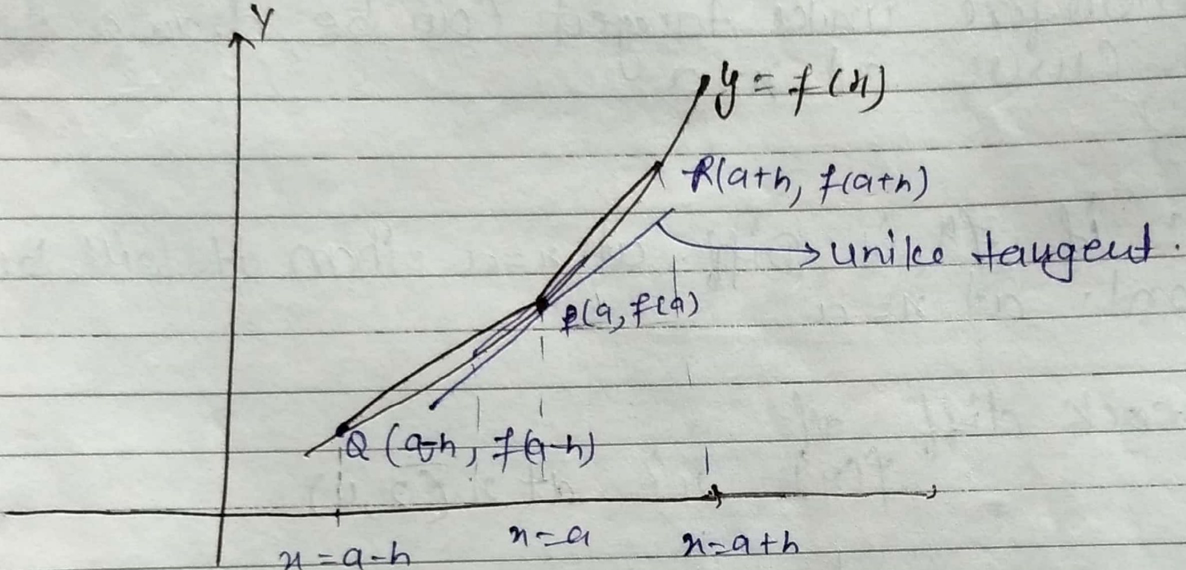
 1 2 3 4 5

06/05/17

chapter-

Differentiability



Differ. of $f(x)$ is cont. at $x = a$

Slope of PQ $= \dfrac{f(a-h) - f(a)}{a-h-a}$

Slope of PR $= \dfrac{f(a+h) - f(a)}{a+h-a}$

LHD $= f'(a^-) = \lim\limits_{h \to 0} \dfrac{f(a-h) - f(a)}{-h}$

RHD $= f'(a^+) = \lim\limits_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$

$\boxed{\text{if LHD = RHD = finite quantity}}$

then f^n is diff. at $x=a$

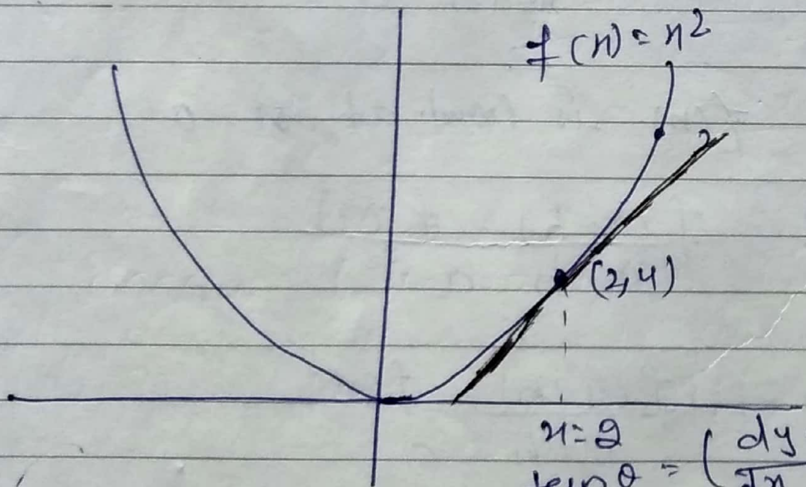
therefore unice target can be drawn to the curve at $x=a$

07/06/17

Note! if f^n is Diff. at $x=a$ than it will be cont. at $x=a$

Ex: Cheack diff of

$f(x) = x^2$ at x (2,4)



$f(x) = x^2$ at (2,4)

$x=2$

$\tan\theta = \left(\frac{dy}{dx}\right)_{x=2} = 4$

$$R.H.D = f'(2^+) = \lim_{h\to 0} \frac{f(2+h)^2 - f(2)^2}{\cancel{2}+h-\cancel{2}}$$

$$L.H.D \quad \cancel{f(2^-)} \lim_{h\to 0} \cancel{h} = \frac{f(2) - f(2)}{h} =$$

$$L.H.D = \frac{f(2-h) - f(2)}{-h} =$$

$$\lim_{h\to 0} = \frac{f(2)(- f(2)}{-h} = -2$$

$$\lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h} + h^2 + 4h - 4}{h} \quad \left(\frac{0}{0}\right)$$

$$\lim_{h \to 0} (h+4)$$

$$= 4$$

$$L.H.D = f'(2^-) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$$

$$\lim_{h \to 0} \frac{(2-h)^2 - 2^2}{-h}$$

$$\lim_{h \to 0} \frac{\cancel{4} + h^2 - 4h - 4}{-h} \quad \left(\frac{0}{0}\right)$$

$$\lim_{h \to 0} \frac{\cancel{h} - 4}{-1}$$

$$= 4$$

→ Becaus LHD = RHD = 4 = finite quantity therefore fn $f(x) = x^2$ is Diff. at $x = 2$. Hence unik tangent will be drawn to the curve at $x = 2$.

