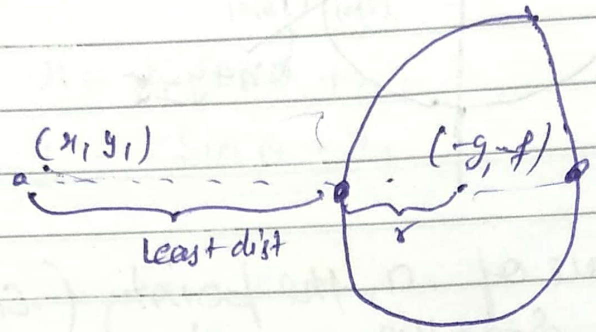
CIRCLE

* Position of a point w.r.t a circle.

$$S \equiv x^2+y^2+2gx+2fy+c$$



If Point P is outside the circle

$$OP > r$$
$$OP^2 > r^2$$

$$(x_1+g)^2+(y_1+f)^2 > g^2+f^2-c$$

$$x_1^2+y_1^2+2x_1g+2y_1f+g^2+f^2 > g^2+f^2-c$$

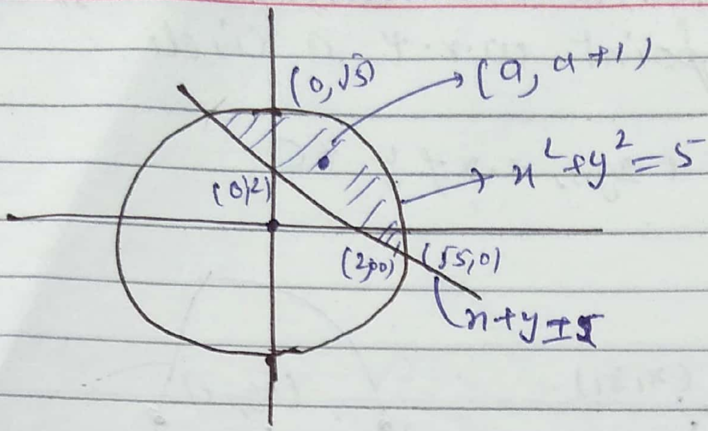
$$x^2+y_1^2+2gx+2fy_1+c>0 \qquad S_1>0$$

If P is inside the circle then

$$\boxed{S_1<0}$$

max. dist. from circle $= |OP+r|$
min " " " $= |OP-r|$

Q



for what value of a, the point (a, a+1) lie inside the smaller regio bounded by circle $x^2+y^2=5$ and line $x+y=2$

$$S = x^2+y^2-5=0$$

$S_1 < 0$

$a^2+(a+1)^2-5<0$

$a^2+a^2+2a+1-5<0$

$2a^2+2a-4<0$

$a^2+a-2<0$

$(a+2)(a-1)<0$

$a \in (-2,1)$

$x+y-2=0$

$0+0-2<0$

$a+a+1-2>0$

$2a>1$

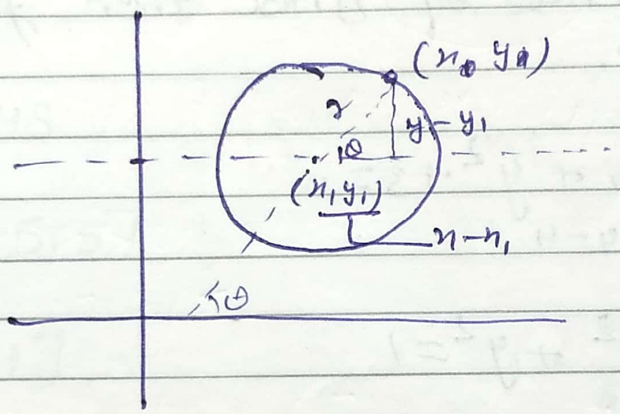
$a>\frac{1}{2}$

$a \in (\frac{1}{2},1)$

* Parametric eq^n of Circle :-

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$$x = r\cos\theta + x_1$$
$$y = r\sin\theta + y_1$$



$$\sin\theta = \frac{y-y_1}{r}$$

$$\cos\theta = \frac{x-x_1}{r}$$

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

r is constant
θ is parameter.

$x = r\cos\theta + x_1$
$y = r\sin\theta + y_1$

Q. $x^2+y^2-4x+3=0$
find range of

(i) x^2+y^2
(2) x^2+y^2+2y
(3) $3x+4y$

Q. If $A(\cos\theta_1, \sin\theta_1)$, $B(\cos\theta_2, \sin\theta_2)$, $C(\cos\theta_3, \sin\theta_3)$ are the vertices of ΔABC then find Orthocentre.

Ans:-1 $x^2-4x+y^2+3=0$
$+4-4$

$(x-2)^2+y^2=1$

$(x-x_1)^2+(y-y_1)^2=r^2$

$x-x_1=r\cos\theta$
$y-y_1=r\sin\theta$

$x=x_1+r\cos\theta$
$y=y_1+r\sin\theta$

$x=2+\cos\theta$
$y=0+\sin\theta$

(1) x^2+y^2
$4+c^2+4c+s^2$ $\qquad c^2 \to \cos^2\theta$
$5+4\cos$ $\qquad (-1,1)$
$[1 \quad 9]$

(2) x^2+y^2+2y

$5+4c+2s$

$5+[-2\sqrt{5}, 2\sqrt{5})$

$[5-2\sqrt{5} \quad 5+2\sqrt{5}]$

$a\cos\theta + b\sin\theta$

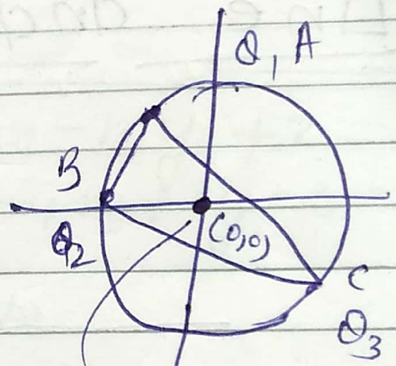
$\in(\sqrt{a^2+b^2} \quad \sqrt{a^2+b^2}]$

(3) $3x+4y$

$6+3c+4s$

$6+[-5,5]$

$[1, 11]$



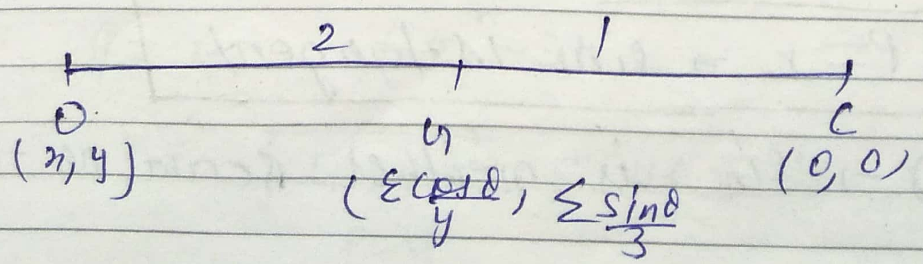
Circumcentre $=(0,0)$

Ans x^2+y^2+1

$x=\cos\theta$

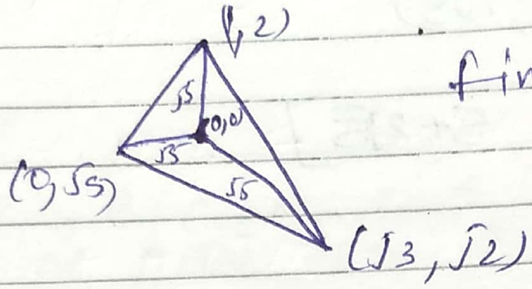
$y=\sin\theta$

centroid $= G = \left[\dfrac{\cos\theta_1+\cos\theta_2+\cos\theta_3}{3}, \dfrac{\sin\theta_1+\sin\theta_2+\sin\theta_3}{3}\right]$



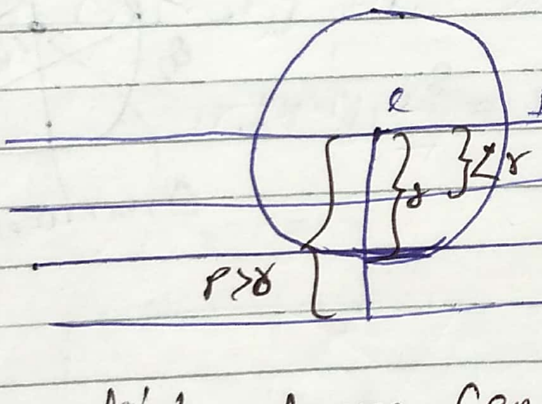
$\frac{x}{3} = \frac{\Sigma \cos\alpha}{3}$

$\frac{y}{3} = \frac{\Sigma \sin\alpha}{y}$ $\quad (x, y) = (\Sigma \cos\theta, \Sigma \sin\theta)$



find orthocentre

Line and Circle :-



$p \rightarrow$ dist. from Centre

$p = 0 \Rightarrow$ line is diameter

$p < r \Rightarrow$ line is secant

$\boxed{p = r \Rightarrow \text{line is tangent}}$

$p > r \Rightarrow$ line is neither secant nor tangent

$x+y=0$

$x^2+y^2=1$

$x^2+(1-x)^2=1$

$D=0 \rightarrow$ tangent

$D>0 \rightarrow$ secant

$D<0 \rightarrow$ neither secant nor tangent

Q. Two Coincident Point

Tangent

Q.1 for what value of m, $3x-my+z=0$ intersect circle $x^2+y^2-4x+ry-3=0$

at two coincident points.

Q.2 If there are 3 distinct point on $3x+by+7$ which form a right angle $\triangle$ with two point $A(0,0)$ and $B(0,4)$ then find b.

Q.3) Line $3x+4y+\lambda=0$ neither touches nor cuts circle $(x-1)^2+(y-2)^2=9$ then find λ.

