{\pi}{4} + \frac{\pi}{6} - \frac{\pi}{4} + \frac{\pi}{3} + \frac{\pi}{3} - \frac{\pi}{3} + \pi - \frac{\pi}{3}$$

$$= \frac{3\pi}{2} + \frac{\pi}{6}$$

$$= \frac{10\pi}{6} = \frac{5\pi}{3}$$

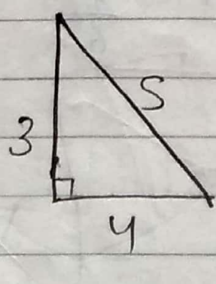
Que: 2: Solve $\left(2\sin^{-1}\frac{3}{5}\right) = 2\sin\theta\cos\theta$

$= 2 \times \frac{3}{5} \cdot \frac{4}{5}$

$= \frac{24}{25}$

$\sin^{-1}\frac{3}{5} = \theta$

$\sin\theta = \frac{3}{5}$

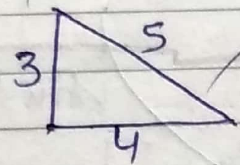


Que: $\sin\left(\underbrace{\arcsin \frac{3}{5}}_{\theta_1} - \underbrace{\arccos \frac{3}{5}}_{\theta_2}\right)$

$= \sin(\theta_1 - \theta_2)$ ~~sin~~

[$\arcsin \frac{3}{5}$

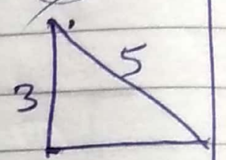
$\cancel{\sin\theta} = \frac{3}{5}$



$\cos\theta_1 = \frac{4}{5}$

$\arccos \frac{3}{5}$

$\cos = \frac{3}{5}$



]

$= \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$

$\frac{3}{5} \cdot \frac{3}{5} - \frac{4}{5} \cdot \frac{4}{5}$

$= \frac{-7}{25}$

Que: 3) $\sin(2\sin^{-1} \frac{3}{5})$

E: $\sin\left(\tan^{-1} \cos \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right)$

[$\cancel{\tan \cos \frac{\pi}{3}}$ $-\cot($]

$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \cot^{-1}\frac{1}{\sqrt{3}}$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\sin\left(\tan^{-1}\left(\underbrace{\cos \frac{2\pi}{3}}_{-\frac{1}{2}}\right)\right)$ ←

$\sin \tan^{-1}\left(-\frac{1}{2}\right) = \sin\left(-\left(\tan^{-1}\frac{1}{2}\right)\right)$

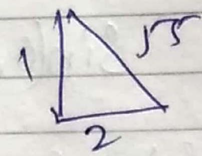
[1,2)

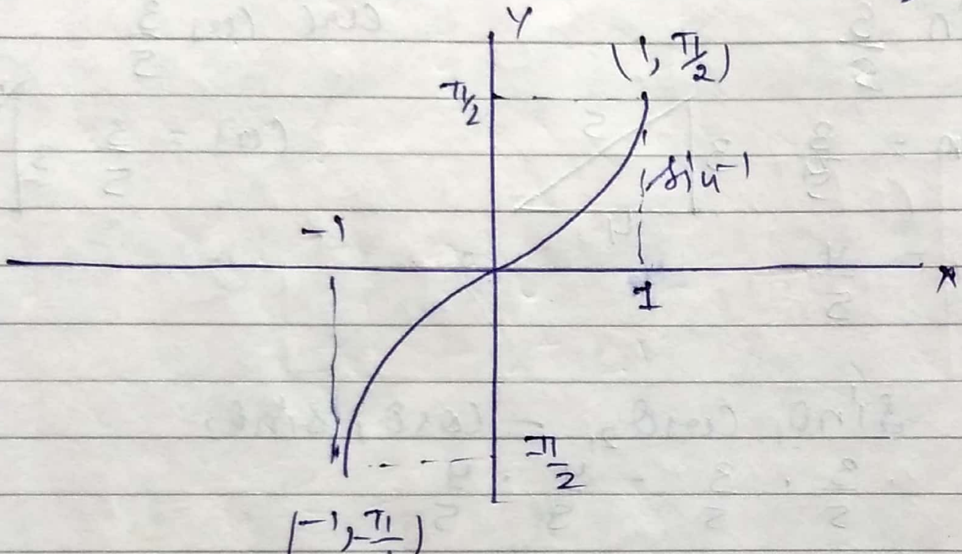
p
h

$-\sin\left(\tan^{-1}\frac{1}{2}\right)$ $\qquad$ $\tan^{-1}\frac{1}{2} = \theta$

$= -\sin\theta$ $\qquad$ $\tan\theta = \frac{1}{2}$

$= -\frac{1}{\sqrt{5}}$





Que: find domain of $\sin^{-1}(x^2)$ $\qquad$ $a^2 \le x^2 \le b^2$

$x \in (-b, -c) \cup \{a, b\}$

$|x^2| \le 1$

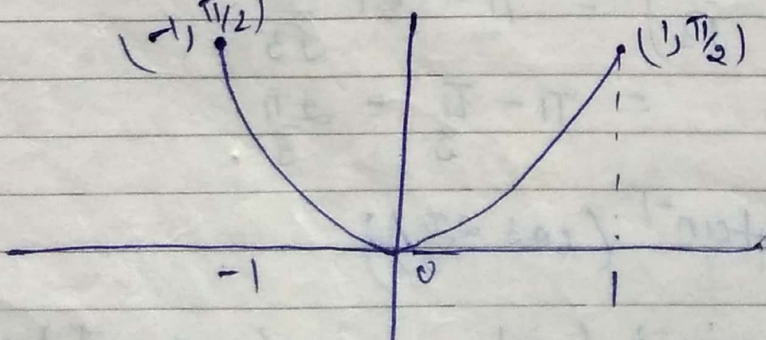
or

$-1 \le x^2 \le 1$

$0 \le x^2 \le 1 \Rightarrow x \in (-1, 0] \cup [0, 1]$

Domain $\in [-1, 1]$

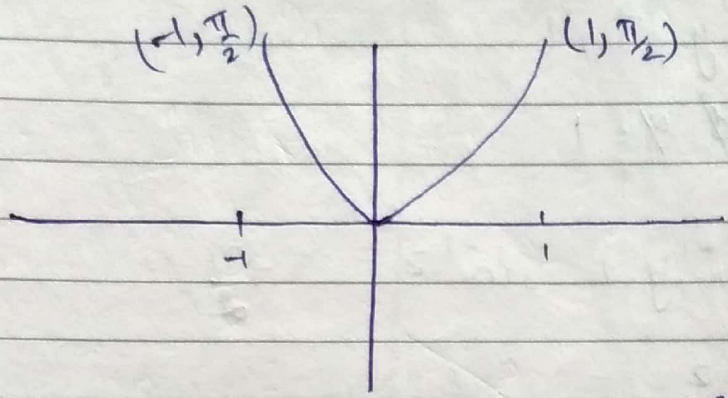
Range $\Rightarrow \left[0, \frac{\pi}{2}\right]$



$\{0\} = [0,1]$

[redacted]

Que:- find D/R $y = \sin^{-1}[x]$



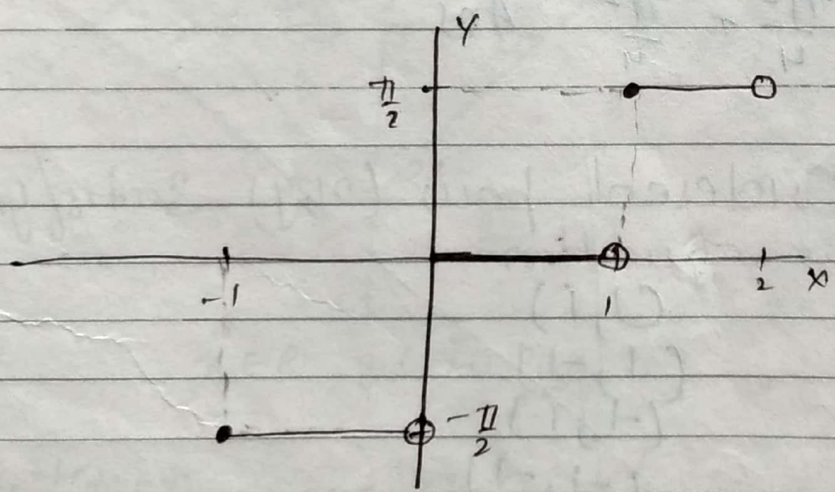
$y = \sin^{-1}[x]$ Range: $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

$-1 \le [x] \le 1$

$[x] = -1, 0, 1$

Domain - $\boxed{-1 \le x < 2}$

Que! 1



Que:- $\sin^{-1}\{x\}$

Domain - R. $[D \in R]$

sheet: 0-1, 1, 2, 3, 4, 10,
(5-1) 1, 2(a)(b) (3)

H.W = 55 Page
Figure.

Boundry uc solues

Boundry Value Problem:-

* If $\sin^{-1}x + \sin^{-1}y = \pi$

$= \; x = 1 \;\&\; y = 1$

* If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = -\frac{3\pi}{2}$

$x = -1 = y = z$

* If $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 = \frac{\pi^2}{2}$

$x = \pm 1 \;,\; y = \pm 1$

$\frac{\pi^2}{4} + \frac{\pi^2}{4}$ Ans

* Find Ordered pair (x,y) satisfying above equation.

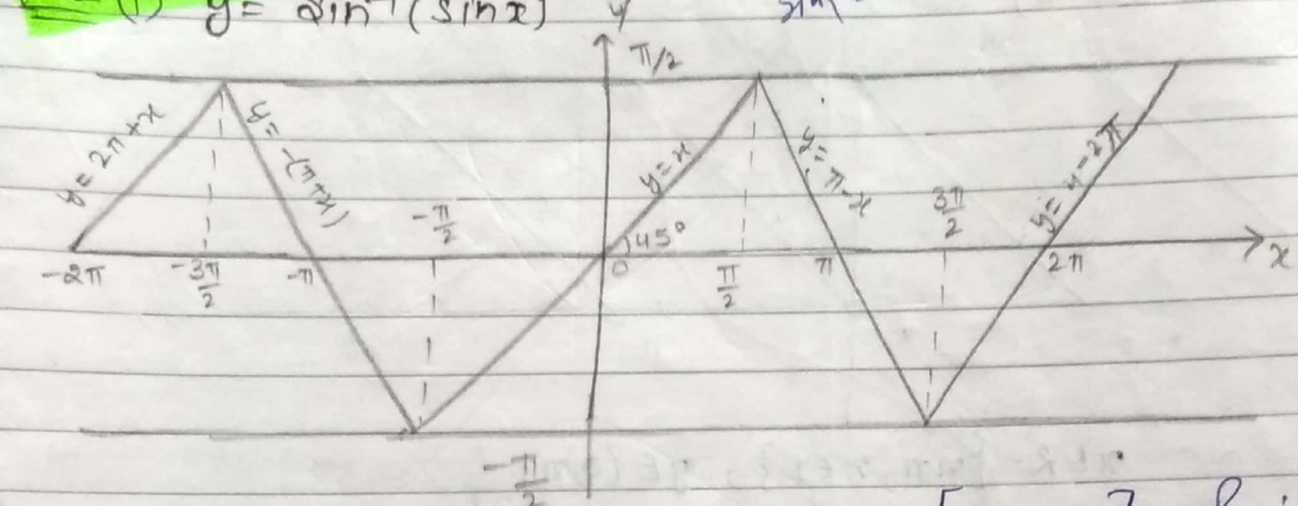
(x,y) $(1,1)$
$(1,-1)$
$(-1,1)$
$(-1,-1)$

* If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$x = y = z = -1$

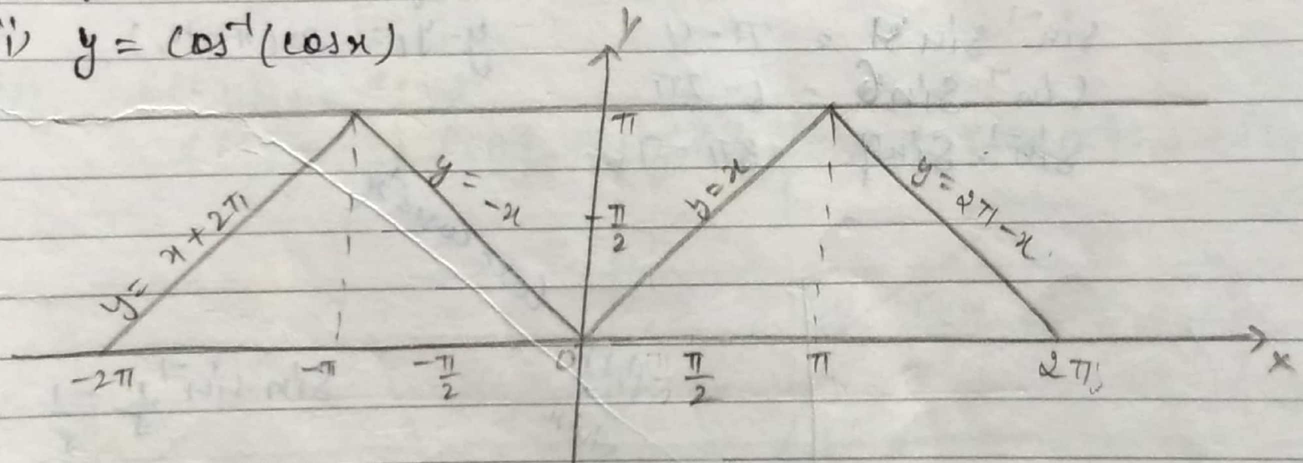
$$y - y_1 = m(x - x_1) \qquad y - 0 = 1(x - 2\pi)$$