Note! $f(x) = x^2$

$$f'(x) = 2x$$

$$f'(x)\Big|_{x=2} = \left(\frac{d f(x)}{dx}\right)_{x=2} \quad f'(2) = 4$$

Que: Check differ. of f^n $f(x) = |x|$ at $x=0$

Ans

$$LHD = \lim_{h\to 0} \frac{f(0-h) - f(h)}{-h}$$

$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$\frac{f(-x) - f(-x)}{h}$$

$$= -x + x = 0$$

$$R.H.D = \lim_{h\to 0} f(0+h) - f(h)$$

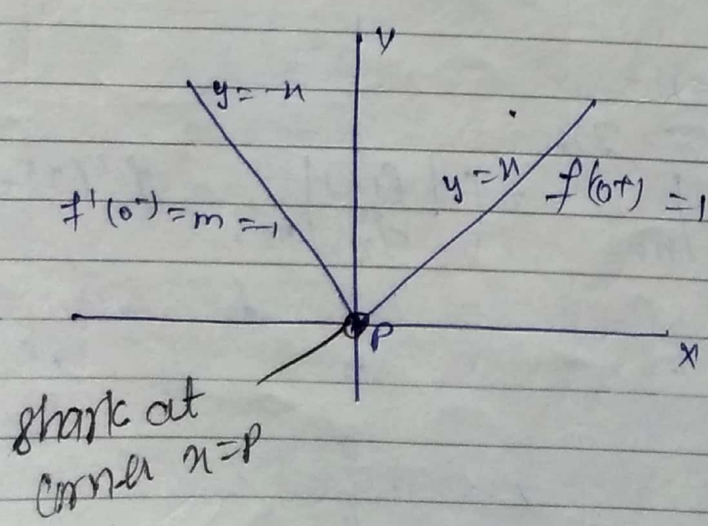
$$\frac{f(h) - f(-h)}{-h}$$

$$= \frac{f(x) - f(-0)}{-x} =$$

Ans: $R.H.D = f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{(0+h) - 0}{h} = 1$

$$f'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{x\to 0} \frac{-(0-h) - 0}{-h} = \frac{h}{-h} = -1$$

Becaus $L.H.D \neq RHD$ therefore (But both limit quantity are finite quantity) therefore f^n is not diff at $x=0$ although it is continuous.



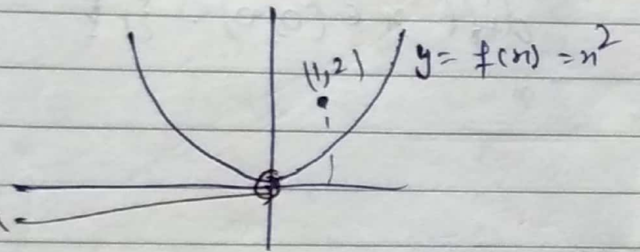
Second Method. 2 to check Diff.

$$f'(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

$f'(0^+) = 1 = $ R.H.D

$f'(0^-) = -1 = $ L.H.D.

*



$\frac{dy}{dx} = f'(x) = 2x$

$\left(\frac{dy}{dx}\right)_{(1,2)} = 2 \times 1 = 2$ Does not Valid.

this is wrong, Point not lie on the curve

~~this~~

* Note: if function is Diff at $x=a$ then it will be cont. at $x=a$ i.e diff $\Rightarrow$ conti.

$\therefore$ But cont. $\not\Rightarrow$ diff.

2) Non - Darivable $\neq$ discontinuous.

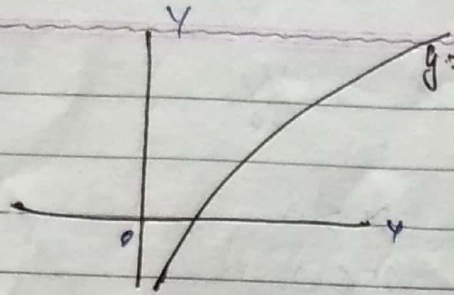
3) $f'(a^+) = p$ (finite)

$f'(a^-) = q$ (finite).

i) if $P = Q$ then fn is diff. at $x = a$

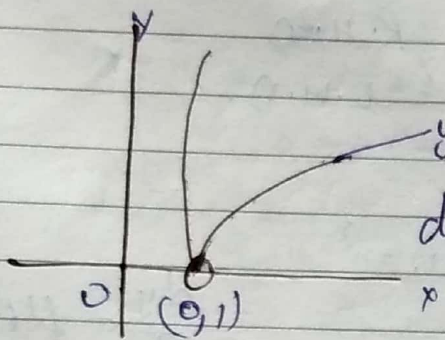
ii) if $P \neq Q$ cont. but not diff. at $x = a$.

iii) if fn is cont. at $x = a$ but not diff. at $x = a$ then geometricaly curve represent sharp corner at $x = a$.



$y = \ln(x)$

diff. at $\forall x \in (0, \infty)$



$y = |\ln x|$

diff $x \in (0,\infty) - \{1\}$

$*$ Diff. in open interval (a,b)

$y = f(x)$ diff. in (a,b) if and only if it is derivable at each and every point of the interval. also

$f'(a^+)$ & $f'(b^-)$ = finite quantity.

$y=f(x)$	$y=g(x)$	$f+g$	$f-g$	$f\cdot g$	f/g
D	D	D	D	D	D
D	ND	ND	ND	CNS	CNS
ND	D	ND	ND	CNS	CNS
ND	ND	CNS	CNS	CNS	CNS

D = Differ.
N.D = Not Diff.

CNS => cannot say.

Que: $f(x) = \begin{cases} \frac{\sin x^2}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

~~Chek~~: check its Diff.

$L.H.D = f'(0^-) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$

$$\lim_{h \to 0} \frac{\frac{\sin (0-h)^2}{0-h}}{-h} = \lim_{h \to 0} \frac{\sin h^2}{h^2} = 1$$

$R.H.D = f'(0^+) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$$\lim_{h \to 0} \frac{\frac{\sin (0+h)^2}{0+h} - 0}{h} = \lim_{h \to 0} \frac{\sin h^2}{h^2} = 1$$

Que: $f(x) = [x] \sin \pi x$
Fin L.H.D at $x = 2$

Ans: $L.H.D = f'(2^-) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$

$$= \lim_{h \to 0} \frac{[2-h] \sin \pi (2-h) - [2] \sin 2\pi}{-h}$$

$$= \lim_{h \to 0} \frac{1 \cdot (-\sin \pi h) \pi}{-\pi h} = \lim_{h \to 0} \left(\frac{\sin \pi h}{\pi h} \right) \cdot \pi = \pi \text{ A}$$

Que: $f(x) = \begin{cases} ax+b & x \leq -1 \\ ax^3+x+2b & x > -1 \end{cases}$

if fn is diff for all x. then find a, b.

