01/07/17

chapter: SBG STUDY

Definite Integration

* if $\int f(x)\,dx = F(x) + c$

then $\int_{x=a}^{x=b} f(x)\,dx = \int_a^b f(x)\,dx = \left[F(x)+c\right]_a^b$

$= \left[F(b)+c) - (F(a)+c)\right]$

$F(b) - F(a)$

Que! $\int_{\pi/4}^{\pi/3} \cos x\,dx = \left[\sin x\right]_{\pi/4}^{\pi/3}$

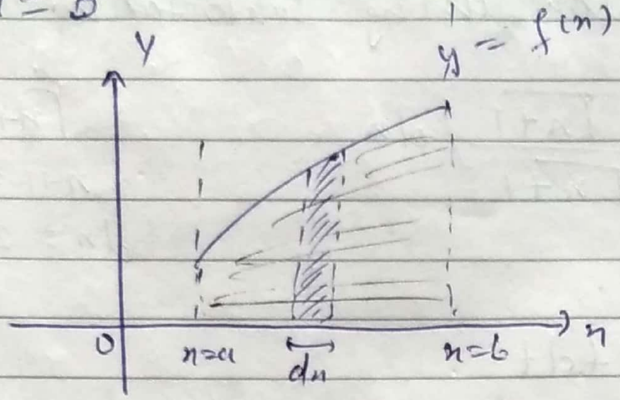
$= \sin\frac{\pi}{3} - \sin\frac{\pi}{4}$

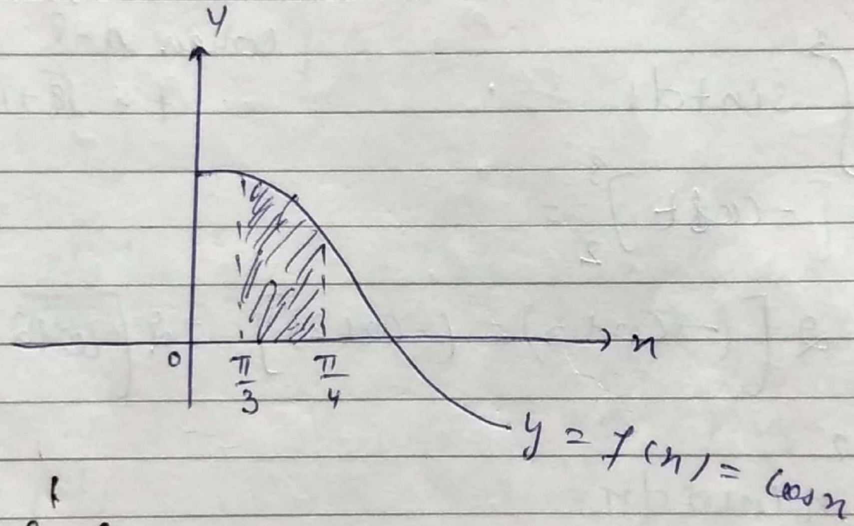
$= \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$

* Geometrical meaning of definite integration

$\int_{x=a}^{x=b} f(x)\,dx = \int_a^b f(x)\,dx =$ algebraic sum of area bounded by curve $y = f(x)$, x axis and ordinate $x = a$

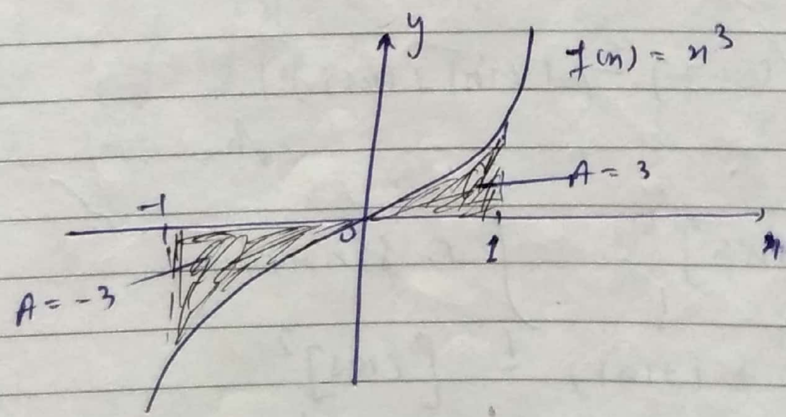
and $x = b$





Que: $\int_{-1}^{1} x^3 \, dx =$

$= \left[\frac{x^4}{4}\right]_{-1}^{1} = \left[\left(\frac{1}{4}\right) - \left(\frac{(-1)^4}{4}\right)\right] = \frac{1}{4} - \frac{1}{4} = 0$



Definate Integration using Substitution

Que! $I = \int_3^8 \frac{\sin\sqrt{x+1}}{\sqrt{x+1}}\,dx$

$\sqrt{x+1} = t$

$x+1 = t^2$

$dx = 2t\,dt$

$= \int_2^3 \frac{\sin t}{\cancel{t}}\, 2\cancel{t}\,dt$

when $x = 3$

$t = \sqrt{3+1} = 2$

when $x = 8$

$t = \sqrt{8+1} = 3$

~~when~~

$2\int_2^3 \sin t\,dt$

$2\left[-\cos t\right]_2^3 =$

$2\left[(-\cos 3) - (-\cos 2)\right] = 2\left[\cos 2 - \cos 3\right]$ Ans

Que!! $\int_1^2 \underset{\text{I}}{x} \underset{\text{II}}{\cos x}\,dx$

$= x\sin x - \int 1\cdot \sin x\,dx$

$\left[x\sin x + \cos x\right]_1^2$

$= \left[2\sin 2 + \cos 2\right] - (1\sin 1 + \cos 1))$

Ans

Method, 2 $\Rightarrow$ $\left[x\sin x\right]_1^2 - \int_1^2 1\cdot\sin x\,dx$

$= (2\sin 2 - 1\sin 1) - \left[\cos\right]_1^2$

$= \underline{\qquad\qquad} - \left[\cos 2 - \cos 1\right)$ Ans

Que 1: $\int_0^{\pi/4} \cos 2x \sqrt{4-\sin 2x}\, dx.$

$\sqrt{4-\sin 2x} = t$
$4-\sin 2x = t^2$
$-2\cos 2x\, dx = 2t\,dt$
$\cos 2x\, dx = -t\,dt$

$x=0,\ t=\sqrt{4}=2$
$x=\frac{\pi}{4}\ \ t=\sqrt{3}$

$\int_2^{\sqrt{3}} -t\cdot t\,dt = -\int_2^{\sqrt{3}} t^2\,dt$

$= -\frac{1}{3}\cdot\left[t^3\right]_2^{\sqrt{3}}$

$= \frac{-1}{3}\left[3\sqrt{3}-8\right]$ Ans.

(2) $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}\,dx$

$\sin^{-1}x = P$

$\frac{1}{\sqrt{1-x^2}}\,dx = dP$

$x=0,\ P=0$
$x=1,\ P=\pi/2$

$\int_0^{\pi/2} P\,dP$

$= \int \frac{1}{2}\cdot\left[P^2\right]_0^{\pi/2} = \frac{\pi^2}{8}$

Que: $\int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}}\,dx.$

$x = \alpha\cos^2\theta + \beta\sin^2\theta$

$(x-\alpha) = (\alpha\cos^2\theta + \beta\sin^2\theta - \alpha)$
$= \beta\sin^2\theta - \alpha(1-\cos^2\theta)$

$\sqrt{\frac{x-\alpha}{\beta-x}} = \tan\theta$ $\quad = \beta\sin^2\theta - \alpha\sin^2\theta$
$= (\beta-\alpha)\sin^2\theta$

$\beta - x = \beta - \alpha\cos^2\theta - \beta\sin^2\theta$
$= \beta\cos^2\theta - \alpha\cos^2\theta$
$= (\beta-\alpha)\cos^2\theta$

$x = \alpha = \alpha\cos^2\theta + \beta\sin^2\theta$

$\alpha(1-\cos^2\theta) = \beta\sin^2\theta$

$\alpha\sin^2\theta = \beta\sin^2\theta$

$\sin^2\theta\cdot(\alpha-\beta) = 0$

$\therefore \alpha-\beta \neq 0 \therefore \sin^2\theta = 0 = \theta = 0$

$x = \alpha\cos^2\theta + \beta\sin^2\theta$

$dx = (-\alpha\sin 2\theta + \beta\sin 2\theta)\,dx$

$= (\beta-\alpha)\sin 2\theta\, d\theta$

$x = \beta$

$\beta = \alpha\cos^2\theta + \beta\sin^2\theta$

$\beta\cos^2\theta = \alpha\cos^2\theta$

$(\alpha\cdot\beta)\cos^2\theta = 0$

$\cos^2\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

$$= (\beta-\alpha)\int_0^{\pi/2} \tan\theta\cdot\sin 2\theta\, d\theta$$

Ans.

Que! assume $x'' =$ cont.

and $f(1) = 3$

$f'(1) = 2$

$\int_0^1 f(x)\,dx = 5$ $\quad = x^2.$

then find $I = \int_0^1 \underset{I}{x^2}\, \underset{II}{f''(x)}\, dx$

$= \left[x^2 f'(x)\right]_0^1 - 2\int_0^1 \underset{I}{x}\, \underset{II}{f'(x)}\, dx$

$= (1 f'(1) - 0) - 2\left[\left[x f(x)\right]^1 - \int_0^1 f(x)\,dx\right]$

$(2-0) - 2\left[\{1 f(1) - 0\} - 5\right]$

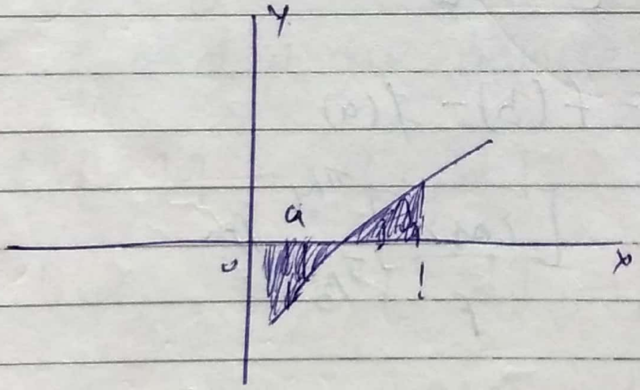
$2 - 2[(3-0) - 5]$
$= 2 + 2 \cdot 2$
$= 2 + 4 = 6$ A

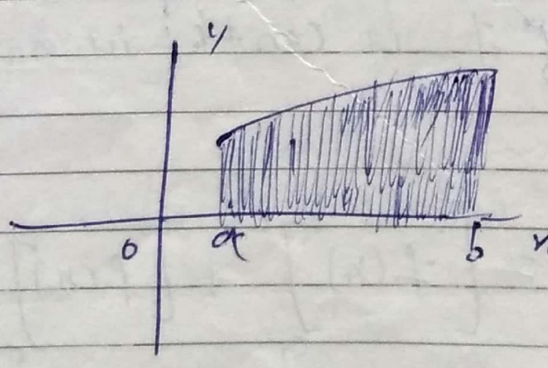
<u>Note</u>:

(1) $\int_a^a f(x)\,dx = 0$

(2) if $\int_a^b f(x)\,dx = 0$ and $f(x)$ is cont. f^n

then $y = f(x)$ must have at least one root in (a, b)





(3) $$\int_a^b f(x)\cdot d(g(x)) = \int_{g^{-1}(a)}^{g^{-1}(b)} f(x)\cdot g'(x)\,dx.$$

Ex: $$\int_4^9 x\,d(x^2) = \qquad (x>0)$$

$d(g(x)) = g'(x)dx$

$g(x)=a,\ g(x)=b$

$x=g^{-1}(a),\ x=g^{-1}(b)$

$x^2=4 \Rightarrow x=2$

$x^2=9 \Rightarrow x=3$

4) $$\int_a^b \left(\frac{d}{dx}f(x)\right)dx = \left[f(x)\right]_a^b$$

$$= f(b)-f(a)$$

Ex: $$\int_{\pi/3}^{\pi/4}\left(\frac{d}{dx}\cos x\right)dx = \left[\cos x\right]_{\pi/3}^{\pi/4} \text{ Ans}$$

Note: However if f is conti. in (a,b) at $x=c$

$$\int_a^b \left(\frac{d}{dx}f(x)\right)dx = \left[f(x)\right]_a^{c^-} + \left[f(x)\right]_{c^+}^b$$

(5)

Imp

* (i) $\int_0^{\pi/2} \sin x\,dx = \int_0^{\pi/2} \cos x\,dx = 1$

(ii) $\int_0^{\pi/2} \sin^2 x\,dx = \int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4}$

(iii) $\int_0^{\pi/2} \sin^3 x\,dx = \int_0^{\pi/2} \cos^3 x\,dx = \frac{2}{3}$

(iv) $\int_0^{\pi/2} \sin^4 x\,dx = \int_0^{\pi/2} \cos^4 x\,dx = \frac{3\pi}{16}$

If f and g are inverse of each other

$f: [a,b] \to [c,d]$

$f^{-1}(x) = g(x) : [c,d] \to [a,b]$

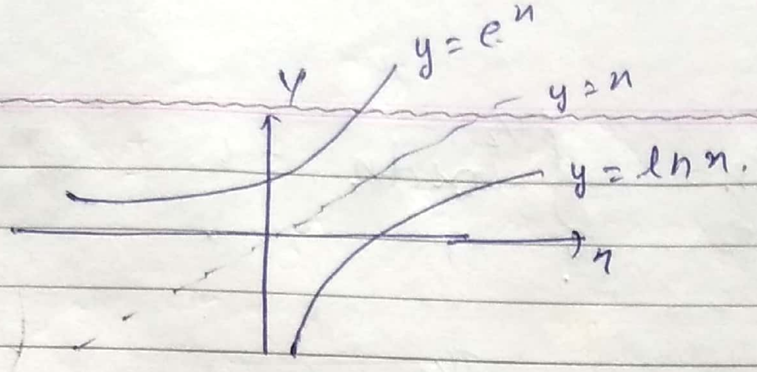
$f(a) = c \qquad f(b) = d$

$g(c) = a \qquad g(d) = b$

$$\int_a^b f(x)\,dx + \int_c^d g(x)\,dx = bd - ac$$ Ans

0-1 full complete. Qu. D.