P-2 (i) $y = \sin^{-1}(\sin x)$



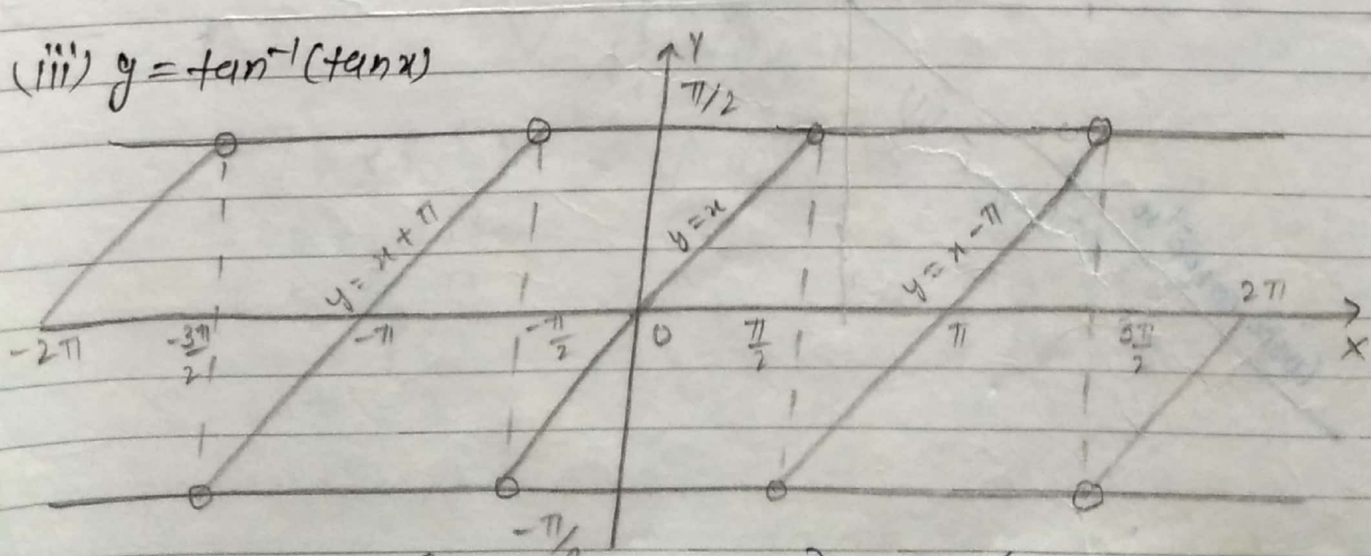
$x \in R, \ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Periodic.

(ii) $y = \cos^{-1}(\cos x)$



$x \in R, \ y \in [0, \pi]$

(iii) $y = \tan^{-1}(\tan x)$



$x \in R - \left\{(2n-1)\frac{\pi}{2}, n \in I\right\}; \ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

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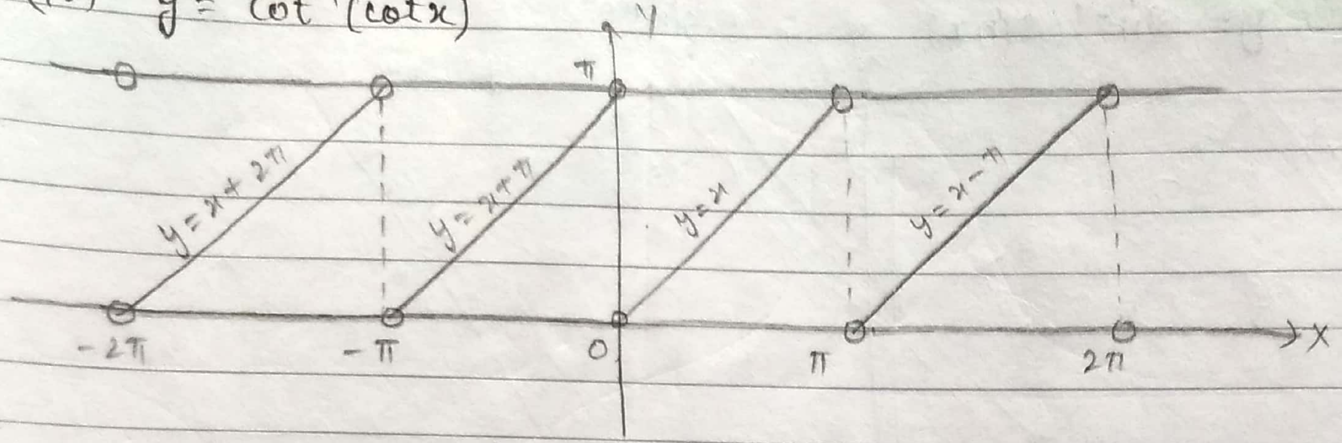
$cosec^{-1} x = \phi$

$\sin^{-1} \sin \frac{1}{5} = \frac{1}{5}$

(iv) $y = \cot^{-1}(\cot x)$



$x \in R - \{n\pi, n \in I\}, \; y \in (0, \pi)$

Properties - 1) $\sin^{-1} \sin 1 = 1$

$\sin^{-1} \sin 2 = \pi - 2$

$\sin^{-1} \sin 4 = \pi - 4$

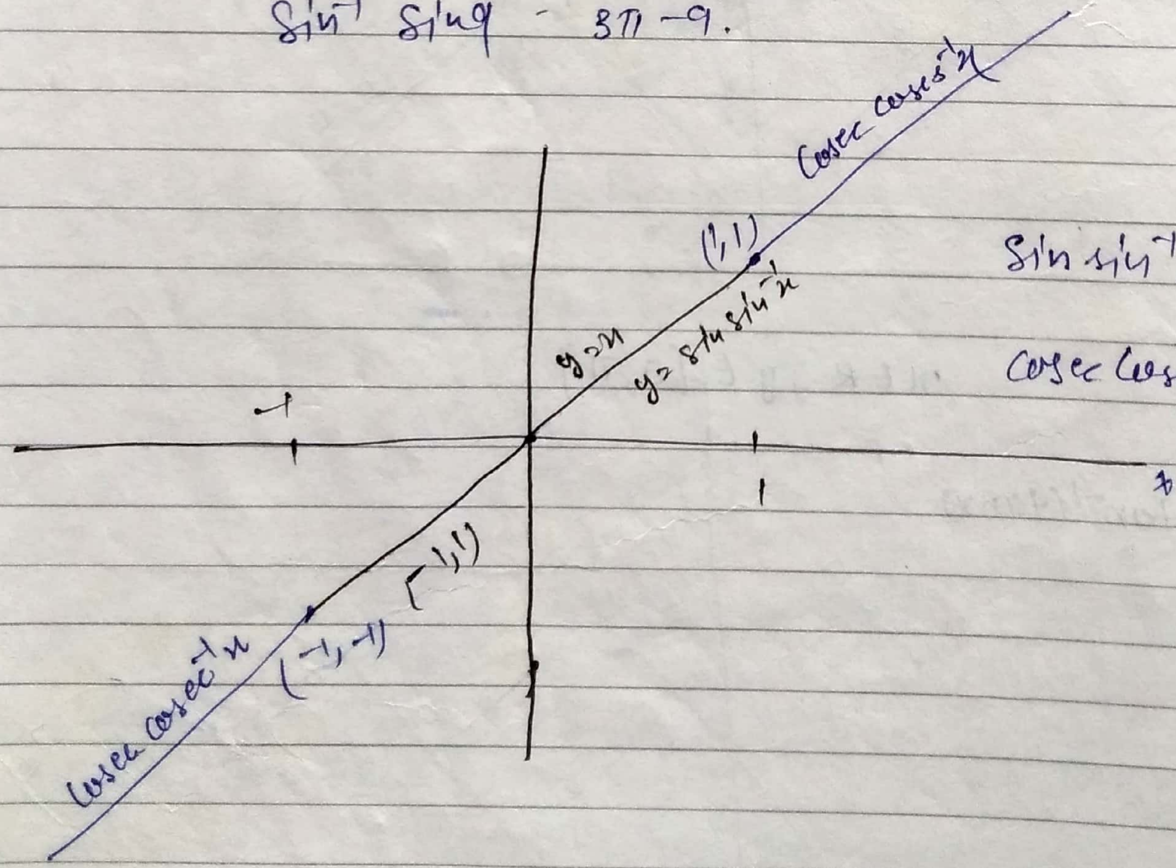
$\sin^{-1} \sin 6 = 6 - 2\pi$

$\sin^{-1} \sin 9 = 3\pi - 9.$

$y - 0 = 1(x - 2\pi)$

$y - 0 = -1(x - \pi)$

$y - y_1 = m(x - x_1)$



$\sin \sin^{-1} \frac{1}{5} = \frac{1}{5}$

$cosec\, cosec^{-1}(-5) = -5$

$\alpha + \frac{1}{\alpha} \in (-\infty, -2) \cup (2, \infty)$

19/05/17 Exercise : 0-1

(8) $\tan^{-1}\left(1 - x^2 - \frac{1}{x^2}\right) + \sin^{-1}\left(x^2 + \frac{1}{x^2} - 1\right)$

$x = \pm 1$

Domain = $\{1, -1\}$.

$\alpha + \frac{1}{\alpha} \in (-\infty, -2] \cup (2, \infty)$

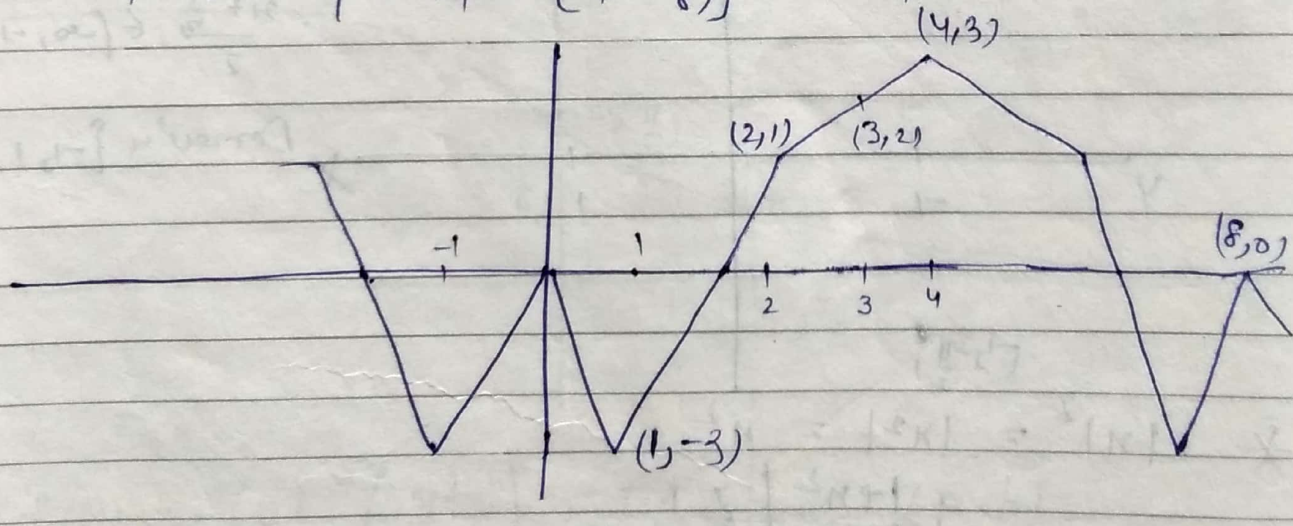
$x^2 + \frac{1}{x^2} \in [2, \infty]$

(10) $\sin\left(\underbrace{2\cos^{-1}\frac{1}{\sqrt{5}}}_{\theta_1}\right) + \cos\left(\underbrace{2\tan^{-1}\left(\frac{1}{3}\right)}_{\theta_2}\right) = \frac{p}{q}$

QS - 1

(1) $f(-x) = f(x)$

$f(3) + 2|f(1)| + \left[f\left(\frac{7}{8}\right)\right] + \cos^{-1} f(2) + f(7) + f(20)$

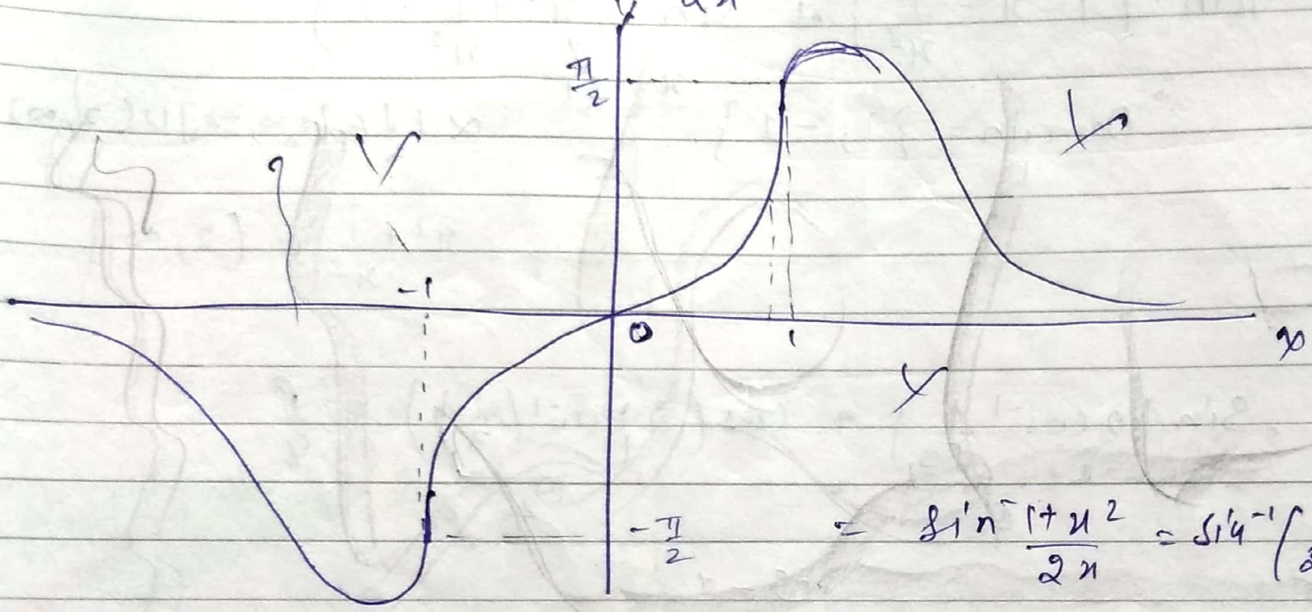


$f(8 \times 2 + 4) = f(4) = 3$

$f(8 + (-1))$
$= f(-1)$
$= f(1)$
$= -3$

$S(24 + x) = S(x)$
$S(24 \times 2 + x) = S(x)$
$f(x + nT) = f(x)$.

