17/08/17 Chapter:-

Determinant

* Determinants:

$$a_1x + b_1y = c_1 \quad \text{--- (i)}$$

$$a_2x + b_2y = c_2 \quad \text{--- (2)}$$

(i) × b_2 — (ii) × b_1 gives

$$x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1} \quad \& \quad y = \frac{a_1c_2 - a_2c_1}{a_1b_2 - a_2b_1}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \therefore \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$x = \frac{D_1}{D} \qquad y = \frac{D_2}{D}$$

$$\begin{array}{cc} & \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} C_1 & C_2 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \end{array} & \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \end{array}$$

no. of Row = no. of Coulmn.
i.e square.

* No. of elements in n^{th} order Det. is $= n^2$

$$A = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \quad a_1, a_2, b_1, b_2 \text{ are elements.}$$

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad a_{ij}$$

i^{th} row, j^{th} column

Name ⇒ Row and then Column.

*

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

* Sign Conviucer!

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Expand along R_1 =

$$a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

⇒ Determinant always gives numerical value

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{22}) +$$
$$a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$= a_{11} a_{22} a_{33} + a_{12} a_{31} a_{23} + a_{13} a_{21} a_{32}) - (a_{11} a_{32} c_{23} + a_{12} a_{21} a_{33}$

$+ c_{13} a_{31} a_{22})$

$$D = \left|\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{array}\right.$$

Sarrus Diagram

Q. Expand along R_1

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 4 & 2 \\ -1 & 1 & 3 \end{vmatrix}$$

$2(12 - 2) - 3(0+2) - 1(0+4)$

$2(10) - 3(2) - 1(4)$

$20 - 6 - 4$

$= 20 - 10 = 10$

Expand along C_2

$-3(0+2) + 4(6-1) + 1(4-0)$

$-6 + 20 - 4$

$20 - 10 = 10$ Ans

* Co-factor or minor :

x minors :

minors of an element is defined as the minor determinant obtained by deleting a particular row and column in which that element lies

Minor of $a_{ij} = M_{ij}$

and Cofactor of $a_{ij} = C_{ij} = (-1)^{i+j} M_{ij}$

$$\begin{vmatrix} 2 & 3 & -1 \\ 0 & 4 & 2 \\ -1 & 1 & 3 \end{vmatrix}$$

minor of $a_{11} = M_{11} = \begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} = 10$

& cofactor of $a_{11} = C_{11} = (-1)^{1+1} M_{11}$

Sign conven

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Minor of $a_{12} = M_{12} = 2$

cofacter $c_{12} = -2$

Que: $D = \begin{vmatrix} 2 & 3 & -1 \\ 0 & 4 & 2 \\ -1 & 1 & 3 \end{vmatrix}$ replace each element by cofactor

$$D_c = D^c = \begin{vmatrix} 10 & -2 & 4 \\ -10 & 5 & -5 \\ 10 & -4 & 8 \end{vmatrix} = 100 = D^2$$

$10(40-20) + 2(-80 + 50) + 4(+40 - 50)$

$\Rightarrow 200 - 60 - 40 = 100$

$D_c = D^2$

Here, by observation $D_c = D^2$

In general $\boxed{D_c = D^{n-1}}$

where n is the order of determinant.

$= D^2$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$D = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{32} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$D = a_{11} A_{11} + a_{12} A_{12} + C_{13} A_{13}$

$D = a_{31} A_{31} + a_{32} A_{32} + a_{33} A_{33}$

$D = a_{12} A_{12} + a_{22} A_{22} + a_{32} A_{32}$

$a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$

$a_{13} A_{11} + a_{23} A_{21} + a_{33} A_{31} = 0$

* ~~Determinant~~:

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

If C_{ij} is co factor of a_{ij}

then find value of Δ a_{ij}, c_{ij}

$$\Delta = \sum_{j=1}^{3} \sum_{i=1}^{3} a_{ij} c_{ij}$$

$$= \sum_{j=1}^{3} (a_{1j} c_{1j} + a_{2j} c_{2j} + a_{3j} c_{3j})$$

$$= (a_{11} c_{11} + a_{21} c_{21} + a_{31} c_{31}) + (a_{12} c_{12} + a_{22} c_{22} + a_{32} c_{32})$$
$$+ (a_{13} c_{13} + a_{23} c_{23} + a_{33} c_{33})$$

$$= D + D + D = 3D$$

* a_{ij} & a_{ji} are conjugate elements (take it from matrix)

Skew symmetric $\Rightarrow$ $a_{ij} + a_{ji} = 0$

* 3rd order skew symmetric = $\begin{vmatrix} 0 & 4 & -3 \\ -4 & 0 & 7 \\ 3 & -7 & 0 \end{vmatrix} = 0$

2nd order skew $\begin{vmatrix} 0 & 4 \\ -4 & 0 \end{vmatrix} \neq 0$

odd order skew sy $\Rightarrow$ Determinant $= 0$
even " $=$ $D \neq 0$

D-1 ⇒ 1, 2, 7, 8, 9, 10, 19, 20

S-1 ⇒ 1(a), 3(a)(b)

* Value of odd order skew symmetric Determinant will be = 0

— even order skew sym. D will not equal to zero.