Ans:) $P=r$ $(2,-3)$

$r=\sqrt{4+9+3}=\sqrt{16}=4$

$P=4$

$$\left|\frac{6+3m+2=0}{\sqrt{m^2-9}}\right|=4$$

$$8+3m=4\sqrt{m^2+9}$$

$$64+9m^2+48m=16m^2+144$$

$$7m^2-48m+80=0$$

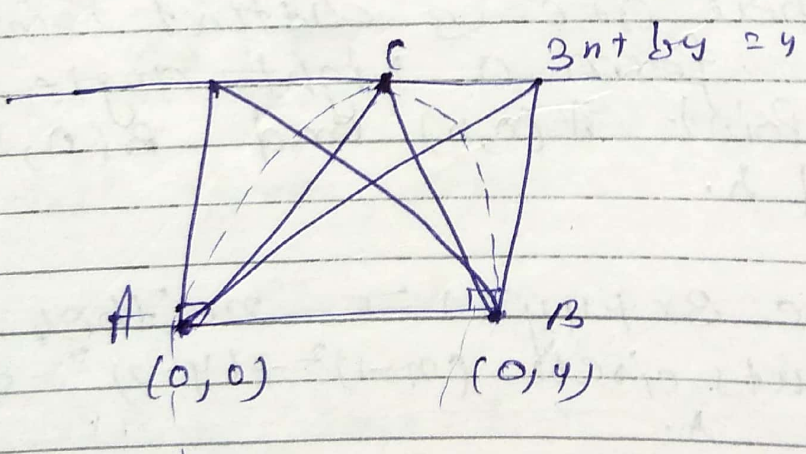
$$7m^2-28m-70m+80=0$$

$$7m(m-4)-20(m-4)=0$$

$$(m-4)(7m-20)=0$$

$$m=4,\ \frac{20}{7}$$

Ans: 2



$$(x-0)(x-0)+(y-0)(y-4)=0$$

$$x^2+y^2-4y=0$$

4-Secant · tangent - 3, ① · ②

$$p = r \qquad (0, 2)$$

$$r = \sqrt{0 + 4 - 0}$$

$$r = 2$$

$$\left| \frac{0 + 2b - 4}{\sqrt{a + b^2}} \right| = 2$$

$$a + b^2 = b^2 + 4 - 4b$$

$$4b = -5$$

$$b = \frac{-5}{4} \text{ Ans.}$$

Ans:- 3 $\quad (1, 2), \; r = 3$

$$3x + 4y + \lambda = 0$$

$$p > r$$

$$p > 3$$

$$\left| \frac{3 + 8 + \lambda}{5} \right| > 3 \qquad |x| > a$$

$$x \in (-\infty, -a) \cup (a, \infty)$$

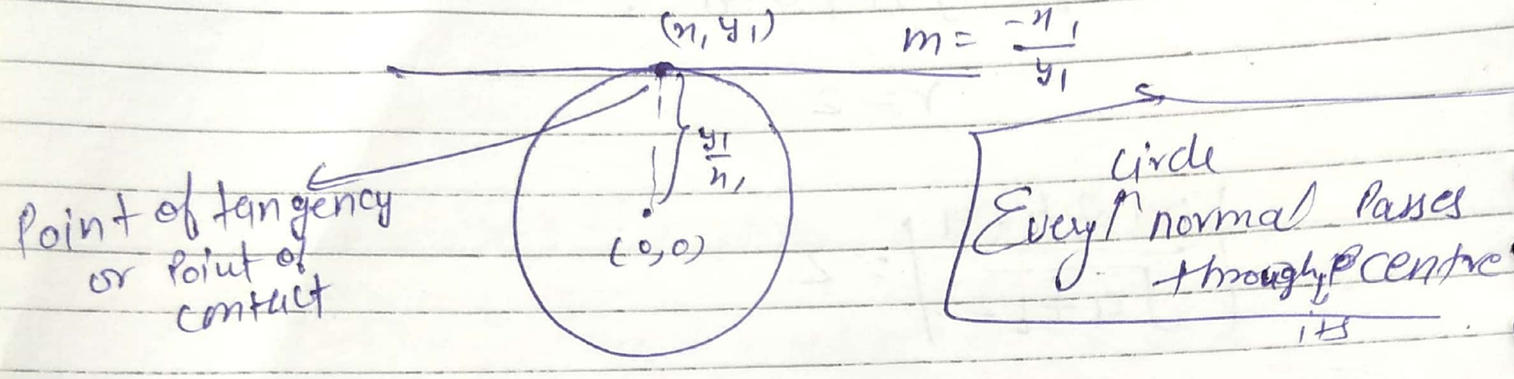
$$|11 + \lambda| > 15 \qquad |x| < a$$

$$a \in (-a, a)$$

$$|11 + \lambda| \in (-\infty, -15) \cup (15, \infty)$$

$$\lambda \in (-\infty, -26) \cup (4, \infty)$$

Tangent of a circle:



$$x^2 + y^2 = a^2$$

$$x_1^2 + y_1^2 = a^2$$

$$y - y_1 = \frac{-x_1}{y_1}(x - x_1)$$

$$y y_1 - y_1^2 = (-x x_1 + x_1^2$$

$$x x_1 + y y_1 = x_1^2 + y_1^2$$

$$\boxed{x x_1 + y y_1 = a^2} \quad \boxed{T = 0}$$

Replace

$x^2 = x x_1$

$y^2 = y y_1$

$2x = x + x_1$

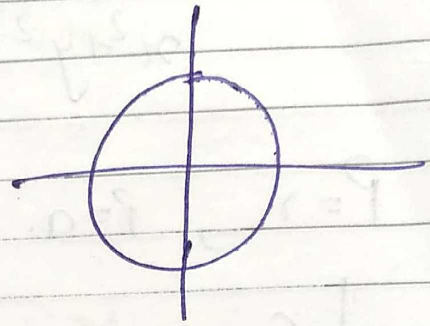
$2y = y + y_1$

$2xy = x y_1 + y x_1$

It is Valid for Every second degree Curve.

Every normal of a circle passes ~~tang~~ through its centre

* $x^2 + y^2 = r^2$
$(r\cos\theta, r\sin\theta)$

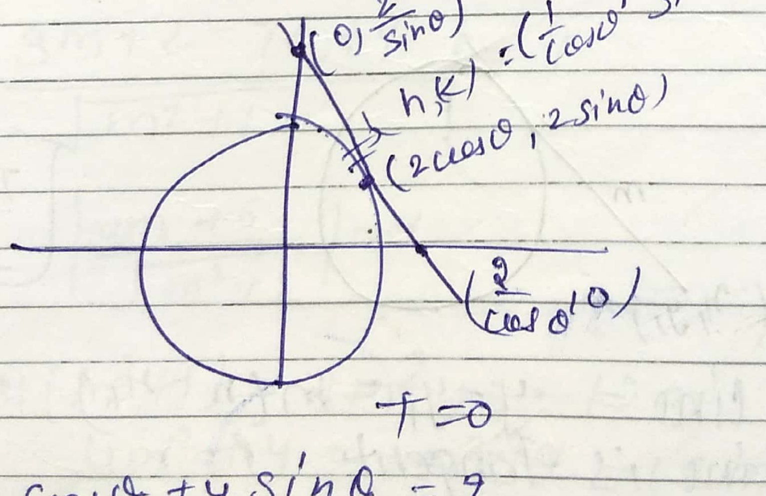


$T = 0$

$x(r\cos\theta) + y(r\sin\theta) = r^2$

$x\cos\theta + y\sin\theta = r$

Que Let there a circle $x^2 + y^2 + 4$ It has a tangent in first Quadrant.
⇒ find the locus of mid point of the portion of ~~th~~ tangent intercept b/w coordinate axis.



$T = 0$

$x\cos\theta + y\sin\theta = 2$

$h = \frac{1}{\cos\theta}$ $\qquad$ $\frac{1}{h^2} + \frac{1}{k^2} = 1$

$k = \frac{1}{\sin\theta}$ $\qquad$ $\frac{1}{x^2} + \frac{1}{y^2} = 1$

Slope form of tangent.

if $y = mx + c$ is tangent of Circle

$$x^2 + y^2 = a^2$$

$(0,0) \Rightarrow r = a$

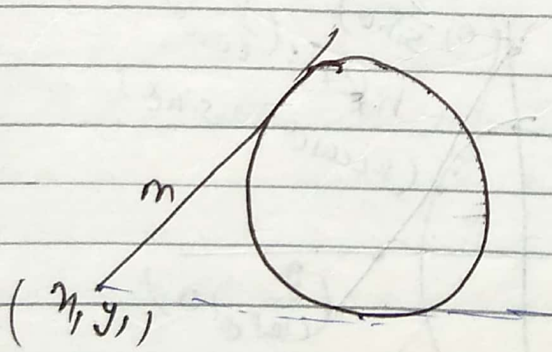
$P = r$, $P = a.$

$$\left| \frac{c}{\sqrt{m^2+1}} \right| = a \qquad |c| = a\sqrt{1+m^2}$$

$$c = \pm a\sqrt{1+m^2}$$

$$y = mx \pm a\sqrt{1+m^2}$$

If a point is given outside the circle.



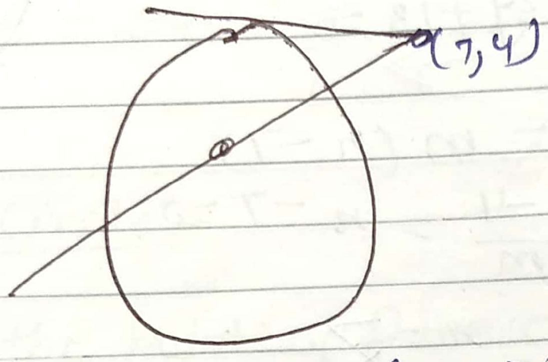
Two Value of m

line = $y - y_1 = m(x - x_1)$

if line is tangent $P = r$

Q. find the eq^n of tangent from (7, 4) to (Circle)

$$x^2+y^2-6x+4y-3=0$$



$$y-4=m(x-7)$$

$$mx-y-7m+4=0$$

$(3, -2)$ $\quad r=\sqrt{9+4+3}=4$

$P=4$

$$\left|\frac{3m+2-7m+4}{\sqrt{m^2+1}}\right|=4$$

$$\left|\frac{-4m+6}{\sqrt{m^2+1}}\right|=4$$

$$4(m^2+1)=4m^2+9-12m$$

$$4m^2+4=4m^2+9-12m$$

$$12m=5$$

$$m=\frac{5}{12}$$

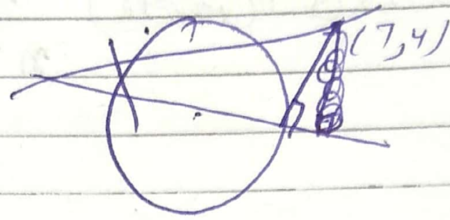
$m \to \infty$.