Que: $f(x) = \sin^{-1} \frac{1+x^2}{2x}$

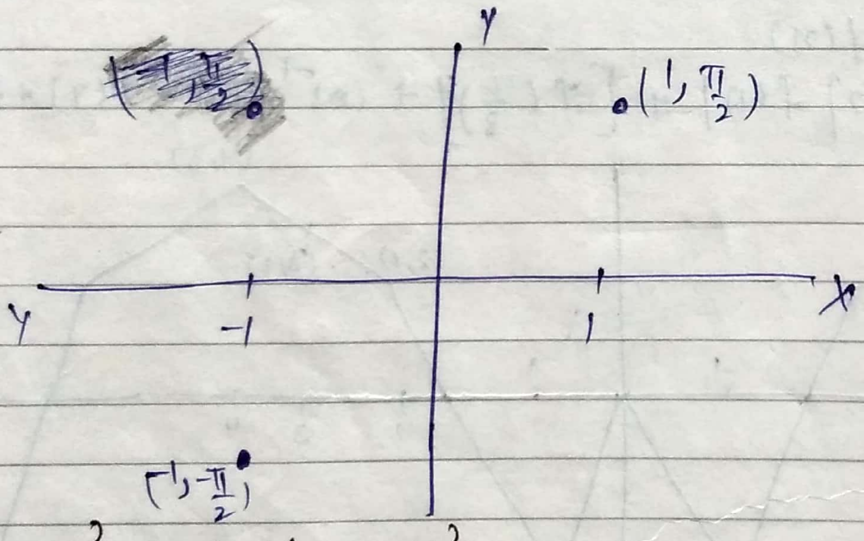


$= \sin^{-1} \frac{1+x^2}{2x} = \sin^{-1}\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$

$x+\frac{1}{x} \in (-\infty, -2) \cup (2, \infty)$

$\frac{x+\frac{1}{x}}{2} \in (-\infty, -1] \cup (1, \infty)$

Domain $\{-1, 1\}$



$* \quad |x|^2 = |x^2| = x^2$

$\left|\frac{1+x^2}{2x}\right| \le 1$

$|1+x^2| \le 2|x|$

$1+x^2 \le 2|x|$

$|x^2| + 1 - 2|x| \le 0$

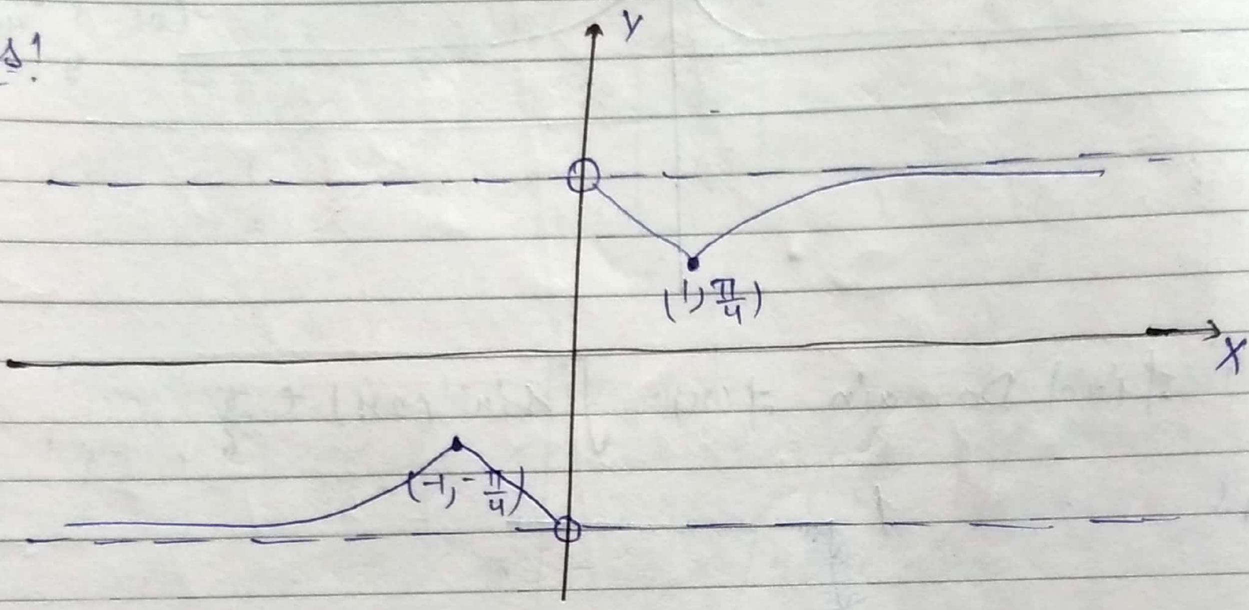
$(|x|-1)^2 \le 0$

$|x| - 1 = 0$

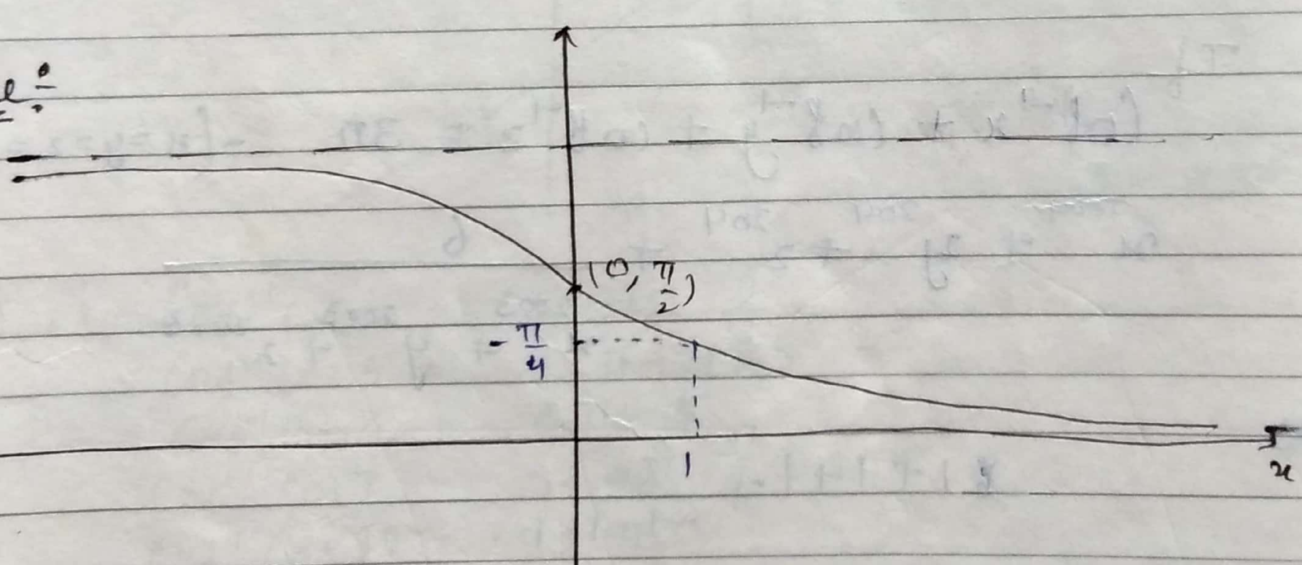
$|x| = 1 \Rightarrow x = \pm 1$ Ans

Que! $f(x) = \tan^{-1}\left(\frac{x^2+1}{2x}\right) = \tan^{-1}\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$

Ans!

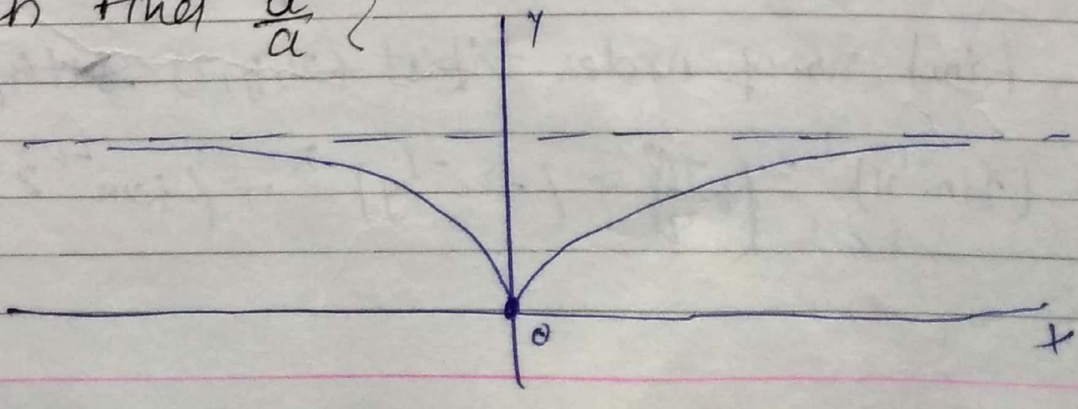


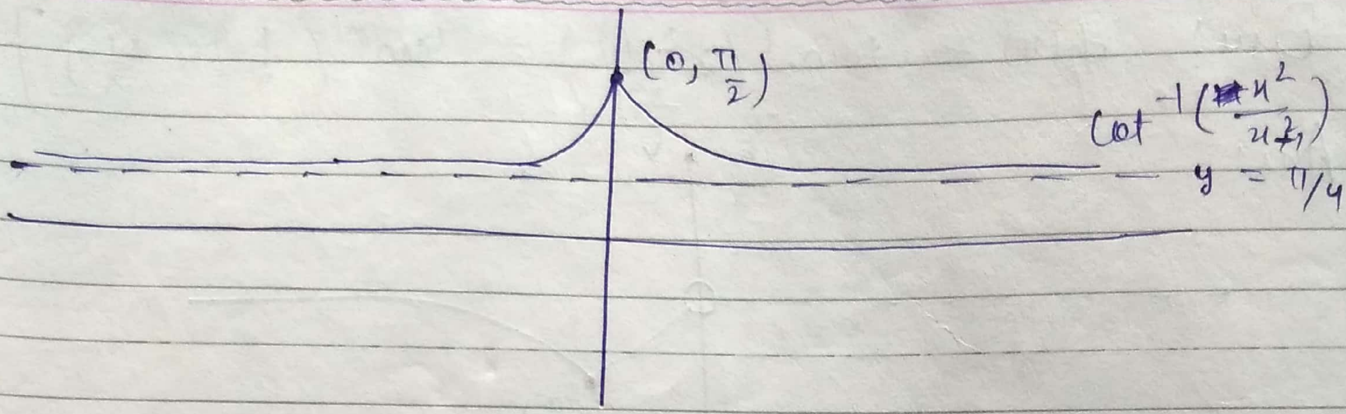
Que:



$f(x) = \cot^{-1}\left(\frac{x^2}{x^2+1}\right)$, If range (a, b)

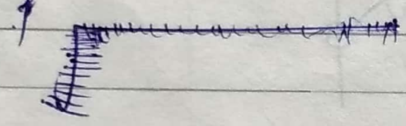
then find $\frac{b}{a}$?





Que: find Domain $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$

Ans



* If

$\cot^{-1}x + \cot^{-1}y + \cot^{-1}z = 3\pi \quad = (x = y = z = 1)$

$x^{2004} + y^{204} + z^{204} + \dfrac{6}{x^{2003} + y^{2003} + z^{2003}}$

Ans

$1 + 1 + 1 + \dfrac{6}{-1 - 1 - 1}$

$= \dfrac{3}{1} - \dfrac{6}{3}$

$= 1$ Ans

Que: find no. of order triplet (x, y, z) satisfying.