* **Properties:-**

(1) The value of Deter. Remains ~~zero~~. unchanged if Row and Columns are interchanged

$$D = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 3 \\ 2 & 1 & 0 \end{vmatrix}$$

$$D' = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 3 & 0 \end{vmatrix}$$

(2) If any two Rows (or Columns) of a determinant be interchanged then value of determinant is changed is sign only

$$D = \begin{vmatrix} 1 & 2 & 3 \\ -1 & 0 & 3 \\ 2 & 1 & 0 \end{vmatrix}$$

$$R_1 \leftrightarrow R_2 \begin{vmatrix} -1 & 0 & 3 \\ 1 & 2 & 3 \\ 2 & 1 & 0 \end{vmatrix} = -D$$

$C_1 \leftrightarrow C_3$

$$\begin{vmatrix} 3 & 0 & -1 \\ 3 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = -(-D) = D$$

(iii) if all the element of the Row (or Column) are zero then value of det. is zero.

(iv) if any two Rows (or Columns) are same (or propotional) the value of det. is zero

$$\begin{vmatrix} 0 & 0 & 0 \\ 1 & 2 & 3 \\ 75 & \frac{1}{76} & -7 \end{vmatrix} = 0$$ (R_1 element are zero)

$$\begin{vmatrix} 1 & 2 & 1 \\ 3 & 4 & 3 \\ -1 & 5 & -1 \end{vmatrix} = 0$$ (C_1 and C_3 are same)

$$\begin{vmatrix} 2 & -4 & 3 \\ 0 & 7 & 9 \\ -4 & 8 & -6 \end{vmatrix} = 0$$ (Here R_1 and R_3 are propotional.)

(v) if all the element of the any row (or columns) are multiply by the same no. then det. is multiply by that no.

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$kD = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ ka_3 & kb_3 & kc_3 \end{vmatrix}$$

$$D = \begin{vmatrix} 3 & -6 & 18 \\ 5 & 4 & 2 \\ 0 & 8 & -6 \end{vmatrix}$$

$$= 3 \cdot 2 \cdot 2 \begin{vmatrix} 1 & -1 & 3 \\ 5 & 2 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

(vii) if each element of any row (or column) is expressed as sum of two terms then det. can be expressed as sum of two or more determinants.

$$D = \begin{vmatrix} a_1+x & b_1+y & c_1+z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

*

$$D = \begin{vmatrix} a_1+x+p & b_1+y+q & c_1+z+r \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} p & q & r \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

* $D = \begin{vmatrix} a_1+x & b_1+y & c+z \\ a_2+p & b_2+q & c_2+r \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2+p & b_2+q & c_2+r \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2+p & b_2+q & c_2+r \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & b_1 & c_1 \\ p & q & r \\ a_3 & b_3 & c_3 \end{vmatrix} +$$

$D = |$ = 1st row 2 = (
2nd row 3
3rd row 4

then = $2 \times 3 \times 4 = 24$ determenant break.

* $D(r) = \begin{vmatrix} f(r) & g(r) & h(r) \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $r \in N$.

$$\sum_{r=1}^{N} D(r) = \begin{vmatrix} \sum_{r=1}^{n} f(r) & \sum_{r=1}^{n} g(r) & \sum_{r=1}^{n} h(r) \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Q. if $D(r) = \begin{vmatrix} r & n & \frac{n(n+1)}{2} \\ 2r-1 & y & n^2 \\ 3r-2 & z & \frac{n(3n-1)}{2} \end{vmatrix}$

then find value of $\sum_{r=1}^{n} D_r =$

$$= \begin{vmatrix} 1 & n & \frac{n(n+1)}{2} \\ 1 & y & n^2 \\ 1 & z & \frac{n(3n-1)}{2} \end{vmatrix} + \begin{vmatrix} n & n & \frac{n(n+1)}{2} \\ 2n-1 & y & n^2 \\ 3n-2 & z & n(3n \end{vmatrix}$$

$$\sum_{r=1}^{n} D_r = \begin{vmatrix} \sum_{r=1}^{n} r & n & \frac{n(n+1)}{2} \\ \sum_{r=1}^{n} (2r-1) & y & n^2 \\ \sum_{r=1}^{n} (3r-2) & z & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{n(n+1)}{2} & \cdot & \cdot \\ n^2 & \cdot & \cdot \\ \frac{n(3n-1)}{2} & & \end{vmatrix}$$