m second value ∞

$$y - 4 = \frac{5}{12}(x - 7)$$

$$12y - 48 = 5x - 35$$

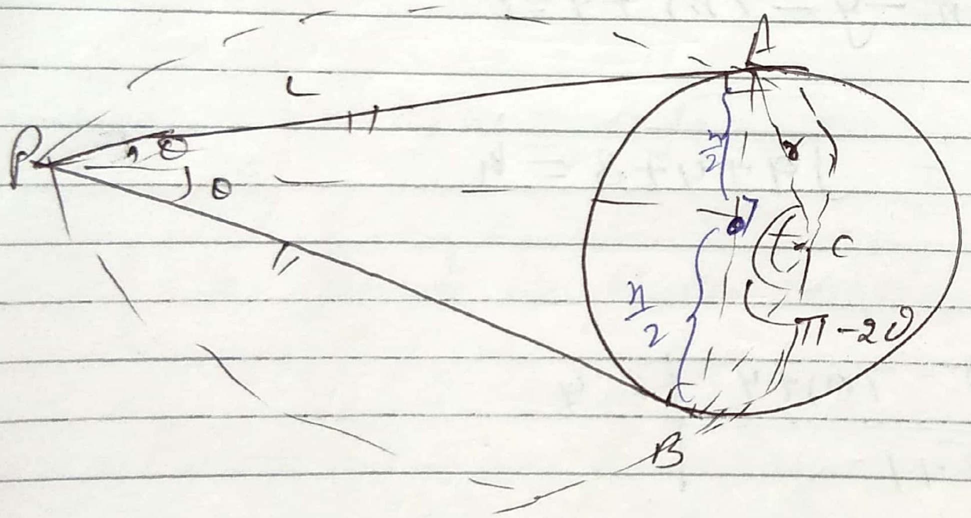
$$5x - 3y + 13 = 0$$



$$y - 4 = m(x - 7)$$

$$\frac{y - 4}{m} = x - 7 = 0$$

$$x = 7$$



PACB → cyclic Quadrilateral with diameter PC

Area of $\triangle APC = \frac{1}{2} \times L \times r$

Area of $PACB = L \times r$

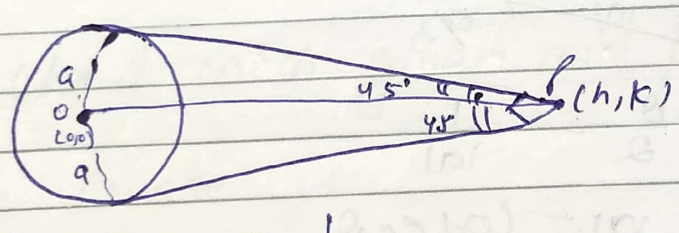
$\tan\theta = \frac{r}{L}$

$\theta = \tan^{-1}\frac{r}{L}$

$2\theta = 2\tan^{-1}\frac{r}{L}$

* **Director Circle :-**

Locus of the point of Intersection of two mutually Perpendicular tangent to a given curve is called Director circle



$\sin 45 = \frac{a}{OP} = \frac{1}{\sqrt{2}}$ $\quad OP^2 = 2a^2$

$OP = \sqrt{2}a$ $\quad h^2+k^2 = 2a^2$

$\boxed{x^2+y^2 = 2a^2} = (\sqrt{2}a)^2$

concentric circle.

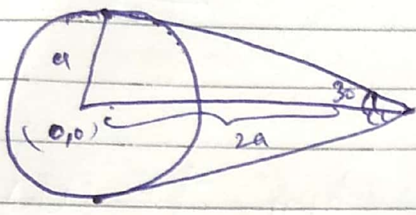
Q. Locus of Point P which moves such that angle made by tangent to circle $x^2+y^2=a^2$ is $60°$

Q. find range of a such that angle θ b/w tangents from $(a,0)$ to circle $x^2+y^2=1$ lying in $(\frac{\pi}{2}, \pi)$

Q. Find locus of centre of Circle touching two mutually ⊥ular Lines from origin and $x+y=1$

Ans:

(1)

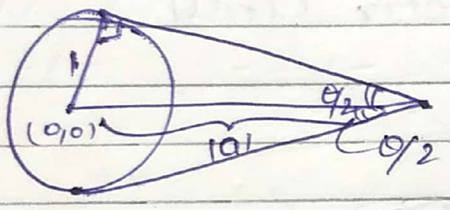


$\sin 30° = \frac{a}{OP} = \frac{1}{2}$

$h^2 + k^2 = 4a^2$

$\boxed{OP = 2a}$

(2)



$\sin\frac{\theta}{2} = \frac{1}{|a|}$

$|a| = \operatorname{cosec}\frac{\theta}{2}$

$\theta \in \left(\frac{\pi}{2}, \pi\right)$

$|a| = \operatorname{cosec}\theta/2$

$|a| \in (1, \sqrt{2})$

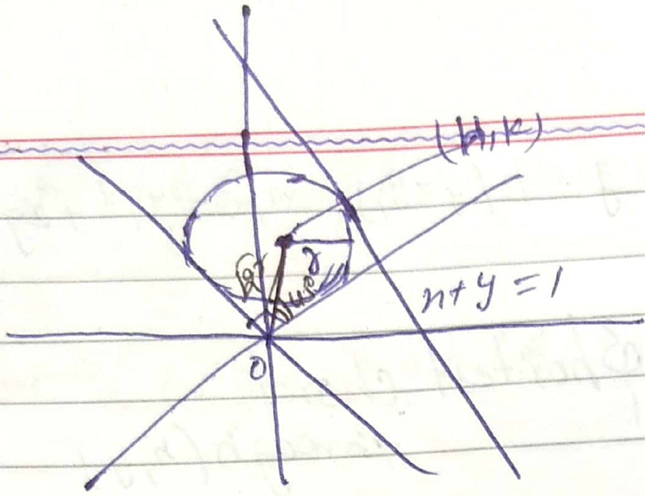
$\sqrt{2} > |a| > 1$

$|a| < \sqrt{2} \Rightarrow a \in (-\sqrt{2}, \sqrt{2})$

$|a| > 1 \quad a \in (-\infty, -1) \cup (1, \infty)$

$a \in (-\sqrt{2}, -1) \cup (1, \sqrt{2})$

3



$OP = \sqrt{2}\, r$

$OP = \sqrt{2}\, r$

$OP^2 = 2r^2$

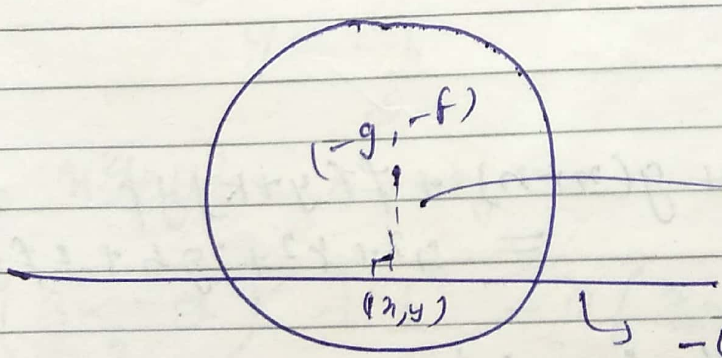
$\cancel{h^2}+\cancel{k^2} = 2\left(\frac{h+k-1}{\sqrt{2}}\right)^2 = \cancel{h^2}+\cancel{k^2}+1+2hk-h-k$

$h+k-2hk=1$

* <u>eqⁿ of chord with given mid point :</u>

Let a circle is

$$x^2+y^2+2gx+2fy+c=0$$



$\text{slope} = \frac{y+f}{x_1+g}$

$\frac{-(x_1+g)}{y_1+f}$

$$y-y_1 = -\frac{(x_1+g)}{(y_1+f)}(x-x_1)$$

$$+yy_1+yf-y_1^2-y_1f = -xx_1+x^2-gx+gx_1$$

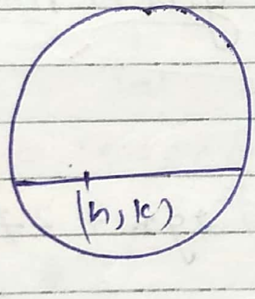
$xx_1 + yy_1 + (x+x_1)g + f(y+y_1) = x_1^2 + y_1^2 + 2gx + 2fy + c$

$\boxed{T = S_1}$ $\rightarrow$ Shortest chord through (x, y)

Q. find locus of mid-point of chords of $x^2 + y^2 + 2gx + 2fy + c = 0$ which Passes through (a, b) lying outside the circle.

(2) If two distinct Chords are drawn from (a, b) to circle $x^2 + y^2 - ax - by = 0$ are divided by x-axis in ratio 2:1 then P.T. $a^2 > 3b^2$

Ans: (1)



$T = S_1$

$hx + ky + g(x+h) + f(y+k) + c$
$= h^2 + k^2 + 2gh + 2fy + c$

Passes through (a, b)

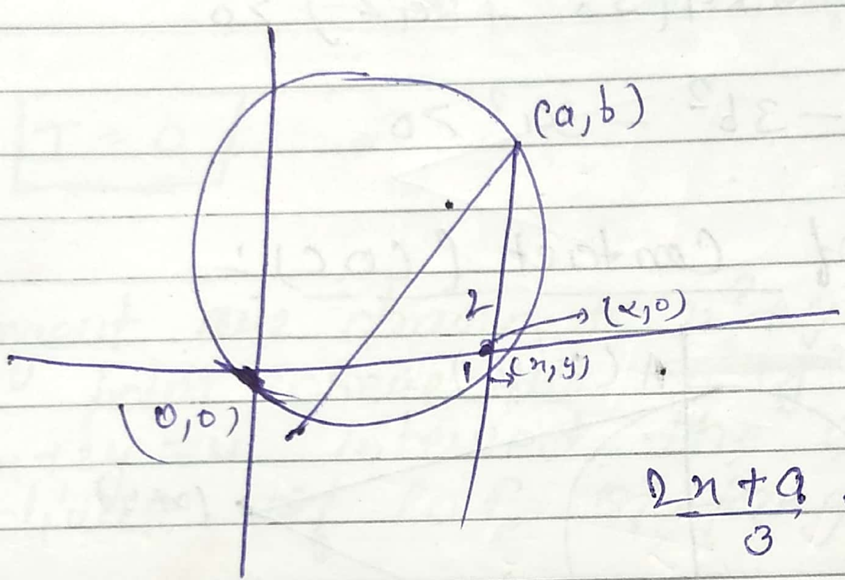
$ah + bk + g(a+h) + f(b+k) = h^2 + k^2 + 2gh + 2fk$