Ans: Check Cont. at $x = -1$

$$f(-1) = f(-1^-) = a(-1)+b = -a+b$$

$$\& \quad f(-1^+) = a(-1)^3+(-1)+2b = -a-1+2b$$

Because cont.

$$-a+b = -a-1+2b \qquad b = 1$$

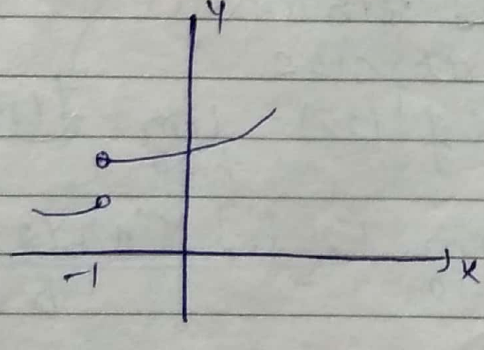
Check ~~Cont~~. diff

$$f'(x) = \begin{cases} a, & x < -1 \\ 3ax^2+1 & x > -1 \end{cases}$$

$$f'(-1^-) = f'(-1^+)$$

$$a = 3a(-1)^2+1$$

$$a = 3a+1 \Rightarrow a = -\frac{1}{2}$$



Que: $f(x) = \cos x + |\cos x|$ $\quad [0, 3\pi]$

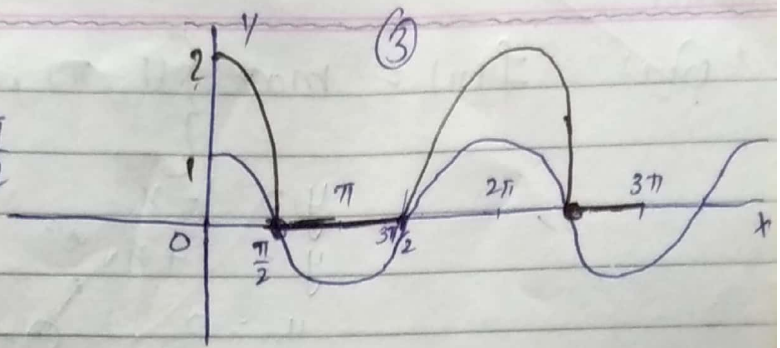
Check non-diff.

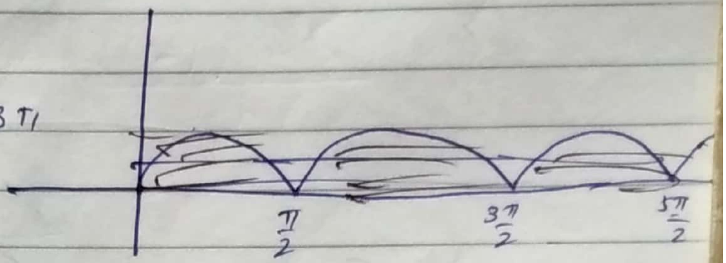
H.W.: O-1: 4, 5, 7, 8, 9, 10, 11, 17

S-1 1, 2, 3, 4, 7, 8

Ans:

$$= \begin{cases} 2\cos x, & 0 \le x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \le x < \frac{3\pi}{2} \\ 2\cos x & \frac{3\pi}{2} \le x < \frac{5\pi}{2} \\ 0 & \frac{5\pi}{2} \le x \le 3\pi \end{cases}$$



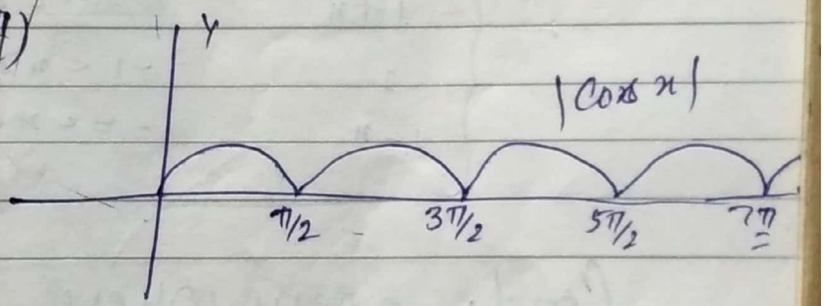


2nd Method:

$$f'(x) = \begin{cases} -2\sin x & x \in [0, \frac{\pi}{2}) \\ 0 & x \in (\frac{\pi}{2}, \frac{3\pi}{2}) \end{cases}$$

$$f'(\tfrac{\pi}{2}^-) = -2$$

$$f'(\tfrac{\pi}{2}^+) = 0 \quad \text{Ans}$$



Que: f^n $f(x)$ is diff. at $x = a$ and $f'(a) = \frac{1}{4}$ then find value of (L-Hospital)

$$\lim_{h \to a} \frac{f(a+2h^2) - f(a-2h^2)}{h^2}$$

Ans:

$-f(a)$, $f(a)$ Add diff $f(a)$ in N^r substract

$$\lim_{h \to 0} \frac{[f(a+2h^2) - f(a)]}{2h^2} - \frac{[f(a-2h^2) - f(a)]}{-2h^2}$$

$$2f'(a^+) + 2f'(0) = 4f'(a) = 4 \times \frac{1}{4} = (1) \text{ Ans}$$

$= 1$ Ans

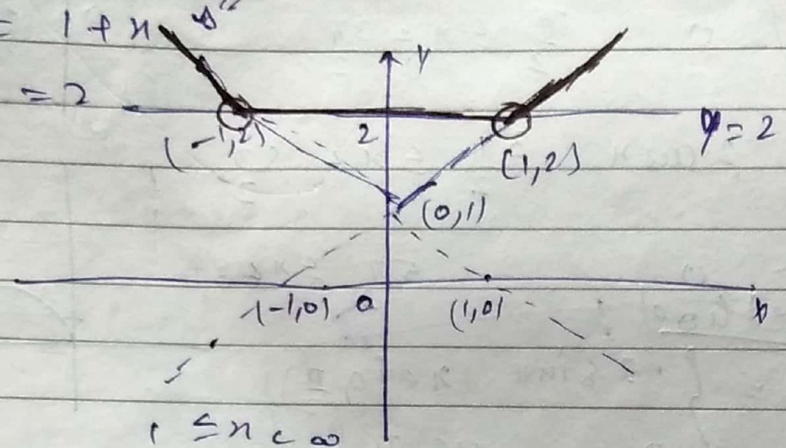
5/06/17

Que: $f(x) = \max\{(1-x), (1+x), 2\}$

$y = 1-x$

$y = 1+x$

$y = 2$



$$= \begin{cases} 1+x & 1 \le x < \infty \\ 2 & -1 < x < 1 \\ 1-x & -\infty < x < -1 \end{cases}$$