$(\sin^{-1}x)^2 = \left(\frac{\pi}{4}\right)^2 + (\sec^{-1}y)^2 + (\tan^{-1}z)^2$

$y - y' = m(x - x_1)$ R.N.

slope value

Boundary Problem Value :

Ans :- $\sin^{-1} x = \frac{\pi^2}{4} + (\sec^{-1} y)^2 + (\tan^{-1} z)^2$

minimum value 0 ; minimum value 0

$\left(\pm \frac{\pi}{2}\right)^2$; $y = 1$; at $z = 0$

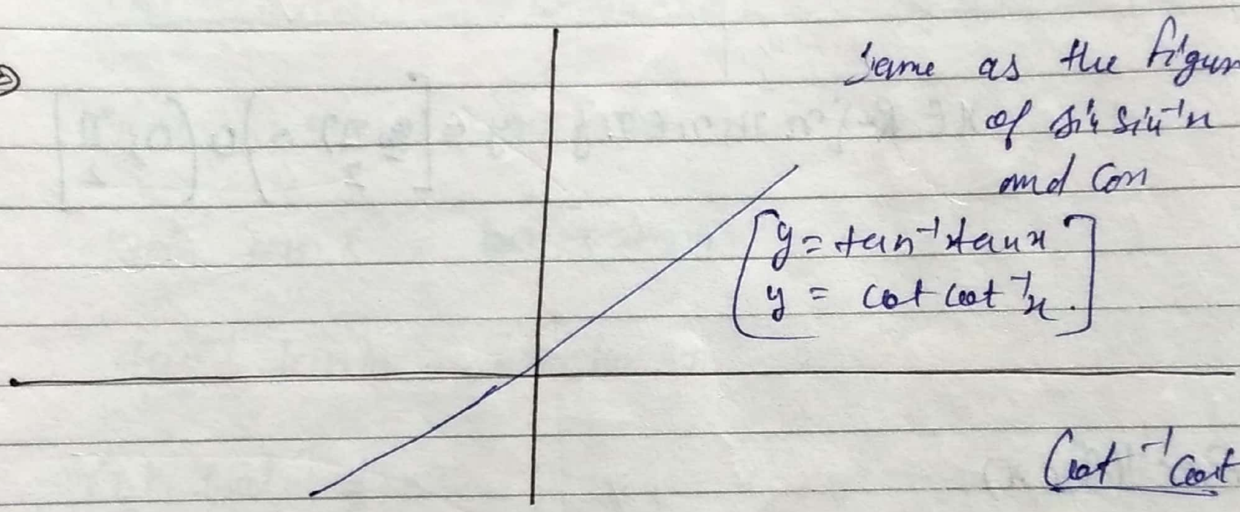
$x = \pm 1$

$z = 0$

x	y	z
1	1	0
-1	1	0

$2 \times 1 \times 1 = 2$

A ⇒



Same as the figure of $\sin \sin^{-1} x$ and cos

$y = \tan^{-1} \tan x$

$y = \cot \cot^{-1} x$.

$\cot^{-1} \cot x$

Que! $\cos^{-1} \cos 2 = 2$

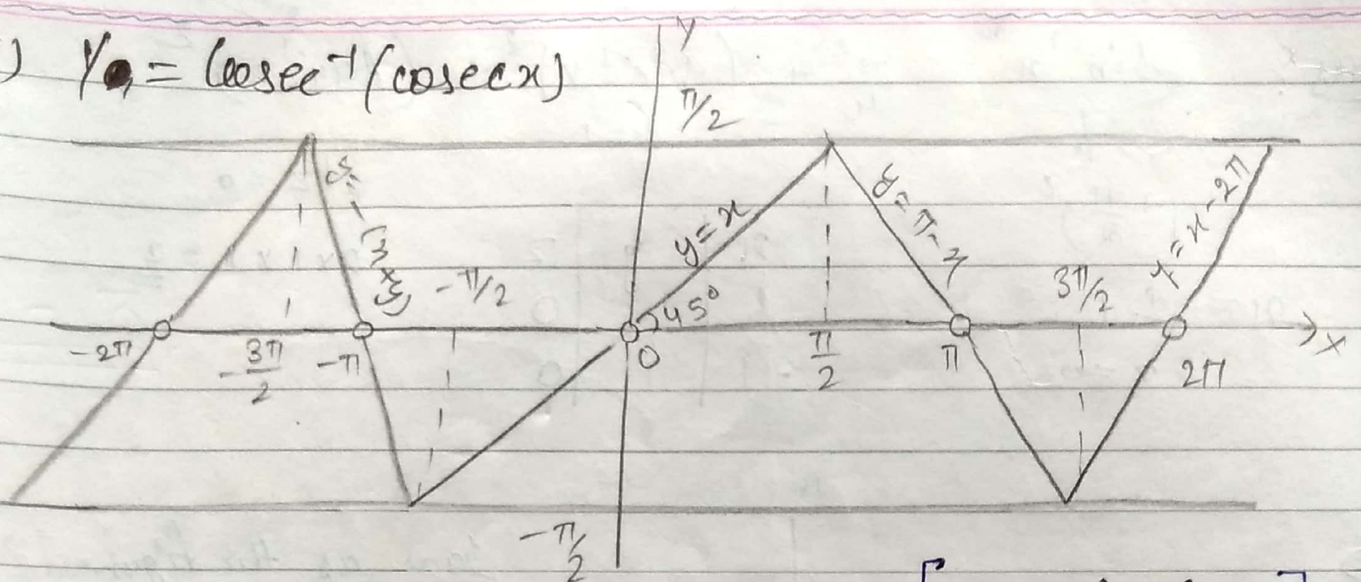
$\cos^{-1} \cos 5 = 2\pi - 5$

$\cos^{-1} \cos 6 = 2\pi - 6$

$\cos^{-1} \cos 7 = 7 - 2\pi$

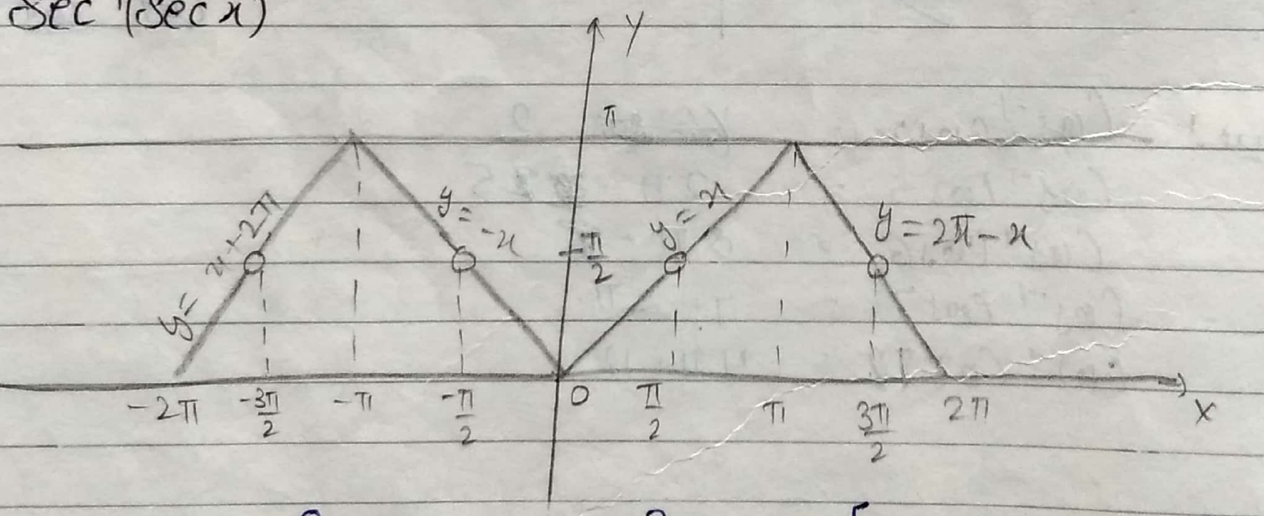
$\cos^{-1} \cos 12 = 4\pi - 12$

(v) $y = \operatorname{cosec}^{-1}(\operatorname{cosec} x)$



$x \in R - \{n\pi, n \in I\}$, $y \in \left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

(vi) $\sec^{-1}(\sec x)$



$x \in R - \left\{(2n-1)\frac{\pi}{2}, n \in I\right\}$, $y \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

Date: 20/05/17

Que: $\sin^{-1}\sin(-2) = \pi + 2$

$\sin^{-1}\sin 4 = \pi - 4$

$\sin^{-1}\sin 10 = 3\pi - 10$

$\cos^{-1}\cos(-10) = 4\pi - 10$

$\cos^{-1}\cos 5 = 2\pi - 5$

$\tan^{-1}\tan 6 = 6 - 2\pi$

$\tan^{-1}\tan 10 = 10 - 3\pi$

$\cot\cot^{-1}\frac{1}{2} = \frac{1}{2}$

$\cot\cot^{-1} 20 = 20$

P-2:

i) $\sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$, $x > 0$ & $x \in$ domain.

ii) $\cos^{-1}x = \sec^{-1}\frac{1}{x}$

(iii) $\cot^{-1}x = \begin{cases} \tan^{-1}\frac{1}{x}, & x > 0 \\ \pi + \tan^{-1}\frac{1}{x}, & x < 0 \end{cases}$

$\text{cosec}^{-1} 2 = \sin^{-1} \frac{1}{2}$

P-3:

(i) $\sin^{-1}(-x) = -\sin^{-1} x$

(ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x$

(iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

P-4:

(i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ $\quad |x| \le 1$ or $-1 \le x \le 1$

(ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ $\quad x \in R$

(iii) $\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$ $\quad |x| \ge 1$ or $x \in (-\infty, -1] \cup [1, \infty)$

P-5:

(i) (a) $\tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) & x>0, y>0, xy<1 \\ \pi + \tan^{-1} \frac{x+y}{1-xy} & x>0, y>0, xy>1 \end{cases}$

Imp (b) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$ $\quad x>0, y>0$

(c) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\dfrac{x+y+z-xyz}{1-(xy+yz+zx)}$

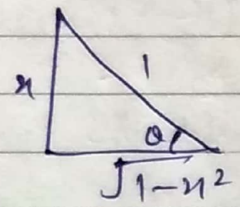
S Cos T
P B P
H H B

Que: If $x>0$, $\sin^{-1}x =$

$$\underbrace{\sin^{-1}x}_{\theta} = \operatorname{cosec}^{-1} = \frac{1}{x}$$
$$= \cos^{-1} = \sqrt{1-x^2}$$
$$= \tan^{-1} = \frac{x}{\sqrt{1-x^2}} \qquad \tan\theta = \frac{x}{\sqrt{1-x^2}}$$
$$\sin^{-1}x = \theta \qquad = \sec^{-1} = \frac{1}{\sqrt{1-x^2}}$$
take sin with both $= \cot^{-1} \Rightarrow \frac{\sqrt{1-x^2}}{x}$

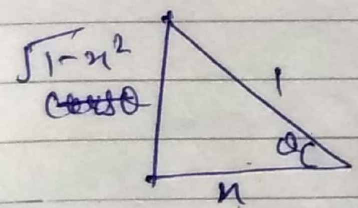
$\sin\sin^{-1}x = \sin\theta$

$x = \sin\theta$



$$\cos\theta = \frac{\sqrt{1-x^2}}{1}$$
$$\theta = \cos^{-1}\sqrt{1-x^2}$$

(2) $x>0$ $\cos^{-1}x = \sin^{-1}$ ~~x~~ $\sqrt{1-x^2}$
$= t$
$\cos^{-1}x = \theta$ $\quad = \tan^{-1} = \frac{\sqrt{1-x^2}}{x}$
$\cos^{-1}\cos\theta = \cos\theta$
$x = \cos\theta$ $\quad = \sec^{-1} = \frac{1}{x}$

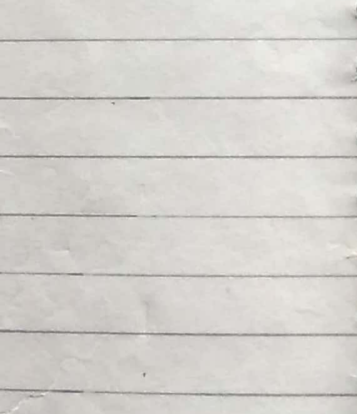


$= \operatorname{cosec}^{-1} = \frac{1}{\sqrt{1+x^2}}$

$= \cot^{-1} = \frac{x}{\sqrt{1+x^2}}$

$\tan = \cos\ \frac{\sqrt{1-x^2}}{x}$

$\sec = \cos = \frac{x}{1}$



(3) $x>0$ $\tan^{-1}x = \sin^{-1} = \frac{x}{\sqrt{1+x^2}}$
$\cos^{-1} = \frac{1}{\sqrt{1+x^2}}$

decreasing $\Rightarrow \frac{dy}{dx} < 0$ d f^n

increasing $= \frac{dx}{dy} > 0$ I f^n

* Inequalities in case of inverse function.

$\sin^{-1}$ is increasing function

$$y = \sin^{-1} x$$
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} > 0$$

$\therefore$ I f^n.

$\cos^{-1} x$ is $\downarrow$ d f^n

$\tan^{-1} x$ increasing function

$\cot^{-1} x$ decreasing function

$\Rightarrow$ $\frac{1}{3} < \frac{1}{2}$

$\sin^{-1}\frac{1}{3}$ $\qquad \cot^{-1}\frac{1}{2} > \cos^{-1}\frac{1}{2}$ $\qquad \tan^{-1}\frac{1}{3} < \tan^{-1}\frac{1}{2}$

$\cot^{-1}\frac{1}{3} > \cot^{-1}\frac{1}{2}$

Que! Solve $\sin^{-1} x > \sin^{-1}\left(\frac{1}{2} - x\right)$

$x > \frac{1}{2} - x$ $\qquad -1 \leq x \leq 1$ — (i)

$2x > \frac{1}{2}$ $\qquad -1 \leq \frac{1}{2} - x \leq 1$

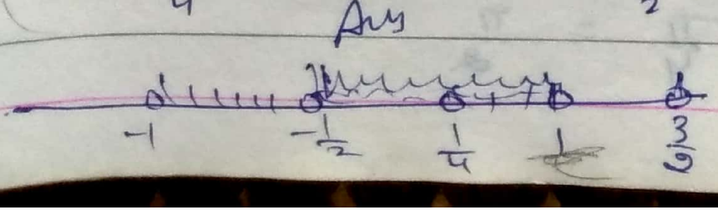
$x > \frac{1}{4}$ $\qquad -1 - \frac{1}{2} \leq -x \leq 1 - \frac{1}{2}$

A $\frac{1}{4} < x \leq 1$ $\qquad -\frac{3}{2} \leq -x \leq \frac{1}{2}$

Ans

$\frac{3}{2} \geq x \geq -\frac{1}{2}$

$-\frac{1}{2} \leq x \leq \frac{3}{2}$ — (ii)



$\cos^{-1} x \geq \cos^{-1}\left(\frac{1}{2} - x\right)$

$-1 \leq x \leq 1$ — (i)

$-1 \leq \frac{1}{2} - x \leq 1$ — (ii)

$\longrightarrow$ $x \leq \frac{1}{2} - x$ Ans

Que: Solve $\cos^{-1} x > \cos^{-1} x^2$

$-1 < x < 1$ $\cancel{-2 \leq x \leq \infty}$

$-1 \leq x < 1$ $x < x^2$

$-1 \leq x^2 \leq 1$

$x\ (-1, 1)$ $x < x^2$

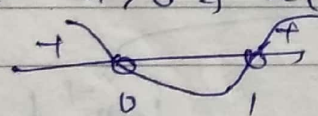
$x\ (\infty -1, \infty)$ $(\cancel{\infty, 1}),$

$-1 \leq x \leq 1$ $\cancel{-1} \leq x^2 \leq 1$

$x < x^2$ $0 \leq x^2 \leq 1$

$x^2 > x$ $-1 \leq x \leq 1$

$x^2 - x > 0 \Rightarrow x(x-1) > 0$



$x \in (-\infty, 0) \cup (1, \infty)$

$x \in [-1, 0)$.