Ex !



$$\int_1^2 e^x dx + \int_e^{e^2} \ln x \, dx = 2e^2 - e \text{ Ans.}$$

$$* \lim_{n\to\infty} \left(\int_a^b f_n(x) dx \right) = \int_a^b \left(\lim_{n\to\infty} f_n(x) \right) dx.$$

Que: Evaluate $I = \int_{-2}^{2} \frac{dx}{4+x^2} = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right)\Big]_{-2}^{2}$

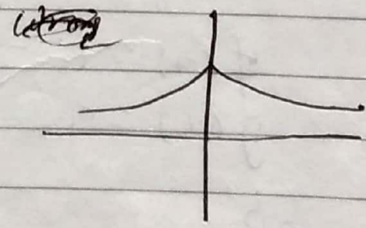
$$= \frac{1}{2}\left[\tan^{-1}(1) - \tan^{-1}(-1)\right] =$$

$$= \frac{1}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{4}$$

wrong M. M-2 Put $x = \frac{1}{t}$

wrong Method

$dx = -\frac{1}{t^2} dt$



$t = \frac{1}{x}$ is dis cont.

$$I = \int_{-1/2}^{1/2} \frac{-\frac{1}{t^2}}{4+\frac{1}{t^2}} dt = -\int_{-1/2}^{1/2} \frac{1}{4t^2+1} dt$$

$$= -\frac{1}{2}\left[\tan^{-1} 2t\right]_{-1/2}^{1/2} = -\frac{1}{2}\left[\tan^{-1} 1 - \tan^{-1}(-1)\right]$$

$$= -\frac{\pi}{4} \text{ Ans}$$

* Substitution must be continuous. $x = \frac{1}{x}$ is dis cont

Que! $$\int_2^4 \frac{\sqrt{x^2-4}}{x^4}\,dx = \int_2^4 \frac{\sqrt{1-\frac{4}{x^2}}}{x^3}$$

$$\frac{1}{8}\cdot\int_0^{3/4} \sqrt{k}\,dk$$

$1-\frac{4}{x^2} = k$

$\frac{8}{x^3}dx = dk$

(2) $$\int_{-1}^{1} x^2\, d(\ln x) = \int_{1/e}^{e} x^2\cdot\frac{1}{x}\,dx = \int_{1/e}^{e} x\,dx.$$

(3) $$\int_1^2 x^2\,dx + \int_1^4 \sqrt{x}\,dx$$

① $= y = x^2$

$x = \sqrt{y}$

② $f^{-1}(x) = \sqrt{x}$

$= \quad [\ \int^{2} \quad \int^{4}$

$2\cdot 4 - 1\cdot 1 = 7$

tan x = t

Que! let $I = \int_0^{\pi/2} \frac{\cos x}{a\cos x + b\sin x}\,dx$.

$J = \int_0^{\pi/2} \frac{\sin x}{a\cos x + b\sin x}\,dx$

$Ia + Ib = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{a\cos x + b\sin x}\,dx$

$Ia + Ib = [x]_0^{\pi/2} = \frac{\pi}{2}$ — (i)

$Ib - Ja = \int_0^{\pi/2} \frac{b\cos x - a\sin x}{a\cos x + b\sin x}\,dx.$

$\frac{d}{dx}(a\cos x + b\sin x)$

$= -a\sin x + b\cos x.$

$= \int_a^b \frac{1}{t}\,dt = \ln\frac{b}{a}$ —— (ii)

~~= la x~~

* Properties of definite Integrals:

*

MT - P
SF - M

Que: let $\frac{d}{dx} f(x) = \frac{e^{\sin x}}{x}$ $\quad x>0.$

If $\int_1^4 \frac{2x e^{\sin x^2}}{x^2}\, dx = f(k) - f(1)$

then find k.

$x^2 = p$
$2x\,dx = dp$

Ans:

$$\int_1^4 \frac{2x e^{\sin x^2}}{x^2}\, dx = f(k) - f(1)$$

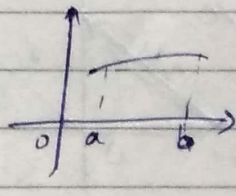
$$\int_1^{16} \frac{e^{\sin p}}{p}\, dp = \left[f(p)\right]_1^{16} = f(16) - f(1)$$

$k = 16.$

~~*~~ Properties:

~~P-1:~~ $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt.$

P-2 $\quad \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$



(P-3) $\quad \int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

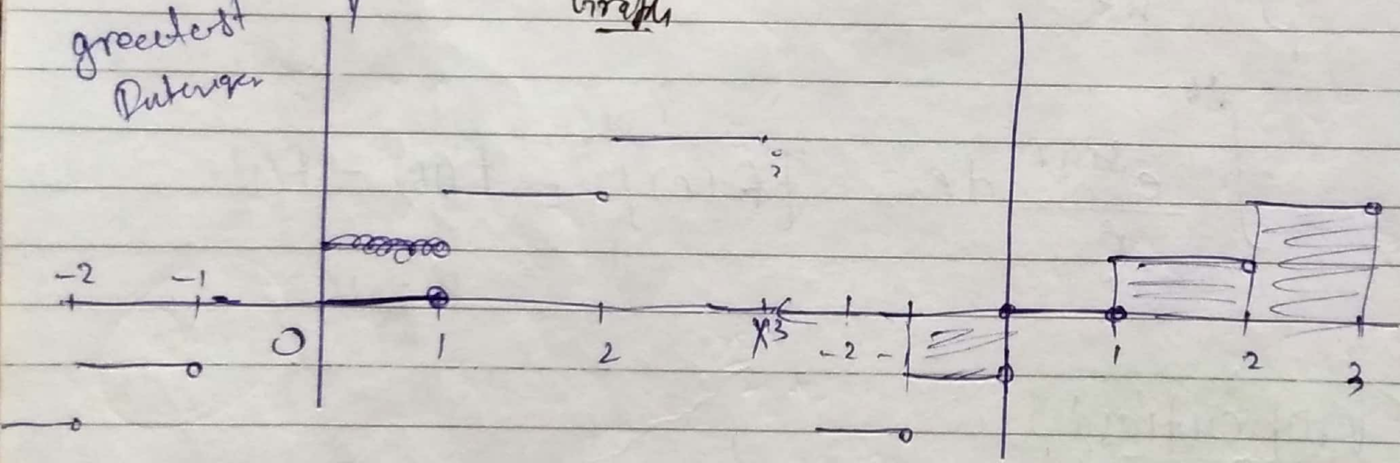
Here f is uniformly not defined in open interval (a, b)

$$f(x) = \begin{cases} \sin x & x \in [0, \frac{\pi}{4}) \\ \cos x & x \in [\frac{\pi}{4}, \frac{\pi}{3}] \end{cases}$$

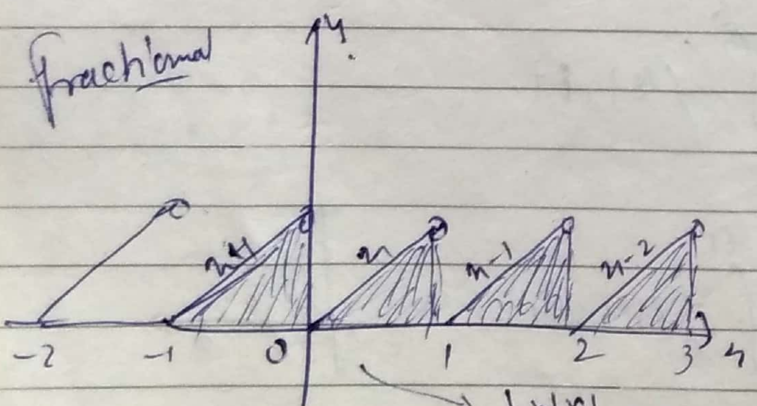
$$\int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/4} \sin x\,dx + \int_{\pi/4}^{\pi/2} \cos x\,dx$$

Graphs

greatest Integer



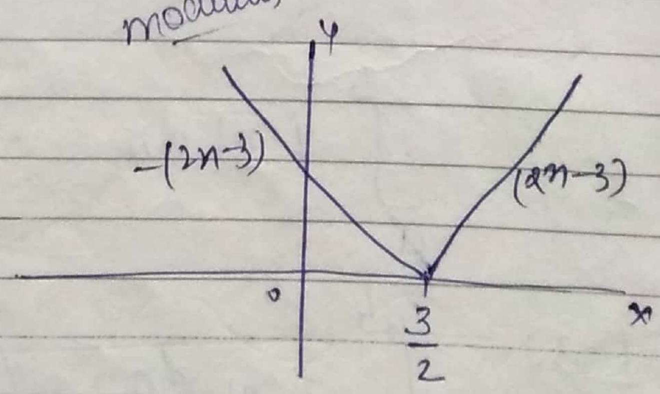
fractional



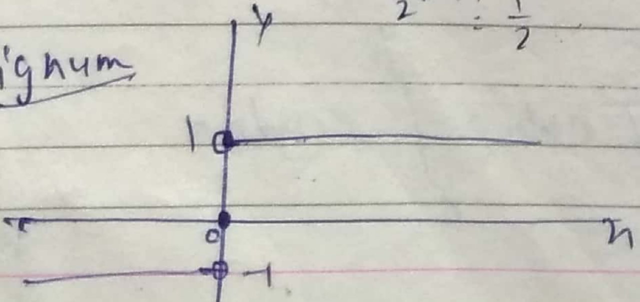
$\frac{1}{2}|x||x| = \frac{1}{2}$

$f(x) = |2x - 3|$

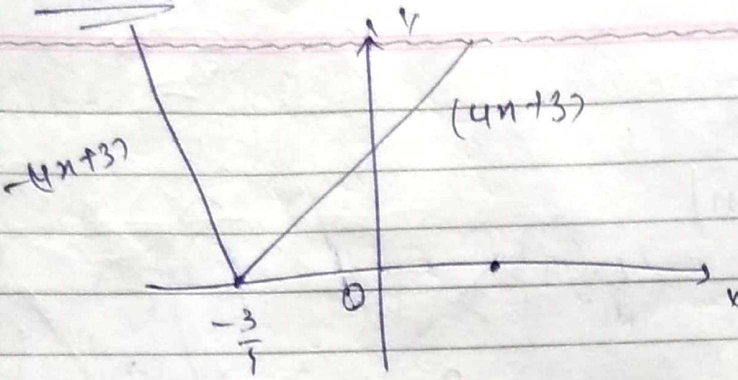
modulus



Signum



modulus.