$\Sigma(2r-1)$
$= 2\Sigma r - \Sigma 1$
$= 2\frac{(n(n+1))}{2} - n$
$= n^2 + n - n$
$= n^2$

$\Sigma(3r-2) = 3\Sigma r - 2\Sigma 1$
$= 3\frac{n(n+1)}{2} - 2n$
$= \frac{3n^2+3n-4n}{2} = \frac{n(3n-1)}{2}$

C_1 and C_3 are same so $D = 0$

$= 0$

* Row Column operation :

The value of det. remain unaltered under a Column operation of the form

$$C_i \longrightarrow C_i + \alpha C_j + \beta C_k \; (i \neq j, k)$$

or row operation

$$R_i \longrightarrow R_i + \alpha R_j + \beta \; (i \neq j, k)$$

By applying this property at least one Row or Column Remains unchanged

$$\Rightarrow \quad D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$C_1 \rightarrow C_1 + 2C_2$

$$\begin{vmatrix} a_1 + 2b_1 & b_1 & c_1 \\ a_2 + 2b_2 & b_2 & c_2 \\ a_3 + 2b_3 & b_3 & c_3 \end{vmatrix} = D$$

$C_2 \rightarrow C_2 - C_1 + 4C_3$

$$D = \begin{vmatrix} a_1 & b_1 - a_1 + 4c_1 & c_1 \\ a_2 & b_2 - a_2 + 4c_2 & c_2 \\ a_3 & b_3 - a_3 + 4c_3 & c_3 \end{vmatrix} = D$$

$R_3 \rightarrow R_3 - 2R_1$ & $R_2 \rightarrow R_2 + R_1$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 + a_1 & b_2 + b_1 & c_2 + c_1 \\ a_3 - 2a_1 & b_3 - 2b_1 & c_3 - 2c_1 \end{vmatrix}$$

* Spacial type of Determinants :-

(1) Cyclic determinant :

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

$= \cancel{-\frac{1}{2}(a+b+c)}$

$$-\frac{1}{2}(a+b+c)\cdot[a^2+b^2+c^2-ab-bc-ca]$$

$$= \frac{-1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$= -\cancel{\frac{1}{2}}(a+b+c)(a+b\omega+c\omega^2)(a+b\omega^2+c\omega)$$

Here ω is Cube roots of.

* $\begin{vmatrix} 0 & -\lambda & \mu \\ \lambda & 0 & v \\ -\mu & -v & 0 \end{vmatrix} = 0$ Skew symmetric

* Other Important Determinants :

(i) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

(ii) $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

Ans.

(iii)

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

Q (iv)

$$\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)(a^2+b^2+c^2 - ab-bc-ca)$$

* <u>factor theorem:</u>

If by putting $x = a$, the value of det. vanishes then $(x-a)$ is factor of the given determinant.

(2 rows are same)

Q:

$$D = \begin{vmatrix} x & 5 & 2 \\ x^2 & 9 & 4 \\ x^3 & 16 & 8 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$$

$x = 2$ $\quad = (x-2).(px^2 + qx + r)$

Factor theorem

$$D = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \lambda(a-b)(b-c)(c-a).$$

* Behave as an Identity

$\therefore$ LHS = RHS

H.W : Learn Proberties
and — — — —
Que. other Book Solve

Hence LHS = RHS $\forall R \in (a, b, c)$

$a = 0,\ b = 1,\ c = -2$
get $d = 1$.

*

$$\begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & b_3 \end{vmatrix} = a_1 b_2 c_3$$

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & 0 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

$$\begin{vmatrix} a_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

* **Differenciation of Determeunant :**

if $f(x) = \begin{vmatrix} f & g & h \\ u & v & w \\ l & m & n \end{vmatrix}$

wher, $f, g, h, u, v, w, l, m, n$ are either f^n of x or constant

$$f'(x) = \begin{vmatrix} f' & g' & h' \\ u & v & w \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u' & v' & w' \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u & v & w \\ l' & m' & n' \end{vmatrix}$$

Q. without expansion P. that

$$\begin{vmatrix} ax & by & cz \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} a & b & c \\ x & y & z \\ yz & zx & xy \end{vmatrix}$$

$$\frac{xyz}{xyz}\begin{vmatrix} a & b & c \\ x & y & z \\ \frac{1}{x}\times xyz & \frac{1}{y}\times xyz & \frac{1}{z}\times xyz \end{vmatrix} =$$

(2) find $D = \begin{vmatrix} a & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix} = 0$