$ah + bk + ga + gh + fb + fk = h^2 + k^2 + 2gh + 2fb$

$h^2 + k^2 + gh + fk - ah - bk - ga - bf = 0$

$$\boxed{h^2 + k^2 + h(g-a) + k(f-b) - bk - bf = 0}$$

(2)



$\frac{2x+a}{3} = \alpha$

$x = \frac{3\alpha - a}{2}$

$\frac{2y+b}{3} = 0$

$y = -\frac{b}{2}$

$x^2 + y^2 - ax - by = 0$

$\left(\frac{3\alpha - a}{2}\right)^2 + \frac{b^2}{4} - a\left(\frac{3\alpha - a}{2}\right) + \frac{b^2}{2} = 0$

$a\alpha^2 + a^2 - 6a\alpha + b^2 - 2a(3\alpha - a) + 2b^2 = 0$

~~eqn of chord with given~~

$D = b^2 - 4ac$

$9x^2 + a^2 - 6ax + b^2 - 6ax + 2a^2 + 2b^2 = 0$

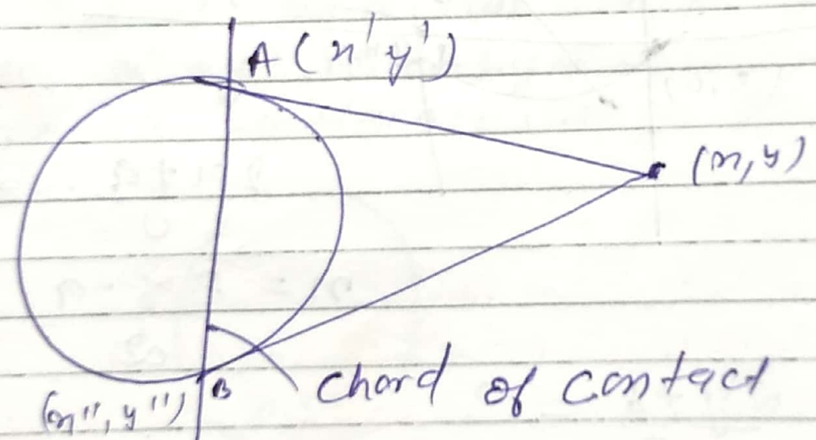
$9x^2 - 12ax + 3b^2 + 3a^2 = 0$

$D > 0$

$144a^2 - 4 \times 9(3b^2 + 3a^2) > 0$

$4a^2 - 3b^2 - 3a^2 > 0$

* chord of contact (COC) :-



$x^2 + y^2 = a^2$

eq^n of tangent passing through A

$T = 0 \Rightarrow xx' + yy' = a^2$

Tangent from B

$T = 0, \quad xx'' + yy'' = a^2$

$T=0$ may be chord of contact to out of circle

Point on curve ⟶ Tangent

Passes through (x_1, y_1)

$$x_1 x' + y_1 y' = a^2$$
$$x_1 x'' + y_1 y'' = a^2$$

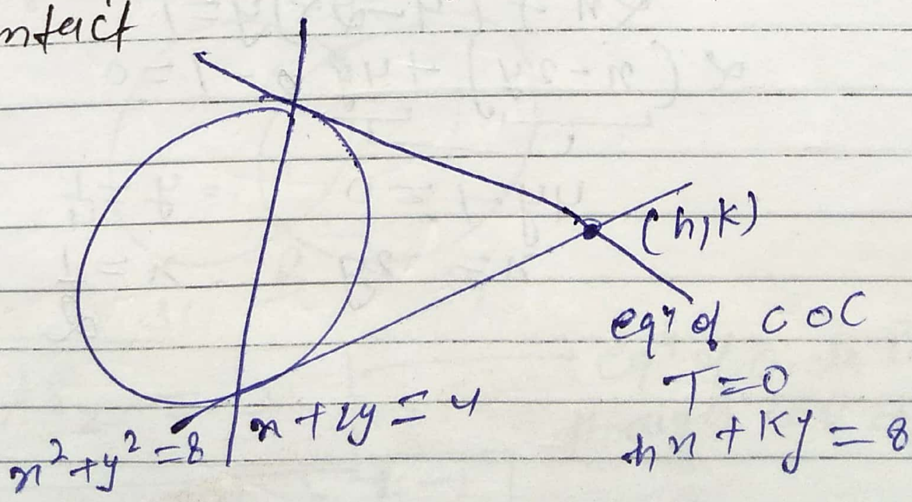
$\rightarrow$ $\boxed{x x' + y y_1 = a^2}$ ⟶ this is the eq^n of chord of Contact

$\boxed{T = 0}$

Q.1 Tangent are drawn to $x^2+y^2=8$ from the point where the ~~x^2+y^2~~ line $x+2y=4$ intersect the circle. find coordinate of POI of tangent.

Q.2 Tangent are drawn to unit circle centred at origin from every point on the $2x+y=4$
(i) chord of contact passes through a fix point, find that point
(2) find the eq^n of locus of middle point of chord of contact

Ans (1)

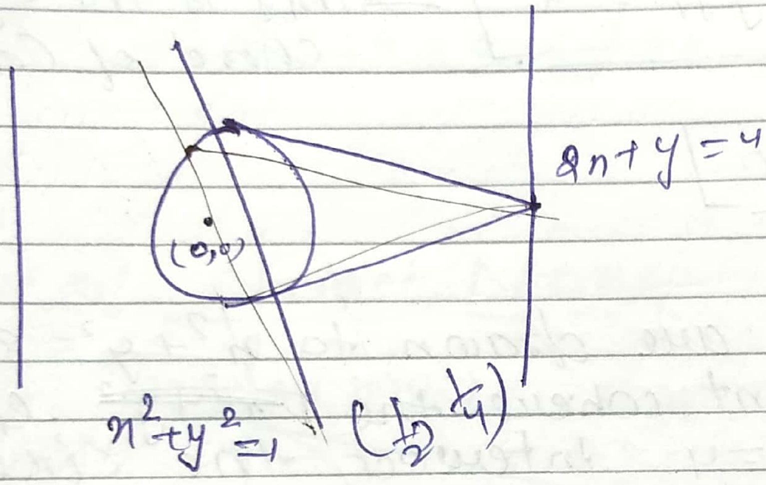


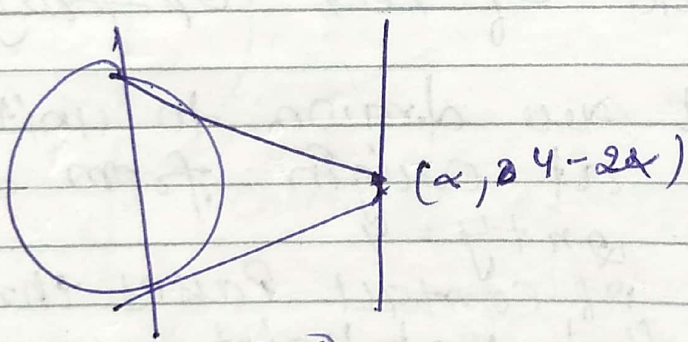
$\frac{h}{1} = \frac{k}{2} = \frac{8}{2}$ compare

$\boxed{\begin{matrix} h=2 \\ k=4 \end{matrix}}$ $(2, 4)$

(2)

(i)





$T=0$

$\alpha x + (4-2\alpha)y = 1$

$\alpha\underbrace{(x-2y)} + \underbrace{4y - 1} = 0$

$4y - 1 = 0 \Rightarrow y = \frac{1}{4}$

$x = 2y \qquad x = \frac{1}{2}$

Point P

(ii) mid-point (h,k)

$$T = S_1$$

$$hx + ky - 1 = h^2 + k^2 - 1$$

$$hx + ky = h^2 + k^2$$

- compare

$$\frac{h}{\alpha} = \frac{k}{4-2\alpha} = \frac{h^2+k^2}{1}$$

$$\alpha = \frac{h}{h^2+k^2} \qquad \frac{k}{h^2+k^2} = 4 - 2\alpha$$

$$\frac{k}{h^2+k^2} = 4 - \frac{2h}{h^2+k^2}$$

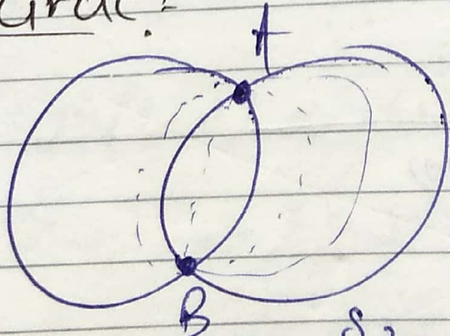
$$k = 4h^2 + 4k^2 - 2h$$

$$4h^2 + 4k^2 - 2h - k = 0$$

* family of Circle:

$S_1 = 0$

$S_2 = 0$



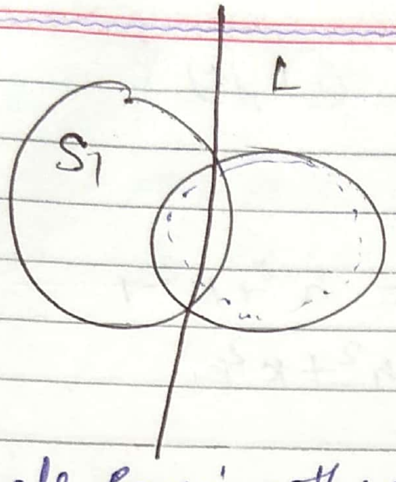
A

B

S_1 S_2

$$\boxed{S_1 + \lambda S_2 = 0} \longrightarrow$$ eq^n of Circle Passing through POI of S_1 & S_2