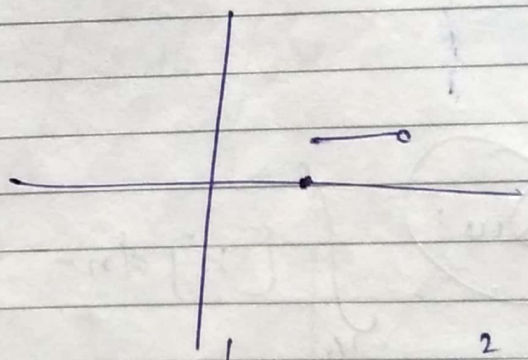
Q. $\int_{-1}^{3} [x]\,dx = \int_{-1}^{0} \cancel{f(x)}\,dx + \int_{0}^{1} \cancel{f(x)} + \int_{1}^{2} 1\,dx$

$+ \int_{2}^{3} 2\,dx$

$= -1[x]_{-1}^{0}$

$= -1 + 0 + 1 + 2[x]_{2}^{3}$

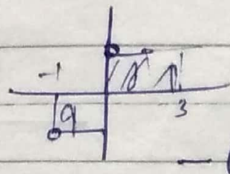
$= 2$. Ans



Que! $\int_{-1}^{3} \{x\}\,dx = \int_{-1}^{0} (x+1)\,dx + \int_{0}^{1} (x)\,dx + \int_{1}^{2} (x-1)\,dx + \int_{2}^{3} (x-2)\,dx$

$= 2$

Q $\int_{-1}^{3} \operatorname{sgn} x\,dx = 2$



$-(1\times1) + 3\times1 =$

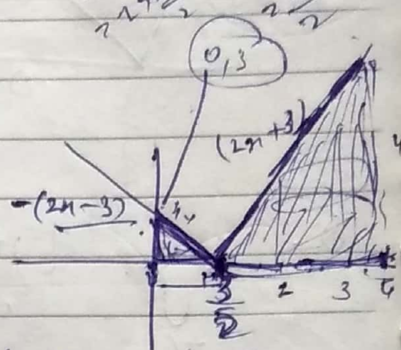
(2)

$2\times4-3$ (5) $\qquad -(2x-3)$ $+3$

Note: $\sqrt{x^2} = |x|$

$\sqrt{\cos^2 x} = |\cos x|$

Que $\displaystyle\int_0^4 |2x-3| \qquad x = \frac{3}{2}$



$= \frac{1}{2}\cdot\frac{3}{2}\cdot 3 + \frac{1}{2}\cdot\frac{5}{2}\cdot 5$

$\frac{1}{2}\cdot\frac{3}{2}\cdot 3 + \frac{1}{2}\cdot\frac{5}{2}\cdot 5$

$= \frac{17}{2}$

M.2 $\displaystyle\int_0^{3/2} -(2x-3)\,dx + \int_{3/2}^4 (2x+3)\,dx.$

$\frac{1}{2}\times\frac{3}{2}\times 3$

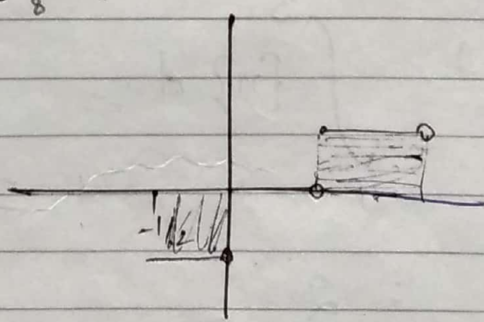
$\frac{1}{2}\times B\times H \qquad \frac{5}{2}$

$\frac{1}{2}\times\frac{5}{2}\times 4$

Que! $\displaystyle\int_{-1/2}^{2} [x]\,dx = \left(\frac{x^2}{2}\right)_{-1/2}^{2}$

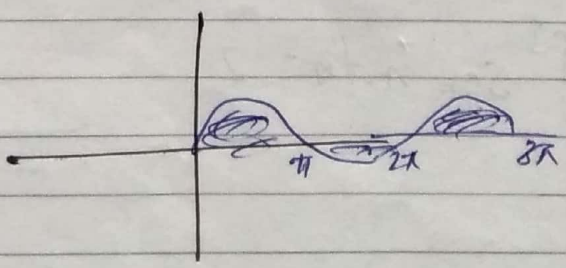
$= 2 - \frac{1}{8} = \frac{5}{8}$

$= 1 + \left(-\frac{1}{2}\right) = \frac{1}{2}$



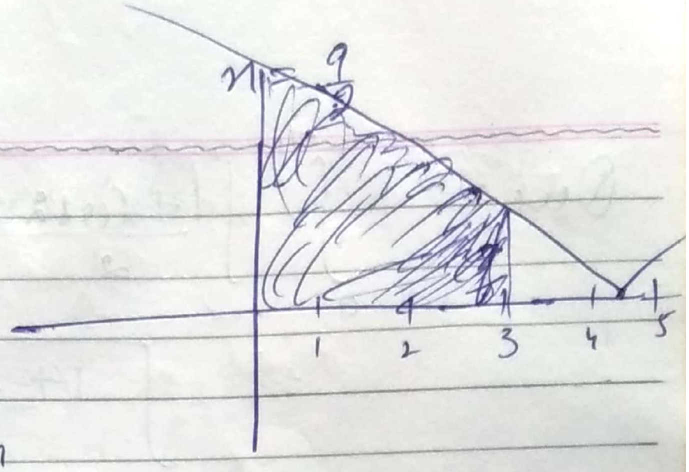
Que $\displaystyle\int_0^{3\pi} \sin x\,dx$

$= 1$



$(\cos x)_0^{3\pi}$ $(\cos 3\pi) = 1$

04/07/17



Que! $\int_0^3 |5x-9|$ =

$$\int_0^{9/5} -(5x-9)\,dx + \int_{9/5}^{3} (5x-9)\,dx$$

Que! $\int_0^2 [x^2]\,dx$

$0 < x^2 < 4$

$0 < x < 16$ +

$\cancel{\int_0^{16} \left(\frac{x^3}{3}\right)} = \cancel{\left[\frac{x^3}{3}\right]_0^{16}} =$

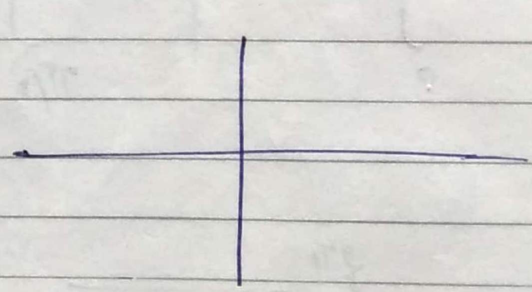
$= \int \cancel{\left[\frac{x^3}{3}\right]_0^{2}}$

$= \cancel{\left(\frac{4}{3} - 0\right)}$

$= \cancel{\frac{4}{3}\ \text{Ans}}$

Ans! $$\int_0^1 0\,dx + \int_1^{\sqrt2} 1\,dx + \int_{\sqrt2}^{\sqrt3} 2\,dx + \int_{\sqrt3}^{2} 3\cdot dx$$

Que! $\int_{-1/2}^{3/2} \left[x+\frac{1}{2}\right] dx$ =



Ans. $\int_0^{3\frac{1}{2}} [k]\,dx = \int_0^{3\frac{1}{2}} [x]\,dx$

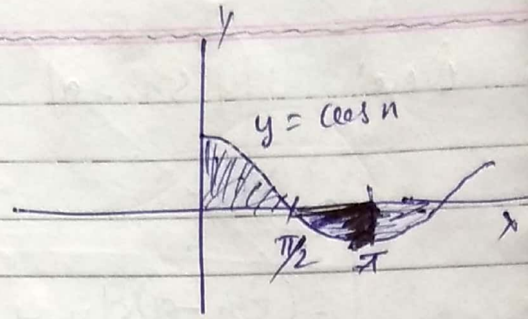
$x + \frac{1}{2} = k$

$dx = dk$

$= 1 + 1\times 2 + \frac{1}{2}\times 3$

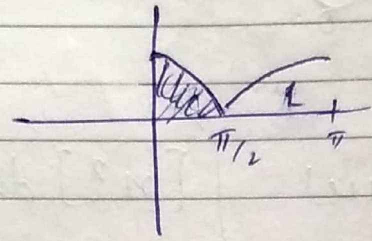
$= 1 + 2 + \frac{3}{2} = 3 + \frac{3}{2} = \frac{9}{2}$ Ans

Que: $\displaystyle\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}\, dx$



$= \sqrt{\frac{1+2\cos^2 x-1}{2}}$

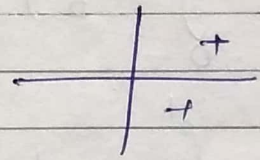
$= \displaystyle\int_0^{\pi} |\cos x|\, dx = 2$



Que: $\displaystyle\int_0^{2\pi} |1-2\cos x|\, dx$

$|1-2\cos x|$
$-1-2\cos$

$1-2\cos x = 0$
$\cos x = \frac{1}{2}$



$x = \frac{\pi}{3},$

$2\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$= \displaystyle\int_0^{\pi/3} (2\cos x - 1)\, dx + \int_{\pi/3}^{5\pi/3} (1-2\cos x)\, dx + \int_{5\pi/3}^{2\pi} (2\cos x - 1)\, dx.$

Que: $\displaystyle\int_0^{2\pi} \sqrt{1-\sin 2x}\, dx$ $\quad x = \frac{\pi}{4}, \frac{5\pi}{4}$ $\quad \sin x = \cos x$, $\tan x = 1$

$\sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x} = \sqrt{(\sin x - \cos x)^2}$

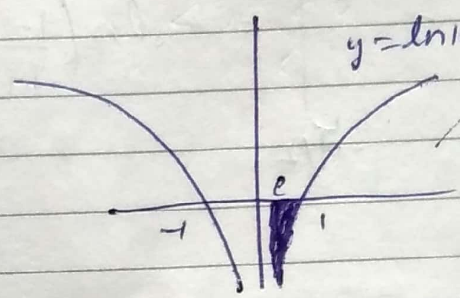
$= |\sin x - \cos x|$

$= \displaystyle\int_0^{2\pi} |\sin x - \cos x|\, dx =$

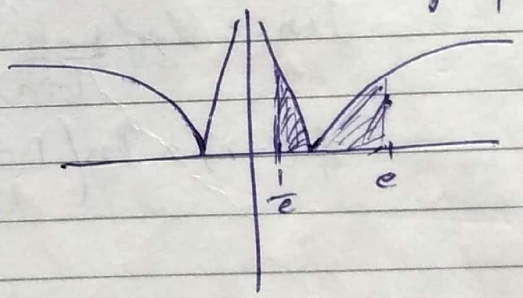
$$= \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{5\pi/4} (\sin x \cdot \cos x)\,dx$$

$$+ \int_{5\pi/4}^{2\pi} (\cos x - \sin x)\,dx.$$

Que: $\displaystyle\int_{e}^{1/e} \left|\ln|x|\right| dx$



$$= -\int_{1/e}^{e} \left|\ln|x|\right| d$$

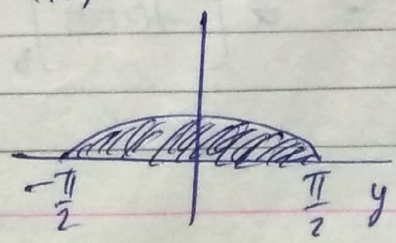


$$= \left[\int_{1/e}^{1} -\ln x\,dx + \int_{1}^{e} \ln x\,dx\right]$$

Property-4 $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ if $f(-x) = f(x)$, f is an even f^n

$= 0$ if $f(-x) = -f(x)$, i.e. f is an odd f^n.

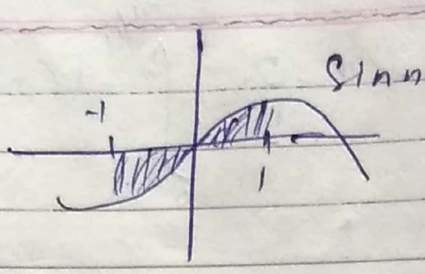
Ex: $\displaystyle\int_{-\pi/2}^{\pi/2} \cos x\,dx = 2\int_0^{\pi/2} \cos x\,dx = 2 \times 1$



O-1 ⇒ 2, 8, 9, 16, 18, 24
S-1 ÷ 1)(i), (6), 18, 24, 23, 37,

Ro:

$$\int_{-1}^{1} \sin x\,dx = 0$$



$f(x) = \sin x$

$f(-x) = \sin(-x) = -\sin x = -f(x)$

Que:

$$I = \int_{-1/2}^{1/2} \ln\left(\frac{1-x}{1+x}\right)dx =$$

$$= 0$$

$= f(x) = \ln\left(\frac{1-x}{1+x}\right)$

$f(-x) = \ln\left(\frac{1+x}{1-x}\right) = \ln\left(\frac{1-x}{1+x}\right)^{-1} = -\ln\left(\frac{1-x}{1+x}\right)$

$= -f(x)$

(2)

$$I = \int_{-\pi/4}^{\pi/4} (\sin x + \sec^2 x - \tan x)\,dx$$

$$= \int_{-\pi/4}^{\pi/4} \sec^2 x\,dx = 2\int_{0}^{\pi/4} \sec^2 x\,dx$$

$$= 2\left[\tan x\right]_0^{\pi/4} = 2(1-0) = 2$$

Que: $\displaystyle\int_{-1/2}^{1/2} [x] + \ln\left(\frac{1+x}{1-x}\right) dx = -\frac{1}{2} + 0$