Q.3 $D = \begin{vmatrix} x & x+a & x+2a \\ x & x+2a & x+4a \\ x & x+3a & x+6a \end{vmatrix}$ P that $D=0$

$$\begin{vmatrix} x & a & x+2a \\ x & 2a & x+4a \\ x & 3a & x+6a \end{vmatrix} + 2\begin{vmatrix} x & a & \frac{2a}{2} \\ x & 2a & \frac{4a}{2} \\ x & 3a & \frac{6a}{2} \end{vmatrix}$$

$$= 0 \qquad : \qquad 2\begin{vmatrix} x & a & a \\ x & 2a & 2a \\ x & 3a & 3a \end{vmatrix}$$

$$\begin{vmatrix} x & a & 2a \\ x & 2a & 4a \\ x & 3a & 6a \end{vmatrix} = 0 + 0 = 0$$

$C_3 \to C_3 - C_1 \quad : \quad C_2 \to C_2 - C_1$

Q. if $f(x) = \begin{vmatrix} 2 & 1 & 3 \\ x & 2 & -1 \\ x^2 & x & 1 \end{vmatrix}$ find $f'(x)$ without expanding

$$f'(x) = \begin{vmatrix} 0 & 0 & 0 \\ x & 2 & -1 \\ x^2 & x & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 3 \\ 1 & 0 & 0 \\ x^2 & x & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 3 \\ x & 2 & -1 \\ 2x & 1 & 0 \end{vmatrix}$$

Que! without expanding proff that

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} + \begin{vmatrix} q & r & p \\ y & z & x \\ b & c & a \end{vmatrix} = 0$$

$= R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a+b+c & b & c \\ x+y+z & y & z \\ p+q+r & q & r \end{vmatrix} + \begin{vmatrix} q+r+p & r & p \\ y+z+x & z & x \\ b+c+a & c & a \end{vmatrix} =$$

$R_1 \longleftrightarrow R_3$

$$= - \begin{vmatrix} p & q & r \\ x & y & z \\ a & b & c \end{vmatrix}$$

$C_1 \longleftrightarrow C_2$

$$= - - \begin{vmatrix} q & p & r \\ y & x & z \\ b & a & c \end{vmatrix}$$

$C_2 \longleftrightarrow C_3 = - - - \begin{vmatrix} q & r & p \\ y & z & x \\ b & c & a \end{vmatrix} =$

then
treat with Identity.

Q. $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$

(i) find $A+B+C+D+E$ $\quad x=1$

$= \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 6 \\ 1 & 1 & 6 \end{vmatrix} = 0$

(ii) find $A-B+C-D+E = 0$ $\quad x=-1$

$= \begin{vmatrix} -1 & 2 & -1 \\ -1 & -1 & 6 \\ -1 & -1 & 6 \end{vmatrix} = 0$

(iii) find E $=$ Put $x=0$

(iv) find D $=$ $= 4Ax^3 + 3Bx^2 + 2Cx + D$.

Differ. and Put $x=0$.

Q find $5A+4B+3C+2D+E$

Ans

multiply by x and diff.

$x\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex$

$\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} + x\left(\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} + \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} + \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}\right) = 5Ax^4 + 4Bx^3 + 3Cx^2 + 2Dx + E$

Q. Prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

$= \quad R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a-b-c+2b+2c & 2a+b-c-a+2c & 2a+2b+c-a-b \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_3 \to C_3 - C_1$ & $C_2 \to C_2 - C_1$

$$(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{vmatrix} =$$

$$(a+b+c)(a+b+c)^2 = (a+b+c)^3.$$

Q. P.T $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a^2+b^2+c^2)(a+b+c)(a-b)(b-c)(c-a))$

$$\Rightarrow \begin{vmatrix} b^2+c^2+2bc & a^2 & bc \\ c^2+a^2+2ac & b^2 & ca \\ a^2+b^2+2ab & c^2 & ab \end{vmatrix}$$

$\cancel{R_1 \to C_1 + R_2}$

$$= \begin{vmatrix} \cancel{a^2+b^2+c^2+2bc} \\ \cancel{a^2+b^2+c^2+} \\ \end{vmatrix}$$

$C_1 \to C_1 - 2C_3 + C_2$

$$(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

Q. $\begin{vmatrix} (\beta+\gamma-\alpha-\delta)^4 & (\beta+\gamma-\alpha-\delta)^2 & 1 \\ (\gamma+\alpha-\beta-\delta)^4 & (\gamma+\alpha-\beta-\delta)^2 & 1 \\ (\alpha+\beta-\gamma-\delta)^4 & (\alpha+\beta-\gamma-\delta)^2 & 1 \end{vmatrix} =$