Cont. every where

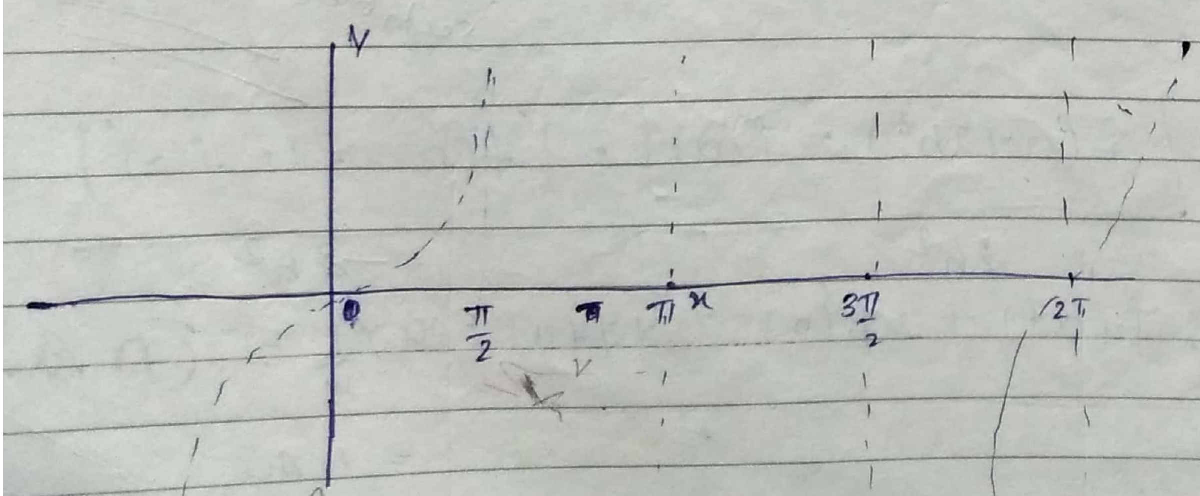
not diff. at $x = 1$ & -1

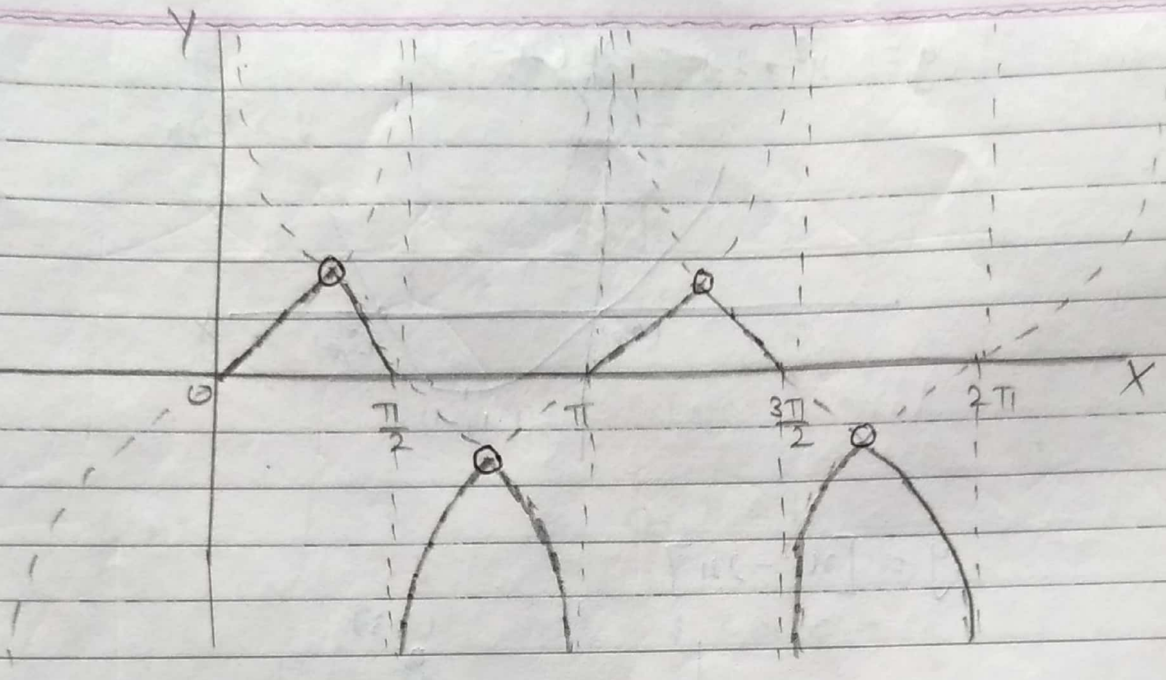
Que: Let m = no. of point of discont. and

n = no. of point of non-diff.

then find $n+m$. For

$f(x) = \min\{\tan x, \cot x\}$ in $x \in (0, 2\pi)$





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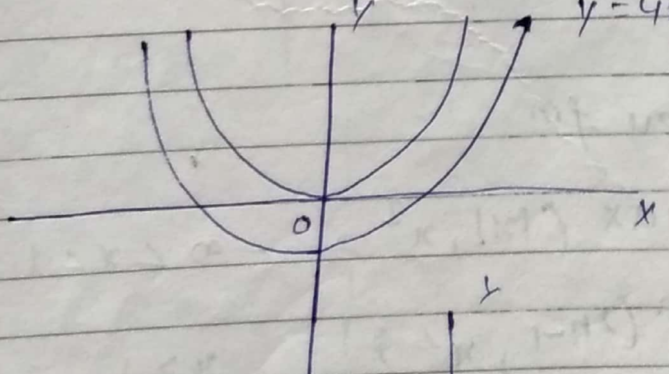
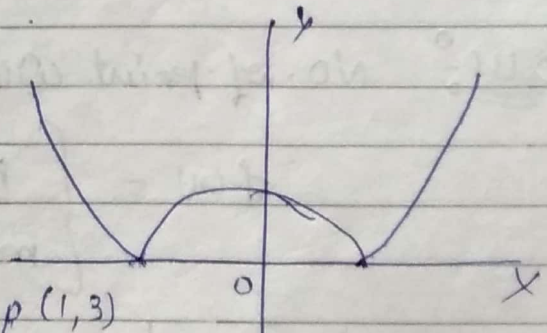
Que: $f(x) = \begin{cases} |1-4x^2| & 0 \le x < 1 \\ [x^2-2x] & 1 \le x \le 2 \end{cases}$