Que: $\underbrace{\tan^{-1} 1 + \tan^{-1} \frac{1}{2}}_{x>0} + \underbrace{\tan^{-1} \frac{1}{3}}_{y>0}$ $xy = \frac{1}{2} \cdot \frac{1}{3} < p$

$= \tan^{-1} 1 + \tan^{-1}\left(\dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right)$

$= \tan^{-1} 1 + \tan^{-1} 1$

$= \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$$\boxed{\frac{\pi}{4} = \tan^{-1} 1}$$

$$\boxed{\cot^{-1} 1 + \cot^{-1} 2 + \cot^{-1} 3}$$

Ans $= \tan^{-1} 1 + \underbrace{\tan^{-1} 2}_{x>0} + \underbrace{\tan^{-1} 3}_{x>0}$

$\cancel{2\cdot3<1}$

$2\cdot3 > 1$

$= \tan^{-1} + \pi + \left(\frac{2+3}{1-6}\right)$

$= \tan^{-1} + \pi + \left(\frac{5}{-5}\right)$

$\tan^{-1} = -1 \quad \tan^{-1} 1 + \pi + \tan^{-1}(-1)$

$= \pi$

Que! $4\sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$

Ans! $4 \cdot \frac{\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{4}$

$= \frac{3\pi}{4} - \frac{2\pi}{2}$

$= \frac{3\pi - 4\pi}{4} = \frac{-\pi}{4}$

Que! $3\sin^{-1} + \sin^{-1} x + \cos^{-1} x = \frac{3\pi}{4}$

$3\sin^{-1} x + \frac{\pi}{2} = \frac{3\pi}{4}$

$3\sin^{-1} x = \frac{\pi}{4}$

$\sin^{-1} x = \frac{\pi}{12} \longrightarrow x = \sin\frac{\pi}{12}$

$\Rightarrow$ take sin both side. $\therefore \sin \sin^{-1} = \sin \frac{\pi}{12}$

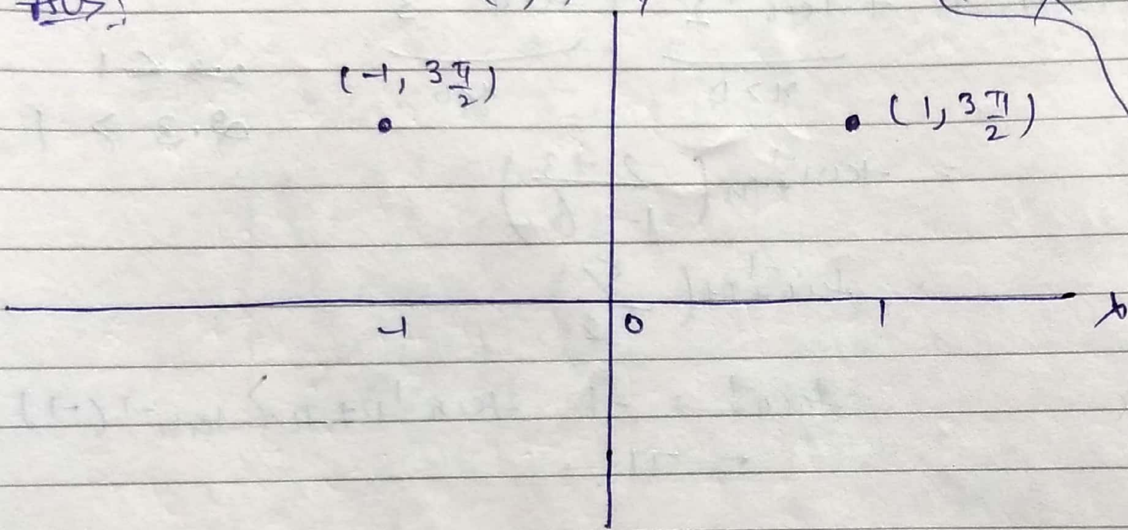
$x = \sin \frac{\pi}{12}$.

R

* Draw $y = \underbrace{\sin^{-1}x + \cos^{-1}x}_{(-1,1)} + \tan^{-1}x + \sec^{-1}x + \csc^{-1}x + \cot^{-1}x$.

$(-\infty,-1] \cup (1,\infty)$

Ans:



$= [-1,1] \quad [-\infty,-1] \cup (1,\infty) \;, R$

Domain $= \{1,-1\}$

$y = \frac{3\pi}{2}$

Que: $y = \underbrace{\sin^{-1} + \cos^{-1}x}_{[-1,1]} + \underbrace{\tan^{-1}x}_{x \in R}$.

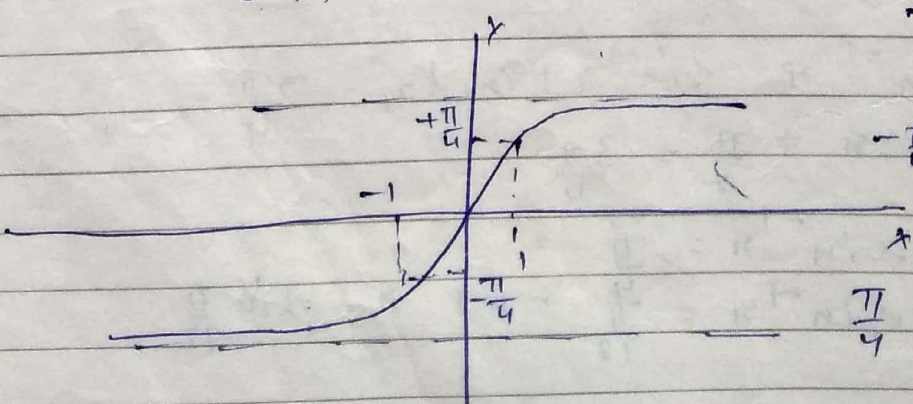
$\therefore$ Domain $= [-1,1]$

$-1 \le x \le 1$

$-\frac{\pi}{4} \le \tan^{-1}x \le \frac{\pi}{4}$

$-\frac{\pi}{4} + \frac{\pi}{2} \le \frac{\pi}{2} + \tan^{-1}x \le \frac{\pi}{2} + \frac{\pi}{4}$

$\frac{\pi}{4} \le y \le \frac{3\pi}{4}$



H.W: O-1, 6, 8, 10, 12, 13, 14, 15,
S-1: 8, 9, 10,
J-M ⇒

Que: Solve $\tan^{-1} x > 2$ $= \phi$

Ans: ~~take~~ $x > \tan 2$ $\tan^{-1} x$ is not greater than 2.

Date: 22/05/17 Ex: O-1

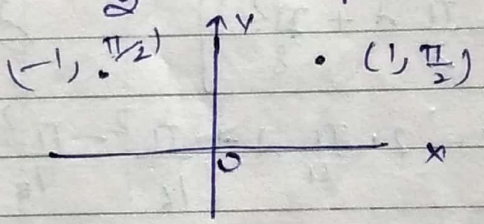
12) $f(x) = \sin^{-1}(\tan x) + \cot^{-1}(\cot x)$

$x = +\frac{\pi}{4}$ $-1 \le \cot x \le 1$ $-1 \le \tan x \le 1$

$\tan = \cot x = 1$ or -1

$f(x) = \sin^{-1} + \cos^{-1} 1 = \frac{\pi}{2}$

$x = \frac{\pi}{4}$ $f(x) = \frac{\pi}{2}$ for $y = f(x)$



(8) $1 + b + b^2 + \cdots \infty = a - \frac{a^3}{b} + \cdots \infty$

$\frac{1}{1-b} = \frac{a}{1+\frac{a}{3}}$

$\frac{1}{1-b} = \frac{3a}{3+a}$

(10)

$\frac{4}{5} + \frac{4}{5} = \frac{p}{q}$ $\frac{p}{q} = \frac{8}{5}$

$p = 8, \quad q = 5$ $(p-q)^{2k+1}$

3^{2k+1}

$3^1 = 3$

$3^3 = 27$

$3^5 =$

22/05/17

Que: 1) $f(x) = (\sin^{-1})^2 + (\cot^{-1}x)^2$, $f(x) = (\sin^{-1}x)^3 + (\cot^{-1}x)^3$,

$f(x) = (\tan^{-1}x)^2 + (\cot^{-1}x)^2$.

Formula

$a^2+b^2 = (a+b)^2 - 2ab$

Formula.

$a^3+b^3 = (a+b)^3 - 3ab(a+b)$.

$= (\sin^{-1}x + \cos^{-1}x)^2 - 2\sin^{-1}\cos^{-1}$

$(\sin^{-1} + \cos^{-1}x)^3$

$= \frac{\pi^2}{4} - 2\sin^{-1}x\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= 2(\sin^{-1}x) - \pi\sin^{-1} + \frac{\pi^2}{4}$

$= 2\left[\lambda^2 - \frac{\pi}{2}\lambda + \frac{\pi^2}{8}\right]$ $\quad \lambda = \sin^2 x$

$= 2\left[\lambda^2 - 2\cdot\frac{\pi}{4}\lambda + \frac{\pi^2}{16} - \frac{\pi^2}{16} + \frac{\pi^2}{8}\right]$

$= 2\left[\left(\lambda - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$

$= 2\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}$

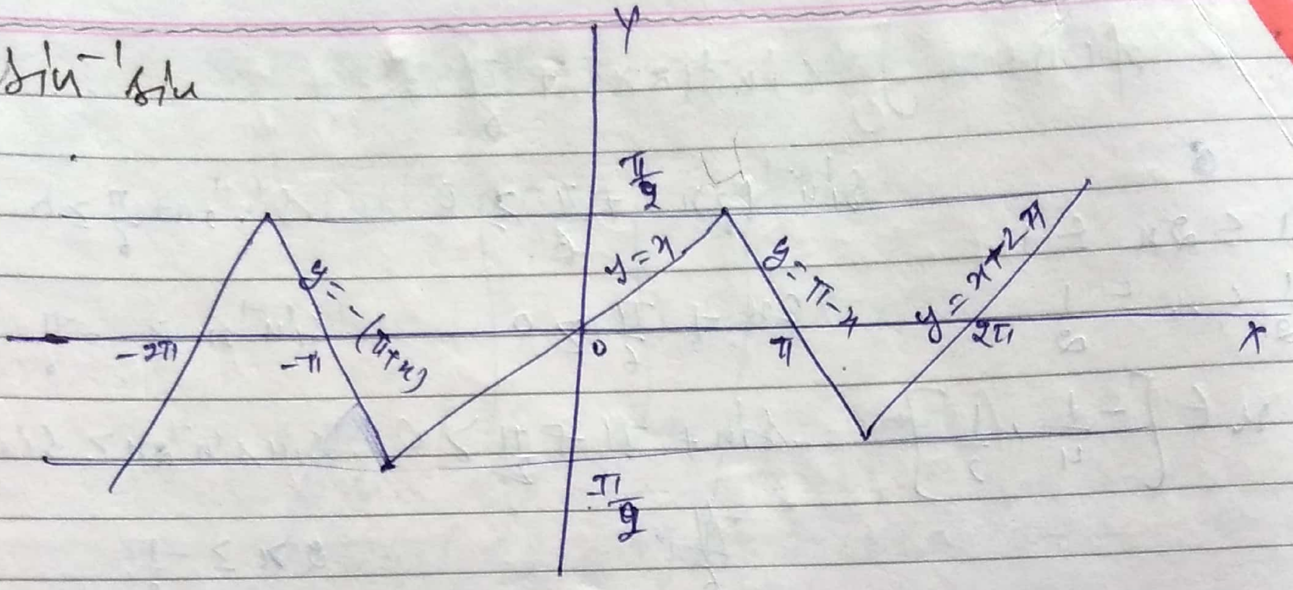
$= 2\left(\sin^{-1} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8}$