$= (\beta+\gamma-\alpha-\delta)^2 = A$

$(\gamma+\alpha-\beta-\delta)^2 = B$

$(\alpha+\beta-\gamma-\delta)^2 = C$

$$\begin{vmatrix} A^2 & A & 1 \\ B^2 & B & 1 \\ C^2 & C & 1 \end{vmatrix} = (A-B)(B-C)(C-A)$$

Q. $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$a+b+c=0$

$R_1 \to R_1+R_2+R_3$

$= \begin{vmatrix} a+b+c-x & c+b+a-x & a+b+c-x \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0 =$

$= (\cancel{a+b+c}-x) \begin{vmatrix} 1 & 1 & 1 \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$

$C_3 \to C_3 - C_1 \quad , \quad C_2 \to C_2 - C_1$

(9)

$D = \begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & bc \\ ac & bc & c^2+\lambda \end{vmatrix} \begin{matrix} \div a \\ \div b \\ \div c \end{matrix}$

$\downarrow a \quad \downarrow b \quad \downarrow c$

$D = abc \begin{vmatrix} a+\frac{\lambda}{a} & b & c \\ a & b+\frac{\lambda}{b} & c \\ a & b & c+\frac{\lambda}{c} \end{vmatrix}$

$= \begin{vmatrix} a^2+\lambda & b^2 & c^2 \\ a^2 & b^2+\lambda & c^2 \\ a^2 & b^2 & c^2+\lambda \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$

$(a^2+b^2+c^2+\lambda) \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+\lambda & c^2 \\ 1 & b^2 & c^2+\lambda \end{vmatrix}$

Que? if $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 5$

then find

$$\begin{vmatrix} bc-a^2 & ac-b^2 & ab-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix} = 5^2 = 25$$

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} bc+ab+ac-a^2-b^2-c^2 & ab+bc+ca-a^2-b^2-c^2 & ab+bc+ac-a^2-b^2-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$$

$$ab+bc+ca-a^2-b^2-c^2 \begin{vmatrix} 1 & 1 & 1 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$$

=

* multiplication of Determinant

R R

$\boxed{R \quad C}$

C R

C C

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & l_2 \\ m_1 & m_2 \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_1l_1 + b_1m_1 & a_1l_2 + b_1m_2 \\ a_2l_1 + b_2m_1 & a_2l_2 + b_2m_2 \end{vmatrix}$$

Q-1 : 11, 12, 13, 17, 18
31 => 1(b), 2, 4, 6(a)
8, 10

Q. $\begin{vmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} 2 & 1 & 0 \\ 0 & -1 & 3 \\ 1 & -1 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 5 \\ 2 & -3 & 5 \\ 7 & 1 & 4 \end{vmatrix}$

Que.

$\begin{vmatrix} 3 & -1 & 2 \\ 0 & 2 & 2 \\ -1 & 0 & 5 \end{vmatrix} \times \begin{vmatrix} 0 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & 0 & 0 \end{vmatrix} =$

$= \begin{vmatrix} -4 & 2 & -1 \\ 2 & 8 & 8 \\ -5 & 4 & -1 \end{vmatrix}$

* $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} =$

$\begin{vmatrix} a_1 l + b_1 l_2 & a_1 m_1 + b_1 m_2 \\ a_2 l + b_2 l_2 & a_2 m_1 + b_2 m_2 \end{vmatrix}$

Cos A Cos B + Sin A Sin

Sin Cos Cos(A-B)

$a_1 b_1$ $a_1 b_1$

Que: P. that

$$\begin{vmatrix} a_1 l_1 + b_1 m_1 & a_1 l_2 + b_1 m_2 & a_1 l_3 + b_1 m_3 \\ a_2 l_1 + b_2 m_1 & a_2 l_2 + b_2 m_2 & a_2 l_3 + b_2 m_3 \\ a_3 l_1 + b_3 m_1 & a_3 l_2 + b_3 m_2 & a_3 l_3 + b_3 m_3 \end{vmatrix}$$

Ans:

$$A \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \times \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ 0 & 0 & 0 \end{vmatrix}$$

OR

$$A \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \times \begin{vmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ 1 & 1 & 1 \end{vmatrix}$$

Que: P. that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

Ans: →

$$\begin{vmatrix} \cos A \cos P + \sin A \sin P & \cos A \cos Q + \sin A \sin Q \\ \cos B \cos P + \sin B \sin P & \cos B \cos Q + \sin B \sin Q \\ \cos C \cos P + \sin C \sin P & \cos C \cos Q + \sin C \sin Q \end{vmatrix}$$

$$\begin{matrix} \cos A \cos R + \sin A \sin R \\ \cos B \cos R + \sin B \sin R \\ \cos C \cos R + \sin C \sin R \end{matrix} \Bigg|$$

a^2+b^2 $(a+b)^{11}$

$$= \begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix} \times \begin{vmatrix} \cos P & \cos Q & \cos R \\ \sin P & \cos Q & \cos R \\ 0 & 0 & 0 \end{vmatrix}$$