$= \dfrac{-1}{2}$

Que $\displaystyle\int_{-\pi/4}^{\pi/4} f(x)\,dx$ where $f(x) = \dfrac{x^7 - 3x^5 + 7x^3 - 8x + 1}{\cos^2 x\,dx}$

$f(x) = \dfrac{(x^7 - 3x^5 + 7x^3 - 8x) + 1}{\cos^2 x}$

$\displaystyle\int_{-\pi/4}^{\pi/4} f(x)\,dx =$

$\displaystyle\int_{-\pi/4}^{\pi/4} \sec^2 x$

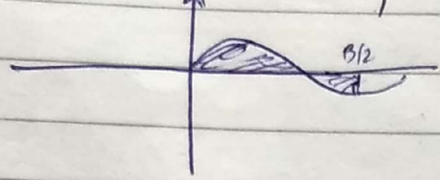
Q. $\displaystyle\int_{-2}^{2} |1-x^2|\,dx = 2\int_{0}^{2} |1-x^2|\,dx$

$= 2\left[\displaystyle\int_{0}^{2} (1-x^2)\,dx = \int_{1}^{2} -(1-x^2)\,dx\right]$

Que: $\displaystyle\int_{-1}^{3/2} |x \sin \pi x|\,dx = \int_{-1}^{1} |x \sin x|\,dx + \int_{1}^{3/2} |x \sin x|\,dx$

$= 2\displaystyle\int_{0}^{1} |x \sin x\pi|\,dx + \int_{1}^{3/2} |x \sin x\pi)\,dx$

$$= 2\int_0^1 x \sin \pi x \, dx + \int_1^3 -(x \sin \pi x)\, dx.$$



Que: $$I = \int_{-2}^{0} \left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right) dx.$$

$x + 1 = k$

$dx = dk$.

$$= \int_{-1}^{1} \left(\left[(k-1)^3 + 3(k-1)^2 + 3(k-1) + 3\right] + k\cos k\right)$$

$$= \int_{-1}^{1} (\underbrace{k^3}_{0} + \underbrace{k\cos k}_{0} + 2)\, dk.$$

$$= 2\int_{-1}^{1} dx = 2 \cdot 2 \int_0^1 dx = 4$$

odd

odd

Que: $$I = \int_{-1}^{1} \frac{2x^{332} + x^{998} + 4x^{1668} \cdot \sin x^{691}}{1 + x^{666}} \, dx.$$

$$= \int_{-1}^{1} \frac{2x^{332} + x^{998}}{1 + x^{666}} \, dx = 2 \int_0^1 \frac{2x^{332} + x^{998}}{1 + x^{666}}$$

$$= 2\int_0^1 \frac{x^{332}\cdot(2+x^{666})}{1+x^{666}}\,dx$$

$x^{333} = k$

$333\cdot x^{332}\,dx = dk$

$$= \frac{2}{333}\cdot\int_0^1 \frac{2+k^2}{1+k^2}\,dk$$

P-5 :- King property :- (Lower limit + upper limit − x)

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Proff :

$a - x = k$

$dx = -dk$

$$I = \int_0^a f(a-x)\,dx$$

$$= \int_a^0 f(k)(-dk) = \int_0^a f(k)\,dk = \int_0^a f(x)\,dx$$

Que :- $I = \int_{50}^{100} \frac{\ln x}{\ln x + \ln 150 - x}\,dx$ $\quad x \to 50 + 100 - x$

$$I = \int_{50}^{100} \frac{\ln(150-x)}{\ln(150-x)+\ln x}\,dx$$

(+)

$$2I = \int_{50}^{100} dx = [x]_{50}^{100} = 100 - 50 = 50$$

$I = 25$

Que: $I = \int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} dx$ — (i)

$x = \frac{\pi}{2} - x$

$I = \int_0^{\pi/2} \frac{f(\sin \frac{\pi}{2} - x)}{f(\sin \frac{\pi}{2} - x) + f(\cos \frac{\pi}{2} - x)}$

$= \frac{f(\sin x)}{f(\sin x + f(\cos x)} dx$

$= \int_0^{\pi/2} \frac{f(\cos x)}{f(\cos x) + f(\sin x)}$ — (ii)

Adding (i) and (2)

$2I = \int_0^{\pi/2} \frac{f(\sin x) + f(\cos x)}{f(\sin x) + f(\cos x)} dx$

$= [x]_0^{\pi/2} = \frac{\pi}{2}$.

Que: $I = \int_{\pi/6}^{\pi/3} \sin 2x . \ln(\tan x) dx$. $x = \frac{\pi}{2} - x$

$= \int_{\pi/6}^{\pi/3} \sin\left(2\left(\frac{\pi}{2} - x\right)\right) . \ln\left(\tan\left(\frac{\pi}{2} - x\right)\right) dx$

$= \int_{\pi/6}^{\pi/3} \sin 2x . \ln(\cot x) dx$.

P-4 and P-5 with learn

$\log 1 = 0$ H.W: Q- 6, 17, 19,

King: 29,

Not lean

S-1 ⇒ 11, 13

J.A ⇒ 12, 14

King

$$2I = \int_{\pi/6}^{\pi/3} \sin 2x\left[\ln(\tan x) + \ln(\cot x)\right] dx$$

$$= \int_{\pi/6}^{\pi/3} \sin 2x \ln(\tan x \cdot \cot x) dx$$

$\log a + \log b \neq l(a+b)$

$= \lg ab.$

$$= \int_{\pi/6}^{\pi/3} \sec x \cdot 0\, dx = 0$$

$$I = 0$$

Q.. $I = \int_{-a}^{a} f(x) dx = \int_{-a}^{a} f(-x) dx$

$x \to -a + a - x$

$x \to -x$

If f(x) not even nor odd.

if P-4 not Apply.

then apply P-5

Que: $I = \int_{\pi/8}^{3\pi/8} \ln\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$

$\log \frac{a}{b} + l \frac{b}{a} = \ln \frac{ab}{ba} = 1$

$x \to \frac{\pi}{2} - x$

$$I = \int_{\pi/8}^{3\pi/8} \ln\left(\frac{4+3\cos x}{4+3\sin x}\right) dx \qquad 2I = \int_{\pi/8}^{3\pi/8} 0$$

$$I = 0$$

Qu $I = \int_0^{\pi/2} \frac{\sin^{208} x}{\sin^{208} x + \cos^{208} x} \, dx$

$$I = \int_0^{\pi/2} \frac{\cos^{208} x}{\sim}$$

$$2I = \int_0^{\pi/2} dx$$

$$= \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Que: Proff that $I = \int_0^{\pi/4} \ln(1+\tan x) \, dx = \frac{\pi}{8} \ln 2.$

$$= \int_0^{\pi/4} \ln\left(1+\tan\left(\frac{\pi}{4}-x\right)\right) dx$$

$1+\tan\left(\frac{\pi}{4}-x\right)$

$= 1 + \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4} \cdot \tan x}$

$= 1 + \frac{1 - \tan x}{1 + \tan x} = \frac{2}{1+\tan x}$

$$I = \int_0^{\pi/4} \ln\left(\frac{2}{1+\tan x}\right) dx$$

$$2I = \int_0^{\pi/4} (\ln 2) \, dx = \ln 2 \cdot \frac{\pi}{4}.$$

$$\Rightarrow I = \frac{\pi}{8} \ln 2.$$

Que:- $I = \int_0^{\pi/2} \frac{x \cdot \sin x}{1 + \sin x} \, dx$

$= \int (1 - \frac{1}{1+\sin x} = \int_0^{\pi} \frac{(\pi - x)\sin x}{1 + \sin x}$

$2I = \pi \int_0^{\pi} \frac{\sin x}{1 + \sin x} = 2I = \pi \int_0^{\pi} \frac{\sin x + 1}{\sin x + 1} - 1$

Que: $I = \int_0^{\infty} \frac{\ln x}{ax^2 + bx + a} \, dx$

$x = \frac{1}{t}$

$dx = \frac{-1}{t^2} dt$

$\int_0^{\infty} \frac{\ln \frac{1}{t}}{\frac{a}{t^2} + \frac{b}{t} + a} \left(-\frac{1}{t^2}\right) dt$

$= \int_{\infty}^{0} \frac{(-\ln t) \left(-\frac{1}{t^2}\right) dt}{\frac{1}{t^2}(at^2 + bt + a)} = + \int_0^{\infty} \frac{\ln t \, dt}{at^2 + bt + a}$

$= \int_0^{\infty} \frac{\ln x \, dx}{ax^2 + bx + c} = -I$

$I = -I$

$2I = 0 \Rightarrow I = 0$

(0) (0) (1)

2 $I = \int_0^{\pi/2} \frac{\sin x - \cos x \, dx}{1 + \sin x \cos x}$

$= \int_0^{\pi/2} \frac{C - S}{1 + S C} \, dx.$

$2I = 0$

$I = 0$

Or.. $I = \int_0^{\pi} \frac{dx}{1 + 2^{\tan x}}$

$1 + 2^{\tan(\pi - x)} = 1 + 2^{-\tan x}$

$= 1 + \frac{1}{2^{\tan x}} = \frac{1 + 2^{\tan x}}{2^{\tan x}}$

$\cancel{1 + 2^{\tan(\pi - x)}}$

$I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} \, dx$

$I = \int_0^{\pi} \frac{2^{\tan x} \, dx}{1 + 2^{\tan}}$

$2I = \int_0^{\pi} dx = \pi$

$I = \frac{\pi}{2}$ Ans

Que! $I = \int_0^1 \frac{\ln(1 + x) \, dx}{1 + x^2}$

$x = \tan\theta$

$dx = \sec^2\theta \, d\theta$

$I = \int_0^{\pi/4} \frac{\ln(1 + \tan\theta)}{\sec^2\theta} \cdot \sec^2\theta \, d\theta$

$\frac{\pi}{8} \ln 2.$

evaluate

Que:- $I = \int_{-2}^{2} (\overset{0}{\cancel{x^3 f(x)}} + \overset{0}{\cancel{x \cdot f''(x)}} + 2)\, dx$

where f is an even or diff. fn

$= 2\int_{-2}^{2} dx$

f - Even
f' - odd
f'' = E

$= 2[x]_{-2}^{2}$

$= 8$

Que! $I = \int_{\pi/2}^{3\pi/2} [2\sin x]\, dx$

$[x] + [-x] = \begin{cases} 0 & x \in I \\ -1 & x \notin I \end{cases}$

$I = \int_{\pi/2}^{3\pi/2} [-2\sin x]\, dx$

$2I = \int_{\pi/2}^{3\pi/2} ([2\sin x] + [-2\sin x])\, dx$

$= -\int_{\pi/2}^{3\pi/2} dx = -\left(\frac{3\pi}{2} - \frac{\pi}{2}\right) = -\pi$

$I = -\frac{\pi}{2}$

$\cos(2\pi - \theta) = \cos\theta$

P – 6 : Queen

$$\int_0^{2a} f(x)\,dx = \begin{cases} 0 & f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx, & f(2a-x) = f(x) \end{cases}$$

Q.1 $I = \int_0^{2\pi} \cos^5 x\,dx$ $\qquad J = \int_0^{2\pi} \sin^4 x\,dx$

$= 2\int_0^{\pi} \cos^5 x\,dx$

$f(x) = \cos^5 x$

$f(2\pi - x) = \cos^5(2\pi - x)$

$= \cos^5 x$

$= f(x)$

$= 0$

$f(\pi - x) = \cos^5(\pi - x)$

$= (-\cos x)^5$

$= -\cos^5 x$

$= -f(x)$

$f(x) = \sin^4 x$

$f(2\pi - x) = (\sin(2\pi - x))^4$

$= (-\sin x)^4$

$= \sin^4 x$

$= f(x)$

$= 2\int_0^{\pi} \sin^4 x\,dx$

$f(\pi - x) = \sin^4(\pi - x)$

$= \sin^4 x$

$= f(x)$

$= 2 \cdot 2 \cdot \int_0^{\pi/2} \sin^4 x\,dx$

$= 2 \cdot 2 \cdot \frac{3\pi}{16}$ ✓

Que: $I = \int_0^{\pi} \frac{\sin x}{\sin 4x}\,dx$

$f(\pi - x) = \frac{\sin^4 x}{\sin 4x\,dx}$

$f(\pi - x) = \frac{\sin^4(\pi - x)}{\sin 4(\pi - x)}$

$= \cancel{\frac{\sin^4 x}{\sin}}$

$= 0$ Ans

$f(\pi - x) = \dfrac{\sin(\pi - x)}{\sin(4\pi - 4x)}$

$= \dfrac{\sin x}{-\sin x}$

$= -f(x)$

~~Ans formula~~ Remember

~~Que~~ Proof that $\boxed{I = \int_0^{\pi/2} \ln(\sin x)\, dx = -\frac{\pi}{2} \ln 2}$