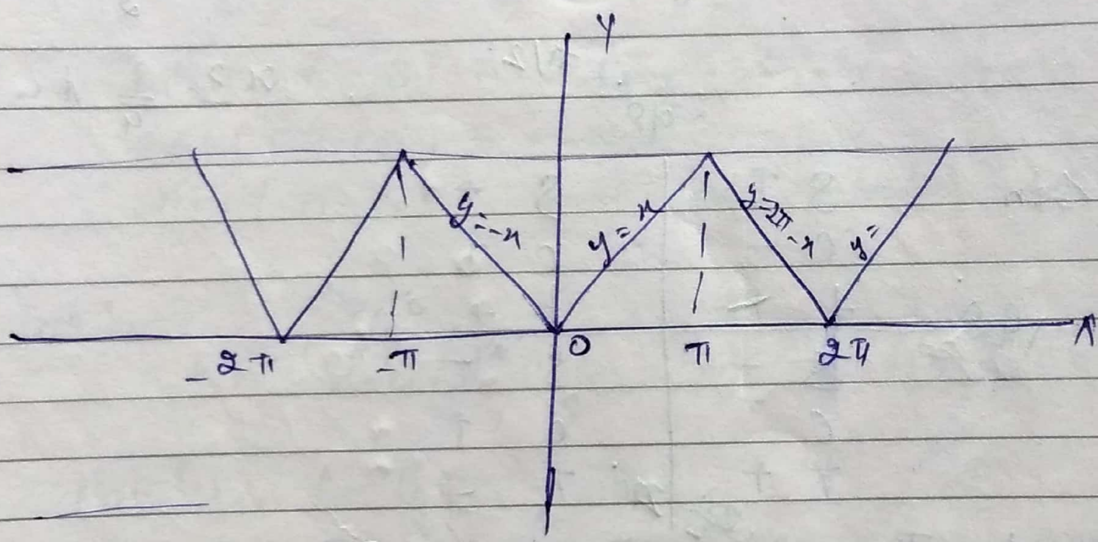
$-\frac{\pi}{2} \le \sin^{-1} \le \frac{\pi}{2}$

$f_{max} = \cancel{2}\{\ 2\cdot\frac{9\pi^2}{16} + \frac{\pi^2}{8} = \frac{10\pi^2}{8} = \frac{5\pi^2}{8}$

$f_{min} = \frac{\pi^2}{8}$ $\quad \sin^{-1}x = \frac{\pi}{4}$

$\sin^{-1}\sin$





Que! Find Domain $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$

(3)

$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

$x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

~~$\sin^{-1}(2x) + \frac{\pi}{6} \ge 0$~~

~~$2x + \frac{\pi}{6} \ge 0$~~

~~$\sin + \pi + \frac{\pi}{2} \ge 0$~~

~~$\frac{\pi}{6}$~~

$\sin^{-1} 2x + \frac{\pi}{6} \ge 0$

$\sin^{-1} 2x \ge -\frac{\pi}{6}$

$\sin \sin^{-1} 2x \ge \sin\left(\frac{\pi}{6}\right)$

$2x \ge -\frac{1}{2}$

$x \ge \frac{1}{4}$ Ans

Que! ~~sin~~

$\pi/2$, 90°

S	+	S	+
C	−	C	+
T	~~+~~	T	+
S	−	S	−
C		C	+
T	+	T	−

π 180° ; 0° start ; x ; $\frac{3\pi}{2}$ 270°

Jump $\frac{\pi}{2} \pm \theta$, $\frac{3\pi}{2} \pm \theta$, $\frac{5\pi}{2} \pm \theta$ –

s $\longrightarrow$ c

c $\longrightarrow$ s

t $\longrightarrow$ cot

(3) Jump $\pi \pm \theta$, $2\pi \pm \theta$, $3\pi \pm \theta$, – –

s $\longrightarrow$ s

c — c

t — t

$$\sin^2\left(\frac{\pi}{2}+\theta\right) = +\cos\theta$$

$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$

$$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$$

$$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$$

$$\tan\left(\frac{3\pi}{2}-\theta\right) = +\cos\theta$$

$$\sin(2\pi-\theta) = +\cancel{\cos} - \sin 2\theta$$
$$\cos(5\pi+\theta) = \cancel{-\sin}\quad -\cos$$
$$\tan(\pi-\theta) = \cancel{\tan}\ \cancel{\cot}\ \cancel{+\cot\theta} - \tan\theta.$$

Que:- $\sin^{-1}\sin\left(\frac{10\pi}{7}\right) = \cancel{\sin^{-1}\left(-\sin\frac{3\pi}{7}\right)}$

$\cos^{-1}\sin\left(-\frac{\pi}{4}\right)$

$\sin^{-1}\left(\cos\frac{33\pi}{10}\right)$

(i) $\sin\frac{10\pi}{7} = \sin\left(\pi+\frac{3\pi}{7}\right) = -\sin\frac{3\pi}{7}$

$\sin^{-1}\left(-\sin\frac{3\pi}{7}\right) = -\sin^{-1}\sin\left(\frac{3\pi}{7}\right) = -\frac{3\pi}{7}$ Ans

(ii) $\cos^{-1}\cos\left(\frac{\pi}{2}-\left(-\frac{\pi}{4}\right)\right) = \cos^{-1}\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)$

$= \frac{\pi}{2}+\frac{\pi}{4}$

(iii)

$\sin(\frac{\pi}{2}-\theta) = \cos\theta$

(iii) $\sin^{-1}\left(\cos\frac{33\pi}{10}\right) = \sin^{-1}\left(\cos 3\pi + \frac{3\pi}{10}\right)$

$= \cos\left(3\pi + \frac{3\pi}{10}\right) = -\cos\frac{3\pi}{10}$

$\sin^{-1}\left(\cos\frac{33\pi}{10}\right) = \sin^{-1}\left(-\cos\frac{3\pi}{10}\right)$

$= -\sin^{-1}\cos\frac{3\pi}{10} = -\sin^{-1}\sin\left(\frac{\pi}{2} - \frac{3\pi}{10}\right)$

$= -\sin^{-1}\sin\frac{2\pi}{10} = -\sin^{-1}\left(\sin\frac{\pi}{5}\right)$

$= -\frac{\pi}{5}$ Ans

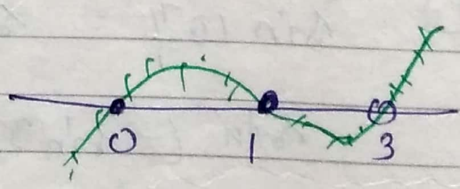
Que: Let, $a < b < c$ be three integer

$<0 \leftarrow \frac{x^2(x-1)}{(x-3)\underbrace{(x^2-2x+5)}_{x>0,\ D<0 \text{ always +ve}}} \le 0$

then find value of

$\cos^{-1}(a) + \sin^{-1}(b) + \sin^{-1}(c) + \operatorname{cosec}^{-1}c\,(b + \tan^{-1}a$

Ans: $\frac{x^2(x-1)(x-0)}{(x-3)}$



$x \in [1,3] \cup \{0\}$

$\therefore$ Integral value =

$a \quad b \quad c$

$\downarrow \quad | \quad |$

$0, \ 1, \ 2$

P B P
H H B

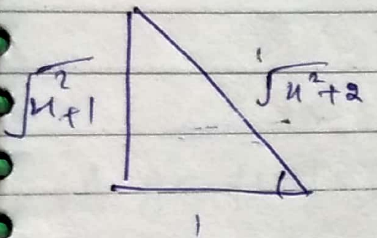
$= \cos^{-1}(0) + \sin^{-1}(1) + \sec^{-1}(2) + \csc^{-1} 1 + \tan^{-1} 0$

$= \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{3} + \frac{\pi}{2}$

$= \frac{3\pi}{2} + \frac{\pi}{3} = \frac{11\pi}{6}$ Ans

Que! Prove that $\cos\tan^{-1}\sin\underbrace{\cot^{-1}x}_{\theta_1} = \sqrt{\frac{x^2+1}{x^2+2}}$

$\sin\theta_1 = \sqrt{\frac{x^2+1}{x^2+2}}$



$\cot^{-1}x = \theta$

$\cot\theta = x$

$\tan\theta = \frac{1}{\sqrt{1+x^2}}$

$= \cos\tan^{-1}\sin\theta$

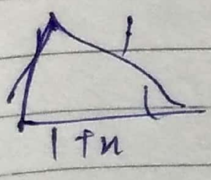
$= \cos\tan^{-1}\frac{1}{\sqrt{4+x^2}}$

$= \cos\theta$

$= \sqrt{\frac{1+x^2}{2+x^2}}.$

Que! $\sin\underbrace{\cot^{-1}}_{\theta_1}(1+x) = \cos\tan^{-1}x$

Ans!



$\cot\theta_1(1+x)$

$\cot\theta = (1+x)$

$\cot\theta =$

P. - Make figure 61 and 62.

Sol: $\sin \underbrace{\cot^{-1}(1+x)}_{\theta_1} = \cos \underbrace{\tan^{-1} x}_{\theta_2}$

$$\sin \sin^{-1} \frac{1}{\sqrt{1^2+(1+x)^2}} = \cos \cos^{-1} \frac{1}{\sqrt{1+x^2}}$$

$$= \frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$1+(1+x)^2 = 1+x^2$$

$$1+x^2+2x = x^2$$

$$x = -\frac{1}{2}$$

Que: $f : R \to \left(-\frac{\pi}{2}, 0\right]$

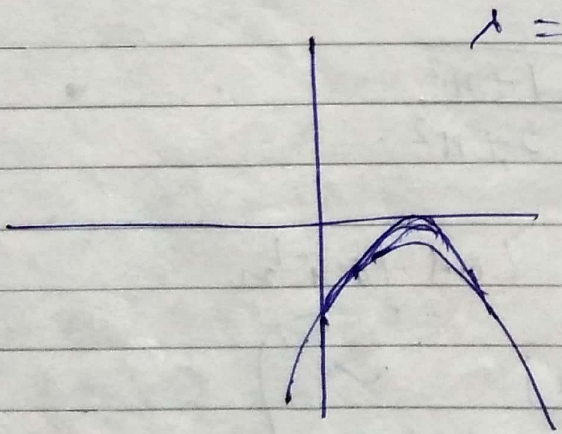
$f(x) = \tan^{-1}(2x - x^2 + \lambda)$

function defined as. if given function is onto

Ans:

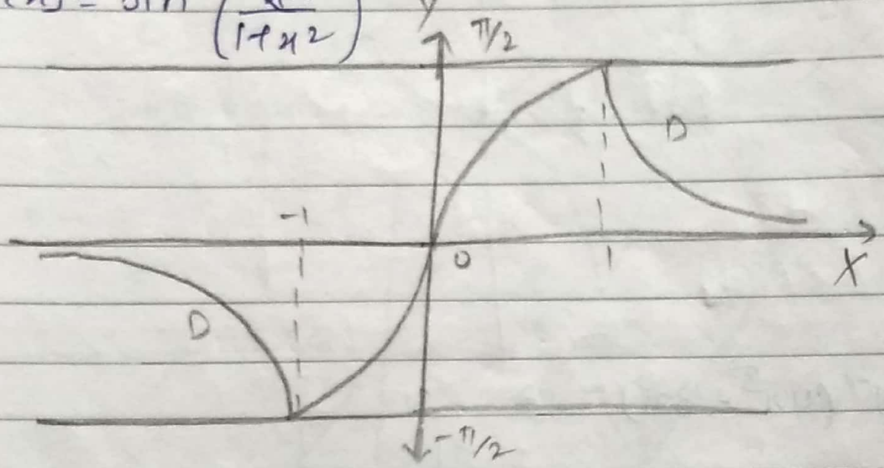
$D = 0 \Rightarrow 4 - 4(-1)\lambda = 0$

$\lambda = -1$

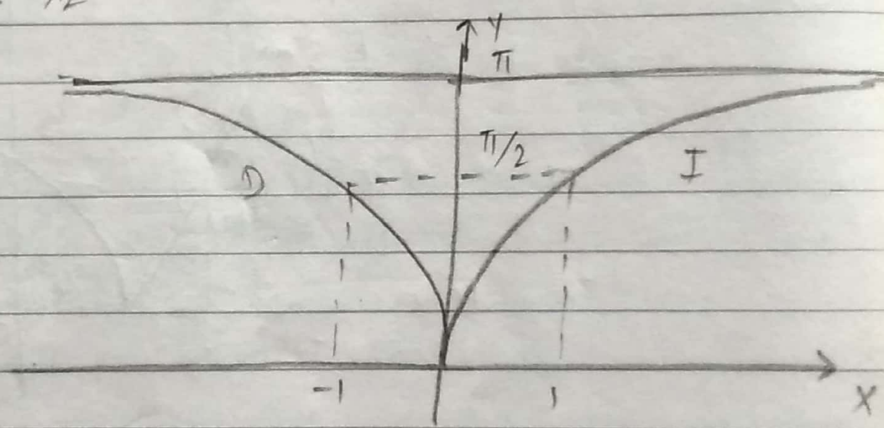


* Simplified ~~bi~~ inverse trigonometric function :

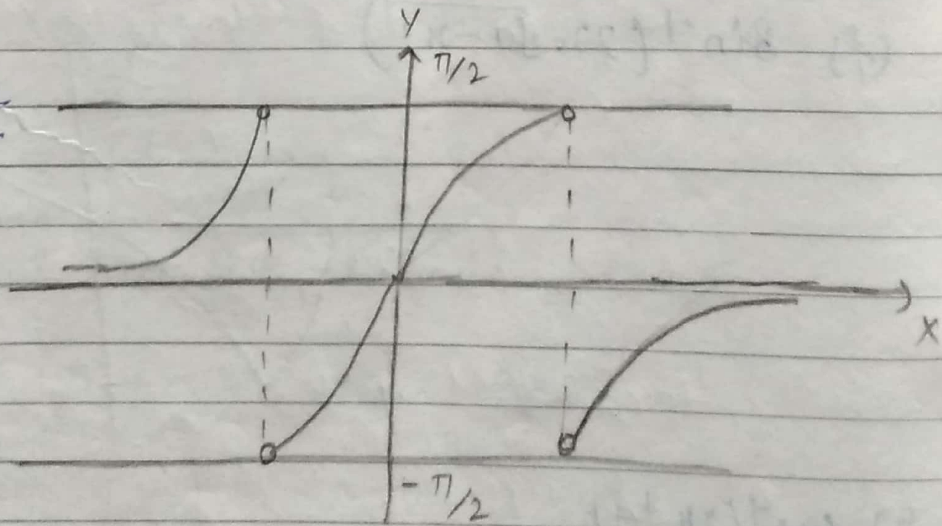
a) $y = f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$