(J.M)

Q.8 if $S_n = \alpha^n + B^n$

and $\alpha - B = 1$

$B - Y = 2$

$Y - \alpha = 3$

then find

$S_1 = \alpha + B$

$S_2 = \alpha^2 + B^2$

$S_3 = \alpha^3 + B^3$

$S_4 = \alpha^4 + B^4$

Ans:-

$$\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$$

$$= \begin{vmatrix} 1+1+1 & 1+(\alpha+B) & 1+\alpha^2+B^2 \\ 1+(\alpha+B) & 1+(\alpha^2+B^2) & 1+\alpha^3+B^3 \\ 1+(\alpha^2+B^2) & 1+(\alpha^3+B^3) & 1+\alpha^4+B^4 \end{vmatrix}$$

$1+\alpha+B$

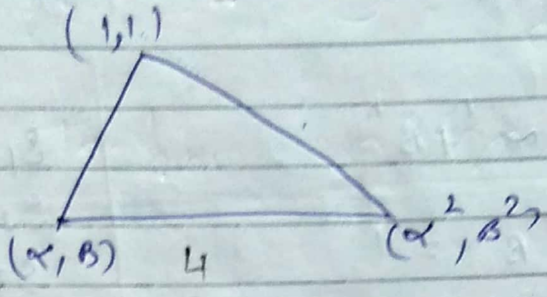
$1+\alpha^2+B^2+\alpha^2$

$$= \begin{vmatrix} 1+1+1 & 1 & 1 \\ 1 & & \\ & & \end{vmatrix} \times \begin{vmatrix} 1 & 1 \\ \alpha+B & \alpha^2+B^2 \\ \alpha^2+B^2 & \alpha^3+B^3 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 \\ & \\ & \end{vmatrix} \times \begin{vmatrix} 1 & & 1 \\ \alpha & b & 1+b \end{vmatrix}$$

$$2 = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \cancel{*} & \cancel{\beta} & \beta' \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$= 16$ Ans



$\Delta = 2$

* System of equation:

(A) In two variables:

$a_1 x + b_1 y = c_1$

$a_2 x + b_2 y = c_2$

$b_1 y = -a_1 x + c_1$

$y = -\frac{a_1}{b_1} x + \frac{c_1}{b_1}$

$x - y = 1$

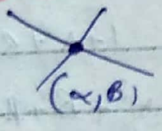
$x + y = 1$

$x = 0$

$y = 1$

Constitent

$\boxed{\frac{a_1}{a_2} \neq \frac{b_1}{b_2}}$



Ex: $x + y = 2$

$x - y = 0$

$(x = y = 1)$

Constent Solution.

Ex: 2 $x + 2y = 0$

$x - y = 0$

$\boxed{x = y = 0}$

Consistent

and Trivial Solution.

(All unknown zero)

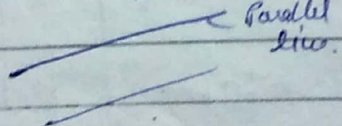
$m_1 = m_2$

$\frac{-a_1}{b_1} = -\frac{c_2}{b_2}$

$\boxed{\text{or } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}}$

Parallel lines.

equations are Inconsistent and no solution.



$\boxed{\text{If } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

Ex: $x + y = 2$

$2x + 2y = 4$

i.e Consided lines Consistant and Infinite many Solution.

$x - y = -1$

$x + y = 1$

$$= \boxed{x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}.}$$

Constiteut and non trivial solution

Que! The no. of values for which the system of eq^n $(k+1)x + 8y = 4k$.

$$kx + (k+3)y = 3k - 1$$

has no solution

Ans:- $(k+1)x + 8y = 4k$

$kx + (k+3)y = 3k - 1$

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$$

$y = \frac{4k - (k+1)x}{8}$.

$y = \frac{3k - 1 - kx}{k+3}$.

$y = \frac{4k}{8} - \frac{(k+1)x}{8}$

$y = \frac{3k-1}{k+3} - \frac{kx}{k+3}$

(B) $a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

3 equation
3 unknown

if atleast one of the $d_1, d_2, d_3 \neq 0$ then system of equation called non-Homogenious equation.

* $$\left.\begin{aligned} a_1x + b_1y + c_1z &= 0 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \end{aligned}\right\}$$ All const. are zero.

Called Homogenious equation.

* Solution Technicge of non-Homogenious eq.

(i) $x = \dfrac{D_1}{D}$, $y = \dfrac{D_2}{D}$, $z = \dfrac{D_3}{D}$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad D_1 = \begin{vmatrix} d_1 \\ d_2 \\ d_3 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \quad D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Non Homogenius equation.