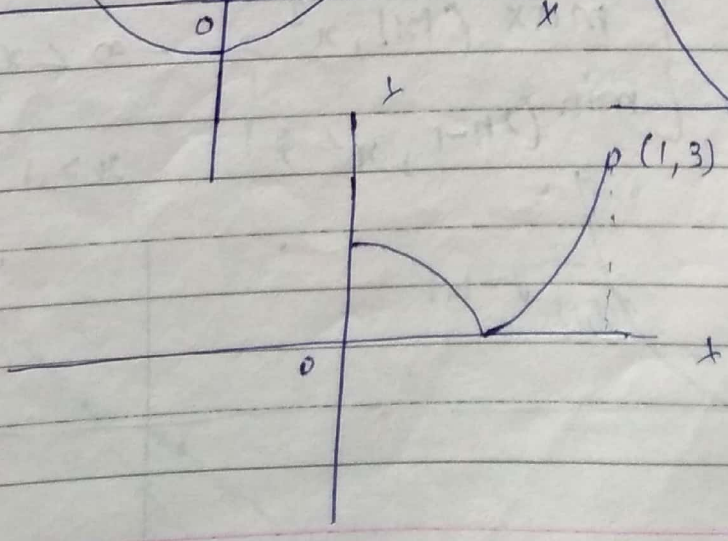
↳ gIf

check its diff. in $(0,2)$

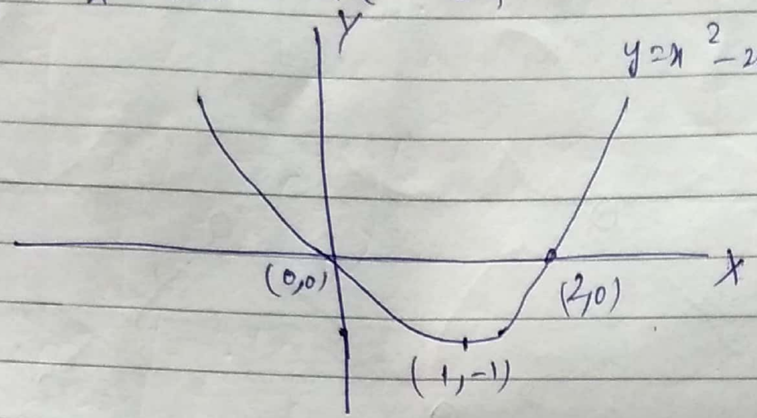
$y = 4x^2 - 1$ at $x=0$ $y=0$



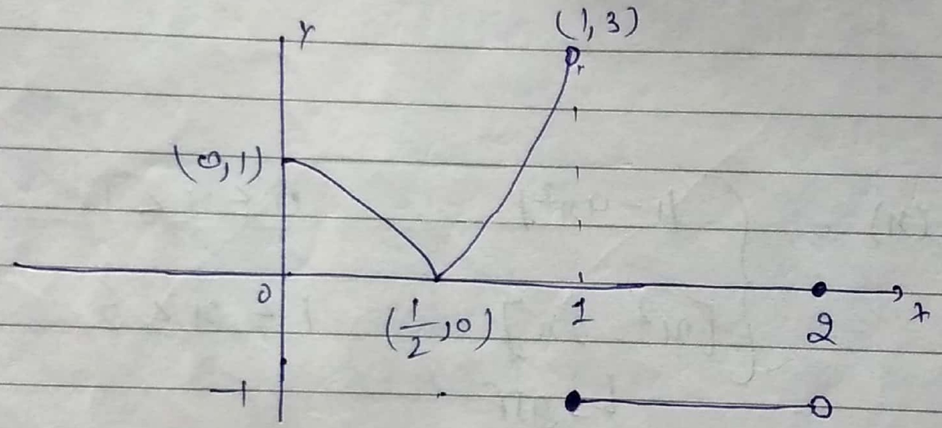




$y = x^2 - 2x = x(x-2)$



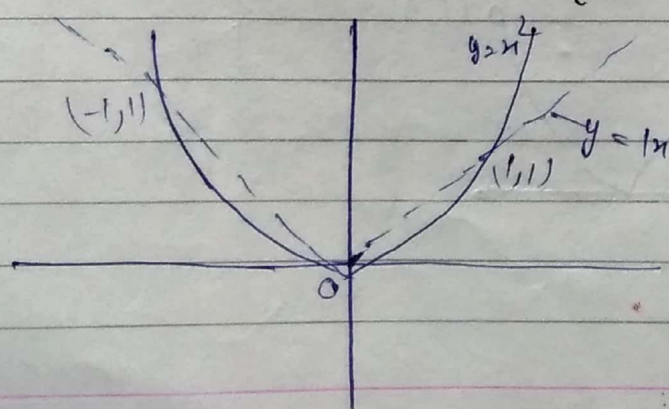
$y = [x^2 - 2x]$

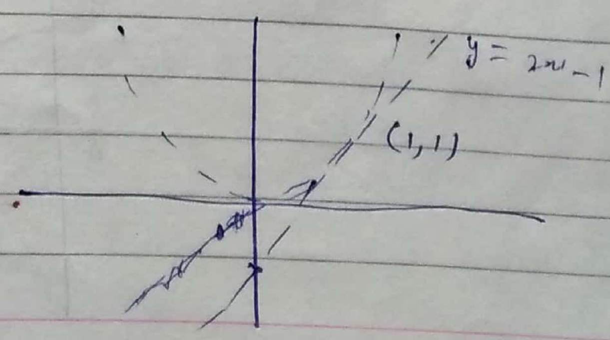


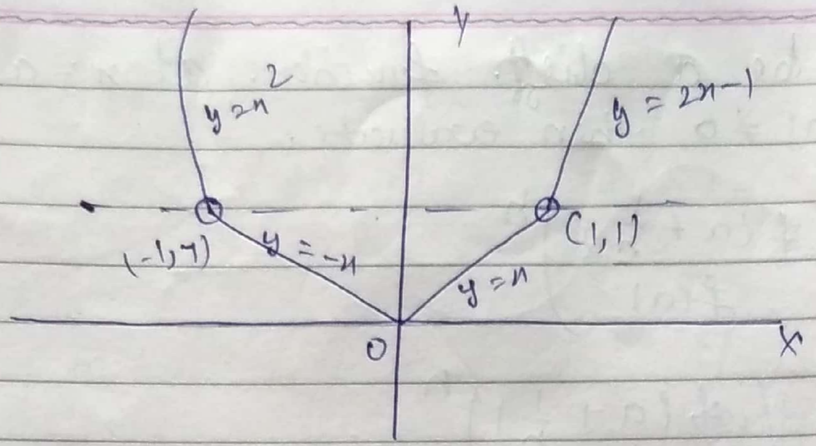
N. Diff. $x = \frac{1}{2}, 1$

Que: No. of point where f^n

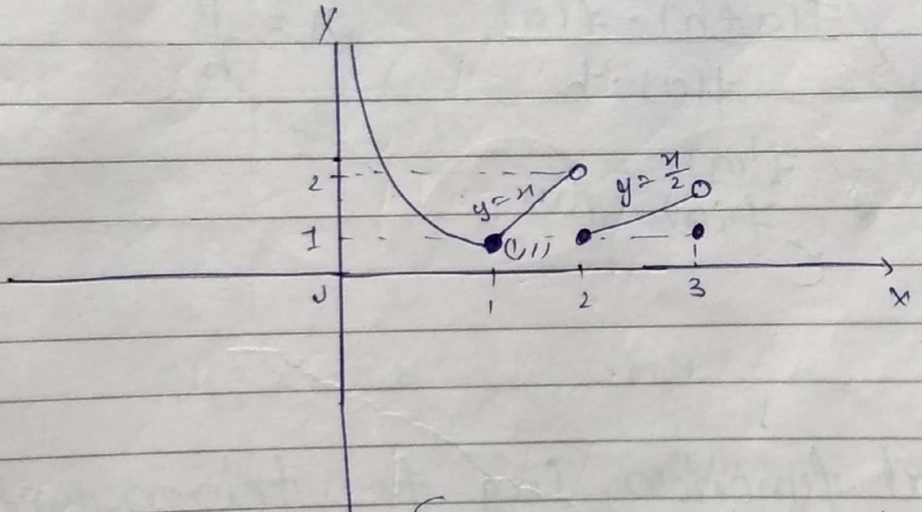
$$f(x) = \begin{cases} \max(|x|, x^2 & -\infty < x < 1 \\ \min(2x-1, x^2), & x \geq 1 \end{cases}$$







Que! $f(x) = \begin{cases} \frac{1}{x} & 0 < x < 1 \\ \frac{x}{[x]}, & 1 \le x \le 3 \end{cases}$



$$\begin{cases} x, & 1 \le x < 2 \\ \frac{x}{2} & 2 \le x < 3 \\ 1 & x = 3 \end{cases}$$

Que: let f be a diff. function at $x = a$ where $f(a) \neq 0$ then evaluate.