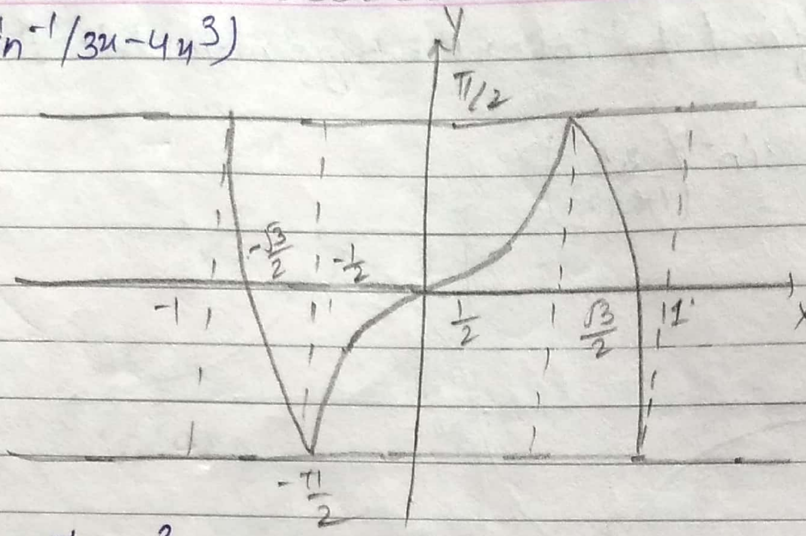
(b) $y = f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$



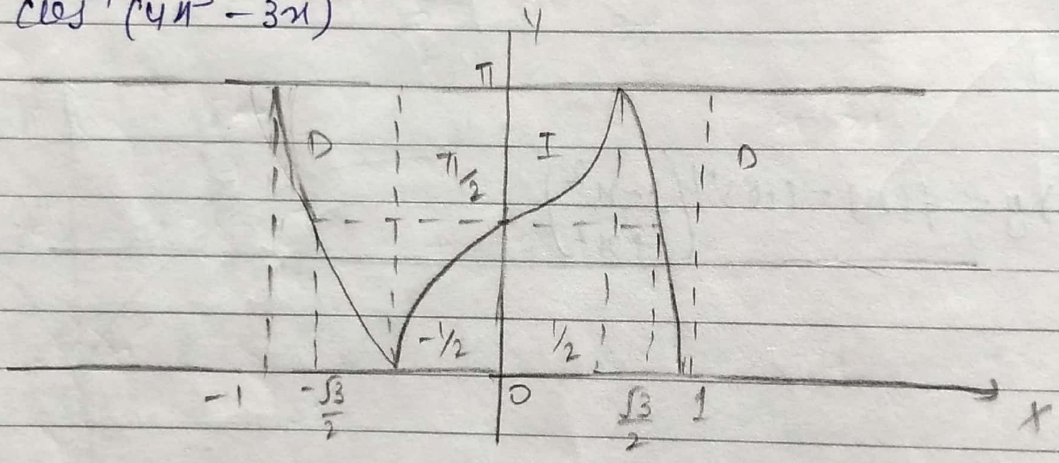
c) $y = f(x) = \tan^{-1}\frac{2x}{1-x^2}$



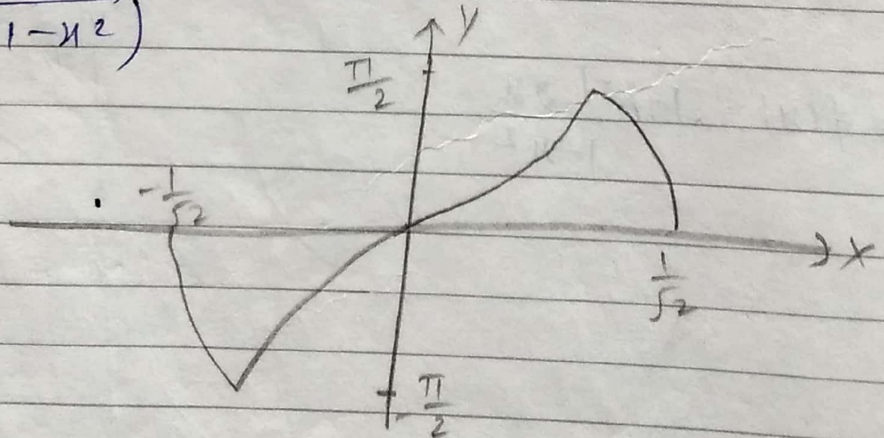
d) $y = f(x) = \sin^{-1}(3x-4x^3)$



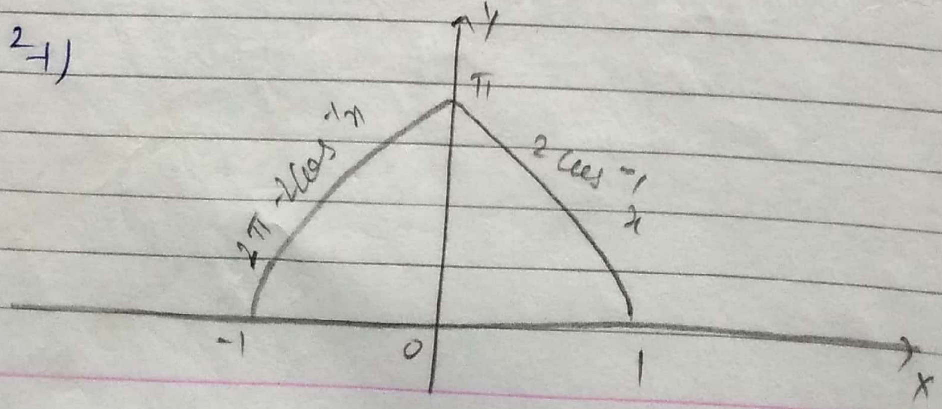
(e) $y = f(x) = \cos^{-1}(4x^3-3x)$



(f) $\sin^{-1}(2x\sqrt{1-x^2})$



g) $\cos^{-1}(2x^2-1)$



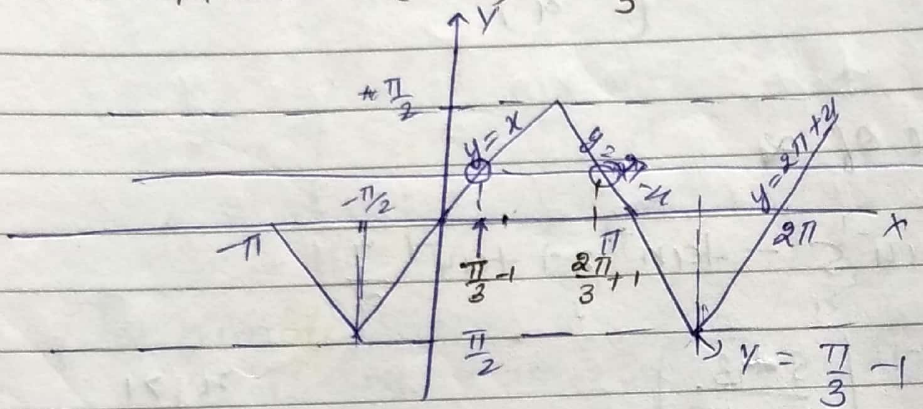
Date: 23/05/17

Que: find no. of points find no. of x. satisfying

$$x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$1 + \sin^{-1}(\sin x) = \frac{\pi}{3}$$

$y = \frac{\pi}{3} - 1$ = small +ve No.



Ans: 2

Method - 2:

Case - I: $-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$ $\quad x = \frac{\pi}{3} - 1$

Case: 2 $\quad \frac{\pi}{2} \le x \le \frac{3\pi}{2}$

$$\pi - x = \frac{\pi}{3} - 1$$

$$x = \frac{2\pi}{3} + 1$$

Que: 2. If $\tan^{-1}\frac{4}{x} + \tan^{-1}\frac{5}{y} = \cot^{-1}\lambda$.

Ans:

$$\pi + \tan^{-1}\left(\frac{4+5}{1-20}\right) \qquad xy > 1$$

$$\pi + \tan^{-1}\left[\frac{9}{-19}\right]$$

~~tan^{-1}~~ ~~π+~~ $\quad \pi + \tan^{-1}\left(-\frac{9}{19}\right)$

$$\cot^{-1}\alpha = \pi - \tan^{-1}\frac{9}{19}$$
$$= \pi - \cot^{-1}\left(\frac{19}{9}\right)$$
$$= \cot^{-1}\left(-\frac{19}{9}\right)$$

Que: find value of α

$$\alpha = \tan^{-1}\underset{x}{5} - \tan^{-1}\underset{y}{3} + \tan^{-1}\frac{7}{9}$$

Ans

$$\cancel{\pi} + \tan^{-1}\left(\frac{5+3}{1+15}\right) \qquad xy>1$$

$$\cancel{\pi + \tan^{-1}\left(\frac{8}{-14}\right)} + \cancel{\tan^{-1}\frac{7}{9}}$$

$$= \tan^{-1}\frac{5-3}{1+5\cdot 3} + \tan^{-1}\frac{7}{9} \qquad \cancel{\pi + \frac{5}{14} + \frac{7}{9}}$$

$$\frac{98+}{12\cancel{6}}$$

$$= \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{7}{9} \qquad \frac{1}{8}\cdot\frac{1}{9} < 1$$

$$= \tan^{-1}\frac{\frac{1}{8}+\frac{7}{9}}{1-\frac{1}{8}\cdot\frac{7}{9}} = \tan^{-1}\frac{65}{65} = \tan^{-1}1$$
$$= \frac{\pi}{4} \text{ Ans}$$

Que: $\tan^{-1}\frac{x+1}{x-1} + \tan^{-1}\frac{x-1}{x} = \tan^{-1}(-7)$

Ans: $$\tan^{-1}\left(\frac{\frac{x+1}{x-1}+\frac{x-1}{x}}{1-\frac{x+1}{x-1}\cdot\frac{x-1}{x}}\right) = \tan^{-1}(-7)$$

$\tan^{-1} 1 = \frac{\pi}{4}$

take tan both side.

$$\tan \tan^{-1}\left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1}\cdot\frac{x-1}{x}}\right) = \tan \tan^{-1}(-7)$$

$$\left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1}\cdot\frac{x-1}{x}}\right) = (-7)$$

After simblification.

$\Rightarrow$ $x^2 - 4x + 4 = 0$ R.H.S

$x = 2$ is -ve.

Now check question.

$\tan^{-1} 3 + \tan^{-1}\frac{1}{2}$ ✗

$\Rightarrow$ L.H.S = +ve.

Que: $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1} x = \frac{\pi}{4}$

Ans $\cancel{\tan^{-1}}\frac{1}{2} + \quad = \cancel{\tan^{-1}}$

$$\cos^{-1} x = \tan^{-1} 1 - \sin^{-1}\frac{1}{\sqrt{5}}$$

$$= \tan^{-1} 1 - \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1} \frac{1 - \frac{1}{2}}{1 + 1\cdot\frac{1}{2}}$$

$$= \tan^{-1}\frac{1}{3}$$

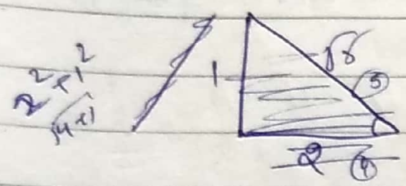
$$= \cos^{-1}\frac{3}{\sqrt{10}} \qquad x = \frac{3}{\sqrt{10}}$$

S C T
P B P
H H B

Que: Find x

$$2\cot^{-1} 2 - \cos^{-1}\frac{4}{5} = \operatorname{cosec}^{-1} x$$

Ans $\cot 2 - 4$



$= \cancel{2}$

$\cancel{\cos = \frac{2}{\sqrt{5}} + \cos^{-1}\frac{4}{5}}$

$$L.H.S = 2\tan^{-1}\frac{1}{2} - \cos^{-1}\frac{4}{5}$$

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2} - \tan\frac{3}{4}$$

$$= \tan^{-1}\frac{\frac{1}{2}+\frac{1}{2}}{1-\frac{1}{2}\cdot\frac{1}{2}}$$

$$= \tan^{-1}\frac{4}{3} - \tan^{-1}\frac{3}{4} = \tan^{-1}\left(\frac{\frac{4}{3}-\frac{3}{4}}{1+\frac{4}{3}\cdot\frac{3}{4}}\right)$$

$$= \tan^{-1}\frac{7}{24} = \operatorname{cosec}^{-1}\frac{25}{7}$$

$$x = \frac{25}{7} \text{ Ans.}$$

Method: 2

$$\Rightarrow \tan^{-1}\frac{1}{2} + \cot^{-1} x = \frac{\pi}{4}$$

take tan both side

$$\tan(\underbrace{\tan^{-1}\frac{1}{2}}_{\theta_1} + \underbrace{\cos^{-1} x}_{\theta_2}) = \tan\frac{\pi}{4}$$

$$\frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\,\cancel{\theta_2}\,\theta_2} = 1$$

$\cos^{-1}x \sin x =$

$$\frac{\tan\tan^{-1}\frac{1}{2} + \tan\cos^{-1}x}{1-(\tan\tan^{-1}\frac{1}{2})(\tan\cos^{-1}x)} = 1$$

$$\frac{\frac{1}{2} + \frac{\sqrt{1-x^2}}{x}}{1-\frac{1}{2}\cdot\frac{\sqrt{1-x^2}}{x}} = 1$$

$$\frac{1}{2} + \frac{\sqrt{1+x^2}}{x} = -1-\frac{1}{2}\frac{\sqrt{1-x^2}}{x}$$

$=$

Que: $\cos^{-1}x - \sin^{-1}x = \cos^{-1}x\sqrt{3}$

Ans! take ~~tan~~ cos both side

$\cos(\cos^{-1}x - \sin^{-1}x) = \cos(\cos^{-1}x\sqrt{3}$

$\cos(\underbrace{\cos^{-1}x}_{\theta_1} - \underbrace{\sin^{-1}x}_{\theta_2}) = x\sqrt{3}$

$\cos(\theta_1-\theta_2) = x\sqrt{3}$

$\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = x\sqrt{3}$

$\cos(\cos^{-1}x)\cos(\sin^{-1}x) + \sin(\cos^{-1}x)\cdot\sin\sin^{-1} = x\sqrt{3}$

$x\sqrt{1-x^2} + \sqrt{1-x^2}\cdot x = x\sqrt{3}$

$2x\sqrt{1-x^2} = x\sqrt{3}$

$x(2\sqrt{1-x^2} - \sqrt{3}) = 0$

$x=0$ or $2\sqrt{(1-x^2)} = \sqrt{3}$

Ans

$$4(1-x^2) = 3$$
$$4 - 4x^2 = 3$$
$$1 = 4x^2$$
$$x = \pm\frac{1}{2} \quad \text{Ans}$$

check :

$x = 0$ satisfy.