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

$\boxed{D \neq 0}$

unike solution

Consistant

at leas one of the d_1, d_2, d_3 is $\neq 0$

non-Trivial solution

$d_1 = d_2 = d_3 = 0$

$\therefore x = y = z = 0$

(Trivial Solution)

$xD = D_1$
$yD = D_2$
$zD = D_3$

$\boxed{D = 0}$

$D_1 = D_2 = D_3 = 0$

(Infinite many Solution)

(consistant)

(zero & non-zero).

at least $d_1, d_2, d_3 \neq 0$

No-Solution

Inconsistant

(ii) Homogenius:-

$$a_1 x + b_1 y + c_1 z = 0$$
$$a_2 x + b_2 y + c_2 = 0$$
$$a_3 x + b_3 y + c_3 z = 0$$

$$D_1 = D_2 = D_3 = 0.$$

$\boxed{D = 0}$

(Infinite many Solution)

(Consistant)

(zero or non-zero)

$\boxed{D \neq 0}$

Trivial Solution.

Q. 13 (a) $x + y + z - 6 = 0$
$2x + y - z - 1 = 0$
$x + y - 2z + 3 = 0$

$x + y + z = 6$
$2x + y - z = 1$
$x + y - 2z = -3$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{vmatrix} = 3$$

Consistent, Non zero.
= unike solution.

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ -3 & 1 & -2 \end{vmatrix} = 3, \quad D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & -3 & -2 \end{vmatrix} = 6$$

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 1 \\ 1 & 1 & -3 \end{vmatrix} = 9$$

$D_1 = \cancel{6(-2+1)}$

(c) $7x - 7y + 5 = 3$
$3x + y + 5z = 7$
$2x + 3y + 5x = 5$

No solution.

$$D = \begin{vmatrix} 7 & -7 & 5 \\ 3 & 1 & 5 \\ 2 & 3 & 5 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 3 & -7 & 5 \\ 7 & 1 & 5 \\ 5 & 3 & 5 \end{vmatrix} =$$

$D_1 \neq 0$

$$D_2 = \begin{vmatrix} 7 & 3 & 5 \\ 3 & 7 & 5 \\ 2 & 5 & 5 \end{vmatrix} =$$

$$D_3 = \begin{vmatrix} 7 & 7 & 3 \\ 3 & 1 & 7 \\ 2 & 3 & 5 \end{vmatrix}$$

Q. Write possible value of P and Q for which system of eqn.

$$2x + py + 6z = 8$$
$$x + 2y + qz = 5$$
$$x + y + 3z = 4$$

have

i) infinite many solution :

$P = 2,\ q = R.$ $\quad = R - \{1, 3\}$

2) unike solution $P \neq 2$, OR $P \in R - \{2\}$ $\quad = 1, 3, 2$

$q \neq 3$ $\quad =$

(3) no solution $q = 3$, $P \neq 2$. $\quad P = 2, 3$

$$D = \begin{vmatrix} 2 & P & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = (P-2)(q-3)$$

$$D_1 = \begin{vmatrix} 8 & P & 6 \\ 5 & 2 & q \\ 4 & 1 & 3 \end{vmatrix} = (4q-15)(P-2) \quad D_2 = \begin{vmatrix} 2 & 8 & 6 \\ 1 & 5 & q \\ 1 & 4 & 3 \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & P & 8 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{vmatrix} = P - 2.$$

$$x = \frac{D_1}{D} = \frac{(4q-15)(P-2)}{(P-2)(q-3)}$$

Q. if a, b, c are non zero Real No. such that

$$(a-1)x = y + z$$
$$(b-1)y = x + z$$
$$(c-1)z = x + y$$

has non - Trial solution then find condition

$$(a-1)x - y - z = 0$$
$$-x + (b-1)y - z = 0$$
$$x + y + (c-1)z = 0$$

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & -(b-1) & -1 \\ 1 & 1 & -(c-1) \end{vmatrix} = 0$$

$D_1 = D_2 = D_3 = 0$ Infinite many Sol^n.

S-1
Q.15

$a(y+z) = x$ $\quad$ $b(z+x) = y$

$c(x+y) = z$ $\quad$ $a, b, c \neq -1$

$$\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} =$$

$\boxed{a(y+z) + b(z+x) + c(x+y)}$

$=$ $D = 0$ $\quad = \quad D = 0$

$$x - ay - az = 0$$
$$bx - y + bz = 0$$
$$cx + cy - z = 0$$

$D \neq 0$

$D = 0$

$$\begin{vmatrix} 1 & -a & -a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$$

(3) $x, y, z \neq 0$ (not all zero)

$x = cy + bz \quad y = az + cx \quad z = bx + ay$

$a^2 + b^2 + c^2 + 2abc = ?$

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

$$D = \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$= 1(1-a^2) + c(-c-ab) - b(ac+b)$

$= 1 - a^2 - c^2 - abc - abc - b^2 = 0$

$= a^2 + b^2 + c^2 + 2abc = +1$

Ans

(5) $4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

$$D = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$8 - k(k-2) + 2(2k-8) = 0$

$8 - k^2 + 2k + 4k - 16 = 0$

$-k^2 + 6k - 8 = 0$

non - zero.