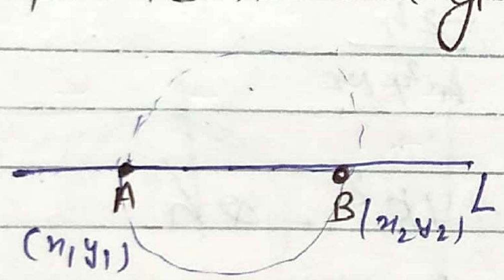
$$\boxed{\lambda \neq -1}$$



eqⁿ of circle passing through PoI of circle S_1 and line L is given by

$$\boxed{S_1 + \lambda L = 0}$$

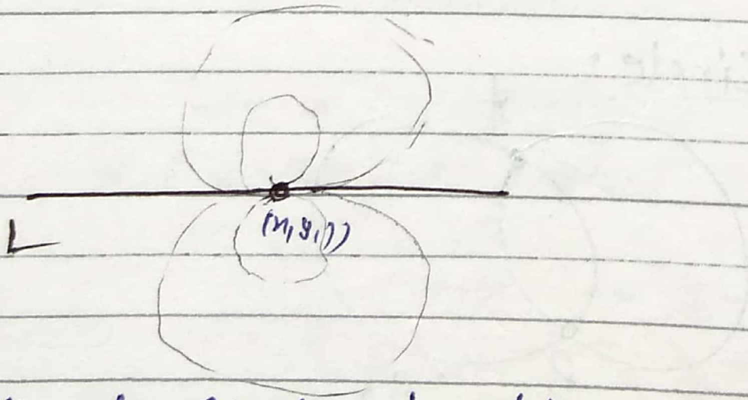
∴ If only two points are given



$S_1 + \lambda L = 0$

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + \lambda L_{AB} = 0$$

*



eqⁿ of circles touching the line L at (x_1, y_1) is given by

$$\boxed{\underbrace{(x-x_1)^2 + (y-y_1)^2}_{S} + \underbrace{\lambda L}_{\lambda L} = 0}$$

Imp.

Q. Find the eqⁿ of circle passing through POI of $x^2+y^2-4x+6y-3=0$ and $x^2+y^2+4x-6y+12=0$ and

① Passes through $(0,0)$

② Centre lie on $y=x$

③ $4x-7y=10$ is normal to circle

Que: the eqⁿ of circle which touches $2x-y=4$ at
② $(1,-2)$ and also passes through $(3,4)$

③ find the eqⁿ of which passes through origin and through the point of contact of the tangent from origin to circle

$$x^2+y^2+11x+13y+17=0$$

① $S_1+\lambda S_2=0$

$$x^2+y^2-4x+6y-3+\lambda(x^2+y^2+4x-6y+12)=0$$

$$x^2(1+\lambda)+y^2(1+\lambda)+x(4\lambda-4)+y(-6\lambda+6)-12\lambda-3=0$$

(i) $12\lambda-3=0 \Rightarrow \lambda=\frac{1}{4}$

$$\frac{5}{4}x^2+\frac{5}{4}y^2-3x-6\left(-\frac{3}{4}\right)=0$$

$$\frac{5}{4}x^2+\frac{5}{4}y^2-3x-\frac{9}{2}=0$$

(2) centre $\left(\frac{-(2\lambda-2)}{\lambda+1}, \frac{3\lambda-3}{\lambda+1}\right)$

$$\frac{3\lambda-3}{\lambda+1} = \frac{-(2\lambda-2)}{\lambda+1}$$

$$3\lambda-3=-2\lambda+2$$

$$5\lambda=5$$

$$\lambda=1$$

$$2x^2+2y^2+9=0$$

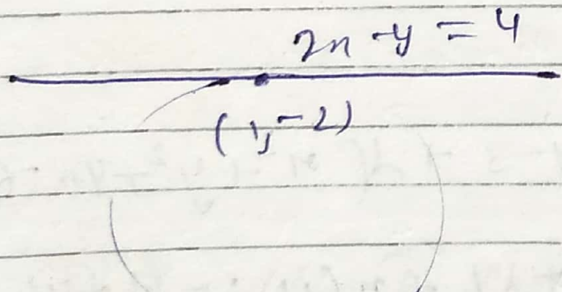
(3) Centre

$$2x-y=4$$

$$4\left(\frac{-(2\lambda-2)}{\lambda+1}\right)-7\left(\frac{3\lambda-3}{\lambda+1}\right)=10$$

$$-\frac{8\lambda-8}{\lambda+1}-\frac{21\lambda+21}{\lambda+1}=10$$

Ans 2

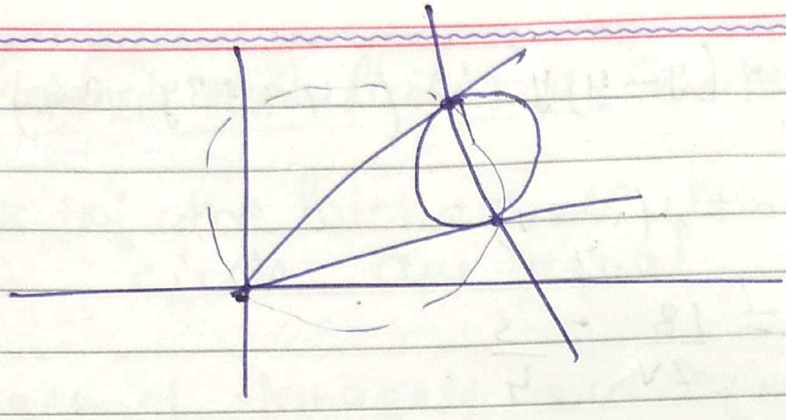


$$(x-1)^2+(y+2)^2+\lambda(2x-y-4)=0$$

$4+36+\lambda(8-4-4)$ $\qquad \lambda=\frac{40}{2}$

$$\boxed{\lambda=20}$$

Ans: (3)



$COC \Rightarrow T = 0$

$0 + 0 + -\frac{11}{2}(x+0) + \frac{13}{2}(y+0) + 17 = 0$

$-11x + 13y + 34 = 0$

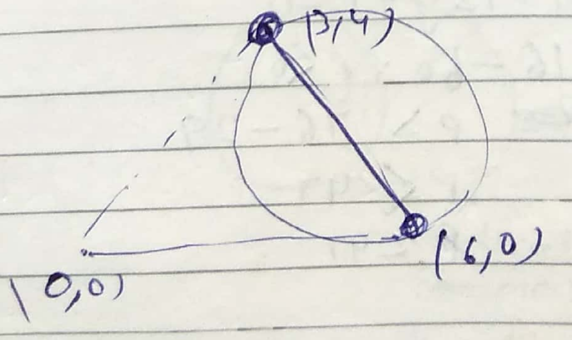
$\phi + \lambda L = 0$

$x^2 + y^2 - 11x + 13y + 17 + \lambda(-11x + 13y + 34) = 0$

$17 + \lambda(34) = 0$

$\boxed{\lambda = -\frac{1}{2}}$

Q. find the eq^n of circle which passes through (0,0) (3,4) (6,0)



$(x-3)(x-6) + (y-4)(y-0) = 0$

$y - 0 = \frac{4}{-3}(x-6)$

$3y = -4x + 24$

$4x + 3y - 24 = 0$

$(x-3)(x-6)+(y-4)y+\lambda(4x+3y-24)=0$

$18+0+\lambda(-24)=0$

$\lambda=\frac{18}{24}=\frac{3}{4}$

* Power of a point :

Square of the length of tangent from point P is called Power of point P w.r.t given circle

Length of tangent $= \sqrt{S_1}$

Power $= S_1$

Que: find the value of P for which Power of point P(2,5) is negative w.r.t circle $x^2+y^2-8x-12y+P=0$ and the circle neither touches nor intersects the y axis.

$x^2+y^2-8x-12y+P=0$

$4+25-16-60+P<0$

$P < 76-29$

$P < 47$

$P < 47$

$2\sqrt{f^2-c}$

$f^2-c<0$

$6^2-P<0$

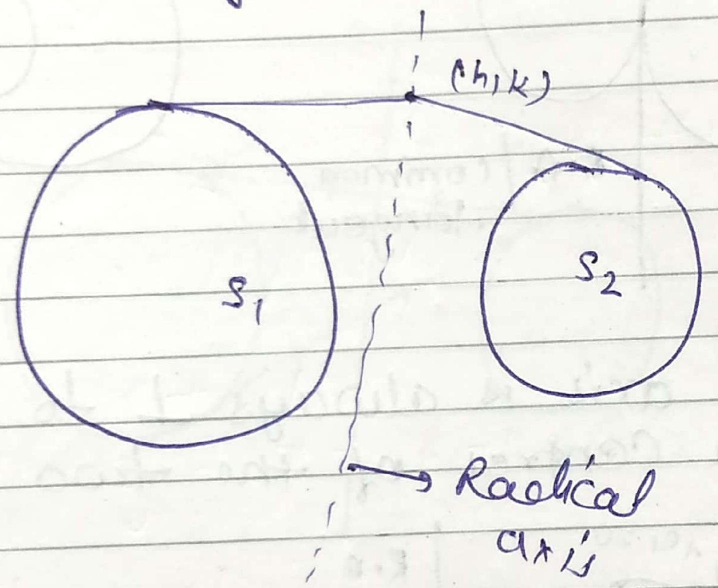
$P>36$

$P\in(36,47)$

* Radical axis and Radical Centre:

Locus of the point whose Power w.r.t the two circle are equal.

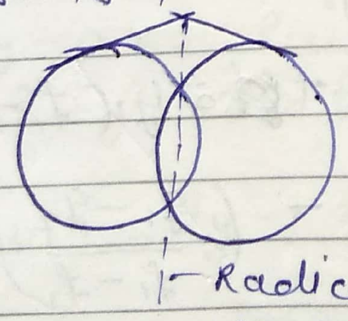
(Length of tangents are equal)



$S_1 = S_2$

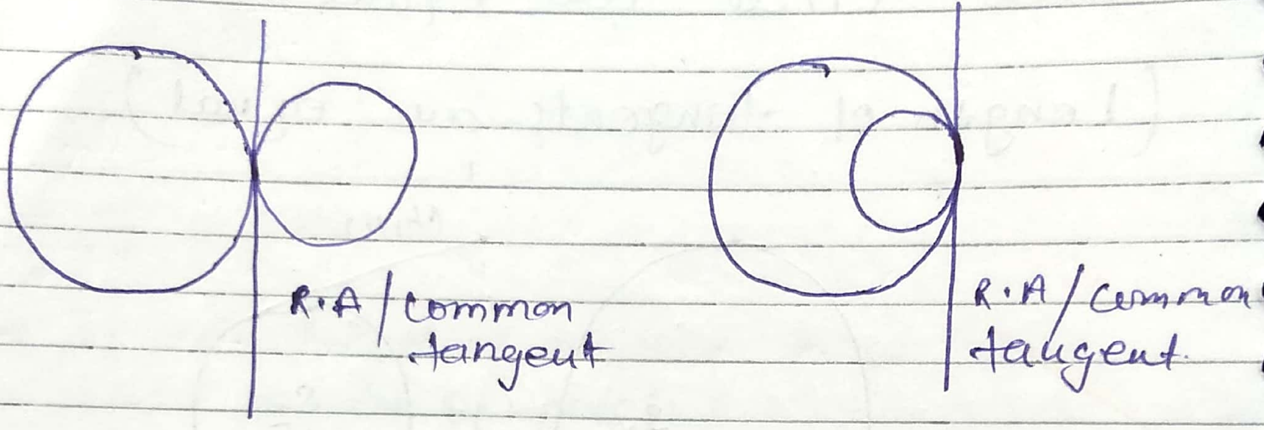
$$\boxed{S_1 - S_2 = 0}$$

1) If two circles intersects then the radical axis is the common chord of two circles.