$= \int_0^{\pi/2} \ln(\cos x)\, dx$

$2I = \int_0^{\pi/2} \ln(\sin x \cdot \cos x)\, dx$

$2I = \int_0^{\pi/2} \ln\left(\frac{2\sin x \cos x}{2}\right) dx$

$= \underbrace{\int_0^{\pi/2} \ln(\sin 2x)\, dx}_{J} - \int_0^{\pi/2}$

$J = \int_0^{\pi/2} \ln(\sin 2x)\, dx$

$f(x)$

$f\left(\frac{\pi}{2} - x\right) = \sin\left(2\left(\frac{\pi}{2} - x\right)\right)$

$= \sin(\pi - 2x)$

$= \sin 2x$

$= f(x)$

$= 2 \cdot \int_0^{\pi/4} \ln \sin 2x\, dx$

$= 2 \cdot \frac{1}{2} \int_0^{\pi/2} \ln(\sin t)\, dt$

$2x = t$

$dx = \frac{1}{2} dt$

$= \int_0^{\pi/2} \ln(\sin x)\, dx$

H.W! (8-1) 1, 22, 23, 28, 27, (P-2)
8-1 2, 3, 4, 5, 7, 8, 9, 10, 12

$$= I - \ln 2 \int_0^{\pi/2} dx$$

$$2I = I - \frac{\pi}{2} \ln 2$$

$$I = -\frac{\pi}{2} \ln 2$$

Que! $\int_0^{\pi} x \ln(\sin x)\, dx$

$$= \int_0^{\pi} (\pi - x) \ln \sin(\pi - x)$$

$$I = \int_0^{\pi} (\pi - x) \ln(\sin x) dx$$

$$2I = \pi \int_0^{\pi} \ln(\underbrace{\sin x}_{f}) dx$$

$$f(\pi - x) = \ln \sin(\pi - x) = \ln(\sin x)$$
$$= f(x)$$

$$2I = \pi \cdot 2 \int_0^{\pi/2} \ln(\sin x) dx$$

$$= \pi \cdot \left(-\frac{\pi}{2} \ln 2\right) = -\frac{\pi^2}{2} \ln 2$$

$\int_0^{\pi/2} \sin^3 x = \frac{2}{3}$

Que: $\int_0^1 \frac{\sin^{-1} x}{x}\, dx$

$= \sin^{-1} x = \theta$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$I = \int_0^1 \frac{\sin^{-1} x\, dx}{x}$

$= \int_0^{\pi/2} \frac{\theta}{\sin\theta} \cos\theta\, d\theta$

$= \int \underset{I}{\theta} \cdot \underset{II}{\cot\theta}\, d\theta = \left[\theta(\ln \sin\theta)\right]_0^{\pi/2} - \int_0^{\pi/2} \ln(\sin\theta)\, d\theta$

$= -\left(-\frac{\pi}{2} \ln 2\right)$

$= + \frac{\pi}{2} \ln 2$

P-7: Suppose $f(x)$ is Periodic with Period T. then.

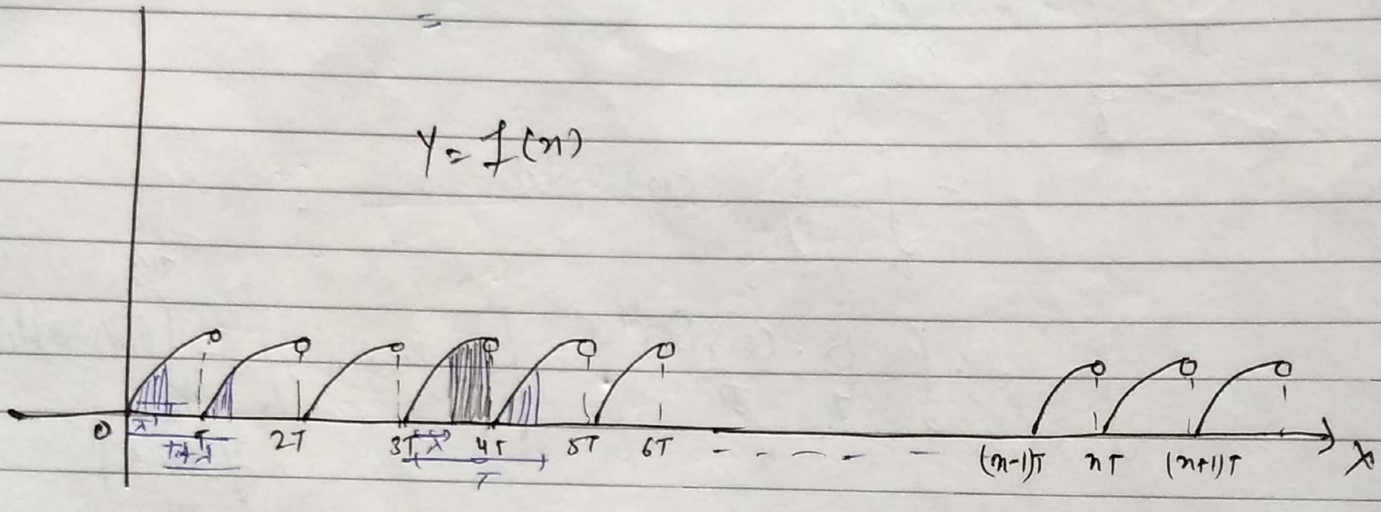
$f(x+T) = f(x)$

(i) $\int_0^{nT} f(x)\, dx = n \int_0^T f(x)\, dx$

(ii) $\int_T^{T+\lambda} f(x)\, dx = \int_0^{\lambda} f(x)\, dx$

(iii) $\int_{\lambda}^{\lambda+T} f(x)\, dx = \int_0^T f(x)\, dx$

(4) $\displaystyle\int_{mT}^{nT} f(x)\,dx = (n-m)\int_{0}^{T} f(x)\,dx.$



$\sin x, \cos x, \sec x, \operatorname{cosec} x = 2\pi$

$\tan x, \cot x = \pi$

$|\sin x|, |\cos x|, |\tan x|, |\sec x|, |\operatorname{cosec} x|, |\cot x| = \pi$

$\{x\} = 1$

$$\sqrt{x^2} = |x|$$

If $Y = f(x)$ has period $= T$

$$y = af(bx+c)+d = \frac{T}{|b|}$$

$\sin x \longrightarrow 2\pi$

$\sin 2x \text{ — } \dfrac{2\pi}{2} = \pi$

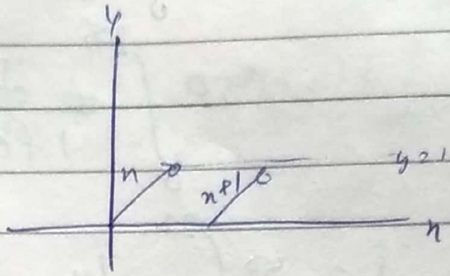
$\sin \dfrac{x}{3} = \dfrac{2\pi}{1/3} = 6\pi.$

②

Que: $I = \int_0^{100} e^{\{2x\}}\,dx$

Sol = $\{x\}$ Period $= 1$

$\{2x\} = \frac{1}{2}$



$$= \int_0^{200(\frac{1}{2})} e^{\{2x\}}\,dx$$

$$= 200\int_0^{1/2} e^{\{2x\}}\,dx = 200\int_0^{1/2} e^{2x}\,dx$$

$$= \frac{200}{2}\left[e^{2x}\right]_0^{1/2}$$

Que: $\int_0^{100\pi} \cancel{\sin x}\sqrt{1+\cos x}\,dx$

$$\int_0 |\sin x|\,dx = 2\int_0^{\pi} |\sin x|\,dx = 2\times 2 = 4$$

$$= \int \sqrt{\cancel{1} + 2\cos^2\frac{x}{2} - \cancel{1}}$$ $|\cos x| \to \pi$

$$= \int \sqrt{2}\cos\frac{x}{2}$$ $|\cos\frac{x}{2}| = \frac{\pi}{1/2} = 2\pi$

$$= \sqrt{2}\cdot 50\int_0^{2\pi} \left|\cos\frac{x}{2}\right|\,dx$$ $\frac{x}{2} = k,\ dx = 2\,dk$

$$I = \int_0^{\pi} \cos\frac{x}{2}\,dx = 2\int_0^{\pi/2} \cos k\,dk = 2\cdot 1 = 2$$

$$= \sqrt{2}\cdot 50\cdot 4$$

$$= 100\sqrt{2}$$

H.W : O-1 14,
S-1 14, 15, 19, 21, 22, 25, 26, 27, 28, 29, 30,

Que: $I = \int_0^{100\pi} \frac{dx}{1+e^{\sin x}}$

$= 50 \int_0^{2\pi} \frac{dx}{1+e^{\sin x}}$

$\sin(2\pi - x) = -\sin x$

$e = \frac{1}{e^{\sin x}}$

$I = 50 \int_0^{2\pi} \frac{dx}{1+\frac{1}{e^{\sin x}}} = 50 \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} dx$

$2I = 50 \int_0^{2\pi} \frac{e^{\sin x}+1}{e^{\sin x}+1} dx$

$= 50 \left[x\right]_0^{2\pi}$

$2I = 50 \cdot 2\pi$

$I = 50\pi$

Que: $\int_0^{100\pi} \left(|\sin x| - \left[\left| \frac{\sin x}{2} \right| \right] \right) dx$

G.I.F

$= \int_0^{100\pi} |\sin x| \, dx$

$-1 \le \sin x \le 1$

$0 \le |\sin x| \le 1$

$0 \le \left|\frac{\sin x}{2}\right| \le 1$

$\left[\left|\frac{\sin x}{2}\right|\right] = 0$

$= 100 \int_0^{\pi} |\sin x| \, dx = 100 \int_0^{\pi} \sin x \, dx = 200$ Ans

* Leibnitz formula for Derivatives or Antiderivatives!

$$\frac{d}{dx}\left(\int_{g(x)}^{h(x)} \frac{f(t)\,dt}{1}\right) = f(h(x))\cdot\frac{d}{dx}h(x) - f(g(x))\frac{d}{dx}g(x)$$

Note that integral $f(x)$ must not contain any function x.

Que) if $\int_{\sqrt{x}}^{x^2} \sin t\,dt$

then find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \left[\sin x^2\left(\frac{d}{dx}x^2\right) - \sin\sqrt{x}\left(\frac{d}{dx}\sqrt{x}\right)\right]$$

$$= 2x\sin x^2 - \frac{1}{2\sqrt{x}}\sin\sqrt{x} \quad \text{An}$$

(2) if $y = \int_{\sqrt{x}}^{x^2} x\sin t^2\,dt$. then find $\frac{dy}{dx}$

$$= x\left(\int_{\sqrt{x}}^{x^2}\sin t^2\,dt\right)$$

$$\frac{dy}{dx} = \int_{\sqrt{x}}^{x^2}\sin t^2\,dt + x\cdot\left[(\sin x^4)\cdot 2x - (\sin x)\cdot\frac{1}{2\sqrt{x}}\right]$$

Que!

$$G(x) = \int_{2}^{x^2} \frac{dt}{1+\sqrt{t}} \quad \text{then find } G'(9)$$

$$= G'(x) = \left(\frac{1}{1+x}\right) 2x - 0$$

$$G'(9) = \frac{18}{10} = \frac{9}{5}$$

2. $$g(x) = \int_{e^{2x}}^{e^{3x}} \frac{t}{\ln t}\, dx \qquad x > 0$$

$$g'(x) = \frac{e^{3x}}{3x} \cdot 3e^{3x} - \frac{e^{2x}}{2x} \cdot e^{2x}$$

3. $$x = \int_{1}^{t^2} \sqrt{z}\, \sin z\, dz$$

$$y = \int_{\sqrt{t}}^{2} z^2 \cos z^2\, dz$$

$$= \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{0 - t\cos t \cdot \frac{1}{2\sqrt{t}}}{(t \sin t^2)\, 2t - 0} =$$

Que!

$$\lim_{x \to 0} \frac{\int_0^{x^2} \cos t^2}{x \sin x} \quad \left(\frac{0}{0}\right)$$

$$\lim_{x \to 0} \frac{(\cos x^4) \cdot 2x}{x \cos x + \sin x}$$

$$\lim_{x\to 0} \frac{2\cos x^4}{\cos x + \frac{\sin x}{x}} = \frac{2}{1+1} = 1 \text{ Ans}$$

Que: $x = \int_0^y \frac{dt}{\sqrt{1+4t^2}}$

if $\frac{d^2y}{dx^2} = ky$ then find k

$$\frac{dx}{dy} = \frac{1}{\sqrt{1+4y^2}}$$

$$\frac{dy}{dx} = \sqrt{1+4y^2}.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\cdot\left(\frac{dy}{dx}\right) = \frac{d}{dx}\sqrt{1+4y^2} =$$

$$= \frac{d}{dy}\cdot\left(\sqrt{1+4y^2}\right)\frac{dy}{dx}$$

$$= \frac{1\cdot 8y}{2\cancel{\sqrt{1+4y^2}}} \cdot \cancel{\sqrt{1+4y^2}} = 4y$$