$$\lim_{x \to \infty} \left(\frac{f(a + \frac{1}{n})}{f(a)} \right)^n$$

$$\lim_{n \to \infty} \left(\frac{f(a + \frac{1}{n})}{f(a)} \right)^n$$

$$e^{\lim_{n \to \infty} \left(\frac{f(a + \frac{1}{n})}{f(a)} - 1 \right) n}$$

$$e^{\lim_{h \to 0} \frac{f(a+h) - f(a)}{f(a) \cdot h}} \qquad n = \frac{1}{h}$$

$$= e^{\frac{f'(a)}{f(a)}} \text{ Ans.}$$

Note:

1) Polynomial function, log fu, trigonometric, expencial function are diff. in their domain

* 2) $f(x)$ is diff. at $x = a$ and we have to consider differenciability of fu $y = |x|$ at $x = a$

$y = f(x)$ at $x = a$

$y = |f(x)|$ at $x = a$

$y = f(x)$ at $x = a$

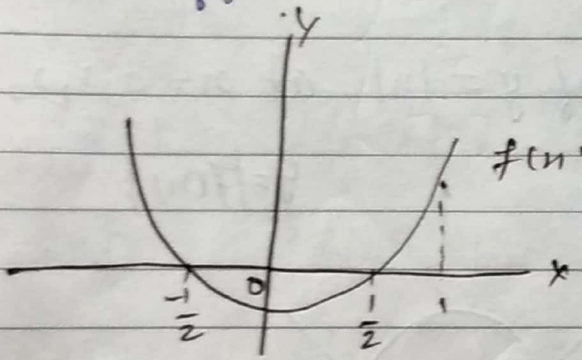
$y = |f(x)|$ at $x = a$

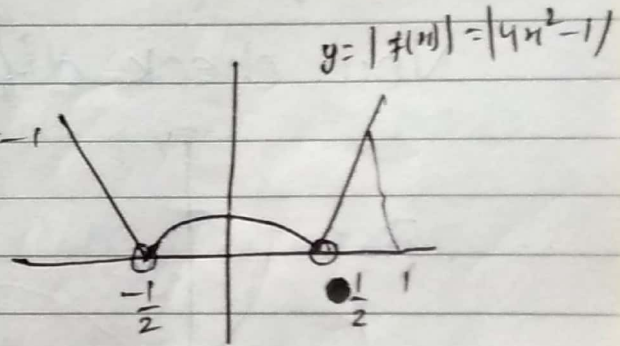
if $f(a) \neq 0$

then $|f(x)|$ must be diff.

if $f(a) = 0$

then $|f(x)|$ may or may not be diff at $x = a$.