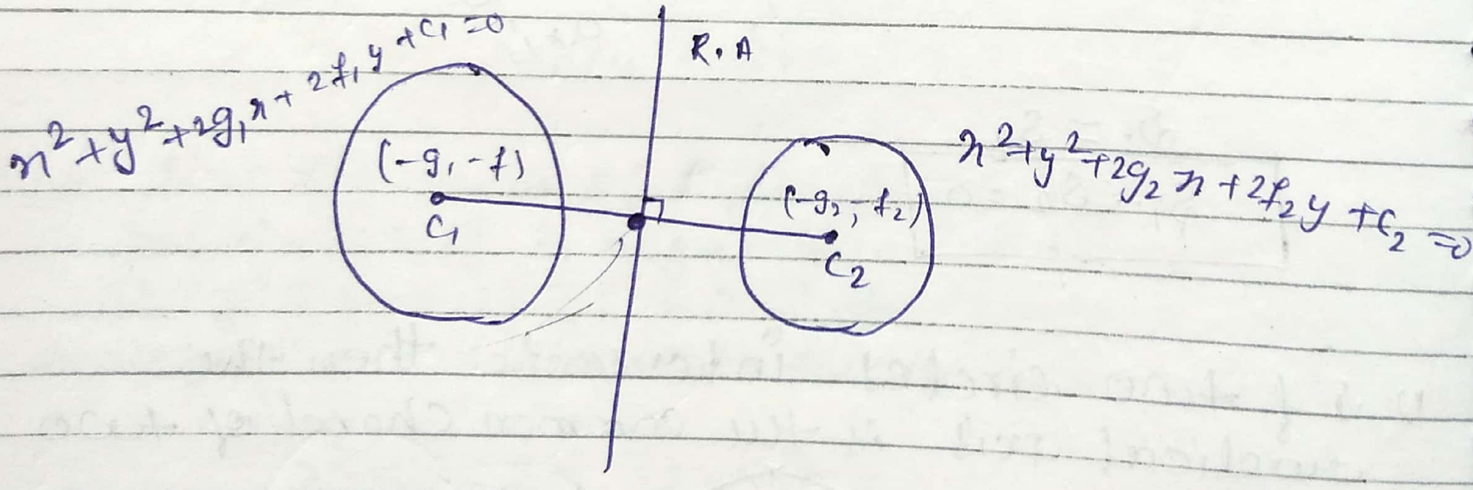


equation of common chord = $\boxed{S_1 - S_2 = 0}$

(2) If two circle touches each other then the radical axis is the common tangent of the two circles at the point of contact.



3) Radical axis is always ⊥ to the line joining the centres of the two circles.



$$2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0$$

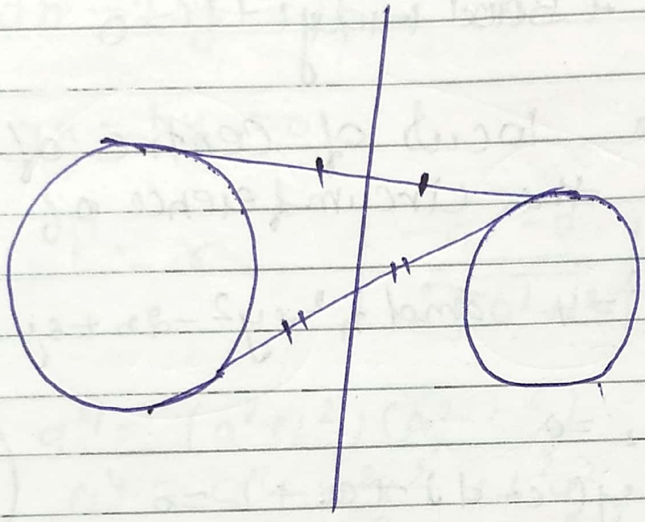
$$m_{R.A}=-\left(\frac{g_1-g_2}{f_1-f_2}\right)=-\left(\frac{g_2-g_1}{f_2-f_1}\right)$$

$$M_{C_1C_2}\cdot M_{RA}=-1$$

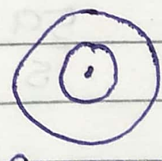
* Radical axis ~~is~~ need not always passes through the mid-point of the line joining of the centres of two circles.

* Radical axis bisect a common tangent b/w the two circles

Given



* Pair of circle which do not have radical axis are Concentric ~~circle~~



Concentric.

Que: No. of real values of a for which line $5x + by - a = 0$ passes through POI of circles $x^2 + y^2 + 2ax + cy + a = 0$ and

$$x^2 + y^2 - 3ax - dy - 1 = 0$$

Q.2 Tangents are drawn to circle $x^2 + y^2 = 12$ at point where it meet $x^2 + y^2 - 5x + 3y - 2 = 0$ find the POI of tangent

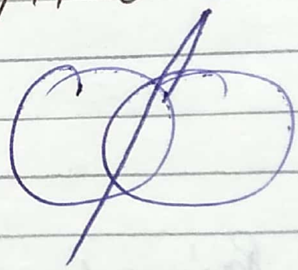
Q. If the circle $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ touch each other. then

P.T $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

Ans: $x^2+y^2+2ax+2dy+c=0$

Qu: find the locus of centre of Circle which bisect the circumference of circle

$x^2+y^2=4$ and $x^2+y^2-2x+6y+1=0$



Ans: 1) $S_1-S_2=0$

$5ax+y(c-d)+a+1=0$

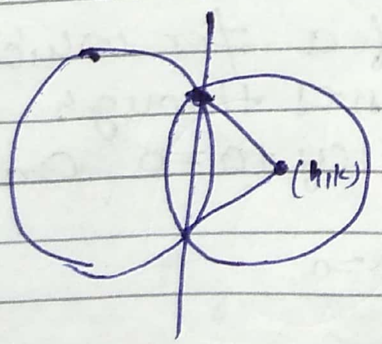
$5x+by-a=0$

$\frac{5a}{5}=\frac{c-d}{b}=\frac{a+1}{-a}$ $\quad -a^2=a+1$

$a^2+a+1=0$

No Real value is possible.

(2)



$S_1-S_2=0$

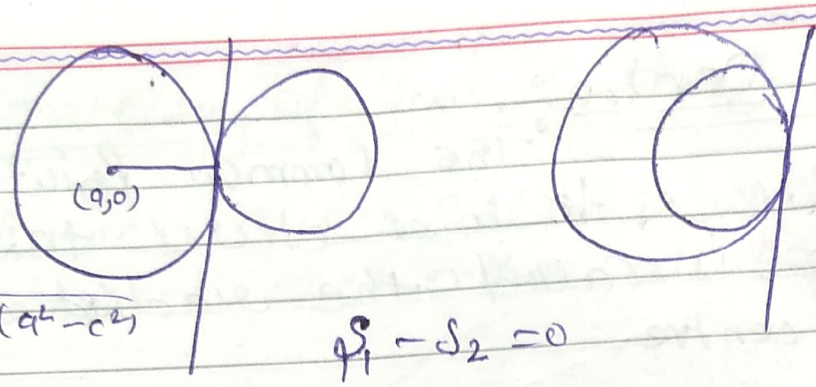
$5x-3y=10$

$T=0 \quad hx+ky=12$

$\frac{h}{5}=\frac{k}{-3}=\frac{12}{10}$ $\quad h=6$

$K=-\frac{18}{5}$

(3)



$r = \sqrt{(a^2-c^2)}$

$S_1 - S_2 = 0$

$2ax - 2by = 0$

$ax - by = 0$

$P = r \quad \left| \frac{-a^2 - 0}{\sqrt{a^2+b^2}} \right| = \sqrt{a^2-c^2}$

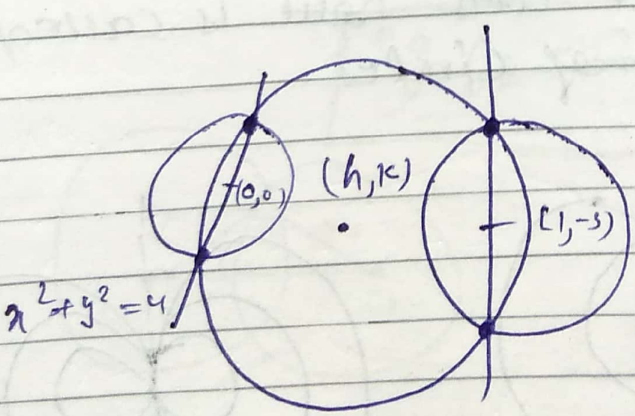
$a^4 = (a^2+b^2)(a^2-c^2)$

$a^4 = a^4 - a^2c^2 + b^2a^2 - b^2c^2$

$a^2c^2 + b^2c^2 = a^2b^2$

$$\boxed{\frac{1}{b^2} + \frac{1}{a^2} = \frac{1}{c^2}}$$

(4)