$k = 4.$

Que: $\lim_{x\to 0} \frac{\int_0^x xe^{t^2}dt.}{1-e^{x^2}} = \lim \frac{x\int_0^x e^{t^2}dt}{1-e^{x^2}}$

$$= \lim_{x\to 0} \frac{\int_0^x e^{t^2}dt + x\cdot e^{x^2}}{-2e^{x^2}} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{e^{x^2} + [e^{x^2} + xe^{x^2}\cdot 2x]}{-2[e^{x^2} + x\cdot 2xe^{x^2})} = \frac{1+1+0}{-2(1+0)} = -1$$

Q. $y = \int_{x}^{x^2} \frac{dt}{(x^2+t^2)}$ $= \frac{1}{x}\left[\tan^{-1}\frac{t}{x}\right]_{x}^{x^2}$

$\frac{dy}{dx}$ $= \frac{1}{x}\left[\tan^{-1}\frac{x^2}{x} - \tan^{-1}\frac{x}{x}\right]$

$y = \frac{1}{x}\left[\tan^{-1}x - 1\right]$ $\int\frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\frac{u}{a}$

Que: $\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t^2}{t^2+1}\,dt$

$= \frac{\int_0^x \frac{t^2}{t^4+1}\,dt}{x^3}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{\frac{x^2}{x^4+1}}{3x^2} = \cancel{0}\,\frac{1}{3}$

Que: $\int_0^x f(t)\,dt = x\cos\pi$ $x>0$

$f(4)$

$= f(x) = \cos\pi x - \pi x\sin x$

$f(4) = \cos 4\pi - \cancel{4\pi\sin 4\pi}\ 0$

$= 1$

$\boxed{x/2 + 3x/2}$

Que! $f(n) = \int_0^n e^t \sin(n-t)\,dt$

and $g(n) = f''(n) - f(n)$

then find range of $g(n)$

$t \to n-t$

$f(n) = \int_0^n e^{n-t} \cdot \sin n\, t\,dt$

$f(n) = e^n \cdot \int_0^n e^{-t} \sin t\,dt.$

$f'(n) = e^n \cdot \int_0^n e^{-t} \sin t\,dt + e^n \cdot e^{-n} \sin n$

$f'(n) = e^n \cdot \int_0^n e^{-t} \sin t\,dt + \sin n.$

$f''(n) = e^n \int_0^n e^{-t} \sin t\,dt + e^n \cdot e^{-n} \sin n + \cos n.$

$f''(n) = f(n) + \sin n + \cos n$

$f''(n) - f(n) = \boxed{g(n = \sin n + \cos n}$

$= \sqrt{2}\left[\frac{1}{\sqrt{2}} \sin n + \frac{1}{\sqrt{2}} \cos n\right] = \sqrt{2}(\sin + \frac{\pi}{4})$

rang $[-\sqrt{2}, \sqrt{2}]$ R

* Wallis Theorem:-

$$\int_0^{\pi/2} \sin^n x \cdot \cos^m x = \frac{[(n-1)(n-3)(\cdots 1 \text{ or } 2.)] \times [(m-1)(m-3) \cdots 1 \text{ or } 2]}{(m+n)(m+n-2)(m+n-4) \cdots, 1 \text{ or } 2} \times k$$

$$k = \begin{cases} \frac{\pi}{2}, & m, n \text{ Both even natural no.} \\ 1, & \text{otherwise} \end{cases}$$

$m, n, \in$ Natural no. non negative no.

Ex: $\int_0^{\pi/2} \sin^6 x\,dx = \frac{(6-1)(6-3)(6-5)}{6\cdot(6-2)(6-4)} \cdot \frac{\pi}{2}$

Que: $\int_0^{\pi/2} \sin^7 x \cos^6 x\,dx$

$= \frac{(7-1)(7-3)(7-5)(6-1)(6-3)(6-5)}{(7+6)(7+6-2)(13-4)(13-6)(13-8)(13-10)(13-12)}$

$= \frac{6\cdot 4\cdot 2\cdot 5\cdot 3\cdot 1}{13\cdot 11\cdot 9\cdot 7\cdot 5\cdot 3\cdot 1}$

$= \underline{\quad 98 \quad}$

105
×9
945
9

OT ⇒ 4, 5, 10, 15, 26, 25, 30,
S-1: 16, 30, Let 31, 36
J.M ⇒ 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12
13, 14, 15, 16.

Q: $I = \int_0^{\pi/6} \cos^4 3x \cdot \sin^2 6x \, dx$

$(\sin x \cos x)^2 = (2 \sin 3x \cos 3x)^2$

$I = 4\int_0^{\pi/6} \sin^2 3x \cdot \cos^6 3x \, dx$

$3x = k$

$dx = \frac{dk}{3}$

$= \frac{4}{3}\int_0^{\pi/2} \sin^2 k \cdot \cos^6 k \, dk = \frac{4}{3}$

$\frac{4}{3}\left[\frac{(2-1)]\,[(6-1)(6-3)(6-5)]}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2}\right.$

Que: $I = \int_0^{2\pi} x \sin^6 x \cos^4 x \, dx$

$4\frac{\pi}{2}$

$I = \int_0^{2\pi} (2\pi - x) \sin^6 x \cos^4 x$

$= 2\,4\int^{\pi/2}$

$2I = 2\pi \cdot 2\int_0^{2\pi} \sin^6 x \cos^4 x = 2\pi \cdot 2\int_0^{\pi} \sin^6 x \cos^4 x$

$= 2\pi \cdot 2 \cdot 2\int^{\frac{\pi}{2}}$

* Determination of function.

find cont. f^n satisfy. $f^2(x) = \int_0^x \frac{f(t) \sin t}{2+\cos t} \, dt$

$$\boxed{f^2(0) = 0 \Rightarrow f(0) = 0}$$

$$2ff' = \frac{f(x)\sin x}{2+\cos x}$$

$$f(x)\left[2f'(x) - \frac{\sin x}{2+\cos x}\right] = 0$$

$$f(x) = 0 \quad \text{or} \quad 2f'(x) = \frac{\sin x}{2+\cos x}$$

$$f'(x) = \frac{\sin x}{2(2+\cos x)} = f(x) = \frac{1}{2}\int \frac{\sin x}{2+\cos x} \, dx + \lambda$$

$$f(x) = -\frac{1}{2} \ln(2+\cos x) + \lambda$$

and put $x=0$

Que: $f(x) = \sin x + \int_0^{\pi/4} f(t) \sin t \, dt$ $\qquad f(x) = \sin x + \frac{1}{4}$ Ans

Imp

$$\boxed{f(x) = \sin x + \lambda}$$

where $\lambda = \int_0^{\pi/2} f'(t) \sin t \, dt$

$$I = \int_0^{\pi/4} \cos t \sin t\, dt$$

$$= \frac{1}{2} \cdot \int_0^{\pi/4} \sin 2t\, dt$$

$$= \frac{1}{2}\left(-\frac{1}{2}\right)\left[\cos 2t\right]_0^{\pi/4} = \frac{-1}{4} \cdot \left[\cos\frac{\pi}{2} - \cos 0\right]$$

$$= \frac{-1}{4}[0-1] = \frac{1}{4}$$

*. $$f(x) = \sin x + \int_0^{\pi/4} x f'(t) \sin t\, dt$$

$$= \sin x + x \cdot \int_0^{\pi/4} f'(t) \sin t\, dt$$

$$f(x) = \sin x + dx.$$

Que: $$(f(x))^{101} = 1 + \int_0^x f(t)\, dt$$

find $(f(101))^{100}$

$$(f(0))^{101} = 1$$

$f(0) = 0$

$$f(0) = 1$$

$$f(x) \cdot \left[101 f(x)^{99} f'(x) - 1\right] =$$

$\because f(x) \neq 0$

$$f^{99} f'(x) = \frac{1}{101}$$

$$f^{99}\frac{dt}{dx} = \frac{1}{101}$$

$$f(x)^{99} \cdot d f(x) = \frac{dx}{101}$$

$$f^{99} \cdot \frac{dt}{dx} = \frac{1}{101}$$

$$f(x)^{99} \cdot d f(x) = \frac{dx}{101}$$

$$\int f^{99} dt = \int \frac{dx}{101} + \lambda$$

$$\frac{f(x)^{100}}{100} = \frac{x}{101} + \lambda$$

$x = 0$ $\quad \frac{1}{100} = \frac{0}{100} + \lambda$

$$\lambda = \frac{1}{100}$$

$$\frac{(f(x))^{100}}{100} = \frac{x}{101} + \frac{1}{100}$$

$x = 101$

$$\frac{(f(101))^{100}}{100} = \frac{101}{101} + \frac{1}{10} = 1 + \frac{1}{10}$$

$$= \frac{101}{100}$$

$$(f(101))^{100} = 101 \quad A$$

Que: if $f(x) = e^x + \int_0^1 t e^{-x} f(t)\, dt$

then find $f(x)$.

$$= e^x + e^{-x} \int_0^1 t f(t)\, dt$$

$$f(x) = e^x + A e^{-x} \quad \text{--- (1)}$$

$$A = \int_0^1 t f(t)\, dt$$

$$A = \int_0^1 t(e^t + Ae^{-t})\, dt$$

$$A = \int_0^1 te^t dt + A\int_0^1 te^{-t} dt$$

$$A\left(1 - \int_0^1 te^{-t} dt\right) = \int_0^1 te^t dt$$

$$= A = \quad\quad \text{Put in eq (1)}$$

Que: if $f(x) = x + \int_0^{\pi/2} \sin(x+y) f(y)\, dy.$

where x and y are independent variable then find $f(x) = ?$

$$= f(x) = x + \int_0^{\pi/2} (\sin x \cos y + \cos x \sin y) f(y) dy$$

$$= x + \sin x \int_0^{\pi/2} \cos y\, f(y)\, dy + \cos x \int_0^{\pi/2} \sin y\, f(y) dy$$

$$f(x) = x + A\sin x + B\cos x$$

$$A = \int_0^{\pi/2} \cos y\, f(y)\, dy \qquad B = \int_0^{\pi/2} \sin y\, f(y)\, dy$$

$$= \int_0^{\pi/2} \cos y\,(y + A\sin y + B\cos y)\, dy \qquad B = \int_0^{\pi/2} \sin y\,[y + A\sin y + B\cos y]\, dy$$

$$A = \int_0^{\pi/2} y\cos y + \frac{A}{2}\int_0^{\pi/2} \sin y\, dy + B\int_0^{\pi/2} \cos^2 y\, dx \quad \text{--- (i)}$$

$$B = \int_0^{\pi/2} y\sin y\, dy + A\int_0^{\pi/2} \sin^2 y\, dy + \frac{B}{2}\int_0^{\pi/2} \sin y\, dx \quad \text{--- (ii)}$$

* Evaluating Integrals dependent on a parameter:

Let $f(x, \alpha)$ be a cont. fn in $[a, b]$ and $\alpha \in [c, d]$

Such that $I(\alpha) = \int_a^b f(x, \alpha)\, dx$.

then $$\frac{dI(\alpha)}{d\alpha} = \int_a^b \left(\frac{\partial f(x, \alpha)}{\partial \alpha}\right) dx$$

Here: α = Parameter

Que: Evaluate: $I(b) = \int_0^1 \frac{(x^b - 1)}{\ln x}\, dx. \quad b > 0$

$$\frac{d}{dx} a^{\overset{\text{var}}{x}} = a^x \ln a$$

const

J-A ≠ 13 Q; 15
full 23 Que.

$$\frac{dI(b)}{db} = \int_0^1 \left(\frac{1}{\ln x} \frac{d}{db}(x^b - 1) \right) dx$$

$$= \int_0^1 \frac{(x^b \cdot \ln x - 0)}{\ln x} dx = \int_0^1 x^b dx = \left[\frac{x^{b+1}}{b+1}\right]_0^1 = \frac{1}{b+1}$$

$$\frac{dI}{db} = \frac{1}{b+1}$$

$$dI = \frac{db}{b+1}$$

$$I = \int \frac{db}{b+1} + d$$

$$I = \ln(b+1) + d \quad \underline{\text{Ans}}$$

Que:

$$I = \int_0^\infty \frac{\tan^{-1}(ax) - \tan^{-1}x}{x} dx$$

where a is perameter.

$$\frac{dI}{da} = \int_0^\infty \frac{\frac{1 \cdot x}{1+a^2x^2} - 0}{x} dx$$

$$= \int_0^\infty \frac{dx}{1+a^2x^2} = \frac{1}{a}\left[\tan^{-1}(ax)\right]_0^\infty = \frac{1}{a}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$= \frac{\pi}{2a}.$$

$$dI = \frac{\pi}{2} \frac{da}{a}$$

$$I = \frac{\pi}{2} \ln a + d.$$

Imp

Ques: If $I_n = \int_0^{\pi/4} \tan^n\theta \, d\theta$

then prove that $I_{n+1} + I_{n-1} = \frac{1}{n}$

$$= \int_0^{\pi/4} \tan^{n-2}\theta \cdot (\sec^2\theta - 1)\, d\theta$$

$$= \int_0^{\pi/2} \tan^{n-1}\theta \sec^2\theta \, d\theta - \int_0^{\pi/4} \tan^{n-2}\theta \, d\theta$$

$\tan\theta = k$

$\sec^2\theta \, d\theta = dk$

$$= \int_0^1 k^{n-2}\, dk - I_{n-2}$$

$$= \left[\frac{k^{n-1}}{n-1}\right]_0^1 - I_{n-2}$$

$$\boxed{I_n + I_{n+2} = \frac{1}{n-1}} \rightarrow \text{Remember}$$

$n \rightarrow n+1$ $\quad I_{n+1} + I_{n-1} = \frac{1}{n}.$

$n = 3$

$I_3 + I_1 = \frac{1}{2}$

$n = 4$, $I_4 + I_2 = \frac{1}{3}$

$n = 5$ $\quad I_5 + I_3 = \frac{1}{4}$

Hence $\frac{1}{I_n + I_{n-2}}$ are in A.P.