$x = \frac{1}{2}, x = 1$

diff at $x = \frac{1}{2}, 1$ $\forall x \in R$.

at $x = 1$

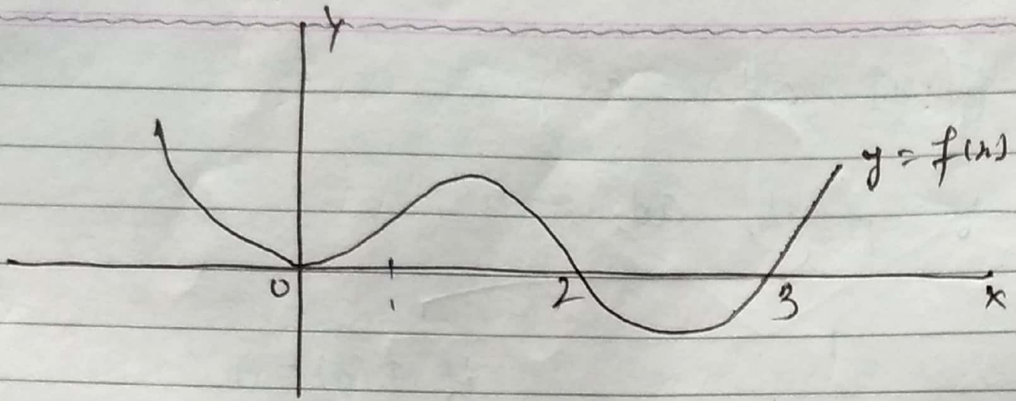
$f(1) \neq 0$

$|f(x)|$ is diff. at $x = 1$

at $x = \frac{1}{2}$

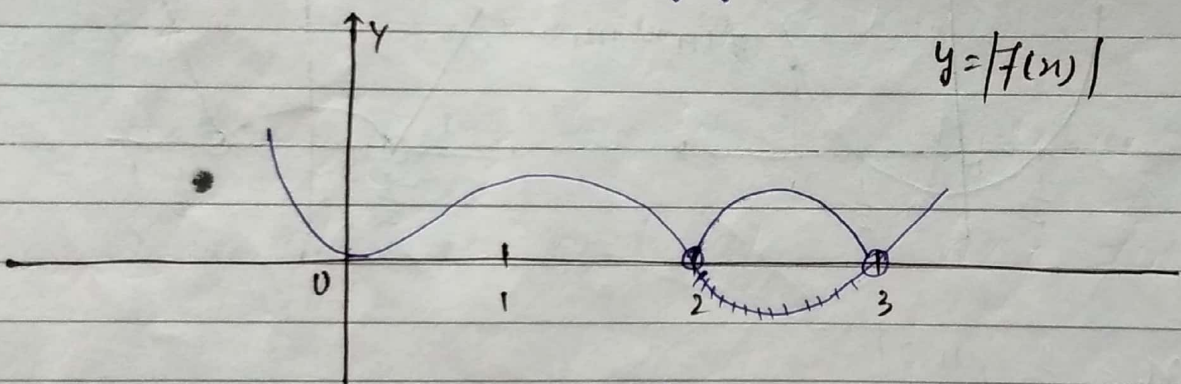
$f(\frac{1}{2}) = 0$

$|f(x)|$ is not diff at $x = \frac{1}{2}$



Polynomial function $y = f(x)$ shown in the fig. is diff at $x = 0, 1, 2, 3$

Now check diff. of $y = |x|$ at $x = 0, 1, 2, 3$



diff at $x = 0, 1$

not diff at $x = 2, 3$

* $f(x)$ is diff. at $x = a$ and $f(a) = 0$ and $g(x)$ is cont at $x = a$ then Product of fn $F(x) = f(x) \cdot g(x)$ will be diff. at $x = a$.

$f(x) = (x-1)$ diff at $x = 1$ ($\because f(1) = 0$)

$g(x) = |x-1|$ is cont. only (at $x = 1$)

$F(x) = (x-1)\,|x-1|$ $\quad F(x) = f(x) \cdot g(x)$

will be diff at $x = 1$

Based on that same Que. → O-1, Q-19, 13,

$h(x) = x|x^2 - x|$

$|x^2 \cdot x| \Rightarrow |x(x-1)| = |x|\,|x-1|$

$= (0, 1)$

$= x\,|x|\,|x-1|$

f / g is cont at $x = 0$

$x = 0$ diff at $f(0) = 0$

f is not diff at $x = 1$ only.

$f(x) = \sin|x|$ not diff at $x = 0$

$f(x) = |\sin x|$ not diff at $x = n\pi$

$f(x) = \cos|x|$ is not diff at no where.

$\cos x$ is not diff. at no where

* Derivative of diff fu need not be a conti. fu

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$f'(0^+) = f'(0^-) = 0 \Rightarrow$ diff. f^n at $x = 0$

$f'(0) =$ NO.

* **Determination of function:**

following steps needed to determine f^n from given functional eq.

Step: 1 Write down $f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

Step - 2: ~~Manipulate~~ manipulate $f(x+h) - f(x)$

according to the functional rule in eq.

Simplify it and get as a function of $f(x)$

Step - 3 Simplify it and integrate with respect to x

Step: 4 Apply boundary Condition.

Qu! Determine f^n which satisfy functional eq.

1) $f(x+y) = f(x) \cdot f(y) \quad \forall\ x, y \in R$

where x is diff. everywhere and $f'(0) = 1$ and $f(0) \neq 0$

Ans) $x = y = 0$

$f(0) = f(0)^2$

$\Rightarrow f(0) [f(0) - 1] = 0$

$f(0) = 0$ (or $f(0) = 1$)

$f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{x \to 0} \frac{f(x+h) - f(x+0)}{h}$

$\lim_{h \to 0} \frac{f(x)f(h) - f(x) f(0)}{h}$

$$= \lim_{h\to 0} f(x)\left(\frac{f(h)-f(0)}{h}\right) = \lim_{h\to 0} f(x)\left[\frac{f(h)-f(0)}{h}\right]$$