$x = \frac{1}{2} \Rightarrow \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$ — satisfy

$x = \frac{-1}{2}$ $\quad \left(\pi - \frac{\pi}{3}\right) + \frac{\pi}{6} = \pi - \frac{\pi}{6}$ satisfy

$$\left\{0, \frac{1}{2}, -\frac{1}{2}\right\}$$

Que:// $\sin^{-1}\underbrace{x}_{\theta_1} + \sin^{-1}\underbrace{2x}_{\theta_2} = \frac{\pi}{3}$

Ans: $\cancel{\sin\theta_2 x}$ $\sin^{-1}(x) + \sin^{-1}(2x) = \frac{\pi}{3}$

take sin both side

$$\sin(\sin^{-1}x + \sin^{-1}2x) = \sin\frac{\pi}{3}$$

$$\sin(\sin^{-1}x)\cos(\sin^{-1}2x) + \cos(\sin^{-1}x)\cdot\sin\sin^{-1}(2x) = \frac{\sqrt{3}}{2}$$

$$x\cdot\sqrt{1-4x} + \sqrt{1-x^2}\,2x = \frac{\sqrt{3}}{2}$$

Method :- 2

$\Rightarrow$ $\sin^{-1}2x = \frac{\pi}{3} - \sin^{-1}$

$$\sin(\sin^{-1}2x) = \sin\left(\frac{\pi}{3} - \sin^{-1}x\right)$$

① $\sin 2\theta$ = [2 tan...] H.W → Jee (M + A)

$$2x = \sin\frac{\pi}{3}\cos(\sin^{-1}x) - \cos\frac{\pi}{2}\sin(\sin^{-1}x)$$

$$= \frac{\sqrt{3}}{2}\cdot\sqrt{1-x^2} - \frac{1}{2}x$$

$$\frac{5x}{2} = \frac{\sqrt{3}}{2}\sqrt{1-x^2}$$

$$25x^2 = 3(1-x^2)$$

$$x = \pm\sqrt{\frac{3}{28}} \qquad -ve\ \times$$

$$x = \sqrt{\frac{3}{28}} \quad \text{Ans}$$

Que :- find all positive Integral solution

$$\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$$

Ans :- $\tan^{-1}x + \tan^{-1}\frac{1}{y} = \sin^{-1}3$

$$\tan^{-1}\frac{1}{y} = \tan^{-1}3 - \tan^{-1}x$$

$$\tan^{-1}\frac{1}{y} = \tan^{-1}\frac{3-x}{1+3x}$$

$$y = \frac{1+3x}{3-x}$$

$$x = \frac{3y-1}{3+y}$$

$x = 1,\ x = 2$

(x, y)

$(2, 7)$

$(1, 2)$

$\theta^{-1} \Rightarrow 2\tan$

formula

(ii) $y = \cot^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$y = \cos^{-1}\cos 2x$

$x = \tan\theta$

$0 \le 2\theta < \pi$

$0 \le \theta < \frac{\pi}{2}$

$x > 0$

$y = 2\theta$

$-\pi \le 2\theta < 0$

$-\frac{\pi}{2} < \theta < 0$

$x < 0$

$y = -2\theta$

$x = d$

$y = \tan^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & x \ge 0 \\ -2\tan^{-1}x & x < 0 \end{cases}$

Que!

$y = f(x) \sin^{-1}\frac{2x}{1+x^2} = \begin{cases} 2\lambda & -1 \le x \le 1 \\ \pi - 2\lambda & x \ge 1 \\ -\pi - 2\lambda & x \le -1 \end{cases}$

$y = f(x) \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} 2\lambda & x \ge 0 \\ -2\lambda & x < 0 \end{cases}$

$y = f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \begin{cases} 2\lambda - \pi & x > 1 \\ 2\lambda & -1 < x < 1 \\ \pi + 2\lambda & x < -1 \end{cases}$

H.W: J.A =

$$f(x)+g(x)+h(x)\begin{cases} 2\lambda & x>0 \\ 6\lambda & 0<x<1 \\ 2\lambda & -1\le x<0 \\ -2\lambda & x<-1 \end{cases}$$

$$f(x)+g(x)=\begin{cases} \pi & x\geq 1 \\ 4\lambda & 0\le x<1 \\ 0 & -1<x<0 \\ -\pi-4\lambda & x<-1 \end{cases}$$

Que: $f(x)=\sin^{-1}\frac{2x}{1+x^2}$, $g(x)=\cos^{-1}$, $h(x)=\lambda^{-1}$

$x\in(-1,1)$

$f(x)+g(x)+h(x)=\frac{\pi}{2}$

Ans:

$-1<x\le 0$

$2\lambda=2\lambda^{-1}=\frac{\pi}{2}$

$\tan^{-1}x=\frac{\pi}{4}$

$x=\tan\frac{\pi}{4}=1$

Reject X

$0<x<1$

$6\lambda=\frac{\pi}{2}$

$\tan^{-1}x=\frac{\pi}{12}$

$x=\tan\frac{\pi}{12}$ ✓

$\left\{\tan\frac{\pi}{12}\right\}$ Ans

No. of sol^n ⇒ ...

$\frac{x}{a} + \frac{y}{b} = 1$ linear eq^n line

Date: 24/05/17 Ex J.A

Main

5) $y - x = z - y = d$ (common difference)

$$\tan^{-1}y - \tan^{-1}x = \tan^{-1}z - \tan^{-1}y$$

$$\frac{\cancel{y-x}}{1+yx} = \frac{z-1}{1+2y}$$

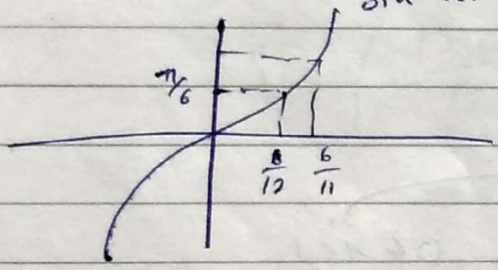
$$1 + yx = 1 + 2y$$

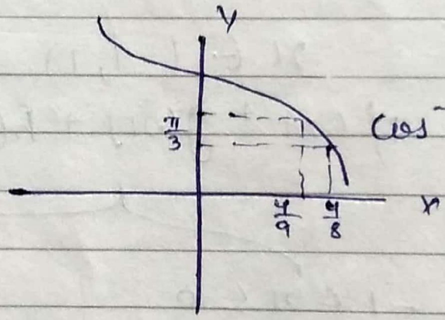
$$y(x-2) = 0$$

$y = 0$ $\quad$ $x - 2 = 0$

$x = 2 = y$

(6)





$\frac{6}{11} > \frac{6}{12}$ $\qquad$ $\frac{4}{8} > \frac{4}{9}$

$\sin^{-1}\frac{6}{11} > \sin^{-1}\frac{6}{12}$ $\qquad$ $\cos^{-1}\frac{4}{8} < \cos^{-1}\frac{4}{9}$

$= \frac{\pi}{6} = 30°$ $\qquad$ $\cos^{-1}\frac{4}{9} > \cos^{-1}\frac{4}{8} = \frac{\pi}{3}$

$\alpha > 90°$

$\alpha = 3\sin^{-1}\frac{6}{11}$ $\qquad$ $\alpha = 3\cos^{-1}\frac{4}{9}$

$\Rightarrow \beta > 180°$

I Create + Product difference = $\tan^{-1}$

Summation of series :-

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}x - \tan^{-1}y$$

Que! Solve $= \underbrace{\tan^{-1}\frac{1}{1+x+x^2}}_{T_1} + \underbrace{\tan^{-1}\frac{1}{x^2+3x+3}}_{T_2} + \underbrace{\tan^{-1}\frac{1}{x^2+5x+7}}_{T_3} + \text{---} \; n \text{ term.}$

Ans!

$$T_1 = \tan^{-1}\frac{1}{1+(x+x^2)} = \tan^{-1}\frac{1}{1\cdot 1\cdot(x+x)^2}$$

$$= \tan^{-1}\frac{(x+1)-x}{1+\underbrace{x}_{x}(\underbrace{x+1}_{y})} = \tan^{-1}(x+y)-x$$

$$T_2 = \tan^{-1}\frac{(x+2)-(x+1)}{1+(x+1)(x+2)}$$

$$T_1 = \tan^{-1}(x+1) - \tan^{-1}x$$
$$T_2 = \tan^{-1}(x+2) - \tan^{-1}(x+1)$$
$$T_3 = \tan^{-1}(x+3) - \tan^{-1}(x+2)$$
$$\vdots$$
$$T_n = \tan^{-1}(x+n) - \tan^{-1}(x+(n-1))$$

$$S = \tan^{-1}(x+n) - \tan^{-1}x$$

similar que. in different form. $\Big\}$ $S = \displaystyle\sum_{n=1}^{n} \tan^{-1}\frac{1}{1+n+n^2}$

this que. is written same as first Que.

$$S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \cdots n$$

or $$\sum_{r=1} \tan^{-1}\frac{1}{1+r+r^2}$$

$$T_r = \tan^{-1}\frac{1}{1+r+r^2} = \tan^{-1}\frac{(r+1)-r}{1+r(r+1)}$$

$$T_r = \tan^{-1}(r+1) - \tan^{-1}r$$

$$T_1 = \tan^{-1}2 - \tan^{-1}1$$
$$T_2 = \tan^{-1}3 - \tan^{-1}2$$
$$\vdots$$

$$\underline{T_n = \tan^{-1}(r+1) - \tan^{-1}n}$$

$$S = \tan^{-1}(n+1) - \tan^{-1}1 \quad \underline{\text{Ans}}$$

*

$$S = \sum_{n=1}^{\infty} \tan^{-1}\frac{1}{1+n+n^2} \quad \text{or} \quad \sum_{r=1}^{\infty} \tan^{-1}\frac{1}{1+n+n^2}$$

$(\tan^{-1}(n+1) - \tan^{-1}1)$ n is very very large no.

$$\lim_{n\to 0}\left(\tan^{-1}(n+1) - \tan^{-1}1\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{\pi}{4} \quad \underline{\text{Ans}}$$

Que: $S = \sum_{n=1}^{n} \tan^{-1}(1+n+n^2)$

$\tan^{-1} x = \frac{\pi}{2} - \cot^{-1} x$

$= \frac{\pi}{2} - \tan^{-1}\frac{1}{x}$

$$\sum_{n=1}^{n} \left(\frac{\pi}{2} - \tan^{-1}\frac{1}{1+n+n^2} \right)$$

$$= \frac{\pi}{2} \sum_{n=1}^{n} 1 - \sum_{n=1}^{n} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)$$

$$= \frac{\pi}{2} \cdot n$$

$$\boxed{\cot^{-1} x = \tan^{-1}\frac{1}{x}, \quad x = 0}$$

Que:

$$S = \sum_{n=1}^{n} \cot^{-1}(2n^2)$$

$$T_n = \cot^{-1}(2n^2) = \tan^{-1}\frac{1}{2n^2}$$

$$= \tan^{-1}\frac{1}{1+(2n^2-1)} = \tan^{-1}\frac{1}{1+(\sqrt{2}n-1)(\sqrt{2}n+1)}$$

$$= \tan^{-1}\left(\frac{(\sqrt{2}n+1)-(\sqrt{2}n-1)}{1+(\sqrt{2}n-1)(\sqrt{2}n+1)}\right) \quad \times$$

$$\times \quad = \tan^{-1}(\sqrt{2}n+1) - \tan^{-1}(\sqrt{2}n-1)$$

$$\left[(n^4+n^2+1) = (n^2+n+1)(n^2-n+1) \right]$$

$$= \tan^{-1} \frac{2}{4n^2}$$

$$= \tan^{-1} \frac{2}{1+(4n^2-1)} = \tan^{-1} \frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}$$

$$T_n = \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

Que:

(i) $$S = \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2n}{2+n^2+n^4}\right)$$

(ii) $$S = \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{4n}{n^4-2n^2+2}\right)$$

(1) $$T_n = \tan^{-1} \frac{(n^2+n+1)-(n^2-n+1)}{1+(n^2+n+1)(n^2-n+1)}$$

$$= \tan^{-1}(n^2+n+1) - \tan^{-1}(n^2-n+1)$$

(2) $$T_n = \tan^{-1} \frac{4n}{1+(n^2-1)^2}$$

$$= \tan^{-1} \frac{(n+1)^2-(n-1)^2}{1+(n-1)^2(n+1)^2}$$

$$= \tan^{-1}(n+1)^2 - \tan^{-1}(n-1)^2$$

SBG STUDY