S-2 Q1 Dw

$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

Que:- $U_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx.$

[Derive Reduction formula:]

$$U_{n+1} = \int_0^{\pi/2} \frac{\sin^2(n+1)x}{\sin^2 x}$$

$$\frac{\pi}{2} = J_n = U_{n+1} - U_n = \int_0^{\pi/2} \frac{\sin^2(n+1)x - \sin^2 nx}{\sin^2 x} dx$$

Ans

$$= \int_0^{\pi/2} \frac{\sin(2nx+x)\sin x}{\sin^{\cancel{2}} x.}$$

$$= \int_0^{\pi/2} \frac{\sin(2n+1)x}{\sin x} dx.$$

$$J_n = \int_0^{\pi/2} \frac{\sin(2n+1)x}{\sin x} dx.$$

$2(n-1)+1$

$2n-1$

$$J_n - J_{n-1} = \int_0^{\pi/2} \frac{\sin(2n+1)x - \sin(2n-1)x}{\sin x}$$

$$= \int_0^{\pi/2} \frac{2\cos 2nx \cdot \sin x\, dx}{\sin x} = 2\int_0^{\pi/2} \cos 2nx\, dx$$

$$= 2\left[\frac{\sin 2nx}{2n}\right]_0^{\pi/2} = 0$$

$$J_1 = \int_0^{\pi/2} \frac{\sin 3x}{\sin x} dx$$

$$= \int_0^{\pi/3} (3 - 4\sin^2 x)\, dx$$

Ques: Determine: $n \leq 5 \in I$ s.t.

$$\int_0^1 e^x (x-1)^n \, dx = 16 - 6e$$

$$I_n = \int_0^1 e^x (x-1)^n \, dx.$$

$$\text{II} \quad \text{I}$$

$$I_n = \left[(x-1)^n \cdot e^x\right]_0^1 - n\int_0^1 (x-1)^{n-1} \cdot e^x \, dx$$

$$\boxed{I_n = 0 - (-1)^n - n I_{n-1}}$$

$$I_1 = \int_0^1 e^x (x-1) \, dx$$

$$= \left[e^x (x-1)\right]_0^1 - \int_0^1 1 \cdot e^x \, dx.$$

$$= \left[0 - e^0 (0-1)\right] - \left[e^x\right]_0^1$$

$$= 1 - \left[e^1 - 1\right]$$

$$\boxed{I_1 = 2 - e}$$

$n = 2$ $\quad I_2 = -(-1)^2 - 2I_1 = -1 - 2(2-e) = -1 - 4 + 2e = 2e - 5$

$n = 3$ $\quad I_3 = -(-1)^3 - 3I_2 = 1 - 3(2e-5) = 1 - 6e + 15 = \boxed{16 - 6e}$

Que: find $\dfrac{5050 \cdot \int_0^1 (1-x^{50})^{100}\, dx}{\int_0^1 (1-x^{50})^{101}\, dx}$.

$I_n = \int_0^1 \underset{II}{1} \cdot \underset{I}{(1-x^{50})^n}\, dx$ $\qquad 1-x^{50}-1$

$= \left[x\,[1-x^{50}]^n\right]_0^1 - \int_0^1 (1-x^{50})^{n-1}(-50x^{49}) \cdot n\, dx$

$= -50n \int_0^1 (1-x^{50})^{n-1}\left[(1-x)^{50}-1\right] dx$

$I_n = -50n \int_0^1 (1-x^{50})^n + 50 \int_0^1 (1-x^{50})^{n-1}\, dx.$

$= -50n\, I_n + 50n\, I_{n-1}$

$$\boxed{(50n+1)\, I_n = 50n\, I_{n-1}}$$

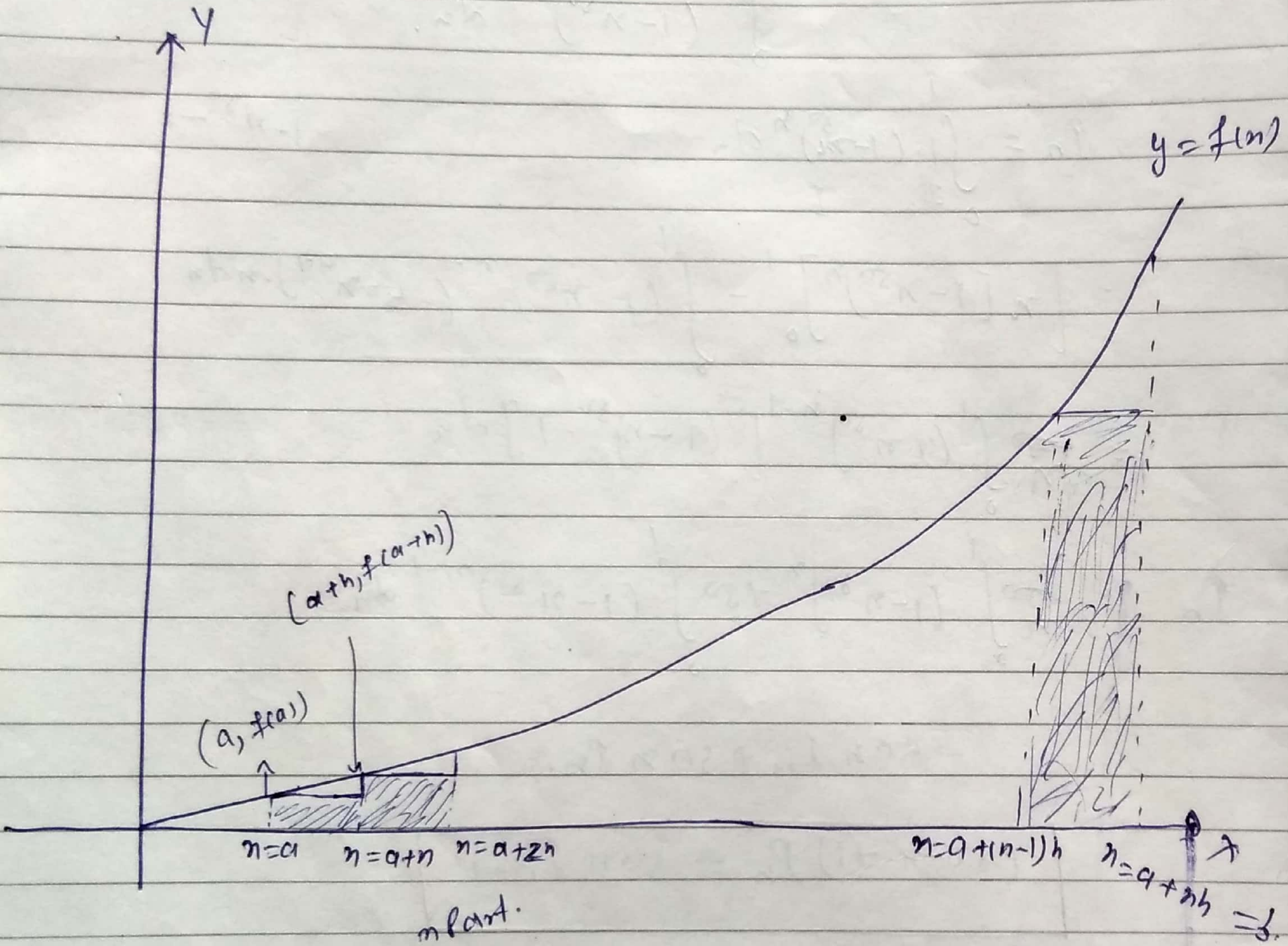
$= 5050 \cdot \dfrac{I_{100}}{I_{101}} = \cancel{5051}$ $\qquad (50 \times 101 + 1)\, I_{101}$

$= 50 \cdot 101 \cdot I_{100}$

$5051\, (I_{101}) = 5050\, I_{100}$

$= 5051$ Ans

* Definite Integral as a limit of sum:



$\Rightarrow$ $b = a + nh$ → thickness of mpart.

$$\int_a^b f(x)\,dx = \lim_{\substack{h\to 0\\ n\to\infty}} h\left[f(a) + f(a+h) + f(a+2h) + \cdots f(a+(n-1)h)\right]$$

$$= \lim_{\substack{h\to 0\\ n\to\infty}} h \sum_{r=0}^{n-1} f(a+rh)$$

definate integral By using 1st Principle
Put $a=0$, and $b=1$.

then $nh=1$

or $h=\frac{1}{n}$.

$$=\lim_{n\to\infty}\sum_{r=0}^{n-1} f\left(\frac{r}{n}\right)\cdot\frac{1}{n}$$

Now Replaced by Integration

(i) Σ by $\int$

(ii) $\frac{1}{n}$ by dx

(iii) $\frac{r}{n}$ by x.

(iv) LL of Integration $=\lim_{n\to\infty}\frac{\text{LL of Sigma}}{n}$

&

UL of integration. $=\lim_{n\to\infty}\frac{\text{UL of sigma}}{n}$.

Que: Evaluate $\lim_{n\to\infty}\frac{n^2}{(n^2+1)^{3/2}}+\frac{n^2}{(n^2+2^2)^{3/2}}+ - -$

$$=\lim_{n\to\infty}\sum_{1}^{n-1}\frac{n^2}{(n^2+r^2)^{3/2}}. \quad - - - \quad \frac{n^2}{[n^2+(n-1)^2]^{3/2}}$$

$$=\lim_{n\to\infty}\sum_{1}^{n-1}\frac{n^2}{n^3\left[1+\left(\frac{r}{n}\right)^2\right]^{3/2}}=$$

$$= \int_0^1 \frac{1}{(1+x^2)^{3/2}} \, dx.$$

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

$$= \lim_{n \to \infty} \frac{n-1}{n} = 1$$

Que: $\lim_{n \to \infty} \left(\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots \frac{1}{4n} \right)$

$$\lim_{n \to \infty} \sum_{r=0}^{3n} \frac{1}{n+r}$$

$$\lim_{n \to \infty} \sum_{0}^{3n} \frac{1}{n+r} = \lim_{n \to \infty} \sum_{r=0}^{3n} \frac{1}{n\left(1+\frac{r}{n}\right)} = \int_0^3 \frac{dx}{1+x}$$