$f(x)\cdot f(0)\cdot =$

$, f'(x) = f(x)$

$$\frac{df(x)}{dx} = f(x) \Rightarrow \frac{df(x)}{f(x)} = dx \Rightarrow \int \frac{df(x)}{f(x)} = \int dx + \lambda \Rightarrow \ln f(x) = x + \lambda$$

$$f(x) = e^{x+\lambda} = e^{x}\cdot e^{\lambda} =$$

$x = 0$

$f(0) = ke^{0}$ | $k=1$

$1 = k\cdot 1$ | $f(x) = e^{x}$

$f(x) = ke^{x}$. Ans

(2) Satisfy functional eq. where f is diff and $x = 0, 1$

$$f(x+y) = f(x) + f(y) \quad \forall\ x, y$$

An $f'(x) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$

$$\lim_{h\to 0} \frac{f(x+h) - f(x+0)}{h} = \lim_{h\to 0} f(x) + f(h) - \left[\frac{f(h) + f(0)}{h}\right]$$

$$= \lim_{h\to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = f'(0) = 1$$

$f'(x) = 1$

$$\frac{df(x)}{dx} = 1$$

$f'(x) = 1$

$$df(x) = dx$$

$$\int df(x) = \int dx + \lambda$$

$$\boxed{f(x) = x + \lambda}$$

$f(0) = f(0) + f(0)$

$f(0) = 0$

$f(0) = 0 + \lambda$

$0 = \lambda$

$$\therefore \boxed{f(x) = x}$$

Method :- 2.

$$f(x+y) = f(x) + f(y) \quad \forall\ x, y \in R$$

$$f'(0) = 1$$

diff given fn ~~with~~ eq with respect to x.
after that put $\frac{dy}{dx} = 0$ (because x and y are independent variable)

After manipulate and integrate.

$y = f(x)$

$$f(x+y) = f(x) + f(y)$$

$$\frac{dy}{dx} = f'(x) = \frac{d f(x)}{dx}$$

$$f'(x+y) \cdot \left[1 + \cancel{\frac{dy}{dx}}\right] = f'(x) + f'(y) \cdot \cancel{\frac{dy}{dx}}$$

$$f'(x+y) = f'(x)$$

Put $x = 0$

$$f'(y) = f'(0) = 1$$

$$\frac{d f(x)}{dy} = 1 \quad , \quad d f(y) = dy$$

$$f(y) = \int dy + d$$

$$f(y) = y + d$$

$$f(y) = y$$

$$f(x) = x.$$

Que! diff f^n x satisfy: $f\left(\frac{x}{y}\right) = f(x) - f(y)$

$\forall\ x, y > 0$

if $f'(1) = 1$

then find $f(x)$.

$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

Ans!

$$\lim_{h \to 0} \frac{f\left(\frac{x+h}{x}\right)}{h} = k$$

$$= \lim_{h \to 0} \frac{f\left(1+\frac{h}{x}\right) - f(1)}{h}$$

$f\left(\frac{x}{y}\right) = f(x) - f(y)$

$x = y = 1$

$f(1) = f(1) - f(1) = f(1) = 0$

$$= \lim_{h \to 0} \frac{f\left(1+\frac{h}{x}\right) - f(1)}{\frac{h}{x} \cdot x}$$

$$= \frac{f'(1)}{x} = \frac{1}{x}$$

$f'(x) = \frac{1}{x}$ $\qquad \frac{dt}{dx} = \frac{1}{x}$

$dt = \frac{dx}{x}$ $\qquad f(x) = \int \frac{dx}{x} + \lambda$

$f(x) = \ln x + \lambda$

~~At~~ $0 = \ln 1 + \lambda \Rightarrow \lambda = 0$

$f(x) = \ln x$

Q. diff f^n f satisfy following f^n Rule.

$$f(x+y) = f(x) + f(y) + x^2y + xy^2 \quad \forall x, y \in R$$

Suppose $\lim_{x \to 0} \frac{f(x)}{x} = 1$ $\quad$ $x = y = 0$

then find $f(x)$. $\quad$ $f(0) = 0$

Method:-1

$$= \lim_{x \to 0} \frac{f(x) + f(h) + (x^2h + xh^2) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(h)}{h} + \lim_{h \to 0} \frac{h(x^2 + xh)}{h} = \underline{1 + x^2}$$

$$f(x) = \int (1 + x^2)\, dx + c$$

$$\boxed{f(x) = x + \frac{x^3}{3} + \cancel{c}} \quad 0$$

Method:-2 $f'(x+y) \cdot \left[1 + \frac{dy}{dx}\right] = f'(x) + f'(y)\frac{dy}{dx} + x^2\frac{dy}{dx}$

$0 \quad 6 \quad 6.$

$$+ 2xy + x \cdot 2y\frac{dy}{dx} + y^2$$

0

$$f'(x+y) = f'(x) + 2xy + y^2$$

$$\lim_{x \to 0} \frac{f(x)}{x} = 1$$

$$\lim_{x \to 0} \frac{f'(x)}{1} = 0$$

$$f'(0) = 1$$

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~~10~~/06/17

$f'(y) = f'(0) + y^2$

$f'(y) = 1 + y^2$ — new value x is x.

Note!

$$\boxed{f(x) = \exp \sin x = e^{\sin x}}$$

Q. $y = \left(\tan e^{(\sin x^2)}\right)^3$

$= \left(\sec^2 e^{\sin x^2}\right)^3 = \sec^{(}$

$= \cancel{\tan e} \frac{d}{dx}^{(\sin} \quad 3 \cancel{\tan} e^{(\sin x^2))^2} \cdot \frac{d}{dx} e^{(\sin x^2))^3}_x$

$y = 3\left(e^{(\sin x^2)}\right)^2 \cdot \left(\sec^2 e^{\sin x^2}\right) \cdot e^{\sin x^2} \cdot \cos x^2 \cdot 2x$

Que: $y = e^{\sqrt{\sin(\ln(x^2+7))^5}}$

Sol: $\frac{d}{dx} e^{\sqrt{\sin(\ln(x^2+7))}^5} \cdot \frac{d}{dx} \sqrt{\sin(\ln(x^2+7))^5}$

$\frac{d}{dx}(\ln(x^2+7))^5 \quad \frac{d}{dx}(x^2+7)^5 \cdot \frac{d}{dx} x^2+7$

$= e^{\sqrt{\sin(\ln(x^2+7))}} \cdot \frac{1}{2}\left(\sin(\ln(x^2+7))^5\right)^{-1/2} \cdot \frac{1}{\ln(x^2+7)^5} \cdot$

$5(x^2+7)^4 \cdot 2x$

$= \dfrac{e^{\sqrt{\sin(\ln(x^2+7))}} \cdot 1 \cdot \cos(\ln(x^2+7))^5 \cdot 1 \cdot 5(\ln x^2+7)^4 \cdot 2x}{2\left(\sin(\ln(x^2+7))^5\right) \cdot (x^2+7)^5}$