$x^2+y^2-2hx-2ky+c=0$

$2hx - 2x + 2ky + 6y + 1 - c = 0$

$2hx + 2ky - c = 4$

$0 + 0 - c = 4$

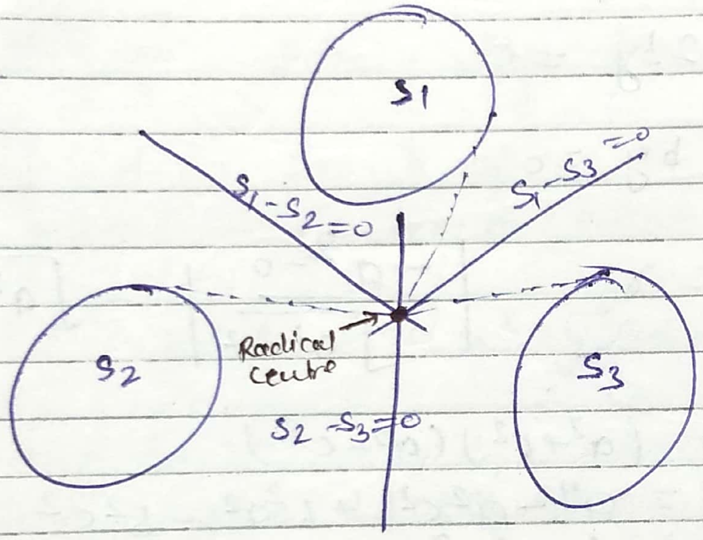
$c = -4$

$2h - 2 - 6k + 18 + 1 + 4 = 0$

$2h - 6k - 15 = 0$

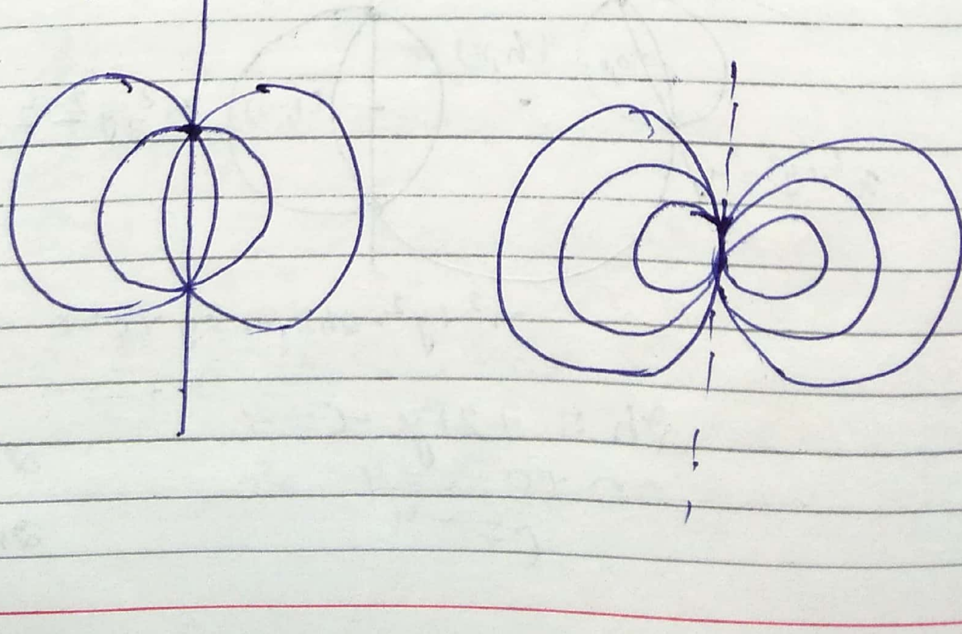
* Radical Centre :

The common point of intersection of the R.A of ~~to~~ three circles taken to at a time is called the ~~radical~~ radical centre of a centre.



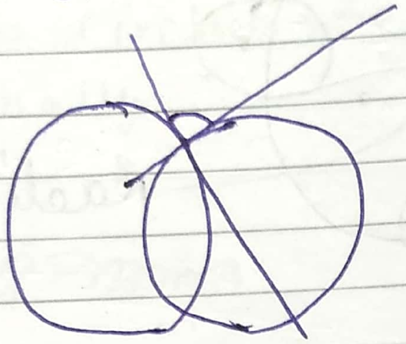
* Co-axial System of Circles :

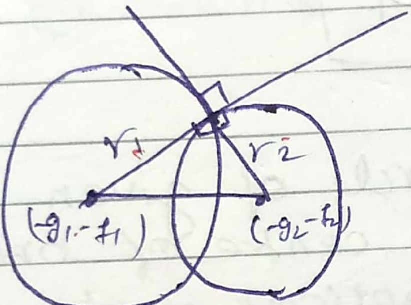
A system of circle every two of which have the same radical axis ~~have~~ is called co-axial system of circles



* Orthogonality of two Circles:

Two Curves are said to be orthogonal if they intersect each other at 90°

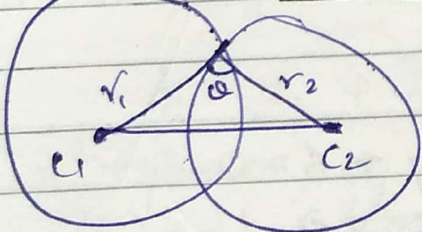




$$r_1^2 + r_2^2 = (g_1 - g_2)^2 + (f_1 - f_2)^2$$

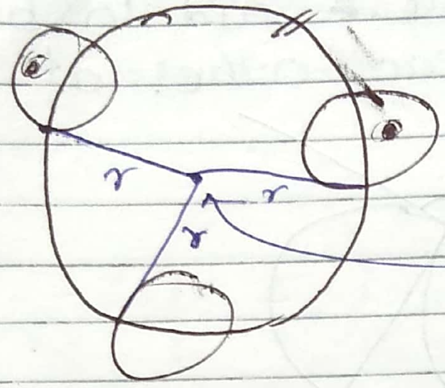
$$g_1^2 + f_1^2 - c_1 + g_2^2 + f_2^2 - c_2 = g_1^2 + g_2^2 - 2g_1g_2 + f_1^2 + f_2^2 - 2f_1f_2$$

$$\boxed{2g_1g_2 + 2f_1f_2 = c_1 + c_2}$$



$$\boxed{\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}}$$

Circle Orthogonal to 3 given Circle



Length of tangent $\boxed{r = \sqrt{S_1}}$

Steps:

* (1) find Radical circle of given circles. It will be the centre of orthogonaly Circle
  ⇒ centre = radical centre.

(2) then find length of tangent from radical axis to any of circle It will be ~~radius~~ ~~of~~ Radius of Orthogonaly Circle

$$\boxed{r = \sqrt{S_1}}$$

Q. $S_1 \equiv x^2 + y^2 + 2x + 2ky + 6 = 0$
$S_2 \equiv x^2 + y^2 + 2ky + k = 0$

If S_1 & S_2 are orthogonal then find k.

Q.2 find the eqn of circle which passes through the PoI of $S_1 = x^2+y^2-4x+6y=0$ and $x^2+y^2-6x+4y=0$ and

cut the circle $x^2+y^2-2x-4y-6=0$ orthogonally.

Ans:-1

$= 2\times(1)(3)+2\times k\times k = 6+k$

$2k^2 = 6+k$

$2k^2-k-6=0$

$(2k+3)(k-2)=0$

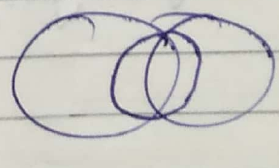
$k=2, -3/2.$

Q.3 find the eqn of circle orthogonally to given circle

$S_1 = x^2+y^2-2x+3y-7=0$

$S_2 = x^2+y^2+5x-5y+9=0$

$S_3 = x^2+y^2+7x-9y+29=0$



Ans:-2) $S_1 + \lambda S_2 = 0$

$x^2+y^2-4x+6y+\lambda(x^2+y^2-(x+4y))=0$

$x^2(\cancel{1+\lambda})+y^2(\cancel{1+\lambda})-x\left(+\frac{(\lambda+4)}{1+\lambda}\right)+y\left(\frac{4\lambda+6}{1+\lambda}\right)=0$

$\cancel{8}\times(-1) - 2\left(\frac{6\lambda+4}{\lambda+1}\right) + 2(-2)\,2\left(\frac{4\lambda+6}{1+\lambda}\right) = -4+0$

$6\lambda+4-8\lambda-12=-4-4\lambda.$

6Q

$2\lambda = 4$

$\lambda = 2$

$x^2 + y^2 - \frac{16}{3}x + \frac{14}{3}y = 0$

Ans - 6

$S_1 - S_2 = -7x + 2y - 16 = 0$

$S_2 - S_3 = -2x + 4y - 20 = 0$

$x - 2y + 10 = 0$

$[7x - 8y + 16 = 0]$

$4x - 8y + 40 = 0$

$3x - 24 = 0$

$x = \frac{\cancel{24}\,8}{\cancel{3}}$

$\left.\begin{array}{l} x = 8. \\ y = 9. \end{array}\right\}$ Radical centre

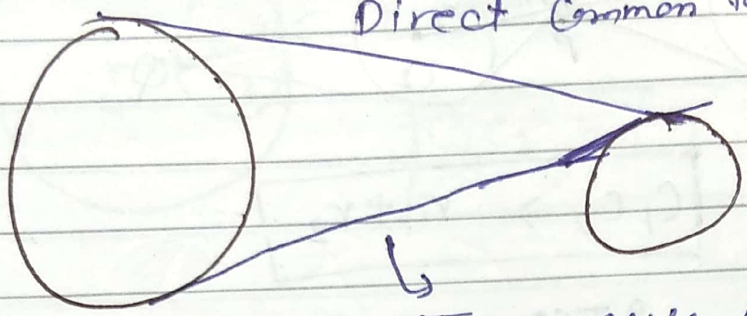
$r = \sqrt{S_1} = \sqrt{8^2 + 9^2 - 16 + 27 - 7}$

$= \sqrt{85 + 64} = \sqrt{149}$

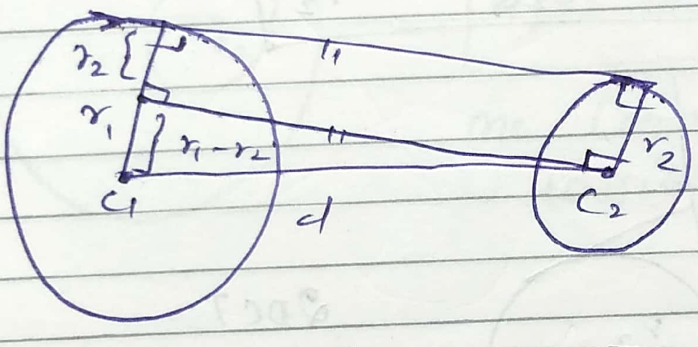
$(x-8)^2 + (y-9)^2 = 149.$

* Common tangent of two given circle:

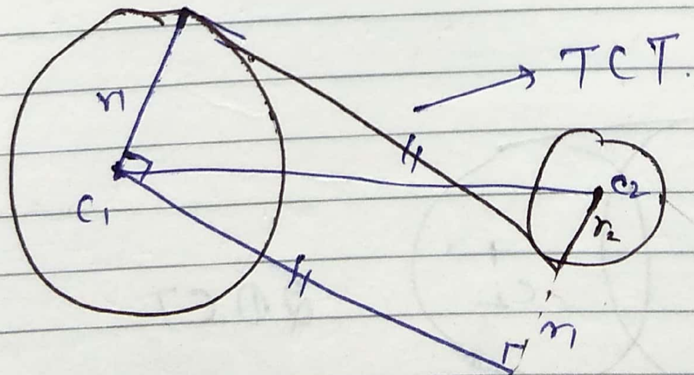
Direct Common tangent (Centre lies on same side)



Transverse Common tangent (TCT)

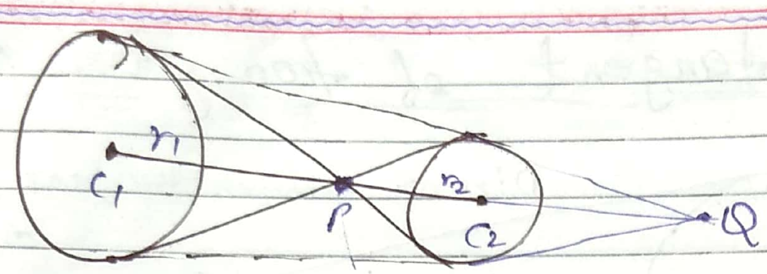


$$\boxed{DCT = \sqrt{d^2 - (r_1 - r_2)^2}}$$



$$\boxed{DCT > TCT}$$

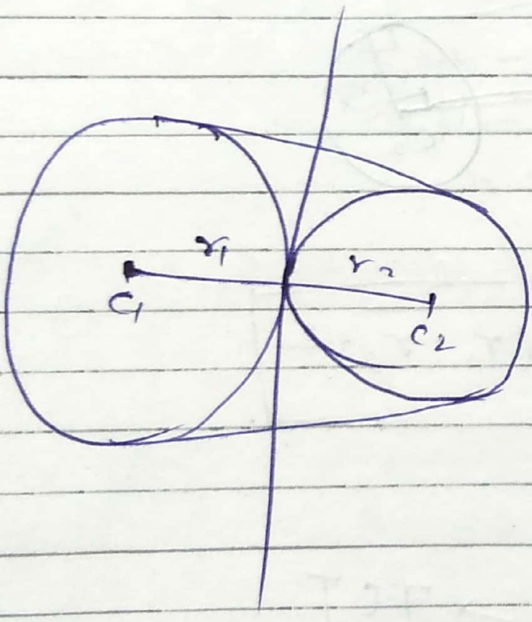
$$\boxed{TCT = \sqrt{d^2 - (r_1 - r_2)^2}}$$



$$\boxed{C_1C_2 > r_1 + r_2}$$

2 TCT , 2 DCT

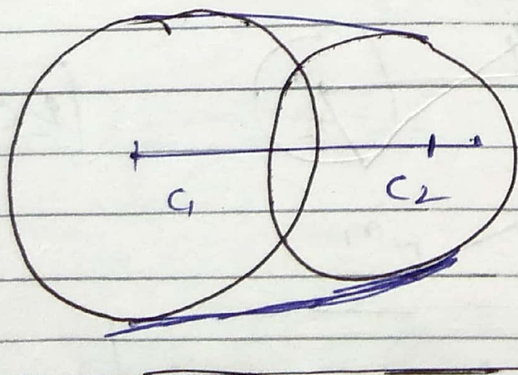
$$\boxed{\frac{C_1P}{C_2P} = \frac{C_1Q}{C_2Q} = \frac{r_1}{r_2}}$$



2 DCT

1 TCT

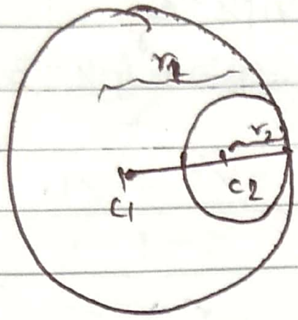
$$\boxed{C_1C_2 = r_1 + r_2}$$



2 DCT

$$\boxed{|r_1 - r_2| < C_1C_2 < r_1 + r_2}$$

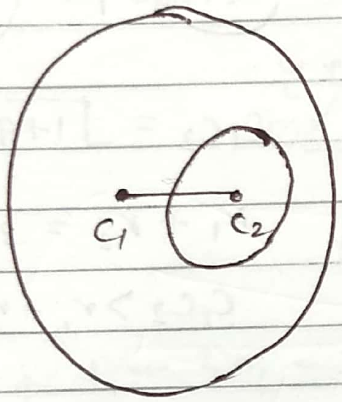
*



1 DCT

$$\boxed{C_1C_2 = |r_1 - r_2|}$$

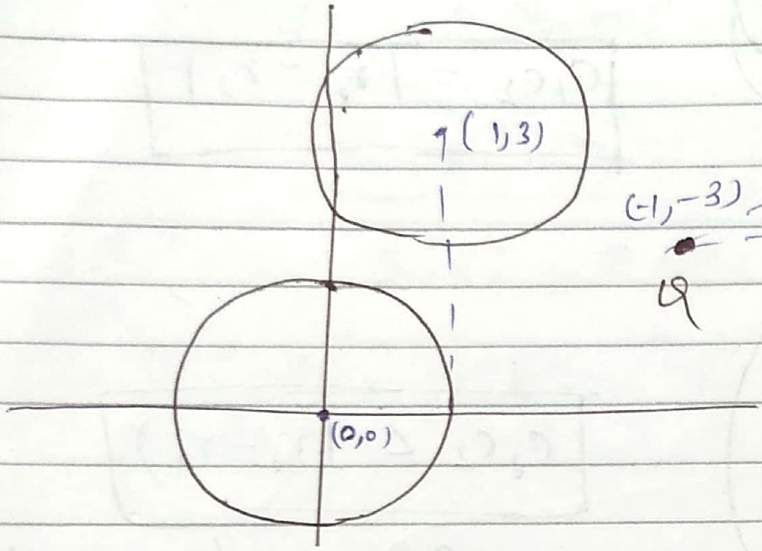
*

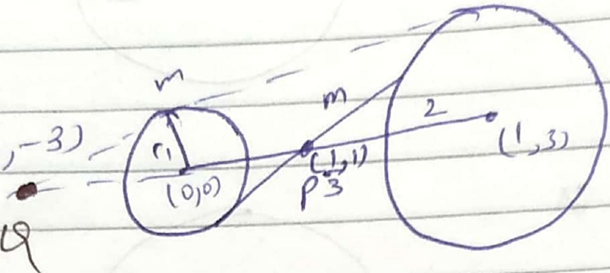


$$\boxed{C_1C_2 < |r_1 - r_2|}$$

no common tangent is possible

Find eq^n of Common tangents of
$x^2+y^2=1$ and $(x-1)^2+(y-3)^2=4$





$C_1C_2=\sqrt{1+9}=\sqrt{10}$

$r_1+r_2=3$

$C_1C_2>r_1+r_2$

$$\frac{C_1P}{C_2P}=\frac{r_1}{r_2}$$

$$P=\frac{2\times 0+1\times 3}{1+2}=\frac{1}{3}$$

$$\frac{3+0}{1+2}=1$$

$y-1=m(x-\frac{1}{3})$

$mx-y-m-m+1=0$

$P=1$

$$\left|\frac{-\frac{m}{3}+1}{\sqrt{m^2+1}}\right|=1$$

$m^2+1=(-m+1)^2$

$\cancel{m^2}+\cancel{1}=\cancel{m^2}+\cancel{1}-2m$ $\quad m\to\infty$

$m=0$

$\boxed{T^2 = SS_1}$

$\boxed{T^2 = SS_1}$ Pair of Tangent.

$y = 1$, $x - 1 = 0$ $x = 1$

~~common tangent~~ (TCT)

$$\cancel{\frac{1\times 1 - 2\times 0}{1-2}}, \cancel{\frac{1\times 3 - 2\times 0}{1-2}} = (-1, -3)$$

$$\cancel{\frac{C_1 Q}{C_2 Q} = \frac{r_1}{r_2}}$$

$$\cancel{y + 3 = m(x+1)}$$

$$\cancel{mx - y + m - 3 = 0}$$

$$\cancel{\left|\frac{m-1}{\sqrt{m^2+1}}\right| = 1}$$

$$\Rightarrow \frac{m^2}{9} + 1 - \frac{2m}{3} = m^2 + 1$$

$$\frac{8m^2}{9} = -\frac{2m}{3} \qquad m = 0$$

$$\frac{8m}{9} = \frac{-2}{3} \qquad m = -\frac{3}{4}$$

$m = 0$ $y = 1$

at $m = -\frac{3}{4}$ common Tangent (TCT)

$$y - 1 = -\frac{3}{4}\left(x - \frac{1}{3}\right)$$

$$m^2 + 9 - 6m = m^2 + 1$$

$m \to \infty$, $6m = 8$

$x + 1 = 0$ $m = \frac{4}{3}$ $y + 3 = \frac{4}{3}(x+1)$

$x = -1$

Q $(x-2)^2+(y+3)^2=1$

$x^2+4-4x+y^2+9+6y=1$

$x^2+y^2-4x+6y+12=0$

then find eq^n of pair of tangent from origin

$(x_1, y_1)=(0,0)$

$T^2=SS_1$

$\{0+0-2(x+0)+3(y+0)+12\}^2=[x^2+y^2-4x+6y+12]\times(12)$

$(2x-3y-12)^2=12(x^2+y^2-4x+6y+12)$

$4x^2+9y^2+\cancel{144}-12xy+\cancel{72y}-\cancel{48x}=12x^2+$
$12y^2-\cancel{48x}+\cancel{72y}+\cancel{144}$

$8x^2+3y^2+12xy=0$

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