$k^2 - 6k + 8 = 0$

$k^2 - 2k - 4k + 8 = 0$

$k(k-2) - 4(k-2)$

$(k-2)(k-4)$

$k = 2, \quad k = 4$ Ans

(11) $x + y + z = 1$

$x + ay + z = 0$

$ax + by + z = 0$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix}$$

$1(a-b) - 1(1-a) + 1(b-a^2)$

Condition $D = 0$

$a - \cancel{b} - 1 + a + \cancel{b} - a^2 = 0$

$-a^2 + 2a - 1 = 0$

$a^2 - 2a + 1 = 0$

$a^2 - a - a + 1 = 0$

$a(a-1) - (a-1)$

$(a-1)(a-1)$

$a = 1$ Hence, a singleton

Ans

10) = $x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

$$D = \begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix}$$

$1(\lambda+1) - \lambda(-\lambda^2+1) - 1(\lambda+1)$

$\cancel{\lambda} + \cancel{1} + \lambda^3 - \cancel{\lambda} - \lambda - \cancel{1}$

$\lambda^3 - \lambda \Rightarrow \lambda(\lambda^2-1)$

$(\lambda-0)(\lambda-1)(\lambda+1)$

$\lambda = 0,\ \lambda = 1,\ \lambda = -1$

exactly three Values of λ.

$\cancel{1(\lambda+1) - \lambda(-\lambda^2+1) - 1(\lambda+1)}$

$\cancel{\lambda + 1 + \lambda^3 - \lambda - \lambda - 1}$

$\cancel{\lambda^3 - \lambda - 1 + 1}$

$\lambda^3 - \lambda =$

$\cancel{\lambda^3 - 1 = \quad \lambda = 1,}$ ~~three Exam Val~~

Q.9:

$(-\lambda+2)x_1 - 2x_2 + x_3 = 0$

$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$

$-x_1 + 2x_2 - \lambda x_3 = 0$

$$D = 0 \Rightarrow \begin{vmatrix} -\lambda+2 & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

Solve them

$(\lambda-1)^2(\lambda+3) = 0$

$\Rightarrow \lambda = 1, -3$ Ans

(S-1) (14)

$x + ky + 3z = 0$ — (i) non trivial Solution

$3x + ky - 2z = 0$ — (ii)

$2x + 3y - 4z = 0$ — (iii)

$$D = \begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\therefore k = \dfrac{33}{2}$

eq^n (1) − (2)

$-2x + 5z = 0$

$x = \dfrac{5}{2}z$ $2x = 5z$

$2x = 5z$ $\cancel{x = \frac{5}{2}}$ in eq^n (3)

$5z + 3y - 4z = 0$

$3y + z = 0$

$y = -\dfrac{1}{3}z$

sol^n $x : y : z :: \dfrac{5}{2}z : -\dfrac{1}{3}z : z$

$\therefore$ Sol^n $\left(\dfrac{5}{2}\lambda, -\dfrac{\lambda}{3}, \lambda\right)$

$\lambda = 1$ $\lambda = 6$

$\left(\dfrac{5}{2}, \dfrac{-1}{3}, 1\right)$ $(15, -2, 6)$

JM (7) $(k+1)x + 8y = 4k$

$kx + (k+3)y = 3k-1$

$D_1 = \begin{vmatrix} (k+1) & 8 \\ k & k+3 \end{vmatrix}$ $D_2 = \begin{vmatrix} 4k & 8 \\ 3k-1 & k+3 \end{vmatrix}$

$D_3 = \begin{vmatrix} (k+1) & 4k \\ k & 3k-1 \end{vmatrix}$

$(k+1)(k+3) - 8k$

$k^2 + 3k + k + 3 - 8k$

$k^2 - 4k + 3$

$k^2 - k - 3k + 3$

$k(k-1) - 3(k-1)$

$k = 3, 1$

(10) $\Big|$

(6) $\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} = 0$

$1(-3+k) + k(-k+3k) + 1(k-9)$

$-3 + k - k^2 + 3k^2 + k - 9$

$2k^2 + 2k - 12$

$k^2 + k - 6$

$k^2 + 3k - 2k - 6$

$k(k+3) - 2(k+3)$ $k = 2, -3.$

for Trivial soln: $R - \{2, -3\}$. Ans