Que: $\lim_{n \to \infty} \left(\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + \frac{n+3}{n^2+3^2} + \cdots \frac{3}{5n} \right)$

$$\lim_{n \to \infty} \sum_{r=1}^{2n} \frac{n+r}{n^2+r^2}$$

$$\frac{n+2n}{n^2+(2n)^2} = \frac{3n}{5n^2}$$

$$= \lim_{n \to \infty} \sum_{r=1}^{2n} \frac{n\left(1+\frac{r}{n}\right)}{n^2\left(1+\frac{r^2}{n^2}\right)} = \int_0^2 \frac{1+x}{1+x^2} \, dx$$

lnI

Que: $\lim_{n\to\infty}\left(\frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+ - - 2\sqrt{n}}{n\sqrt{n}}\right)$

$$\sum_{r=1}^{4n}\frac{\sqrt{r}}{n\sqrt{n}} = \int_0^4 \sqrt{x}\,dx$$

Que: $A = \lim_{n\to\infty}\left[\frac{(n+1)(n+2) - - - (n+n)}{n^n}\right]^{1/n}$

$$A = \lim_{n\to\infty}\left[\left(\frac{(n+1)}{n}\right)\left(\frac{n+2}{n}\right) - - \left(\frac{(n+n)}{n}\right)\right]^{1/n}$$

$$= \ln\left(\frac{n+1}{n}\right)+\ln\left(\frac{n+2}{n}\right) - - \ln\left(\frac{n+n}{n}\right)\Big]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{n+r}{n}\right)$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(1+\frac{r}{n}\right)$$

$$\ln A = \int_0^1 \ln(1+x)\,dx$$

$$A = e^{\int_0^1 \ln(1+x)\,dx}$$

Que Evaluate $y = \lim_{n\to\infty}\left(1+\frac{1}{n^2}\right)^{2/n^2}\left(1+\frac{2^2}{n^2}\right)^{4/n^2} - \left(1+\frac{n^2}{n^2}\right)^{2n/n^2}\Big]$

$$\ln y = \lim_{n\to\infty}\frac{2}{n^2}\ln\left(1+\frac{1}{n^2}\right)+\frac{4}{n^2}\ln\left(1+\frac{2^2}{n^2}\right) - - \frac{2n}{n^2}\ln\left(1+\frac{n^2}{n^2}\right)\Big]$$

$$= \lim \sum_{r=}^{n}\frac{2r}{n^2}\ln\left(1+\frac{r^2}{n^2}\right) =$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} 2\cdot\frac{r}{n}\cdot\frac{1}{n}\cdot \ln\left(1+\left(\frac{r}{n}\right)^2\right) = \int_0^1 2x\ln(1+x^2)\,dx$$

$$= e^{\int_0^1 2x\ln(1+x^2)\,dx}$$

Que: Let $\alpha > -1$, $\beta = -1$

$$\lim_{n\to\infty} n^{\beta-\alpha}\left(\frac{1^\alpha+2^\alpha+\cdots n^\alpha}{1^\beta+2^\beta+\cdots n^\beta}\right)$$

$$\lim_{n\to\infty} \frac{n^\beta}{n^\alpha}\left(\frac{1^\alpha+2^\alpha\cdots\cdots n^\alpha}{1^\beta+2^\beta+\cdots\cdots n^\beta}\right)$$

$$= \lim_{n\to\infty} \frac{\left[\left(\frac{1}{n}\right)^\alpha+\left(\frac{2}{n}\right)^\alpha+\cdots\left(\frac{n}{n}\right)^\alpha\right]\cdot n^\alpha}{\left[\left(\frac{1}{n}\right)^\beta+\left(\frac{2}{n}\right)^\beta+\cdots\left(\frac{n}{n}\right)^\beta\right] n^\beta} \cancel{\frac{n^\beta}{n^\alpha}}$$

$$= \lim \frac{\sum_{r=1}^{n}\left(\frac{r}{n}\right)^\alpha\cdot\frac{1}{n}}{\sum_{r=1}^{n}\left(\frac{r}{n}\right)^\beta\cdot\frac{1}{n}} = \frac{\int_0^1 x^\alpha\,dx}{\int_0^1 x^\beta\,dx}$$

Que: Find $f(x)$ $f(x) = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2x^2}$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{1+\left(\frac{r^2}{n^2}\right)x^2}\cdot\frac{1}{n}$$

$$= \int_0^1 \cancel{\frac{1}{1+x^4}}\,dx \quad \text{wrong}$$

OR $= \int_0^1 \frac{1}{1+t^2x^2}\,dt$ ✓

Now Right

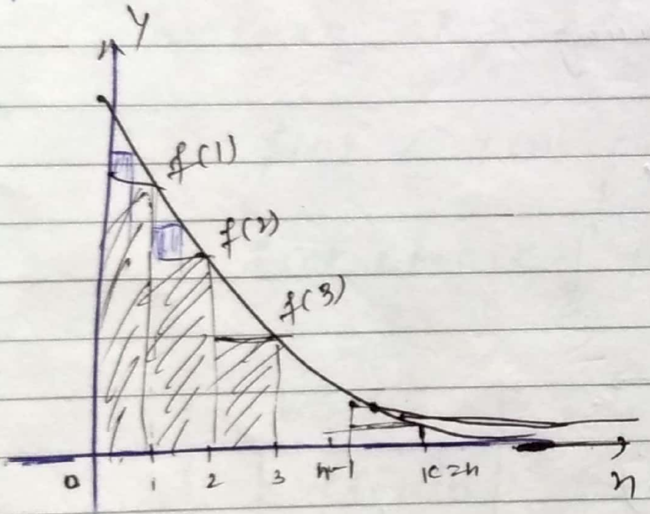
Que: Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+n^2}$

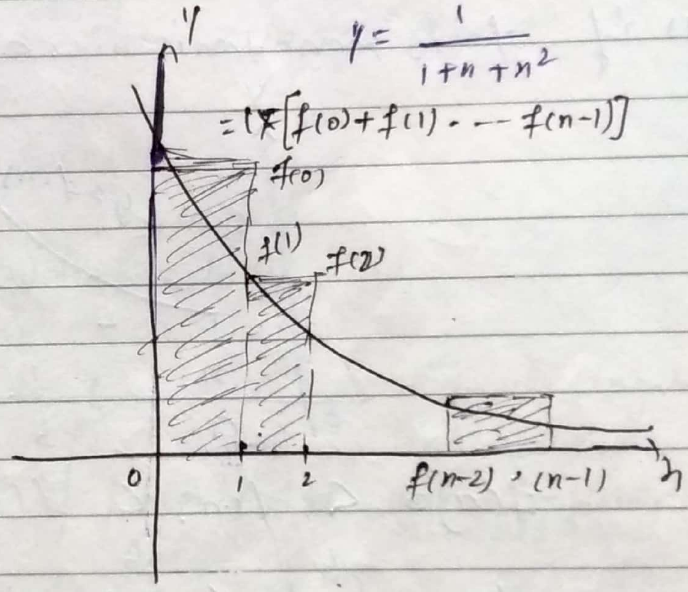
k = 1, 2 - - - -

then chosse correct option.

(i) $S_n < \frac{\pi}{3\sqrt{3}}$ (ii) $S_n > \frac{\pi}{3\sqrt{3}}$ (iii) $T_n < \frac{\pi}{3\sqrt{3}}$ (iv) $T_n > \frac{\pi}{3\sqrt{3}}$

An.





$= [1 \times f(1) + 1 \times (f(2)) + \text{---} \; 1 \times f(n)]$

$S_n <$ Actual Area $< T_n$.

for actual area $A = \lim_{n \to \infty} S_n$

or

$= \lim_{n \to \infty} T_n$

$= \int_0^1 \frac{1}{1+n+n^2} \, dn = \frac{\pi}{3\sqrt{3}}$

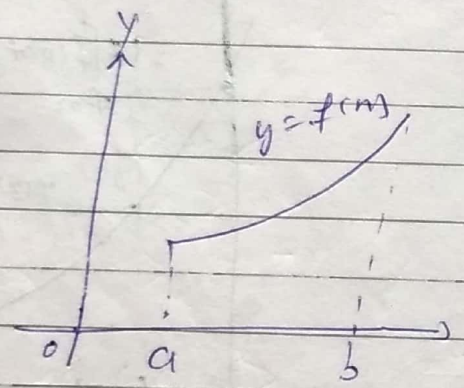
$$S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2} \quad \& \quad T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$$

$$f(1)+f(2)+ \cdots f(n-1) + \frac{1}{3n} \qquad = \frac{1}{n} + f(1) + f(2) + \cdots f(n-1)$$

$$T_n > S_n$$

* Estimation of definate integral:

(1) if f is monotonic increasing

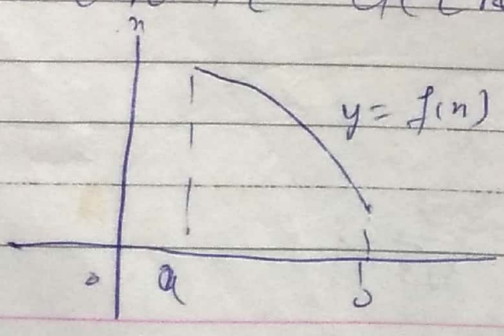


$$f(a) < f(x) < f(b)$$

$$f(a)\int_a^b dx < \int_a^b f(x)\,dx < f(b)\int_a^b dx$$

$$(b-a) f(a) < \int_a^b f(x)\,dx < (b-a) f(b)$$

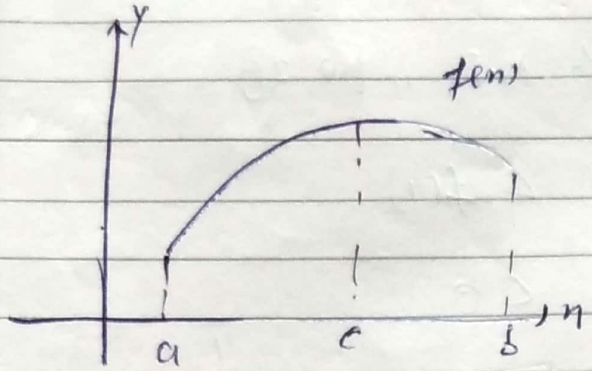
(2) monotonic decreasing



$$f(b) < f(x) < f(a)$$

$$\therefore (b-a) f(b) < \int_a^b f(x)\,dx < (b-a) f(a)$$

③ f is non monotonic in interval $[a, b]$



i.e minima at $x=a$
maxima at $x=c$ as shown in the figure

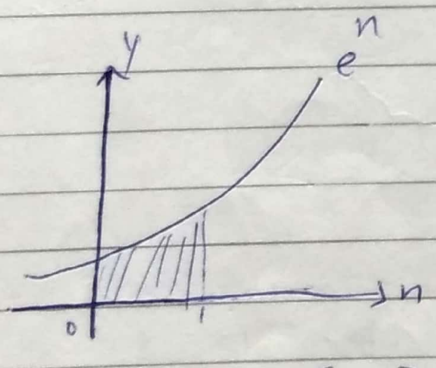
$$f(a) < f(x) < f(c)$$

$$f(a)\cdot(b-a) < \int_a^b f(x)\,dx < (b-a)\,f(c)$$

$$\left|\int_a^b f(x)\,dx\right| \leq \int_a^b |f(x)|\,dx$$

Equal sign inequality hold when a curve completely lie above and ~~held~~ x-axis below x-axis.

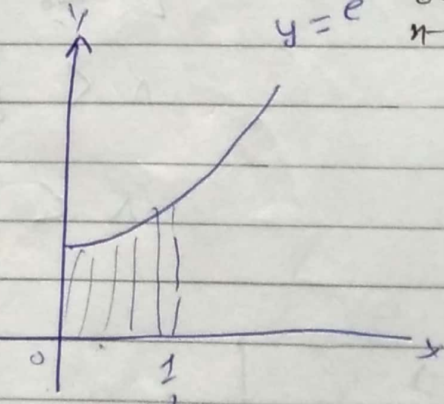
Ex:-



$$\int e^x dx = [e^x]_0^1$$
$$= e - 1$$

accurate Area

$y = e^{x^2}$



$$\int e^{x^2} dx$$

Estimation.

$\int e^{n^2}$

$f(n) = e^{x^2}$ is M.I in $[0, 1]$

$f(0) < f(n) < f(1)$

$1 < e^{n^2} < e$

$1 \cdot \int_0^1 dn < \int_a^1 e^{n^2} dn < e \int_0^1 dn$

$1 < R < e$

Method : 2 $\quad 0 < n < 1$

$n > n^2$

$e^n > e^{n^2}$

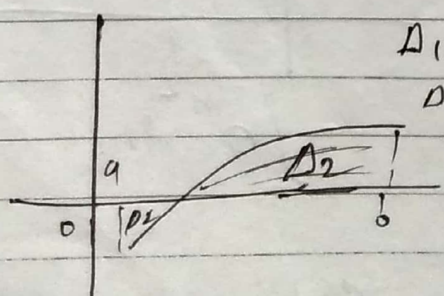
$\int_0^1 e^n dn > \int_0^1 e^{n^2} dn$

$[e^n]_0^1 > R$

$R < [e^1 - e^0]$

$R < (e - 1)$

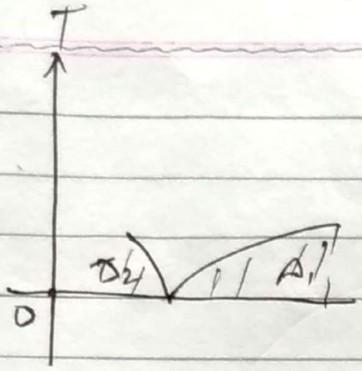
$\Rightarrow$



$\Delta_1 = 10$

$\Delta_2 = 3$

LHS = 10 - 3



$y = f(x)$

$R.H.S = \Delta_1 + \Delta_2$

$= 10 + 3$

$LHS < RHS.$